SIMULATION OF A 1-D MUSCLE MODEL IN SIMULINK

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SIMULATION OF A 1-D MUSCLE MODEL IN SIMULINK

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ABSTRACT

SIMULATION OF A 1-D MUSCLE MODEL IN SIMULINK

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The most basic property of a muscle is its ability to contract and produce force when stimulated. A muscle is mainly composed of cells consisting of myofibrils with its basic unit called as a sarcomere. A sarcomere is composed of actin and myosin responsible for the muscle contraction. The Hill-type muscle model is the most commonly used model to simulate the behavior of a muscle. A muscle can produce its maximum force at isometric conditions. The level of force produced in the muscle is determined by the the frequency of the signals from the CNS. The force production is also a function of force-muscle current velocity and force-muscle current length relations. A muscle contains two types of sensors; i.e. muscle spindle and golgi tendon organ, which give rise to the feedback control of the muscle length and muscle contraction velocity. In this study a 1-D model of a muscle is formed step by step in Simulink. In the models the muscle mechanics has been investigated and the results are compared with the previous works.

Key Words: Hill Type Muscle Model, Simulink, Stretch Reflex, Quick Release Experiment Simulation

ÖZ

BİR BOYUTLU KAS MODELİNİN SIMULINK ORTAMINDA SİMÜLASYONU

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Bir kasın en önemli özelliği uyarılınca kasılması ve kuvvet üretmesidir. Bir kas temel ünitesi sarkomer olan miyofibrillerin oluşturduğu hücrelerden oluşur. Hill tipi kas modeli kas davranışını simüle etmek için en çok kullanılan kas modelidir. Bir kas üretebileceği maksimum kuvveti izometrik koşullarda üretebilir. Üretilen kuvvetin seviyesi merkezi sinir sistemi tarafından gönderilen sinyal frekansları tarafından belirlenir. Kuvvet üretimi aynı zamanda kuvvet-kas hızı ve kuvvet-kas uzunluğu ilişkilerinin fonksiyonudur. Bir kas iki tip sensör içerir: kas iğciği ve golgi tendon organı. Bu sensörler vasıtasıyla kas uzunluğu ve kasılma hızının geri beslemeli kontrolü yapılır. Bu çalışmada bir boyutlu bir kas modeli Simulink bloklarıyla adım adım modellenmiştir. Modellerde kas mekaniği incelenmiş ve sonuçlar eski çalışmalarla kıyaslanmıştır.

Anahtar Kelimeler: Hill Tipi Kas Modeli, Simulink, Uzama Refleksi, Çabuk Serbest Bırakma Deneyi Simülasyonu

ACKNOWLEDGEMENTS

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NOMENCLATURE

a_f	dimensionless constant
f_{total}	the total force on the contractile element
f_{ce}	the force on the contractile element with damping
V_{max}	the maximum velocity of contraction
f_{ceb}	the force produced by the contractile element (without damping)
v_{ce}	the velocity of contraction of the contractile element
f_b	the damped part of the force on the contractile element
f_v	the force-velocity relation
f_l	the force-length relation
a_t	the muscle activation
F_o	the maximal isometric force
l_o	the contractile element length of the muscle
f_{ce}	the contractile element force
x_p	the length of the contractile element of the spindle
x_s	the length of serial elastic element of the spindle
l_c	the fixed length of the connective tissue
ef	external force
CE	contractile element
SE	serial elastic element
PE	parallel elastic element

CHAPTER 1

INTRODUCTION

1.1. Muscle

Muscle (from Latin musculus "little mouse") is the contractile tissue of the body. It is derived from the mesoderm layer of embryonic germ cells. It is classified as skeletal, cardiac, or smooth muscle [1]. The most basic property of a muscle is its ability to contract and produce power. Muscles exert force and produce movement so can be considered as the basic element of movement mechanics in humans and animals [2]. Much of muscle contraction occurs without conscious thought and is necessary for survival, like the contraction of the heart. Voluntary muscle contraction is used to move the body and can be finely controlled, like movements of the eye. There are two broad types of voluntary muscle fibers, slow twitch and fast twitch. Slow twitch fibers contract for long periods of time but with little force while fast twitch fibers contract quickly and powerfully but fatigue very rapidly [1].

1.2. Types of Muscle

There are three types of muscle:

- Skeletal muscle or "voluntary muscle" is anchored by tendons to bone and is used to affect skeletal movement such as locomotion and in maintaining

posture [1]. The contraction of skeletal muscle is under voluntary control [3]. An average adult male is made up of 40–50 % of skeletal muscle and an average adult female is made up of 30–40 %.

- Smooth muscle or "involuntary muscle" is found within the walls of organs and structures such as the stomach, blood vessels, and unlike skeletal muscle, smooth muscle is not under conscious control.
- Cardiac muscle is also an "involuntary muscle" but is a specialized kind of muscle found only within the heart [1]. Throughout life, it contracts some 70 times per minute pumping about 5 liters of blood each minute [3].

Cardiac and skeletal muscle are striated in that they contain sarcomeres and are packed into highly-regular arrangements of bundles; smooth muscle has neither. While skeletal muscles are arranged in regular, parallel bundles, cardiac muscle connects at branching, irregular angles. Striated muscle contracts and relaxes in short, intense bursts, whereas smooth muscle sustains longer or even near-permanent contractions.

Skeletal muscle is further divided into several subtypes:

- Type I, slow oxidative, slow twitch, or "red" muscle is dense with capillaries and is rich in mitochondria and myoglobin, giving the muscle tissue its characteristic red color. It can carry more oxygen and sustain aerobic activity.
- Type II, fast twitch, muscle has three major kinds that are, in order of increasing contractile speed.
    - Type IIa, which, like slow muscle, is aerobic, rich in mitochondria and capillaries and appears red.
    - Type IIx (also known as type IId), which is less dense in mitochondria and myoglobin. This is the fastest muscle type in humans. It can contract more quickly and with a greater amount of force than oxidative muscle, but can sustain only short, anaerobic

bursts of activity before muscle contraction becomes painful (often incorrectly attributed to a build-up of lactic acid).

- o Type IIb, which is anaerobic, glycolytic, "white" muscle that is even less dense in mitochondria and myoglobin [1].

1.3. Structure of Skeletal Muscles

There are approximately 639 skeletal muscles in the human body [1], which range in size from the eye muscles that control eye movements to the large, powerful leg muscles.

Muscle is mainly composed of muscle cells. Within the cells are myofibrils; myofibrils contain sarcomeres, which are composed of actin and myosin. Muscle spindles are distributed throughout the muscles and provide sensory feedback information to the central nervous system [3]. The muscle is covered by a sheath (= aponeurosis) of connective tissue that penetrates from the surface into the muscle to divide the muscle into columns or bundles. The connective tissue extends beyond the ends of the muscle to form tough, collagenous tendons that attach the muscle to bones. A tendon may be quite long, attaching to bone some distance. These tendons have a white appearance and the central muscle part has a red or pinkish color.

Myofibrils are relatively large, elongated and cylinder-shaped, measuring from 10 to 100 micrometers in diameter and up to 750 mm in length. Muscle fibers are oriented either in the direction of the tendon (i.e. a parallel fiber muscle) or at an acute angle α (=pennation angle) to the tendon (i.e. an unipennated muscle) (Figure 1.1) [4].
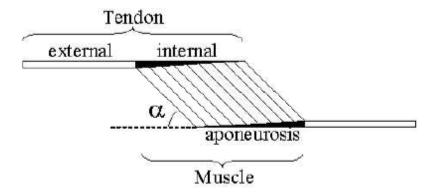
Figure 1. 1- Schematic model of an unipennated muscle. Coupling between muscle fibers and tendon in a pennated muscle. Muscle fibers lie in parallel to each other, have the same length and are oriented at the same angle α to the tendon. Muscle and tendon are linked together at the aponeurosis, or internal tendon. Taken from [4]

1.4. Anatomy of Skeletal muscles

The human skeletal muscle can be organized on different levels of organizations.

1.4.1. Motor Unit

Each motor neuron that leaves the spinal cord usually innervates many different muscle fibers. One motor neuron and all of the muscle fibers that it innervates is called a motor unit [4]. Groups of motor units often work together to coordinate the contractions of a single muscle; all of the motor units that subserve a single muscle are considered a motor unit pool. The muscle fibers that compose a motor unit are dispersed throughout the whole muscle. When a motor unit is activated, all of its fibers contract [5]. Thus, their simultaneous contractions result in an evenly distributed, although weak, contraction of the whole muscle [4].

The number of muscle fibers within each unit can vary: thigh muscles can have a thousand fibers in each unit, eye muscles might have ten [5]. In general, the number of muscle fibers innervated by a motor unit is a function of a muscle's need for the motion. The smaller the motor unit, the more precise the action of the muscle [5]. Thus muscles with large motor units have coarse movements and muscles with small motor units give fine, graded movements [6].
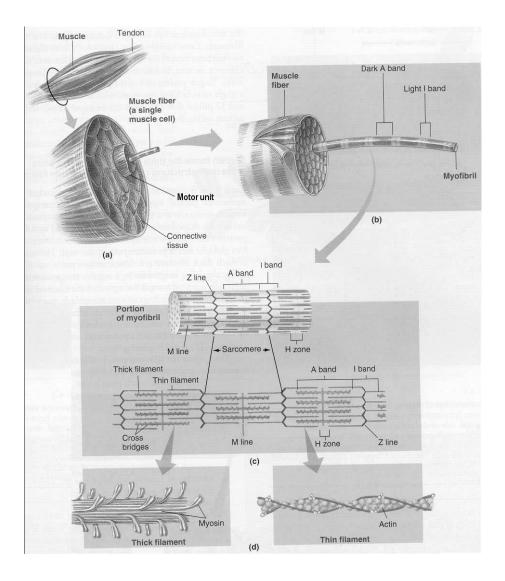
Figure 1. 2 - The different levels of a muscle. Taken from [4]

1.4.2. Myofibril

Every muscle fiber has a presence of numerous myofibrils. These specialized contractile elements are cylinder-shaped intracellular structures 1 mm in diameter that extend the entire length of the muscles fiber. Each myofibril consists of a regular arrangement of highly organized cytoskeletal elements, the thick and thin filaments [5].

The thick filaments, which are 1 to 18 nm in diameter and 1.6 μm in length, are special assemblies of the protein myosin, whereas the thin filaments, which are 5 to 8 nm in diameter an 1.0 μm long, are made up primarily of the protein actin.
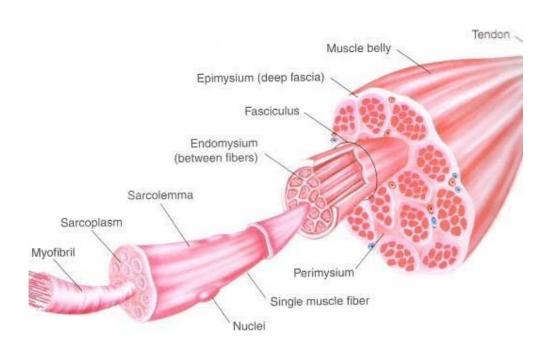
Figure 1. 3 - The different parts of muscle fiber. Taken from [10]
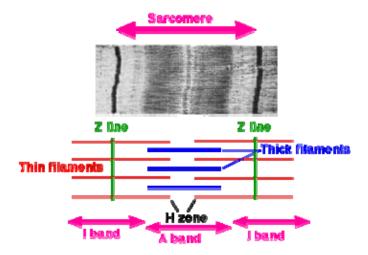
Figure 1. 4 - The sarcomere structure. Taken from [11]

The names of the various sub-regions of the sarcomere are based on their relatively lighter or darker appearance when viewed through the light microscope. Each sarcomere is delimited by two very dark colored bands called Z-discs or Z-lines. These Z-discs are dense protein discs that do not easily allow the passage of light. The area between the Z-discs is further divided into two lighter colored bands at either end called the I-bands, and a darker, grayish band in the middle called the A band. The area between two Z lines is called a sarcomere, which is the functional unit of skeletal muscle [7].

