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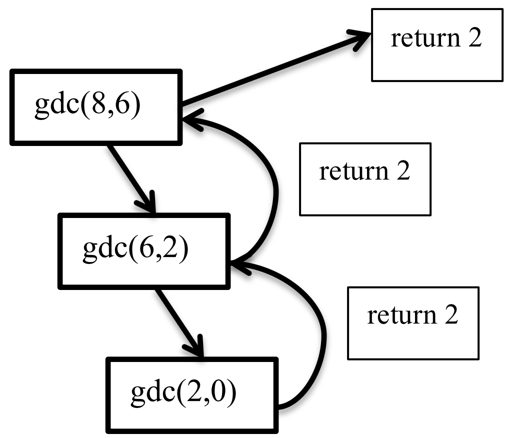
CS 272 Lab 10

Recursive Trace for gdc(8, 6)



Analyzing gdc for correctness:

- Base Case: b == 0, gdc(a, b) = gdc(a, 0) = a
- Inductive Case:
  - a_n and b_n = a and b in gdc(a, b)
  - q and r = the quotient and remainder of dividing b_n by a_n
  - $gdc(a_n, b_n) = gdc(b_n, a_n) = gdc(b_n, qb_n + r) = gdc(b_n, r)$
    - $a_n = qb_n + r$
    - $0 <= r < b_n$
    - $r = a_n \% b_n$
  - Recursive call
    - $gdc(a_{n-1}, b_{n-1}) = gdc(b_n, r)$

Analyzing hanoi for time complexity:

- Time to move n disks = T(n)
- There are two recursive calls for n – 1 disks and one constant
  - Constant = time to move one disk = 1
- T(n) = 2 T(n - 1) + 1

Analysis

- T(1) = 1
- T(2) = (2 *1) + 1
- T(3) = (4 * 1) + (2 *1) + 1
- $T(n) = (2^{n-1} + 2^{n-2} + \ldots + 2^1 + 2^0) * 1 = (2^n - 1) * 1$