Smooth Sliding mode controller design for robotic arm

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***Abstract*—This paper seeks to develop simple yet efficient modelling and control scheme for robotic manipulators in joint space wherein a control law is developed based on sliding mode that provides robustness against uncertainties and perturbations and a stability analysis guarantees the asymptotic stability of origin. Given the bounds of disturbance and a switching manifold that enforces trajectory tracking, a control algorithm is presented to drive the states on the switching surface quickly and maintain them thereafter. Further, the system is analyzed for stability and performances and efforts are made to design a controller with smooth characteristics.**

***Index Terms*—Sliding mode control, nonlinear model, sliding manifold, variable structure, chattering, matched uncertainties, optimal sliding coeficients.**

I. Introduction

In recent years, several developments have been made in design and control of Robotic manipulators for various purposes in order to achieve fast, precise and quality production and several attempts have been made to stabilize the nonlinear model of robotic manipulators. Unlike a linear model, nonlinear models [1] do not have a universal control strategy concerning their equilibrium states and stability but each nonlinear model has its own dynamics of stability and a linear approximation of a nonlinear model is not always feasible. Individual elements comprising the complete system can be described by differential equations that deviate from their linear behaviour due to imperfections that occur either accidently or added intentionally. Various operations in industries require pick and place, part handling and assembly where these manipulators require a proper control strategy to be involved in order to achieve the desired task specifications. In this work, a sliding mode control strategy is proposed for the control of a robotic manipulator described by its nonlinear mathematical model. The controller based on this control is a modified form of ordinary switching controller that is a state dependent feedback controller that intentionally changes the structure of the system. So it is also called a *variable structure* control system [2], [3] composed of independent structures and a switching logic. The overall system behaviour is different from any of its structures. The differential equations of the model are discontinuous on the switching surface. The controller has reaching phase and sliding phase and the system in sliding is not affected by matched uncertainties and order reduction is achieved. However, due to the effects of sampling, switching and delay caused by the components used to implement the controller as well as simulations engines used in modelling, this control suffers from chattering [3]. Chattering results in low control accuracy, high heat losses in electrical power circuits and high wear of moving mechanical parts. It may also excite unmodeled high frequency dynamics, which degrades the performance of the system [4] and may even lead to instability. Here, chattering is avoided by making the signum function continuous by piecewise linear approximation and it is replaced by *saturation* function and performances are investigated.

II. Plant description

A robotic manipulator with *n* rigid degrees of freedom [5] is characterized by a set of n generalized coordinates $q^T = [q_1 \; q_2 \; ... \; q_n]$ called the joint space. The dynamic equation governing the nonlinear model of a n-dof robotic manipulator is given as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{1}$$

where
q : vector of generalized coordinates
$M(q)$: n × n positive definite inertia matrix
$C(q,\dot{q})$: n × n matrix of Coriolis and Centrifugal forces
$g(q)$: n×1 vector of gravitational forces
τ : vector of torques

Representing in state space, $x = [x_1 \; x_2]^T = [q \; \dot{q}]^T$
$\Rightarrow M(x_1)\dot{x_2} + C(x_1,x_2)x_2 + g(x_1) = u$
$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix} = \begin{bmatrix} x_2 \\ M^{-1}(x_1)[u - C(x_1,x_2)x_2 - g(x_1)]\end{bmatrix}$$
$\dot{x} = f(x,u)$ is the general form of nonlinear state space equation where f(.) is a vector of function satisfying mathematical conditions that guarantee the existence and uniqueness of solution but a form that is usually encountered in practice and includes the equations of robotic manipulator is

$$\dot{x} = f(x) + b(x)u + d(x) \tag{2}$$

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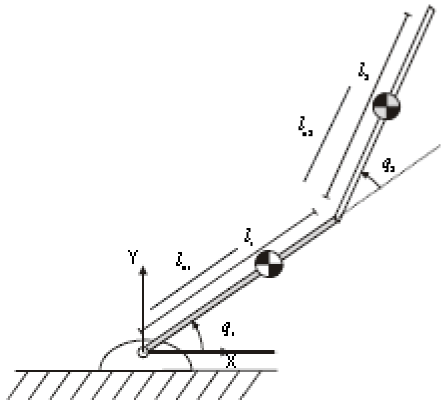

Fig. 1. Two link planar robotic manipulator

where d is the disturbance.