Some other footnotes about a muscle are as follows:

- The sarcolemma is the plasma membrane of a muscle cell.
- The sarcoplasm is the cytoplasm of a muscle cell. The sarcoplasm contains many small nuclei, mitochondria and myofibrils.
- Myofibrils are are located in the sarcoplasm.
- A sarcomere is a repeating pattern of a myofibril. Myofibrils may be thought of as sarcomeres joined end to end.

- Each myosin molecule consists of two twisted protein strands with globular parts called cross-bridges that project outward along their lengths.
- Actin has a binding site to which the cross-bridges of a myosin molecule can attach [11].

1.5. Contraction of Skeletal Muscles

A muscle contraction (also known as a muscle twitch or simply twitch), a complex interaction of several cellular and chemical constituents, [10] occurs when a muscle fiber shortens. The contracting part of a muscle is called contractile element. Contraction is controlled by the central nervous system comprised of brain and spinal cord. The brain controls voluntary muscle contractions, while the spine controls involuntary reflexes.

For voluntary muscles, contraction occurs as a result of conscious effort originating in the brain. The brain sends signals through the nervous system to the motor neuron that innervates the muscle fiber and [8] when enough receptors are stimulated, an action potential is generated and the permeability of the sarcolemma is altered. This process is known as initiation [9]. Involuntary muscles such as the heart contract as a result of non-conscious brain activity.

1.6. Acetylcholine

Acetylcholine (ACh) is the neurotransmitter that motor neurons use to control skeletal muscle. ACh is stored in synaptic vesicles near the distal end of its axon [10]. When a nerve impulse reaches the end of the axon, some of these vesicles release acetylcholine into the synaptic cleft. Acetylcholine diffuses rapidly across the synaptic cleft, combines with ACh receptors on the motor endplate, and stimulates the muscle fiber. The response is a muscle impulse, an electrical signal

that is very much like a nerve impulse [10]. A muscle impulse changes the muscle cell membrane in a way that transmits the impulse in all directions along and around the muscle cell, into the transverse tubules, into the sarcoplasm, and ultimately to the sarcoplasmic reticulum [11].

1.7. Interaction of Skeletal Muscles

Skeletal muscles almost always function in groups. As a result, when a particular body part moves, a person must do more than contract a single muscle; [10] instead, after learning to make a particular movement, the person wants the movement to occur, and the nervous system stimulates the appropriate group of muscles.

1.8. Contraction Types

1.8.1. Isometric Contraction

An isometric contraction of a muscle generates force without changing length [8]. An example can be found in the muscles of the hand. When the forearm holds an object; the joints of the hand do not move but muscles generate sufficient force to prevent the object from being dropped.

1.8.2. Isotonic Contraction

An isotonic contraction occurs when the muscle develops a constant force and the muscle shortens (concentric) or lengthens (eccentric) depending on the external force [4].

Isotonic contraction takes care of the movement of body parts as isometric contraction is more for posture tasks or hold an object at a certain height. The contraction can require a combination of these two contractions.

1.9. Major Events of Muscle Contraction and Relaxation

To summarize what happens during muscle contraction and relaxation the major events throughout the processes are listed.

1.9.1. Muscle Fiber Contraction

1. The distal end of a motor neuron releases acetylcholine.
2. Acetylcholine diffuses across the gap at the neuromuscular junction.
3. The sarcolemma is stimulated, and a muscle impulse travels over the surface of the muscle fiber and deep into the fiber through the transverse tubules and reaches the sarcoplasmic reticulum.
4. Calcium ions diffuse from the sarcoplasmic reticulum into the sarcoplasm and bind to troponin molecules.
5. Tropomyosin molecules move and expose specific sites on actin filaments.
6. Actin and myosin filaments form linkages.
7. Actin filaments are pulled inward by myosin cross-bridges.
8. Muscle fiber shortens as a contraction occurs.

1.9.2. Muscle Fiber Relaxation

1. Acetylcholinesterase decomposes acetylcholine, and the muscle fiber membrane is no longer stimulated.
2. Calcium ions are actively transported into the sarcoplasmic reticulum.

3. ATP causes linkages between actin and myosin filaments to break without ATP breakdown.

4. Cross-bridges recock.

5. Troponin and tropomyosin molecules inhibit the interaction between myosin and actin filaments.

6. Muscle fiber remains relaxed, yet ready until stimulated again [10].

1.10. Time Constants of Muscle Contraction

Although a single action potential in a skeletal muscle fiber lasts only 1 to 2 msec, the onset of the resultant contractile response lags behind the action potential. The action potential is completed before the contractile machinery even becomes operational. This time delay of a few milliseconds is known as the latent period [4].

Time is also required for the generation of tension within the muscle fiber produced by means of the sliding interactions between the thick and thin filaments. The average contractile time is between 15 and 50 msec and depends on the type of muscle fiber. This contractile response does not stop untill all of the Ca^{2+} ions released in response to the action potential is taken up [10].

1.11. Muscular Responses

One way to observe muscle contraction is connect the muscle to a device that senses and records changes in the fiber's length [10]. An electrical stimulator is usually used to promote muscle contraction.

1.12. Threshold Stimulus

When an isolated muscle fiber is exposed to a series of stimuli of increasing strength, the fiber remains unresponsive until certain strength of stimulation is applied. This minimal strength required to cause contraction is called the threshold stimulus [10]. An impulse in a motor neuron normally releases enough ACh to bring the muscle fibers in its motor unit to threshold.

1.13. Recording a Muscle Contraction

The response of a single muscle fiber to the ACh released by a single action potential is called a twitch. A twitch involves a period of contraction, when the fiber pulls at its attachments, followed by a period of relaxation, during which pulling force declines. These events can be recorded, and the resulting pattern is called a myogram. Note that a twitch has a brief delay between the time of stimulation and the beginning of contraction. This is the latent period, which in human muscle may be less than 0.01 seconds. If a muscle fiber is exposed to two stimuli (of threshold strength or greater) too quickly, it may respond with a twitch to the first stimulus but not to the second. This is because it takes an instant following a contraction for muscle fibers to become responsive to further stimulation [10]. Thus, for a very brief moment following stimulation, a muscle fiber remains unresponsive. This time is called the refractory period.

A resting muscle fiber that is not brought to threshold will not contract. If a threshold strength or above stimulus is applied to a resting muscle fiber, enough calcium ions are released from the sarcoplasmic reticulum to fully activate the action. The actual force generated by that fiber depends on its length when stimulated, but at any given length, it will either contract or not. This has been termed the all-or-none response [11].

To record how a whole muscle responds to stimulation, a skeletal muscle can be removed from a frog or other small animal and mounted in a special device. The muscle is then stimulated electrically, and when it contracts, it pulls on a lever. The lever's movement is recorded as a myogram.
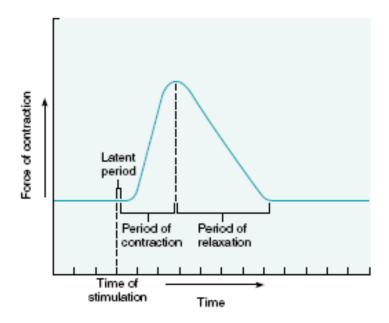
Figure 1. 5 - A myogram of a single twitch. Taken from [10]

1.14. Electromyography (EMG)

Electromyography (EMG) is a technique for evaluating and recording physiologic properties of muscles at rest and while contracting. EMG is performed using an instrument called an electromyograph, to produce a record called an electromyogram. An electromyograph detects the electrical potential generated by muscle cells when these cells contract, and also when the cells are at rest. An Electromyogram is the most common form of biofeedback measurement [10].

1.15. Recruitment of Motor Units

When the motor units are stimulated, the movements that result are gradual. Since the muscle fibers within a muscle are organized into motor units and each motor unit is controlled by a single motor neuron, all the muscle fibers in a motor unit are stimulated at the same time. Therefore, a motor unit also responds in an all-or-none manner [10]. A whole muscle, however, does not behave like this, because it is composed of many motor units controlled by different motor neurons, some of which are more easily stimulated than others.

The total muscle contraction force occurs in two different ways:

1. By increasing the number of motor units contracting simultaneously. Motor units with small number of muscle fibers are far more easily excited than the ones with the large number of muscle fibers, and this affects the gradation of muscle strength during weak muscle contraction to occur in very small steps. The steps become greater as the intensity of contraction increases (the larger motor units begin to contract), a phenomenon known as motor unit recruitment.

2. By increasing the rapidity of contraction of individual motor units. Even though a single action potential in a muscle fiber produces only a twitch, contractions with longer duration and greater tension can be achieved by repetitive stimulation of the fiber. When a muscle is stimulated at greater frequency - strength of contraction is getting bigger [4].

1.16. Summation

The force that a muscle fiber can generate is not limited to the maximum force of a single twitch. A muscle fiber exposed to a series of stimuli of increasing frequency reaches a point when it is unable to completely relax before the next stimulus in the series arrives. When this happens, the individual twitches begin to combine, and the

muscle contraction becomes sustained. In such a sustained contraction, the force of individual twitches combines by the process of summation. When the resulting forceful, sustained contraction lacks even partial relaxation, it is called a tetanic contraction (tetanus) [10].
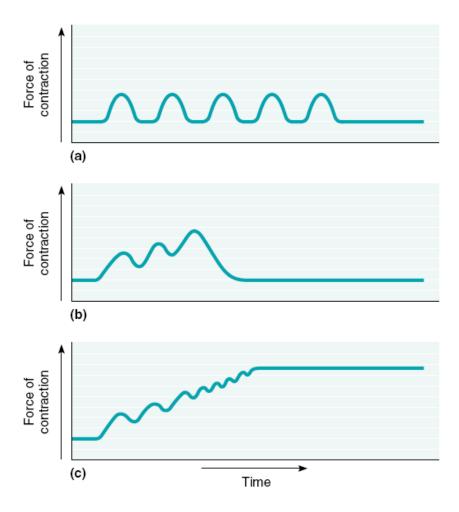
Figure 1. 6 - Myograms of (a) a series of twitches, (b) summation, and (c) a tetanic contraction. Note that stimulation frequency increases from one myogram to the next. Taken from [10]
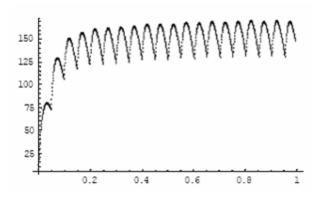
Figure 1. 7 - One sample tetanus figure for frog gastrocnemius muscle. Y axis denotes tension in grams, X axis denotes time in seconds. Taken from [13]

1.17. Muscle Tone (Tonus)

Even when a muscle appears to be at rest, a certain amount of sustained contraction is occurring in its fibers. This is called muscle tone (tonus), and it is a response to nerve impulses originating repeatedly in the spinal cord and traveling to a few muscle fibers [10].

