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Robust Motion Tracking Control of Robotic Arms Based on the Generalized Energy Accumulation Principle

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1. Objective

Consider a rigid-link robot with the dynamic model

$$\tau = H(q;p)\ddot{q} + C(q,\dot{q};p)\dot{q} + G(q;p) + \mathcal{N}(t)$$

where $\mathcal{N}(.)$ denotes a bounded external disturbance (definitions of other terms and variables can be found in the literature). The objective addressed herein is to find a control strategy that exhibits the following features: (1) simple to implement, (2) easy to code for program and (3) robust to possible time-varying uncertainties

2. Results

Define tracking error ϵ as $\epsilon = q - q^*$, where q^* denotes the desired trajectory. Also let $W = \dot{\epsilon} + D\epsilon$, $y_s = \dot{q}^* - D\epsilon$, and $x_s = \ddot{q}^* - D\dot{\epsilon} - \dot{\kappa}\kappa^{-1}W$, where $D = D^T > 0$ and $\kappa(t)$ is one of the rate functions which is introduced to adjust rate-of-convergence (ROC) (see [4]).

Definition 1

Let $\nu(t)$ be defined on $[t_0, \infty)$. $\nu(t)$ is in the class $\mathcal{V}_I$ if $\nu(t)$ is nonnegative constant or in $\mathcal{V}_{II}$ if $\nu(t)$ is bounded, positive, and decreasing for all $t \in [t_0, \infty)$.

The robust control law is given by

$$\tau = H_s(q^*;p^*)x_s + C_s(q^*,\dot{q}^*;p^*)y_s + G_s(q^*;p^*) - KW + U_a, \tag{1}$$

where $K = K^T > 0$, and $H_s(.)$, $C_s(.)$, and $G_s(.)$ are simplified versions of H, C, and G, respectively. U_a is an auxiliary control defined by

$$U_a = -\frac{W\eta^2}{\|W\|\eta + \nu(t)}, \tag{2}$$

where $\nu(t) \in \mathcal{V}_I$ or $\mathcal{V}_{II}$ and η is a nonnegative scalar nonlinear function defined as

$$\begin{aligned} \|H_s - H\|\|x_s\| \quad + \quad & \|C_s - C\|\|y_s\| + \|G_s - G - \mathcal{N}\| \\ \leq \quad & \alpha_0\|x_s\| + \alpha_1\|\dot{q}\|\|y_s\| + \alpha_2 \triangleq \eta. \end{aligned}$$

In this equation, α_i are constants representing bounds on the modelling errors. There are many possible choices for $\nu(t)$ in U_a, and different choices leads to different tracking properties. The $\nu(t)$ defined by

$$\nu(t) = \frac{1}{2}v_1(1 + t^m)^\rho e^{-v_2 t^n} \tag{3}$$

are in $\mathcal{V}_I$ or $\mathcal{V}_{II}$ if m, n, and ρ are chosen properly (v_1 and v_2 are appropriate positive constants). Specifically, if $\rho = -1$, $m, n = 0$, one gets $\nu(t) = v_1 e^{-v_2} \triangleq \mu_0$, which gives, $U_a = -\frac{W\eta^2}{\|W\|\eta + \mu_0}$. This is called saturation (or boundary layer) controller [1] and has been widely used to achieve bounded stability. Another specific choice for $\nu(t)$ ($\rho = -1$, $m = 0$, $n = 1$) is $\nu(t) = v_1 e^{-v_2 t}$, which gives the strategy proposed in [2], $U_a = -\frac{W\eta^2}{\|W\|\eta + v_1 e^{-v_2 t}}$. An extreme case, $n \to \infty$, $m \to \infty$ and $\rho = -1$, gives $\nu(t) = 0$, which corresponds to the variable structure control [3], $U_a = -\frac{W}{\|W\|}\eta$. As is shown later, $\nu(t) = 0$ gives the fastest convergence, while $\nu(t) = \mu_0$ gives the slowest. However, due to physical limitations, "too fast" could lead to chattering. Hence, the choice of $\nu(t)$ depends on the requirements for ROC, transient response, and steady state performance.