The dynamics of a two link manipulator [6] involves the following matrices

$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$

where $M_{11} = m_1 l_{c1}^2 + m_2(2l_1 l_{c2} c_2 + l_1^2 + l_{c2}^2) + I_1 + I_2$

$M_{12} = l_{c2}^2 m_2 + l_1 l_{c2} m_2 c_2 + I_2$

$M_{21} = l_{c2}^2 m_2 + l_1 l_{c2} m_2 c_2 + I_2$

$M_{22} = l_{c2}^2 m_2 + I_2$

$C(q,\dot{q})^T = \begin{bmatrix} -m_2 l_1 l_{c2} s_2 \dot{q_2}^2 - 2m_2 l_1 l_{c2} \dot{(q_1)} \dot{q_2} & m_2 l_1 l_{c2} s_2 \dot{q_1}^2 \end{bmatrix}$

where $c_i = \cos q_i$ and $s_i = \sin q_i$

III. Problem formulation

The set point control objective is to find τ such that $\lim_{t\to\infty} q(t) = q_d$.

$q_d \in R^n$ is a given constant vector representing desired set point.

$\Rightarrow \lim_{t\to\infty} \tilde{q}(t) = 0$

$\tilde{q}(t) \in R^n$ stands for joint position errors.

$\tilde{q}(t) = q_d - q(t)$

For practical purposes, it is desirable that the controller does not depend upon joint acceleration $\ddot{q}$.

The feedback controller can be designed using the equation

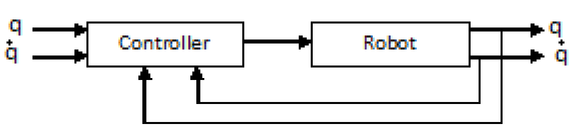

Fig. 2. Closed loop system for set point control

(2) along with a suitable choice of sliding manifold.

IV. Sliding Mode Control Algorithm

A. Sliding mode control

Sliding mode control is a *variable structure system* [2]. It is a dynamical system whose structures change in accordance with the current value of the state. A variable structure system can be viewed as a system composed of independent structures together with a switching logic between each of the structures. This control algorithm focuses on stabilizing the state $x(t)$ of an uncertain system whose only bounds of uncertainty are known. Some suitable manipulation is done in order to make the sign of the derivative of the state, i.e, $\dot{x}(t)$ opposite to that of the state itself that is to be stabilized. If $x(t) > 0$, then $\dot{x}(t) < 0$ and vice versa. Thus, if the initial condition $x(0) > 0$, then $\dot{x}(t) < 0$ and $x(t)$ tends to decrease and reach $x(t) = 0$. Similarly for $x(t) < 0$, its derivative $\dot{x}(t) > 0$ and approaches $x(t) = 0$ [7]. In either case, the state is moving towards $x(t) = 0$ irrespective of any initial condition. Due to this nature, any moment the trajectory crosses $x(t) = 0$, it is reforced towards it. This requires an essentially very high switching(infinite frequency switching) to consistently maintain the state there [7]. The motion of the system while confined to the switching line or a hyperplane is referred to as sliding. A sliding mode will exist if in the vicinity of the switching surface if the state velocity vectors are directed toward the surface. In such a case, the switching surface attracts trajectories when they are in its vicinity; and once a trajectory intersects the switching surface, it will stay on it thereafter. A surface $\sigma(x) = 0$ is attractive if

- any trajectory starting on the surface remains there, and
- any trajectory starting outside the surface tends to it at least asymptotically.

For a sliding motion to occur,

$$\lim_{\sigma\to 0^+} \dot{\sigma} < 0 \text{ and } \lim_{\sigma\to 0^-} \dot{\sigma} > 0 \Rightarrow \sigma\dot{\sigma} < 0 \tag{3}$$

Designing a sliding mode controller can be done in two steps:

- Defining the sliding mode. This is a surface that is invariant of the controlled dynamics, where the controlled dynamics are exponentially stable, and where the system tracks the desired set-point.
- Defining the control that drives the state to the sliding mode in finite time.