Muscle tone is particularly important in maintaining posture what makes the tonus work against gravity. Tautness in the muscles of the neck, trunk, and lower limbs enables a person to hold the head upright, stand, or sit. If tone is suddenly lost, such as when a person loses consciousness, the body will collapse. Muscle tone is maintained in health but is lost if motor nevre axons are cut or if diseases interfere with conduction of nerve impulses.

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1.18. Origin and Insertion of a Muscle

Usually one end of a skeletal muscle is usually fastened to a relatively immovable or fixed part, and the other end is connected to a movable part on the other side of a joint [10]. The immovable end is called the origin of the muscle, and the movable end is called its insertion. Some muscles have more than one origin and insertion. When a muscle contracts, its insertion is pulled toward its origin. The head of a muscle is the part nearest its origin.

CHAPTER 2

MUSCLE MECHANICS and HILL MODEL

The studies of A.V. Hill and colleagues during the 1920's and 30's resulted in what has become known as the Hill muscle model and still widely used.

2.1. Hill Model Structure

One key finding of Hill and colleagues was the observation that for a given sustained level of neural excitation n, a sudden change in force (or length) would result in nearly instantaneous change in length (or force). This suggests the relationship of a spring:

$$k = \frac{\Delta f}{\Delta l} \tag{2.1}$$

where k is often called the spring constant. Hill also noted that activated muscles produce more force when held isometrically (i.e. at a length fixed) than when they shorten. When muscles shorten, they appear to waste some of their active force in overcoming an inherent resistance. So Hill thought of this resistance as another kind of passive force in the muscle. He found that the faster a muscle shortens, the less total force it produces. Assuming a constant active force, Hill concluded that the faster shortening leads to a larger resistive force.

Hill drew an analogy between the resistive force and a shock absorber (Figure 2.1). A piston in a viscous fluid exemplifies a simple shock absorber, also known as a damper. If you push on its piston, a shock absorber will resist by a tension T (equivalent to a force) that depends on the viscosity b of the fluid in its cavity. The faster you try to push the piston, the stronger the fluid resists. To account for the fact that muscle produces less force when it shortens, Hill proposed that this viscous element lies in parallel with the contractile element. Accordingly, this component can be called a parallel elastic element.
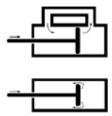
Figure 2. 1 - A simple shock absorber (a damper). The force on the piston can not change the position of the piston instantaneously. It must push against a liquid that has a certain viscosity. The relation between the force on the piston and the speed with which the position of the piston changes is governed by the following equation: Force (or tension) equals viscosity times speed $T = b\dot{x}$. Taken from [13]

To investigate the properties of this viscous element, Hill and his colleagues [41] performed a simple experiment. They attached a muscle to a bar that pivoted around a point as shown in Figure 2.2.

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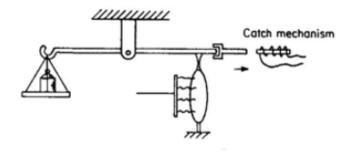
Figure 2. 2 - Catch mechanism. Taken from [13]

One end of the bar had a catch mechanism that they could release at any time. A basket held a weight on the other end of the bar. When Hill released the catch, this weight would pull on the muscle by a force T as shown in Figure 2.3. The experiment began with the catch in place and the muscle stimulated maximally. The stimulation resulted in the production of force T_o in the muscle. Because the muscle pulled on a bar that could not move, the force that the muscle produced did not change the muscle's length.
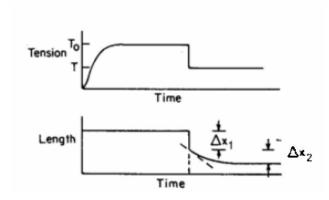
Figure 2. 3- Tension and Length vs. time scopes during the experiment. Taken from [13]
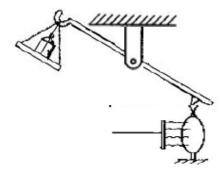
Figure 2. 4 - Catch mechanism released. Taken from [13]

At this point, the experimenters released the catch as shown in Figure 2.4. Note how the length of the muscle suddenly shortened and how the force dropped to T from T_o (Figure 2.3). After this rapid phase of shortening, it is seen how the muscle continued to shorten, but now gradually. The fact that the muscle immediately shortened by amount Δx_1 and reduced its force from T_o to T suggests that something in the muscle acted like a spring. If you put tension on a spring by pulling it, then suddenly release it, the spring will rapidly shorten. This spring is the series elastic element shown in Figure 2.5. It is known that stiffness relates changes in force (or tension) to changes in length. After the immediate change in muscle length and force, a slow, gradual change in length developed, without any change in force. Whereas a part of the muscle's mechanism changed length rapidly in response to the force change, another part did not change as quickly—as if a "shock absorber" acted on the "spring," slowing its response to the force change. The parallel elastic element, referred to above, represents this second passive element in the muscle as shown in Figure 2.5 [13].

The muscle's viscosity, the parallel elastic element and the series elastic element compose the passive components of an elementary model muscle. These all together

form the model of a muscle. In Figure 2.5, the length of the series elastic element is x_1 and the length of the parallel elastic element is x_2.
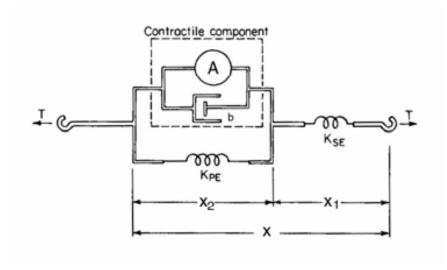
Figure 2. 5 - Hill muscle model. Taken from [13]

2.2. Parts of the Muscle Model

The Hill muscle model mainly consists of the contractile element and the elastic elements now discussed in detail.

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2.2.1. The Contractile Element (CE)

The contractile element is the "active" element in muscle motor unit. It corresponds to the role played by voltage or current sources in an electric circuit. CE responds to motoneuron inputs by contracting. Thus, the tension it produces always acts to try to shorten the muscle. CE is incapable of producing an extension force. The tension developed by CE increases in response to greater motoneuron activity as mentioned, reaching a maximum value that depends on the particular type of muscle (fast/slow twitch type), the effective area of a motor unit (the area per fiber times the number of fibers) and the inherent properties of the muscle (force-length and force-velocity relations).

2.2.2. The Elastic Elements (SE & PE)

As mentioned in section 2.1 a muscle when passively stretched exhibits an elastic restoring force that tends to return the muscle to its original length. In part this force is due to stretching the connective tissue that surrounds the muscle fibers. In part it may be due to stretching the tendons which terminate muscle tissue and attach it to the bone. There is reason to believe that the muscle fibers themselves are at least partly elastic. It is this elastic restoring force that is represented by the elastic elements (springs) in the Hill model. It is not completely correct to assign these elements to any one particular physical arrangement within a muscle, but for our purposes we may regard the PE as being mostly due to the connective tissues and the SE as being primarily dominated by tendon fibers terminating specific motor units [14].

As a result we have the classic Hill model for muscle with springs both in series (SE) and in parallel (PE) with CE .

We also know that for spring-like elements in:

- *series*: forces are the same, and extensions add,

- *parallel*: extensions are the same, and forces add [12].

There exists two forms of Hill model, type A and B. These two formsa re shown in Figure 2.6.
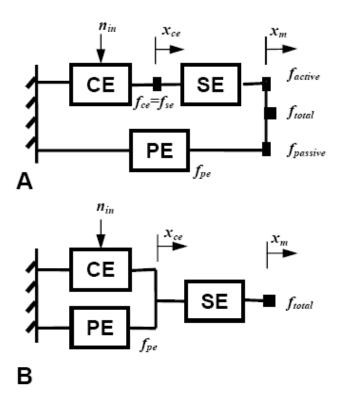
Figure 2. 6 - A. Most common form of the Hill muscle model, with CE (representing the active contractile machinery) bridged by light-damped springs both in series (SE) and in parallel (PE)
B. Alternative form. Note that the constitutive relations for SE and PE differ for the two cases (e.g., with passive stretch where the force across CE is zero, only PE is stretched in A, yet both in B). Taken from [14]

Most modelers use the form A. The first reason is for lengths below the muscle rest length, f_{pe} is essentially zero, and thus SE can be directly obtained. Secondly, when the muscle is not excited, i.e. n_{in} =0 and the force across CE is zero. Since CE and SE are in series, this implies that the force across SE is also zero, i.e. $f_{ce} = f_{se} = f_{active}$ = 0.

2.2.3. Force-velocity Relation

At the beginning of this century, it was recognized that the efficiency of human movement varied as a function of movement speed. At that time, it was found that for a given amount of work, the energy used (efficiency measure) increased with increasing speeds of muscular contraction(i.e. efficiency decreased) [2]. In his classical study Hill worked on the heat production in the frog skeletal muscle [41]. While working, he and his colleagues observed that the force produced by a muscle is a function of its velocity. He focused on shortening muscle, and found that the greater the load, the lower the contraction velocity is. He fit his data with a hyperbolic function that has become known as Hill's equation. This classic force-velocity constitutive relation was mathematically described by Hill [41]. Referring to Figure 2.6B and its terminology, the relation can be given by Equation 2.2 (using f_{total} and v as our variables where f_{total} is the force measured on the serial elastic element and v is the velocity of contraction) :

$$(f_{total} + a_f f_{ce})(v + a_f v_{max}) = const = a_f f_{ce} v_{max} (1 + a_f) \quad (2.2)$$

where af is a dimensionless constant (Hill typically found a value of about 0.25), f_{total} is the force measured on the serial elastic element, f_{ce} is the force produced by the contractile element shown in Figure 2.5, and v_{max} is the maximum velocity of contraction (often called the unloaded maximum velocity). Hill was able to relate af to a thermodynamic entity, the heat of shortening [12].

Through mathematical manipulation, Hill's equation can also be written:

$$f_{ceb} = f_{ce} - f_b = f_{ce} - \left[\frac{f_{ce}(1 + a_f)}{v_{ce} + a_f v_{max}} \right] v_{ce} \tag{2.3}$$

where f_{ceb} is the force on the contractile element excluding the damping and v_{ce} is the velocity of contraction of the contractile element. Here we see more explicitly that the Hill-based CE force-velocity relation can be viewed as the actual CE force f_{ceb} being difference between the force on the contractile element f_{ce} and the velocity-dependent viscous force f_b. Another observation is that the equation shows the relation between f_{ceb}, f_{ce}, v_{max} and v_{ce}.

The Figure 2.7 implies that velocity of muscle contraction is inversely proportional to the load. Thus a large force can not be exerted in very rapid movements that the greatest velocities are attained under conditions of low loading, and that the intermediate values of force and velocity depend on the maximal force [22].
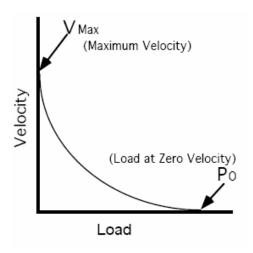
Figure 2.7 - Typical force-velocity curve of a muscle during contraction. Modified from [47]

Again looking at Figure 2.7, it is observed that the load (external force applied) is high when the velocity of contraction is low and that the load decreases with increasing velocity of contraction. Thus, when the load is high, the active muscle force is increased to the required level by reducing the speed of shortening. Conversely, when the load is low, the active force can be made correspondingly small by increasing the speed of shortening [23].

When more muscle fibers are activated than are needed to overcome the load, the excess force is converted into increasing velocity and therefore greater distance of movement. A commonly experienced example is the exaggerated movement, which occurs when one lifts a light object expected to be much heavier [21].