Tracking stability results based on the so-called generalized energy accumulation principle [4] are given next.

Theorem 1: *Given (1) and (2), if $\nu(t)$ and $\kappa(t)$ are chosen such that*

$$\int_{t_0}^{t} \nu(\tau)\kappa^2(\tau)d\tau \leq C_2^2 < \infty \qquad \forall t \in [t_0, \infty), \tag{4}$$

then stable path tracking is ensured and the rate-of-convergence is at least $\kappa^{-1}(t)$.

Proof (outline): The closed-loop model is governed by

$$\begin{aligned} H(q;p)(\dot{W} + \dot{\kappa}\kappa^{-1}W) + C(q,\dot{q};p)W = -KW \\ + \quad \delta H(q;p)x_s + \delta C(q,\dot{q};p)y_s + \delta\mathcal{G}(q,t;p) + U_a. \end{aligned}$$

where $\delta H(.) = H_s - H, \delta C(.) = C_s - C, \delta\mathcal{G}(.) = G_s - G - \mathcal{N}$. Introducing the transformation $\Psi = \kappa W$ gives

$$\begin{aligned} H(q;p)\dot{\Psi} \quad + \quad & C(q,\dot{q};p)\Psi = -K\Psi + \delta H(q;p)x_s\kappa \\ + \quad & \delta C(q,\dot{q};p)y_s\kappa + \delta\mathcal{G}(q,t;p)\kappa + U_a\kappa. \end{aligned}$$

According to the criteria in [4], boundedness of the accumulated generalized energy, $\int_{t_0}^{t} \Psi^T K \Psi d\tau$, proves tracking stability. In fact,

$$
\begin{aligned}
J^c &= \int_{t_0}^{t} \Psi^T K \Psi d\tau \\
&= -\int_{t_0}^{t} \Psi^T H(q;p)\dot{\Psi} d\tau - \int_{t_0}^{t} \Psi^T C(q,\dot{q};p)\Psi d\tau \\
&+ \int_{t_0}^{t} \Psi^T \{\delta H(q;p)x_s\kappa + \delta C(q,\dot{q};p)y_s\kappa + \delta \mathcal{G}(q,t;p)\kappa\} \\
&+ \int_{t_0}^{t} \Psi^T U_a \kappa d\tau.
\end{aligned}
$$

The symmetric positive definite property of $H(.)$ and the skew-symmetric property of $\dot{H}(.) - 2C(.)$ yields,

$$J^c \leq C_v^2 + \int_{t_0}^{t} \|\Psi\|\eta\kappa d\tau + \int_{t_0}^{t} \Psi^T U_a \kappa d\tau \qquad (5)$$

where $C_v^2 = \frac{1}{2} W^T H W \kappa^2|_{t=t_0}$. Inserting (2) into (5),

$$
\begin{aligned}
J^c &\leq C_v^2 + \int_{t_0}^{t} \|\Psi\|\eta\kappa d\tau - \int_{t_0}^{t} \Psi^T \kappa \frac{W\eta^2}{\|W\|\eta + \nu(\tau)} d\tau \\
&= C_v^2 + \int_{t_0}^{t} \nu(\tau)\kappa^2 \frac{\|W\|\eta}{\|W\|\eta + \nu(\tau)} d\tau \\
&\leq C_v^2 + \int_{t_0}^{t} \nu(\tau)\kappa^2(\tau) d\tau. \qquad (6)
\end{aligned}
$$

Condition (4) implies J^c is bounded. The result follows [4].

Theorem 2: *Given (1) and (2), if $\nu(t)$ and $\kappa(t)$ are chosen such that*

$$\limsup_{t\to\infty} \frac{1}{t - t_0} \int_{t_0}^{t} \nu(\tau)\kappa^2(\tau) d\tau \leq C_2^2 < \infty \qquad \forall t \in [t_0, \infty),$$

then stable path tracking is also ensured.