The motion consists of a reaching phase during which trajectories starting off the manifold $\sigma(x) = 0$ move toward it and reach it in finite time, followed by a sliding phase during which the motion is confined to the manifold and the dynamics of the system are represented by a reduced-order model [2] with exponentially stable error dynamics.

B. Formulation of the control law

In order to force the system to stay on the sliding mode, we choose u(t) such that $\dot{\sigma} = 0$. The sliding surface is chosen as

$$\sigma(x) = c_1 x_1 + c_2 x_2 = p^T x \tag{4}$$

$p = [c_1 \; c_2]^T$.

c_1 and c_2 are the weighting parameters that affect the states and the trajectory and thus are responsible for any effect

in system's performance. Hence, an optimal choice of these parameters [8] is done by minimizing the quadratic index.
Let us consider the integral that describes the performance index

$$J = \int_{t_s}^{t} x(t)^T Q x(t)\, dx \tag{5}$$

where t_s denotes the start time of the sliding motion and $Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$ is a positive definite symmetrical matrix satisfying the relations $q_{12} - q_{21} > 0$ and $q_{11}q_{22} - q_{12}^2 > 0$.
Introducing an auxiliary variable v such that it satisfies

$$v = x_2(t) + \frac{q_{11}}{q_{22}} x_1(t) \tag{6}$$

the performance index can be rewritten as

$$J = \int_{t_s}^{t} q_{11}^* x_1(t) + q_{22} v^2(t) \tag{7}$$

where $q_{11}^* = q_{11} - q_{12}^2/q_{22}$. Also, by the definition of v, we have $\dot{x_1} = -a_1^* x_1 + v$ where $a_1^* = q_{12}/q_{22}$.
Now, the optimal control law for the dynamic equation given above with a performance index descibed as in equation (7) is

$$v = \frac{-g}{q_{22}} x_1(t) \tag{8}$$

where g is the positive root of the polynomial $g^2 + 2a_1^* q_{22} g - q_{22} q_{11}^* = 0$,i.e, $g = -q_{12} + \sqrt{q_{11}q_{22}}$. Putting the parameters of equation(6) into the above optimal solution, we get the optimal sliding coefficients as

$$c_1 = -g + q_{12} \tag{9}$$

and

$$c_2 = q_{22} \tag{10}$$

A possible choice of Q is taken as $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ which in turn yields $c_1 = 1$ and $c_2 = 1$ and thus $p^T = [1\ 1]$.
For suitable design of sliding mode controller, the parameters f, b, c_1, c_2 must be known in priori. Ref. equation (2).
Thus,

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & f \end{bmatrix} x + \begin{bmatrix} 0 \\ b \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d \tag{11}$$

$$\dot{x} = a + su + hd \tag{12}$$

where

$$a = \begin{bmatrix} 0 & 1 \\ 0 & f \end{bmatrix} x = \begin{bmatrix} x_2 \\ f x_2 \end{bmatrix}, s = \begin{bmatrix} 0 \\ b \end{bmatrix}, h = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The plant is put in its controller companion form which serves dual purpose of qualitative analysis and computer simulations of various results.
We choose a control law such that $\dot{\sigma}(x) = 0$. Let the control law be given as

$$u = \frac{-p^T a}{p^T s} - \frac{p^T h d_{max} + \mu}{p^T s} sgn(\sigma(x)) \tag{13}$$

$$sgn(x) = \begin{cases} 1, if x > 0 \\ 0, if x = 0 \\ -1, if x < 0 \end{cases} \tag{14}$$

V. STABILITY ANALYSIS

The origin is always a stable equilibrium. If this is not the case, we can translate the coordinate axes to suit the requirement. The system is affected by a bounded disturbance d such that $|d| \leq d_{max}$. The objective is to stabilize the origin. Let us propose a Lyapunov candidate of $\frac{1}{2}\sigma^2(x)$ which represents the distance of point from the sliding mode. To guarantee stability in Lyapunov sense, the derivative of the Lyapunov candidate must be negative definite. We now show that $\dot{V}(x) = \sigma\dot{\sigma} < 0$. Ref. equation (3).