Research for the force-velocity relation to be implemented to the models showed that much work has been done for force-velocity relation of contraction. But how the muscle produces more force in stretch than it is capable of doing so in contraction seems to be not very clear (Figure 2.8).
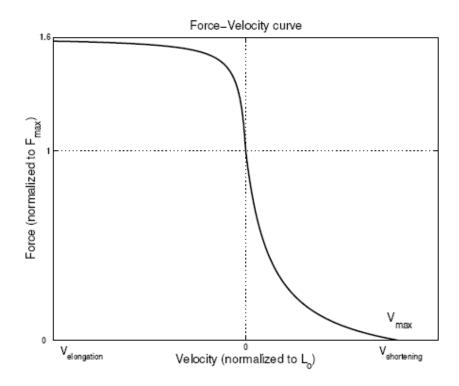
Figure 2. 8 - Force-velocity curve of a muscle (both shortening and elongation). Taken from [4]

When a muscle is stretched at a given speed, its force exceeds the maximal isometric force, F_o and reaching an asymptotic value of about $1.55F_o$ at speeds of stretching much lower than the maximal velocity of shortening [2].

On the other hand, the maximal contraction velocity of the muscle differs according to the type of fiber. The maximum velocities of slow twitch and fast twitch fibers are calculated as

$V_{max}=6.l_o/s$ for slow twitch fibers and

$V_{max}=16.l_o/s$ for fast twitch fibers where l_o is the contractile element length of the muscle in cm and s is the time unit of second [2].

The contraction side of force-velocity relation (positive velocity side) indicates the "absorbed" force production amount of the contractile element under different loads. This means that the force production is less than the maximum that the muscle is capable of during contraction and (the coefficient less than 1) indicates the size of force production loss.

2.2.4. Force-length Relation

The initial length of a muscle, i.e. its length at the time of stimulation, influences the magnitude of its contractile response to a given stimulus. A stretched muscle contracts more forcefully than when it is unstretched at the time of activation. Force capability is less at shorter and longer lengths than the optimal length. Therefore, a muscle can exert the greatest force or sustain the heaviest load when the body position is such as to bring it to its optimal length.

The relationship of force to muscle length may be presented graphically in the form of a force-length curve, shown in Figure 2.9, in which force in an isolated muscle are plotted against a series of muscle lengths from less to greater than its optimal length [21].

As we have seen previously the force that can be produced by a given muscle depends on the length of the muscle. This can be seen in Figure 2.9 along with a diagram of the overlapping actin and myosin filaments that explain why it happens.
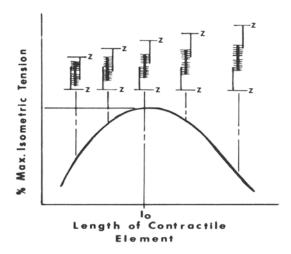
Figure 2. 9 - Typical force-length curve of a muscle during contraction and elongation. Taken from [12]

To interpret it physically, maximal contractile force is developed when sarcomere lengths are such that maximal single overlap of actin and myosin filaments exists. At greater lengths the number of cross-links diminishes as overlap decreases, and at shorter lengths double overlap results in reduced force [24].

2.2.5. Force-length and Force-velocity Relations' Implementation to the Model

The contractile element of the muscle's force production is calculated as follows:

$$f_{ce} = F_{max} \cdot f_v \cdot f_l \cdot a_t \qquad (2.4)$$

where f_{ce} is the contractile element force, F_{max} is the maximum isometric force, f_v is the force-velocity relation, f_l is the force-length relation, a_t is the muscle activation. Muscle activation is the activation level of the muscle determined by the frequency of the signals from the central neural sytem(CNS). It determines directly the force

production of the muscle and the scaling of the force-length and force-velocity relations. The force-length and force-velocity relations are in product to the maximum force (at isometric conditions) and are updated accroding to the activation level. The multiplicative structure shows that the three factors F_{max}, f_v and f_l are linearly independent [4].

2.3. Quick Release Experiment

The quick release experiment is used to determine the force-velocity relation of the contractile element. The process is indeed what is mentioned in section 2.1. Referring to Figures 2.3,2.4 and 2.5, the experiment is overviewed as follows:

- Muscle length is held fixed with the catch.
- Muscle is stimulated to produce steady peak (isometric) force T_0.
- Catch is instantly released.
- At the instant of release, muscle force is reduced to a value T where $T < T_0$ and it depends on the weight in pan [38]. T is called the post-release force.
- The post-release force T is varied (but kept smaller than steady peak isometric force) in repeated runs of the experiment.
- The total muscle length vs. time graphs are drawn. The post-release slope in the graphs is the shortening velocity of the muscle. The initial post-relase slope gives the maximum shortening velocity.
- The initial post-release slope (shortening velocity) against the post-release force is plotted and both axes are normalized to the initial values.
- The force-velocity relation for the contraction of the muscle is obtained [43].

Here the post release force is a constant external force that tries to stretch the muscle. The post-release force is always smaller than the steady peak isometric

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force so that the muscle is kept in contraction. One sample of such experimental resulting graph is shown in Figure 2.10.
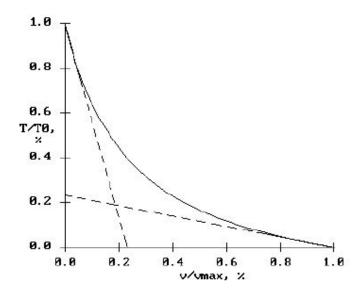
Figure 2. 10 - Force-velocity curve obtained by quick release experiment (both axes normalized to the initial values). Taken from [35]

The resulting graph as depicted in Figure 2.10 is the force-velocity relation of the contractile element of the muscle (contraction side) in normalized conditions. Questions may appear why we did not work on muscle's contractile element and the answer is simple because the data can only be obtained from the tendonuous region during the experiments.

One quick release experimental data of muscle length when the catch is released is found from [44] and shown in Figure 2.11.
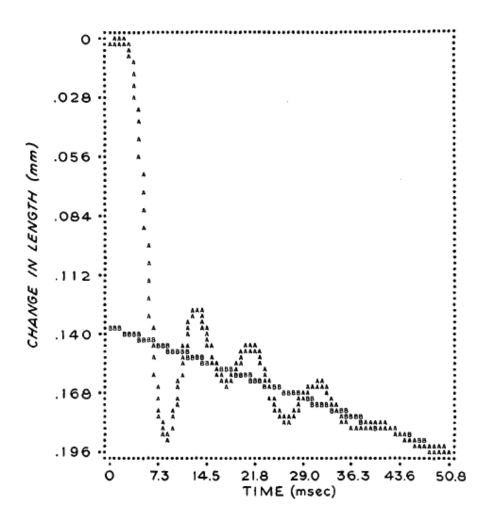
Figure 2. 11 - The change in muscle length vs. time scope when the catch is released in quick release experiment. Taken from [43]

The data in Figure 2.11 is oscillatory. It is unlike the one in Figure 2.3 keeping in mind that Figure 2.3 is a sketch not the experimental data itself. This tells us that if one wants to find the force-velocity relation of the muscle, he should determine some points on the oscillatory data and then should find the best curve fit of this data. The initial slopes (contraction velocities) and afterwards by repeating the experiment the force-velocity relation can then be obtained.

CHAPTER 3

MUSCLE RECEPTORS

Muscle contains two important mechanoreceptors which transduce mechanical stimuli into neural activity. The muscle spindle's functions are to send proprioceptive information about the muscle to the central nervous system, and to respond to muscle stretching. The muscle spindle receptors are innervated by group Ia (large myelinated) and group II (small myelinated) afferent fibers while the golgi tendon organs are innervated by group Ib [4]. The process of sensory transduction occurs in several successive stages. First, the mechanical stimulus causes deformation of the terminal membranes of the receptor, producing conductance changes which give rise to the receptor potential. The receptor potential is then encoded into nerve impulse activity at a nearby impulse-initiating site [15].

3.1. Muscle Spindles

Muscle spindles are found within the belly of muscles and run in parallel with the main muscle fibres. The spindle senses muscle length and changes in length. It has sensory nerve terminals whose discharge rate increases as the sensory ending is stretched. The terminals are wrapped around specialized muscle fibres that belongs to the muscle spindle (intrafusal fibers) and are quite separate from the fibres that make up the bulk of the muscle (extrafusal fibers) [16].

There are 25000 – 30000 muscle spindles in the human body, including about 4000 in each arm and 7000 in each leg. Muscle spindles are encapsulated structures. Each spindle has three main components: a group of specialized muscle fibers, sensory axons that terminate on the muscle fibers and motor axons that regulate the sensitivity of the spindle. The center of the spindle is enclosed by a connective tissue capsule filled with a gelatinous fluid that facilitates sliding of the muscle fibers within it. Thus, the spindle is slightly swollen in the center, and the ends are tapered, giving it fusiform or spindle like shape [4].
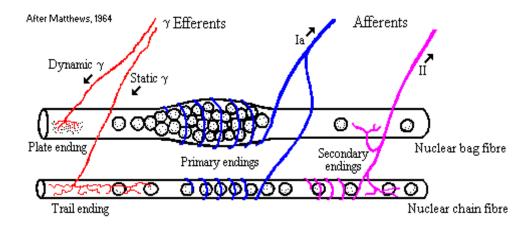
Figure 3. 1 - Muscle spindle endings on fibers. Group Ia and group II fibers can be seen. Taken from [44]

Spindle fibers are generally a few mm in length, with large ones sometimes exceeding 10 mm, and they are attached to in parallel to the extrafusal fibers. They are thus subject to the same stretch (or contraction) as the whole muscle. There is a ratio between the extrafusal fiber and the intrafusal fiber according to the type of the fiber assuming that the extrafusal fibers contract and lengthen uniformly [17]. The portion of the spindle containing the receptor makes up about one to two tenths of the total resting length of the spindle [14].
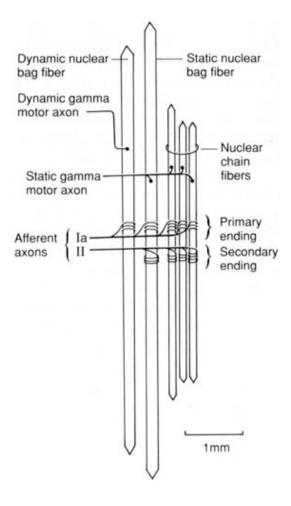
Figure 3. 2 - Muscle spindle types. Taken from [4]

Intrafusal fibers are as mentioned smaller than extrafusal muscle fibers and do not contribute significant force to the muscle contraction. They are thought to contribute to muscle tonus [4].

There are three types of intrafusal muscle fibers [4]:

1. Nuclear chain fibers

2. Static nuclear bag fiber

3. Dynamic nuclear bag fiber

Nuclear chain fibers are short and slender; their nuclei lie in single file within the fiber. Nuclear bag fibers are thicker in diameter and have nuclei clustered in their central regions, which thus appear slightly swollen. The different properties of the three types of intrafusal fibers play a major role in determining the firing characteristics of the sensory endings of the spindle.