Proof: The proof follows the approach used in [4].

Corollary *If $\kappa(t)$ and $\nu(t)$ are chosen such that*

$$\kappa^2(t)\nu(t) \leq C_s^2 < \infty, \qquad (7)$$

then stable path tracking is ensured.

Proof: Under the condition of the Corollary, it is seen that $\int_{t_0}^{t} \nu(\tau)\kappa^2(\tau)d\tau \leq C_s^2(t - t_0)$. Therefore, $\limsup_{t\to\infty} \frac{J^c}{t-t_0} = \limsup_{t\to\infty} \frac{C_v^2}{t-t_0} + C_s^2$.

Several observations are made. First H_s, C_s and G_s are not based on q, $\dot{q}$ and p, but on the desired path $\{q^*, \dot{q}^*\}$ and parameters p^* which can be precomputed off-line. Second, one does not need to re-organize the robotic dynamics (so as to isolate unknown parameters) before calculating the control torque. Also a simple way to get $H_s(.)$, $C_s(.)$, and $G_s(.)$ is to set, $H_s = 0$, $C_s = 0$, and $G_s = 0$, the control torque reduces to $\tau = -KW + U_a$. This gives the same structure as in [2]. However, since $\|U_a\| \leq \eta$, $H_s = 0, C_s = 0$, and $G_s = 0$ leads to a larger U_a which could require more control energy.

3. Synthesis Examples

Example 1: (Natural Rate-of-Convergence)

Assume that a natural ROC is sufficient ($\kappa(t) = 1$). Then

$$J^c \leq C_v^2 + \int_{t_0}^{t} \nu(\tau) d\tau.$$

Suppose that $\nu(t)$ is chosen such that $J^c \leq C_v^2 + \int_{t_0}^{t} \nu(\tau)d\tau \leq J^*$, where J^* is a design specification. If $\nu(t) = v_1 e^{-v_2 t}$, with $v_1 > 0, v_2 > 0$, then $J_A \leq C_v^2 + \frac{v_1}{v_2} e^{-v_2 t_0}$. In order to meet the specification, v_1 and v_2 are determined such that $C_v^2 + \frac{v_1}{v_2} e^{-v_2 t_0} \leq J^*$. Suppose $t_0 = 0$ and the initial condition is such that $C_v^2 = 10$. If the performance specification is $J^* = 12$, then $\frac{v_1}{v_2} \leq 2$. So by choosing $v_2 > 0$ and $0 < v_1 \leq 2v_2$, $J_A \leq J^*$.

Example 2: (Variable Structure Control)

For any $\kappa(t)$, J^c is ensured to be less than or equal to C_v^2 (see (6)) by choosing $\nu(t) = 0$. This implies that the ROC can be arbitrarily fast and the accumulated tracking error is smaller than any other choice of ν. So one might conclude that variable structure control gives the best control performance and the greatest ROC. However, it is its fast speed that causes chattering. So from a practical point of view, one should not require too large a ROC over the entire period of operation. A piecewise ROC may be useful. This can be achieved by methods similar to those given in [4].

4. Comment

Clearly ν plays an interesting role in these robust control strategies. First, ν is related to the overall tracking performance in that the bound on J^c depends on the choice of ν. Second, ν specifies the boundary layer in the strategies. Since ν is time varying, the boundary layer is also varying. This property can be used to retain the merits of the VSC strategy and avoid the problem of chattering.

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References

[1] J. J. Slotine and W. Li, Applied Nonlinear Control, 1991.

[2] D. Dawson, Z. Qu and S. Lim, "Re-thinking the Robust Control of Robot Manipulators," *Proc. of IEEE Conf. Decision and Control, 1991.*

[3] V. I. Utkin, Control System with Variable Structure, 1976.

[4] Y. D. Song and J. N. Anderson, "System Stability Analysis Based on Generalized Energy Accumulation: Part I — Criteria Development," *Proc. of IEEE Conf. Control Application*, 1992.