Math 275 Notes

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1.1 Slope (Direction) Fields

Equations containing derivatives are differential equations.
A differential equation that describes some physical process is often called a mathematical model of the process.

Definition 1.1: Slope (direction) field

Given the differential equation $dy/dt = f(t,y)$. If we systematically evaluate f over a rectangular grid of points in the ty-plane and draw a line element at each point (t,y) of the grid with slope $f(t,y)$, then the collection of all these line elements is called a slope (direction) field of the differential equation $dy/dt = f(t,y)$

We can graph undefined slopes in a slope field by using a vertical line; horizontal for 0.

Question 1.1: pg. 8: #6

Write down a differential equation of the form dy/dt = ay + b whose solutions diverge from y = 2.

Solution:
Given that the equation is dependent on just y, we can say that it is an autonomous differential equation (DE). We can subsequently solve for the DE by setting $\frac{dy}{dt} = f(y) = 0$, such that its solution is its equilibrium solution. Since $y = 2$, $\boxed{\frac{dy}{dt} = y - 2}$. Thus, any number that is less than 2 will result in the solution's phase portrait diverging (-) from $y = 2$ — vice versa for numbers greater than 2.

1.2 tfi

1.3 Classification on DEs

The order of a DE is the order of the highest derivative that appears in the equation.

Definition 1.2: Linear ODE

The ODE $F\left(t, y, y', \ldots, y^{(n)}\right) = 0$ is said to be linear if F is a linear function of the variables $y, y', \ldots, y^{(n)}$. Thus the general linear ODE of order n is $a_0(t)y^{(n)} + a_1(t)y^{(n-1)} + \ldots + a_n(t)y = g(t)$.

Question 1.2: pg. 22

Determine the order of the given DE; also state whether the equation is linear or nonlinear:
1) $t^2y'' + ty' + 2y = \sin t$
2) $\left(1 + y^2\right)y'' + ty' + y = e^t$

Solution:
1) The order of the DE is 2, and it is linear: noticed how all the coefficents of n derivatives of y are functions of t.
2) The order of the DE is 2, and it is nonlinear: noticed how the coefficents of n derivatives of y are functions of t and y.

Definition 1.3: Nth-order IDE solutions

Any function h, defined on an interval and possessing at least n derivatives that are continuous on this interval, which substituted into an nth-order ODE reduces the equation to an identity, is said to be a solution of the equation on the interval.

Question 1.3: pg. 22: #9)

Verify that the functions $y_1(t) = t^{-2}$ and $y_2(t) = t^{-2}\ln t$ are solutions of the DE

$$t^2y'' + 5ty' + 4y = 0$$

Solution:
We can verify that $y_1(t)$ and $y_2(t)$ are solutions of the DE by substituting them into the DE and verifying that the equation holds true.

$$\begin{aligned} y_1 &= t^{-2} \\ y_1' &= -2t^{-3} \\ y_1'' &= 6t^{-4} \end{aligned}$$

Substituting: $t^2y'' + 5ty' + 4y = t^2(6t^{-4}) + 5t(-2t^{-3}) + 4t^{-2} = 0$
For homework 1, we can verify that $y_2(t)$ is a solution of the DE.

Question 1.4: #12

Determine the values of r for which $y'' + y' - 6y = 0$ has solutions of the form $y = e^{rt}$

Solution:
We can solve for the values of r by substituting $y = e^{rt}$ into the DE and solving for r. $y = e^{rt}$, $y' = re^{rt}$, and $y'' = r^2e^{rt}$. Substituting: $r^2e^{rt} + re^{rt} - 6e^{rt} = 0$. Factoring out e^{rt}, we get $e^{rt}(r^2 + r - 6) = 0$. Thus, $r^2 + r - 6 = 0$, and $\boxed{r = 2, -3}$.

Question 1.5: #14

Determine the values of r for which $t^2y'' + 4ty' + 2y = 0$ has solutions of the form $y = t^r$ for $t > 0$

Solution:
$y = t^r$, $y' = rt^{r-1}$, and $y'' = r(r-1)t^{r-2}$. Substituting: $t^2(r(r-1)t^{r-2}) + 4t(rt^{r-1}) + 2t^r = 0$. Factoring out t^r, we get $t^r[r(r-1) + 4r + 2] = 0$. $t > 0$ such that $t^r \neq 0$, and $r(r-1) + 4r + 2 = 0$. Thus, $r^2 + 3r + 2 = 0$, and $\boxed{r = -1, -2}$.