The myelinated sensory axons enter the capsule in its central part and terminate on or near the central portions of the intrafusal fibers. Most afferent endings spiral around individual intrafusal fibers. When the fibers are stretched often referred to as loading the spindle, the endings increase their firing rate. This happens because stretching of the spindle lengthens the central region of the intrafusal fibers around which the afferent endings are entwined. The resulting elongation of the afferent endings activates stretch-sensitive channels that depolarize the membrane and generates action potentials. When the stretch is released, referred to as unloading, the intrafusal fibers slacken and the firing rate in the afferent endings decreases [18].
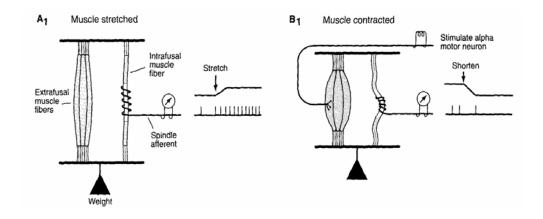
Figure 3. 3 - The response of the spindle in two different cases. Taken from [45]

There are two types of sensory innervation to intrafusal muscle fibers, primary endings (group Ia), which are large, fast-conducting nerve fibers, and secondary endings (group II) which are smaller slower-conducting nerve fibers.

The primary endings of the muscle spindle are highly sensitive to the rate of change of muscle length, a property referred to as velocity sensitivity. The increase in firing rate in primary endings during stretch reflects the rate of change in muscle length-higher rates occur during faster stretches. Because of their high degree of dynamic sensitivity, primary endings respond with bursts of firing to transient stimuli, such as brief taps or vibration of the muscle. Secondary endings in contrast are relatively unaffected by such phasic stimuli because the changes in muscle length occur too quickly to be reflected in the steady-state discharge of these endings [18]. Thus the Ia endings have a larger velocity and acceleration sensitive components of response while the II afferents are more sensitive to low displacements and velocity [4].

Two important factors that affect firing rate of the spindle are:
— Primary endings are most sensitive to small changes in muscle length (less than 0.1 mm) This sensitivity is often reflected in a transient increase in firing rate at the beginning of stretch.
— Primary endings are able to reset their responsiveness to very small stretches after they come to a new length. Consequently they are able to sense small changes in length regardless of the steady-state length of the muscle.
Thus, the relationship between the rate of spindle firing and rate of change in length is not linear and depends in complex ways on other factors, most notably the initial length and the recent history of spindle firing. That is it is not possible to obtain a discharge rate linearly dependent on the length changes of the spindle but rather nonlinear responses of firing are obtained. This issue will be investigated further in the stretch reflex subject.

Velocity of stretch is important in determining the firing rate also. During a rapid stretch, the primary muscle-spindle afferent will fire much more than during a slow stretch. This shows that the spindle senses not only length changes but also velocity changes meaning acceleration sensitive [21].

3.1.1. Model of Muscle Spindle

The physical model of the muscle spindle is in Figure 3.4. As seen it is possible to extend the model of the extrafusal fibers to intrafusal fibers (muscle spindle). This is due to the fact that spindles are initially the contractile fibers but evolve into their new form (muscle receptors) some time after. Thus the spindle has its own contractile element, force-length and force-velocity relations.

The contractile element of the spindle lies at the pole region, which receives synaptic inputs from γ-motor neurons and sensory innervation from a secondary (Group II) muscle-spindle afferents. The central region—called the nuclear bag region—lacks contractile properties and receives sensory innervation from the primary (Group Ia) muscle-spindle afferents. Forces that stretch the muscle spindle result in length changes in the nuclear bag and pole regions, and the muscle-spindle afferents transduce this length change into firing rate. In Figure 3.4 "g" indicates the contractile element of the spindle [13].

The sensory region (nuclear bag region) is represented by the series elastic element and the pole region by the contractile element in parallel with the parallel elastic and viscous elements [4].
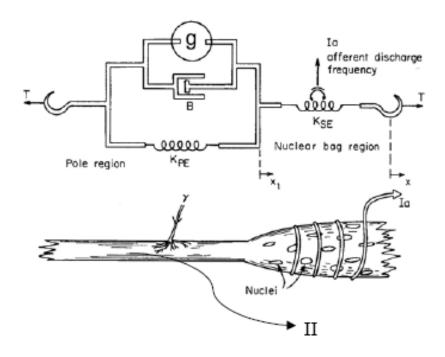
Figure 3. 4 - Muscle spindle model. Taken from [13]

The length of the series elastic element (SE) represents the length of the nuclear bag region. Here x signifies how much the entire intrafusal muscle extends beyond its resting length. The contractile component lies in the pole region and it corresponds to the length of the parallel elastic (PE) element: x_1 signifies how much it extends beyond its resting length and $x-x_1$ to signify how much the series elastic element extends beyond its resting length.

Although it would be natural to assume that the mathematical model of the intrafusal fiber is exactly the same as the model of the extrafusal fiber, this would be misleading and not strictly true. For one thing, in a muscle at its normal resting position the intrafusal fibers are already in extension beyond their slack length. Thus, both the SE and the PE are taut at the normal resting length of the muscle, and therefore they both contribute to tension in the fibers. For another thing, SE and PE

have more clear-cut physical interpretations in the spindle. The PE is associated with the nuclear fibers (bag or chain), while the SE is associated with the receptor area containing the sensory nerve endings. This is the main reason we use the second canonical form of the Hill model for the intrafusal muscle spindle. A third difference is that the length of the spindle is not necessarily the same as the overall muscle length, although it is certainly related to it. The difference is made up by the connective tissue that attaches them at both ends to the main extrafusal muscle mass. We regard the stiffness of this connective tissue as being much greater than that of the spindle fibers, and therefore as not contributing to the dynamics of the spindle [37]. Indeed the connective tissue is the one forming the tendons of the extrafusal muscle, besides the covering sheath of the muscle. We can therefore say that muscle length and spindle length are related as:

$$l = l_c + x_p + x_s \tag{3.1}$$

where x_p is the contractile element length (also the length of parallel elastic element PE), x_s is the length of serial elastic element (SE), l_c is the fixed length of the connective tissue. The addition of all of them gives the total length of the spindle in resting conditions.

As mentioned the spindle springs have slack lengths, the length beyond which they stay in the spindle. The slack lengths are defined as 1/8 of the SE and PE lengths separately. This length is defined as the length over which the springs produce force. When the spindle tries to go down under this length, the active part of the spindle produces the force g(t) so that SE does not become slack. This element's length is important in the sense that it has the path for the major neural feedback (Ia).

3.1.2. Stretch Reflex

A reflex is often defined as a simple, stereotypical behaviour which is classified in relation to the strength of the initiating stimulus. A stretch of the muscle causes a stretch of the sensory regions of spindles. The muscle spindle records the change in length (and how fast), output from spindles increases as a result and this signal is passed along the Ia afferent to the spinal cord. A monosynaptic connection is made by the Ia afferent with the motoneuron which innervates the muscle which contained the responding spindles. Thus, a stretch of the muscle causes a reflexive contraction [19]. This contraction automatically resists the change in muscle length. It also helps to maintain muscle tone and to protect the body from injury.

One of the reasons for holding a stretch for a prolonged period of time is that as you hold the muscle in a stretched position, the muscle spindle habituates (becomes accustomed to the new length) and reduces its signaling. It is possible to gradually train the stretch receptors to allow greater lengthening of the muscles.

Another reason for holding a stretch for a prolonged period of time is to allow this lengthening reaction to occur, thus helping the stretched muscles to relax. It is easier to stretch a muscle when it is not trying to contract.

3.1.3. Components of the Stretch Reflex

The stretch reflex has both a dynamic component and a static component. The static component of the stretch reflex persists as long as the muscle is being stretched. The dynamic component of the stretch reflex lasts for only a moment and is in response to the initial sudden increase in muscle length. The reason that the stretch reflex has two components is because nuclear chain fibers are responsible for the static

component; and nuclear bag fibers are responsible for the dynamic component mostly. The Ia response can be seen in Figure 3.5.
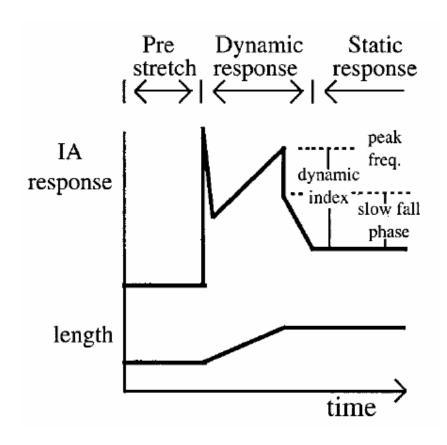
Figure 3. 5 - Definition of measures describing Ia response. Taken from [17]

Nuclear chain fibers are long and thin, and lengthen steadily when stretched. When these fibers are stretched, the stretch reflex nerves increase their firing rates (signaling) as their length steadily increases. This is the static component of the stretch reflex [17].

Nuclear bag fibers bulge out at the middle, where they are the most elastic. The stretch-sensing nerve ending for these fibers is wrapped around this middle area,

which lengthens rapidly when the fiber is stretched. The outer-middle areas, in contrast, act like they are filled with viscous fluid; they resist fast stretching, then gradually extend under prolonged tension. So, when a fast stretch is applied, the middle takes most of the stretch at first; then, as the outer-middle parts extend, the middle can shorten. So the nerve that senses stretching in these fibers fires rapidly with the onset of a fast stretch, then slows as the middle section of the fiber is allowed to shorten again. This is the dynamic component of the stretch reflex: a strong signal to contract at the onset of a rapid increase in muscle length, followed by slightly "higher than normal" signaling which gradually decreases as the rate of change of the muscle length decreases [20].

3.1.4. Sensory Transduction

When an intrafusal fiber is stretched, the sensory endings undergo a mechanical deformation that results in a receptor potential. Under the driving force of the receptor potential, the impulse-initiating region of the nerve membrane produces a sequence of nerve impulses (action potentials) that are conducted along the sensory nerve fiber to the spinal cord (afferent transmission). The mechanical properties of intrafusal fibers determine the proportion of an applied stretch that will act on the sensory endings and consequently how the stretch will be transduced into nerve impulses. The sensitivity of primary and secondary sensory endings appears to be directly related to local stretch in the intrafusal fibers and can be altered by changes in intrafusal fiber stiffness. The polar zones (regions near the ends) of an intrafusal fiber are considerably stiffer than the sensory zones, which are located near the center of the fiber. Therefore, when a muscle spindle is stretched most of the change in length takes place in the sensory zone. Stimulation of a motoneuron produces localized contraction of an intrafusal fiber (usually near the poles). The contraction stretches the sensory zone, which is more compliant, and also enhances its sensitivity to stretch since the polar zones become relatively stiffer and hence more resistant to stretch [21].

3.1.5. Stretch Reflexes Regulate Muscle Tone

Reflex circuit can be viewed as a loop in which the regulated variable is the muscle length. This feedback system counteracts deviations of the regulated variable from desired value.
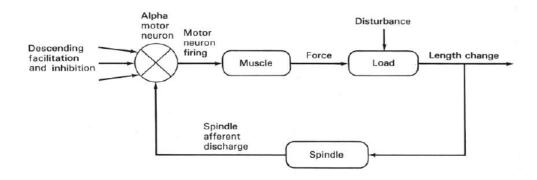
Figure 3. 6 – Reflex circuit. Taken from [46]

The desired value of the muscle length is determined by the sum of the descending excitatory and inhibitory influences on the motor neuron. Deviations from the desired muscle length are sensed by the muscle spindles and fed back to the motor neurons. The motor neurons in turn signal the level of force needed by the muscle to change its length to the desired value. If an external disturbance, such as an increase in load, lengthens the muscle, the discharge rate of the spindle afferents increase. This causes the muscle to contract, counteracting the stretch produced by the load. The stretch reflex loop acts continuously to keep muscle length close to a set value.