$$\begin{aligned} \dot{V}(x) &= \sigma\dot{\sigma} \\ &= \sigma p^T \dot{x} \\ &= \sigma(p^T a + p^T s u + p^T h d) \\ &= \sigma(p^T a + p^T s[\frac{-p^T a}{p^T s} - \frac{p^T h d_{max} + \mu}{p^T s} sgn(\sigma(x))] + p^T h d) \\ &= \sigma(p^T h d - (p^T h d_{max} + \mu) sgn(\sigma)) \\ &= \sigma p^T h d - |\sigma|(p^T h d_{max} + \mu) \\ &= -p^T h(|\sigma| d_{max} - \sigma d) - \mu|\sigma| \end{aligned} \tag{15}$$

Since $|d| \leq d_{max}$
$\Rightarrow |\sigma| d_{max} \geq \sigma d$ for any value of σ
$\Rightarrow \dot{V}(x) < 0$ for all $\mu > 0$
From the above proof, it is clear that the convergence of $V(x)$ to the origin in finite time is guaranteed and stability in Lyapunov sense is achieved. The trajectory reaches the manifold in finite time and stays on it as seen from the inequality above.
On the switching manifold, the dynamics of the system is determined by the value of p^T.

$$\sigma = p^T x = c_1 x_1 + c_2 x_2$$

$$\Rightarrow \sigma = c_1 x_1 + c_2 \dot{x_1} = 0 \tag{16}$$

Solving the differential equation for $x_1(t)$, we have the following solutions

$$x_1(t) = \exp(-\frac{c_1}{c_2}) x_1(0) \tag{17}$$

$$\text{and } x_2(t) = -\frac{c_1}{c_2} \exp(-\frac{c_1}{c_2}) x_1(0) \tag{18}$$

As long as $c_1 c_2 > 0$, the state x will converge to zero exponentially, irrespective of any initial condition $x(0)$. Alternatively we may say that exponential convergence to zero will occur when all the roots of the polynomial $P(s) = c_1 + c_2 s$ lie in the negative half plane.

VI. Controller design and simulations

The design analysis is done on normalized parametric values of f, b, c_1, c_2 and the disturbance is randomly generated but $d \leq 0.9$. μ is taken to be 0.5 for simplicity.

A. Traditional Controller Design

Fig. 3 shows the phase portrait of the model. It is clear that the trajectory emanates from the initial condition and moves towards the manifold in finite time and then it slides on the surface to reach origin, which is stable equilibrium. The farther the initial condition, the longer it takes to reach the sliding surface. The main drawback of this controller is that

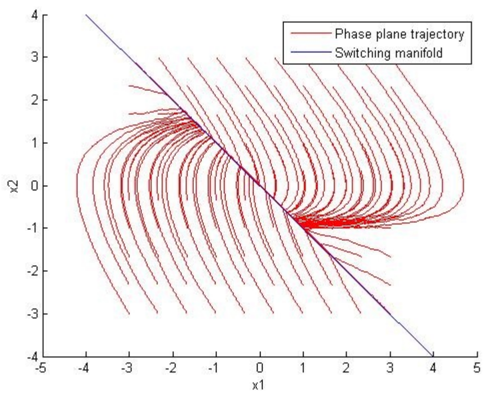

Fig. 3. Phase portrait of the model

it suffers from an undesirable effect called *chattering* which is basically caused due to the electromechanical relays or different hardware used in realizing the controller. Sometimes, numerical quantization errors of a digital signal processor can also cause chattering. Chattering is also caused by the non idealities and imperfections in sensing and actuating elements which may be present or added intentionally for some purpose. This chattering is evident from the plots. Mathematically, this

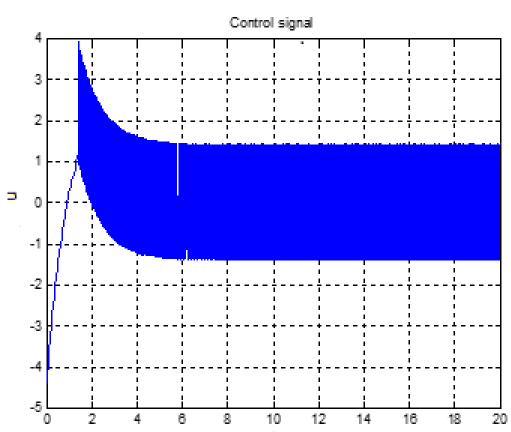

Fig. 4. Control signal

is due to the *signum* function in the control law. However, this problem of chattering can be improved by modifying the control law.

