Practice Problems for chap. 3.

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1. Let $I=[a_1,b_1]\times[a_2,b_2]$, a closed interval (i.e. rectangle) in $\mathbb{R}^2$. Given $\varepsilon>0$, construct an interval I_ε such that

(i) $I \subset \mathring{I}_\varepsilon$ (the interior of I_ε),

(ii) $v(I_\varepsilon)-v(I)<\varepsilon$.

2. Use the result in Prob. 1 to show that

Given a sequence of intervals $\{I_k\}_{k=1}^{\infty}$ in $\mathbb{R}^2$ and $\varepsilon>0$, there exists a sequence of intervals $\{I_k^\varepsilon\}_{k=1}^{\infty}$ such that

(i) $I_k \subset \mathring{I}_k^\varepsilon$, $k=1,2,\dots$

(ii) $\displaystyle\sum_{k=1}^{\infty} v(I_k^\varepsilon) < \sum_{k=1}^{\infty} v(I_k)+\varepsilon$.

3. Use the definition of the outer measure to show:

(i) Let E be a countable set in $\mathbb{R}^2$. Then

$$|E|_e=0.$$

(ii) Let E be the edge (4 boundaries) of an interval $I \subseteq \mathbb{R}^2$. Then

$$|E|_e = 0.$$