Two important considerations of the feedback are the gain and loop delay. The gain of a feedback system refers to its strength and its effectiveness. The larger the gain

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of the stretch reflex, the greater will be the muscle force that results from an imposed change in length. The gain of the feedback system can be changed by the central nervous system.

The loop delay in a feedback system is the time between a disturbance and the compensatory response. For the stretch reflex, the total loop delay is the sum of the sensory and motor conduction times, synaptic delays and the time required for excitation-contraction coupling. The slow mechanical response of the muscle adds significant delay to the compensation for a disturbance. Such delays are obviously undesirable if a disturbance is counteracted rapidly. Muscle spindles by virtue of their velocity sensitivity enhance the responsiveness of the feedback system by contributing a measure of the rate of change of the disturbance to the feedback. This makes the feedback signal larger and more effective when the regulated variable changes rapidly.

The stretch reflex is the only known monosynaptic reflex in the mammalian nervous system. Because the afferent and efferent axons have large diameters   rgansa among the most rapidly conducting neurons in the nervous system, the stretch reflex pathway is adapted for speed of operation.

3.2. Golgi Tendon Organ

The Golgi tendon organ is a slender capsule approximately 1 mm long and 0.1 mm in diameter. Each organ is in series with about 15–20 extrafusal skeletal muscle fibers that enter the capsule through a tight-fitting collar [21]. Golgi tendon   rgansa re not located within the tendons but they are mostly found in the junction between muscular fibers and tendinous tissue [28]. When the skeletal muscle fibers contract, they cause the collagen bundles to straighten, causing compression on the afferent fibers and thereby, their firing. Thus, the organization of the collagen fiber bundles gives them an advantage in sensitivity to small changes in muscle tension. The

afferent fiber responds to multiple motor units and so can register the effects of recruitment. Muscle stretch receptors convey information about muscle length, tension, and velocity of stretch. When the muscle was made to contract actively while still stretched, then the tendon organ farther increased its discharge, but the spindle discharge decreased or ceased altogether. He explained this difference by the anatomical difference: the spindle rgansa re arranged in rgansa with the extrafusal muscle fibers, whereas the Golgi tendon rgansa re arranged in series with the extrafusal fibers. Loading of the muscle activates both the tendon organ and the muscle spindle receptors. Contraction further stretches the tendon organ. However, active contraction of the extrafusal muscle fibers makes the intrafusal fibers go slack, unloading the spindle so that it is no longer stretched [21].

CHAPTER 4

DEVELOPMENT OF A MUSCLE MODEL

In this chapter the stages for developing the muscle model is explained and the model has been introduced step by step as it has beeen created. Each step added to the model introduces another property of the muscle modeled.

We begin with the software choice and then pass on to the stages of the model development.

The models are created in MATLAB 7.0.4 with Simulink/SimMechanics blocks. The blocks can be used for modelling and simulation purposes. With the blocks a wide range of mechanisms can be modeled easily as it contains the simplest form of the mechanical elements.

To mention about the muscle model itself, an initial observation will show that the blocks are good for a simple design. Considering the Hill muscle model for example, the force elements under SimMechanics library would be very useful (i.e. body spring and damper, joint spring and damper) to use. Besides, bodies, joints, sensors and actuators are helpful for building the muscle model and for obtaining the results such as the forces, velocities, motion characteristics without having to derive the equations recalling that there are some nonlinearities in the muscle model such as the force-length and the force-velocity relations.

4.1. Simulink

Simulink is a platform for multidomain simulation and Model-Based Design of dynamic systems. It provides an interactive graphical environment and a customizable set of block libraries that let you accurately design, simulate, implement, and test control, signal processing, communications, and other time-varying systems.

Add-on products extend the Simulink environment with tools for specific modeling and design tasks and for code generation, algorithm implementation, test, and verification.

Simulink is integrated with MATLAB, providing immediate access to an extensive range of tools for algorithm development, data visualization, data analysis and access, and numerical computation.

Key Features

- Extensive and expandable libraries of predefined blocks.
- Interactive graphical editor for assembling and managing intuitive block diagrams
- Ability to manage complex designs by segmenting models into hierarchies of design components
- Model Explorer to navigate, create, configure, and search all signals, parameters, and properties of your model
- Ability to interface with other simulation programs and incorporate hand-written code, including MATLAB algorithms
- Option to run fixed- or variable-step simulations of time-varying systems interactively or through batch simulation
- Functions for interactively defining inputs and viewing outputs to evaluate model behavior

- Graphical debugger to examine simulation results and diagnose unexpected behavior in your design [31].

4.2. SimMechanics

SimMechanics allows engineers to model and simulate mechanical systems in the Simulink environment. With SimMechanics, you can directly model mechanical components (bodies and joints), simulate their motion, and analyze the results without having to derive the mathematical equations describing the system [4].

Technically, the system is characterized by the following properties:
- Provides a modeling environment for building three-dimensional rigid-body mechanical systems
- Includes a variety of simulation techniques for analyzing motion and sizing mechanical components
- Enables the visualization and animation of mechanical system dynamics using either Virtual Reality Toolbox or MATLAB graphics.
- Enables the implementation of high-fidelity, nonlinear plant models in Simulink to support the development and testing of control systems [26].

4.3. The Development Stages

The muscle model development study mainly concentrates on the behavior of the frog gastrocnemius muscle [32],[33],[34]. Some parameters are obtained from literature while some of them are assumed. The assumptions and the steps are mentioned in detail in the work which makes it useful both to learn muscle mechanics and the general Simulink/SimMechanics blocks.

4.3.1. Initial Studies

In chapter 2, great detail has been given about the Hill model. The model is introduced schematically again below in Figure 4.1.
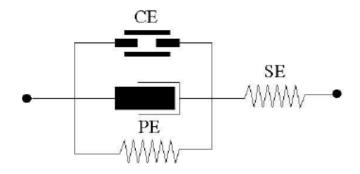

Figure 4. 1 - Hill muscle model

Because there have existed two types of muscle models, firstly we have had to select for the type of model. Firstly, this study does not contain an experimental work in a laboratory but rather collects data from the previous experimental results of literature. Secondly, we have conducted no simulations where the muscle is not excited. These two conditions give us the sufficient reason to build our muscle model with Hill muscle model type B shown in Figure 2.6.

After searching for the blocks in SimMechanics, we see that the parallel elastic element (PE) and the damper in parallel to the conractile element as shown in Figure 4.1 can be modeled with Body Spring & Damper block. See Figure 4.2. Within the block the user can define spring stiffness, damping value and initial length of the spring values where the spring and damper are in parallel (Figure 4.3).
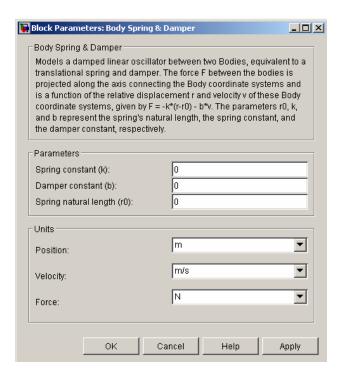
Figure 4. 2 - Body Spring & Damper Block

Figure 4. 3 - Body Spring and Damper Block parameters

The muscle has its own mass and inertia. Besides, it is a 3 dimensional body. The body block in SimMechanics can help to define the mass and inertia of the muscle (Figure 4.4). The inertia is defined with a 3x3 matrix. The body block also features the end points of the body in 3 dimensions. Besides, the connection points, so called CS, can be increased and the orientation of the body can also be defined (Figure 4.5). For the purposes of this study, the muscle is decided to be a 1-D structure.
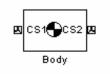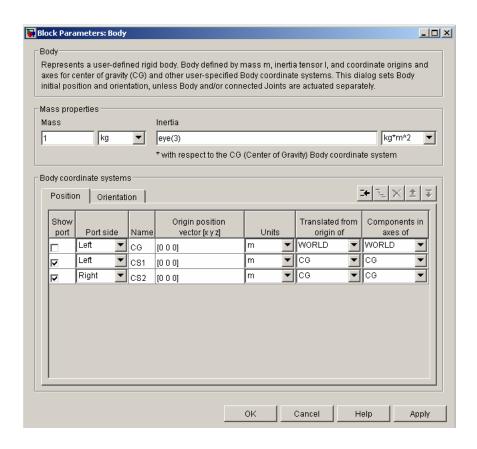
Figure 4. 4 - Body Block

Figure 4. 5 - Body Block parameters. Mass and inertia of the body and the coordinate points can be defined. CG refers to the center of gravity. Orientation tab is next to position tab.

Bodies are connected by means of joints. Model is defined in a 1-D environment so all joints are prismatic joints (Figure 4.6).

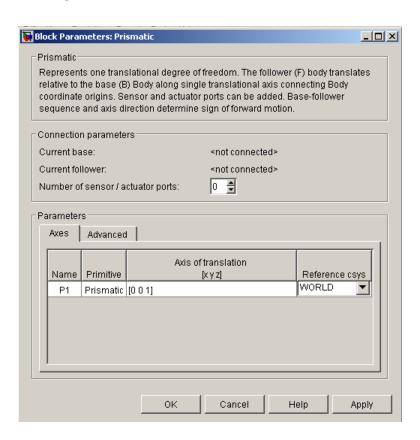Figure 4. 6 - Prismatic Joint. B is base, F is follower.

Figure 4.7 - Prismatic Joint Block parameters

By the help of joints it is possible to define in which direction(s) the motion is to take place. For instance, an axis of translation of [0 0 1] tells the motion can take place in z± directions. For all Body and Prismatic Joint blocks in the model, x± direction is where the motion has been defined. This definiton allows the muscle to move only in one direction.

According to the muscle model defined the Body Spring & Damper block should be connected parallel to the body on each side by means of joints. Therefore, the mass of the muscle is divided into two parts. The two body masses must have the total muscle mass distributed equally and the length between the bodies defines the contractile element length.

To define the ends of the muscle two Ground blocks are used as shown in Figures 4.8 and 4.9. This block is immovable thus helps to define the total muscle length.

Figure 4. 8 - Ground Block

Figure 4. 9 - Ground Block parameters

At least one Ground block must be connected to a Machine Environment for the valid SimMechanics model. If there are more than one Ground blocks in a model, still one Machine Environment Block is enough and can only be connected to Ground block as shown in Figures 4.10 and 4.11. Within Machine Environment Block gravity is defined in z direction as -9.81 m/s^2. In the model the effects of the gravity is somewhat cancelled in the sense that the muscle moves only in one direction. The analysis type is Forward Dynamics.