B. Modified Controller Design

The smooth characteristics of hardware can prevent this effect. Let us modify the control law by introducing *saturation* function in place of *signum* function in the law.
Saturation function is piecewise linear approximation of the discontinuous *signum* function.
Saturation function is given as

$$sat(\sigma, \epsilon) = \begin{cases} 1, if \sigma > \epsilon \\ \sigma/\epsilon, if - \epsilon \leq \sigma \leq \epsilon \\ -1, if \sigma < -\epsilon \end{cases} \tag{19}$$

where ϵ is *boundary layer thickness* design parameter used to adjust the accuracy of the approximation.
The control law now becomes

$$u = \frac{-p^T a}{p^T s} - \frac{p^T h d_{max} + \mu}{p^T s} sat(\sigma(x), \epsilon) \tag{20}$$

The magnitude of chattering is dependent on the parameter ϵ. As the value of ϵ is increased, the control signal gets smoother and although chattering is not eliminated completely but its magnitude is reduced to such a great extent that it can be assumed to be negligible.

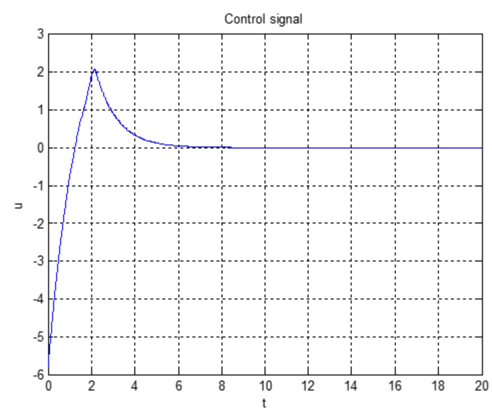

Fig. 5. Smooth Control signal with larger value of ϵ

The plot shown above shows the smooth control signal with values of ϵ centered around 1 and the magnitude of chattering is of the order 10^{-3} and can be assumed to be 0. For lower positive values of ϵ, the magnitude of chattering is smaller compared to that in the controller with *signum* function but it can't be neglected, as clear from fig. 6 below.
Lowering the value of ϵ drives the trajectory to the surface faster but performance of the system is affected. So a proper choice of the parameter is required to make the control signal smooth and to minimize the effect of chattering.
Let us consider $\sigma \leq |\epsilon|$. Then, the Lyapunov function for the modified controller is described by

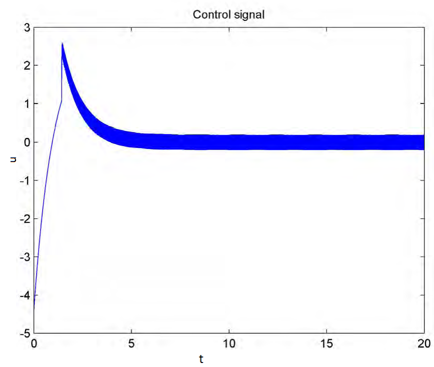

Fig. 6. Smooth control signal with lower value of ϵ

$$\begin{aligned}\dot{V}(x) &= \sigma(p^T hd - (p^T hd_{max} + \mu) sat(\sigma, \epsilon)) \\ &= \sigma p^T hd - \frac{\sigma^2}{\epsilon}(p^T hd_{max} + \mu)\end{aligned} \tag{21}$$

Thus, for smaller values of ϵ, the value of $\dot{V}(x)$ is more negative and the trajectory is driven to the surface quickly.

VII. CONCLUSIONS

We can design a controller that pushes the states towards the equilibrium point on a manifold, thereby guaranteeing stability in Lyapunov sense. However, this controller doesn't possess smooth characteristics, so a slight modification in the control law makes the characteristics smoother. Although the problem of chattering can't be eliminated completely, we can control the magnitude of it by suitably adjusting the design parameter ϵ in the modified control law.

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