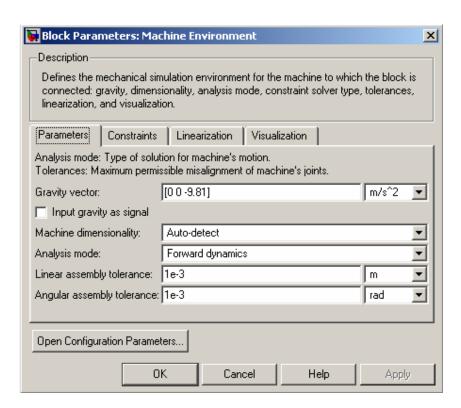

Figure 4. 10 - Machine Environment Block

Figure 4. 11 - Machine Environment Block parameters

Similar to the mass, serial elastic element is distributed on each side of the bodies. Thus the muscle model have two tendons represented by SE (serial elastic element). When two ground blocks are used on both ends of the muscle, Joint Spring & Damper Block is connected to the prismatic joint (Figures 4.12 & 4.13).
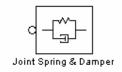

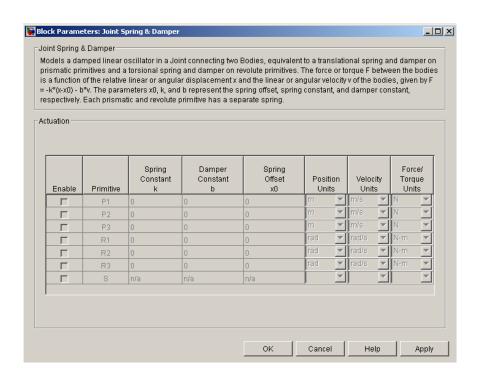Figure 4. 12 - Joint Spring and Damper Block

Figure 4. 13 - Joint Spring and Damper Block parameters

While defining the joint spring and damper block, there should be no damper constant but rather the spring constant. Spring offset should be defined as zero. The initial length of the serial elastic element is defined by the space it occupies, i.e. the space between the bodies and the grounds. If there were no Ground blocks at both ends of the model but instead there were a body, it would also be possible to use the Body Spring & Damper block with the initial length of the serial elastic element. Except the definition of the contractile element, the model mentioned up to this point can be seen in Figure 4.14.
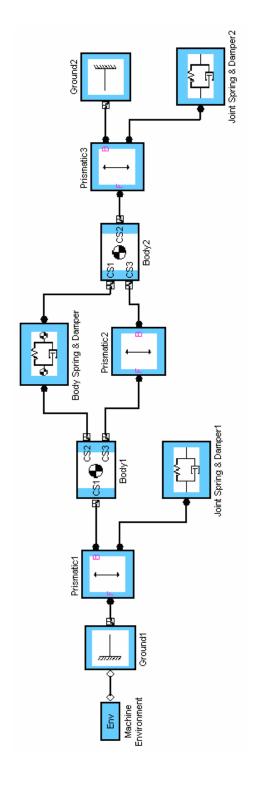
Figure - 4.14 – Initial model

Because it is not possible to find all data for a muscle from one work, the parameters of the model obtained from various literature are listed below:

Ground 1 block Location:

[0 0 0] mm

Body spring & damper block:

Kpe = 196 N/m [33]

b = 49 N.s/m [33]

Spring natural length = 0,040 m [34]

Ground 2 block Location:

[50 0 0] mm

Body1 block:

Mass = 10 g [33]

Inertia = [0,00002 0 0;0 0,00001 0;0 0 0,00001] g.cm^2 [32]

CS1 = [5 0 0] mm

Body2 block:

Mass = 10 g [33]

Inertia = [0,00002 0 0;0 0,00001 0;0 0 0,00001] g.cm^2 [32]

CS1 = [45 0 0] mm

Joint spring & damper blocks:

Kse = 1500 N/m

Joint spring & damper blocks' stiffness values are the same on both sides. The estimated values are approximately 7.5 times the stiffness of the PE which is in accordance with the literature indicating that SE stiffness is about 1 to 10 times more than that for PE [12].

The inertia values of the model are small enough (as for dump bodies) to help the definition of the body blocks.

The length of the muscle is 50 mm while the active part's length is 40 mm. The total mass of the muscle is 20 g [33].

In the model, the contractile element is located at the center of the whole muscle and extending to both sides from that point. Besides, the muscle is defined as parallel fiber muscle.

The body spring and damper is in parallel with a prismatic joint making the force transmission between the bodies along the direction indicated in the joints.

Now the contractile element should be implemented into the model. The active force function (produced by the contractile element) is generated outside and exerted to the muscle model as the simulation begins. Physiologically, the force production occurs as an external force or an electrical twitch acts on the muscle. The active force function implemented in the model is as follows:

$$h(t) = 48144 \exp(-t/0.0326) - 45845 \exp(-t/0.034)$$ (4.1)[13]

where t is the time in seconds, h(t) is the time dependent active force function in grams. This function can be seen in Figure 4.15.
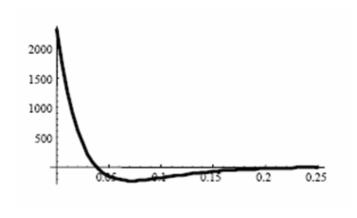

Figure 4. 15 - Single twitch output. Y axis-Force in grams(g), X axis-Time in seconds(s). Taken from [13]

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The function h(t) is of double exponential type and is the response of the muscle to a single twitch under isometric conditions where both ends of the muscle are fixed [13]. As seen in Figure 4.22 there is a negative part of force production. How the muscle produces a negative force is still unknown in literature [48].

A single twitch is the response of a muscle to an electrical twitch. Knowing that the muscle can produce maximum force under isometric conditions, this response can be viewed as the maximum force produced by a single twitch. The time constants of the Equation 4.1 are 32,6 ms and 34 ms which are within the values depicted in section 1.9.

The function h(t) is applied to the prismatic joint Prismatic2 by the Joint Actuator block. The gain block after the single twitch block is for conversion from grams to Newtons. This part of implementation can be seen in Figure 4.16.
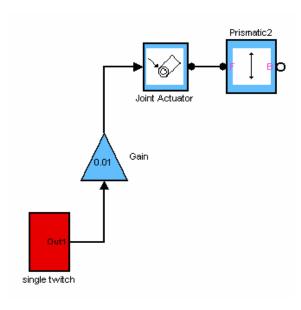

Figure 4. 16 - Implementation of single twitch

Prismatic2 is in parallel to the Body Spring and Damper block so that the contractile element is in parallel with the damper and the parallel elastic element.

The force produced by the contractile element can be defined as single twitch (only once) or in a series of impulses in the model. This "series of impulses" corresponds to the frequency of the signals from the CNS.

Another important point to mention is that the muscle is not observed at the level of one fiber but rather as a lumped mass. Every fiber contracts at the same time and contributes equally to the contraction. Thus the function h(t) can be viewed as the total response of the fibers that make up the muscle.

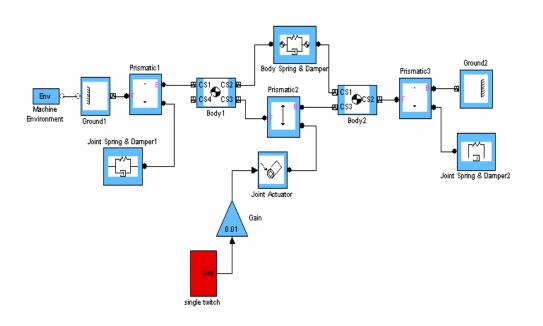The initial sample model can be seen in Figure 4.17.

Figure 4. 17 - Sample model

From this point on the model development stages are discussed in detail. All models except Model 8 are simulated with ode45 variable step solver.

4.3.2. Models

4.3.2.1. Model 1 - Isometric Contraction

Model 1 is for observing the isometric contraction. For the purposes of this type of contraction the model's both ends need to be fixed which has been done already.

At this point the single twitch force production need to be defined in a series of impulses. For this purpose the model in Figure 4.18 is constructed.
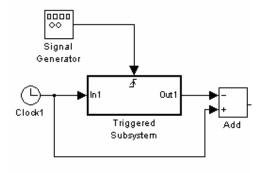

Figure 4. 18 - Triggered subsystem

Here the Signal Generator sends sinusoidal waves at a frequency triggering the subsystem whose output is subtracted from the output of the clock block as shown in Figure 4.18. The trigger type is chosen as rising and the zero crossing detection is enabled in the Trigger block. By these definitions the model is triggered firstly

during the first quarter and secondly during the third quarter of the sine wave. This pattern goes on till the simulation stops.

The output of the Add block in Figure 4.18 is shown in Figure 4.19. The x axis depicts time, y axis shows the output of Figure 4.18.
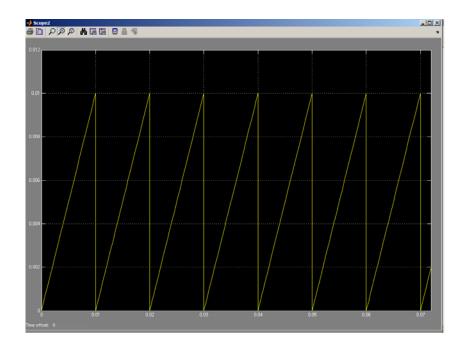

Figure 4. 19 - The output of Figure 4.18. Y axis shows the output of Figure 4.18 and X axis depicts time in seconds.

The output of Figure 4.18 is triangular time samples as shown in Figure 4.19. Within these samples the single twitch function h(t) is applied. All of these details are embedded in a subsystem called Force where the force produced by the contractile element is in a series of impulses.

The frequency of the function Force should be determined by the user. When it is determined, the output of the Force block begins from 2300 grams going down to

some value which changes according to the frequency of the model. It is clear that as the frequency increases, the upper value (2300 g) will remain the same but the lower value of the output of the Force block will increase. This means that for an amount of time the area below the Force block (force x time=impulse) will increase meaning the function be more "impulsive".

A joint sensor attached to the serial elastic elements of the model helps to read the tension in units of N. An initial guess before going into the simulation is that if the model had high enough frequencies, the "tetanus" condition mentioned in section 1.14 would be observed.

The results are discussed in Chapter 5.1.

4.3.2.2. Model 2 - Force-velocity Relation and Its Implementation

The aim of this model is to investigate the effect of the force-velocity relation on the model mentioned in section 2.2.3. Here the isometric conditions are abandoned and the length of the model is made variable. For this purpose Ground1 block is left fixed and Ground2 is replaced by a prismatic joint and a body as shown in Figure 4.20. The Body3 represents the bony structure that the muscle is attached to on one side. A constant external force in N is applied to this body in +x direction to cause stretch so that the muscle can both contract and stretch. The value of the external force is again determined by the user.
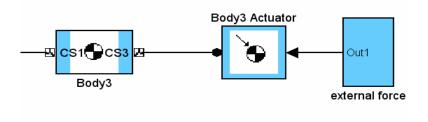
Figure 4. 20 - Ground2 is replaced by the new configuration

The Body3 has the physical properties from [34] and is located at the same point with the Ground2 block.

Body3
Mass=30 g
Ixx=922 g.cm^2
Iyy=1822 g.cm^2
Izz=1252 g.cm^2
The inertia matrix is [922 0 0;0 1822 0;0 0 1252] g.cm^2.

The point of interest for the relation is the velocity of the contractile element of the muscle. Looking at Figure 4.21 the velocity definition can be explained. Since the bodies on both ends of the muscle define the boundaries of the contractile element, the velocity of the contractile element is obtained by subtracting the velocities of those bodies meaning a relative velocity. Velocities are obtained from the body sensors in units of cm/s. The motion is in ±x direction so the y and z components of the velocity are terminated.
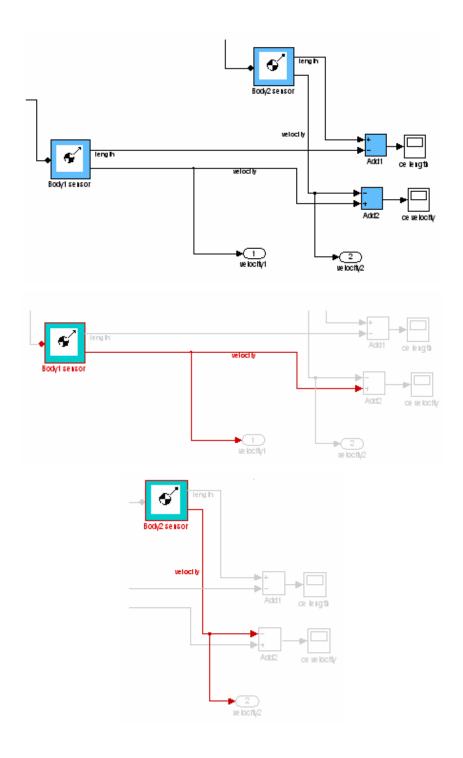
Figure 4. 21 – The definition of the contractile element velocity in three detailed views. Signals highlighted.

Here the relative velocity is adjusted such that for contraction the velocity is positive and for stretch it is negative.

For any value of the external force and the frequency determined by the user, the resulting velocities never have a smooth shape, can cause ripple effect and thus lead to erroneous conclusions. To filter the velocities weighting moving average and zero-order-hold blocks are implemented as seen in the Figure 4.22. The input signal to this filtering diagram is the velocity of the contractile element. This issue is discussed in Appendix.
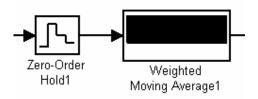

Figure 4. 22 - Filtering diagram

After the velocity and its filtering method is introduced, the force-velocity relations can be implemented. The relations here are updated according to the frequency as mentioned in section 2.2.3. The implementation steps are mentioned as follows:

i) Firstly, the user should determine the frequency. The frequencies are constrained to some values, let us say possible values for the muscle. These values are accepted as 100 Hz,80 Hz,60 Hz,40 Hz and 20 Hz in the model. There is no other intermediate frequency and 100 Hz is the maximum frequency for force production. The force-velocity relations are all defined with lookup tables and for each frequency a different lookup table is used. The main objective of using some determined values is to decrease the number of lookup tables.

ii) Secondly, maximum velocity definition (V_{max}) should be made. The maximum contraction velocity of a muscle is up to its nature of being either fast twitch or slow twitch fiber type as depicted in Chapter 2.2.3. Since every muscle consists of both fiber types, the maximum velocity of contraction is calculated as:

V_{max}=maximum contraction velocity=(16+6)/2 x L_o/s =11 x L_o/s=44 cm/s

where the maximum contraction velocity is assumed to be the summing average of both fiber types' maximum contraction velocities and L_o=4 cm (=40 mm) here. When the Model 2 is tested under isotonic conditions that can give the maximum velocity (100 Hz (maximal activation) and 0 N (constant external force)), maximum velocity is 44.2 cm/s which makes the assumption a good enough approach.

iii) Thirdly, a subsystem called Frequency Switch shown in Figure 4.23 is created and frequency is masked within. The frequencies are the ones listed in part i.
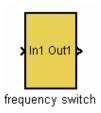

Figure 4. 23 - Frequency Switch

This block accepts the velocity signal as input. The frequency switch blocks are in Figure 4.24.
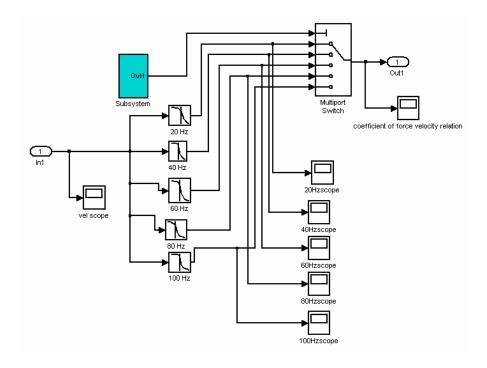
Figure 4. 24 - The Frequency Switch definition

The Subsystem in Figure 4.24 has an embedded function inside and accepts the input as frequency. This embedded function's code is written between [ ] and is as follows:

```
[
function y = fcn(u)
% This block supports an embeddable subset of the MATLAB language.
% See the help menu for details.
y=5;
if u==20
y=1;
elseif u==40
y=2;
elseif u==60
y=3;
elseif u==80
y=4;
elseif u==100
y=5;

end
]
```

The multiport switch is a bridge for multiinputs as the case here, has 5 input ports and does not use zero-based indexing, i.e. 1, 2, 3, 4, 5.

There are 5 different lookup tables all taken from [12]. Within these lookup tables the lookup method is Interpolation-Extrapolation. These tables consist of input (velocity in cm/s) vs. output (force-velocity relation coefficient) values. The out-of-range values for the output are obtained from extrapolation.

The input and output values of the lookup tables are listed:

<u>100 Hz:</u>

input= [-44 -33 -22 -11 -5.5 0 5.5 11 16.5 22 27.5 33 38.5 44]

output= [1.3 1.3 1.3 1.29 1.25 1 0.5 0.3 0.2 0.15 0.10 0.06 0.0357 0]

<u>80 Hz:</u>

input= [-44 -33 -22 -11 -5 0 5.5 11 16.5 22 27.5 33 38.5]

output= [1.05 1.05 1.05 1.025 0.97 0.8 0.3667 0.2335 0.1556 0.116675 0.07334 0.02778 0]

<u>60 Hz:</u>

input= [-44 -33 -22 -11 -5.5 0 5.5 11 16.5 22 27.5 33]

output= [0.80 0.80 0.80 0.78 0.75 0.6 0.31 0.17 0.09 0.0555 0.032 0]

<u>40 Hz:</u>

input= [-44 -33 -22 -11 -5.5 0 5.5 11 16.5 22 27.5]

output= [0.53 0.53 0.53 0.525 0.5 0.4 0.17 0.0905 0.055 0.0225 0]

<u>20 Hz:</u>

input= [-44 -33 -22 -11 -5.5 0 5.5 11 16.5 22]

output= [0.26 0.26 0.26 0.25 0.23 0.2 0.08 0.043 0.019 0]

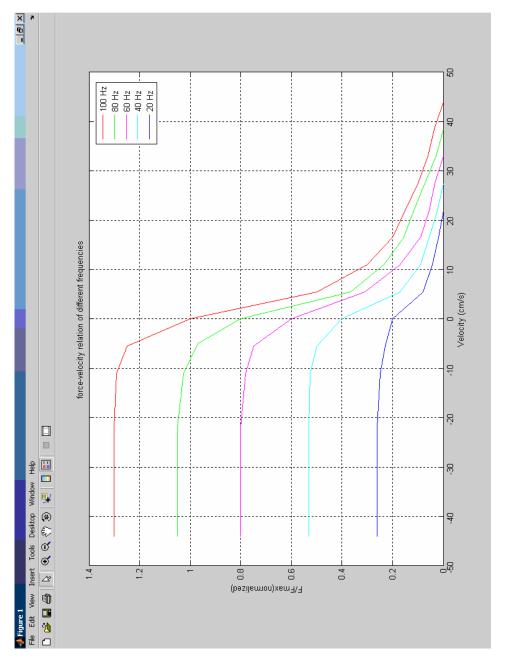All scopes are drawn on the same chart below in Figure 4.25.

Figure 4. 25 - Force-velocity relations for all frequencies

The force-velocity relation may seem to be ambiguous. For instance, the relation can give values greater than 1 as a result of instantaneous velocity no matter the muscle is in contraction or stretch. We know that a real muscle has its own filter and has contraction velocities for contraction and stretch velocities for stretch. The reason of filtering applied to the model is to make the behavior of the model closer to the real case.

The output of the force-velocity relations are in product to the Force block. This makes the maximum force produced by the muscle in isometric conditions change with the relation instantaneously.

The results are in Chapter 5.2.

4.3.2.3. Model 3 - Force-length Relation and Its Implementation

The aim of this model is to investigate the effect of the force-length relation on the model. For this part of the simulation only the force-length relation is added to Model 1. The total length of the muscle is again variable. For applying the external force and making the end of the muscle free, the same prodecure with Model 2 is followed.

For implementing the force-length relation a subsystem called Force-length relation is created as shown in Figure 4.26. Similiar to force-velocity relations, force-length relations are updated according to the frequency.
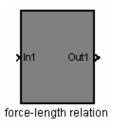
Figure 4. 26 - The force-length relation subsystem

The force-length relation subsystem blocks are shown in Figure 4.27.
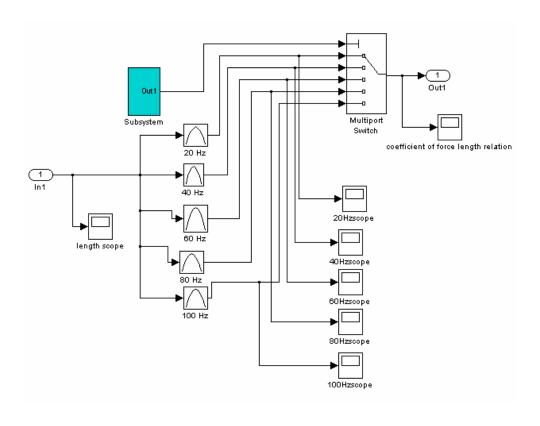

Figure 4. 27 - The force-length relation subsystem definition

The force-length relation for 100 Hz can be written as:

$$FL(l_{muscle}) = -6{,}25 \times (l_{muscle}/l_o)^2 + 12{,}5 \times (l_{muscle}/l_o) - 5{,}25 \qquad (4.2)$$

where lmuscle is the instantaneous length during motion and lo is the optimal length of the contractile element of the muscle. This optimal length is assumed to be the rest length (=initial) length of the muscle (40 mm in this case) [41]. If attention is paid, it is easy to see that Equation 4.2 gives a parabola and the working range of the muscle is roughly between 0,6.lo and 1,4.lo. Thus the range is applicable both in contraction and elongation. The out of range values are all zero [4].

In the lookup tables the input indicates (the muscle length)/(optimal muscle length)=(l_{muscle}/l_o) while the output indicates (instantaneous produced force)/(maximum produced force)=(F/F_{max}). Below the input and output values of the lookup tables are listed:

100 Hz:

input= [0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4]

output= [0 0.4375 0.75 0.9375 1 0.9375 0.75 0.4375 0]

80 Hz:

input= [0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4]

output= [0 0.35 0.6 0.72 0.8 0.72 0.6 0.35 0]

60 Hz:

input= [0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4]

output= [0 0.2625 0.45 0.5625 0.6 0.5625 0.45 0.2625 0]

40 Hz:

input= [0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4]

output= [0 0.175 0.3 0.36 0.4 0.36 0.3 0.175 0]

20 Hz:

input= [0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4]

output= [0 0.0875 0.15 0.177 0.2 0.177 0.15 0.0875 0]

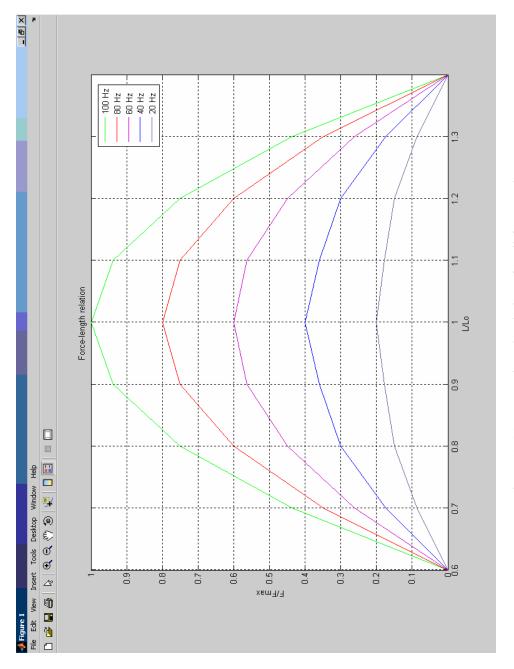All scopes are drawn on the same scope in Figure 4.28.

Figure 4. 28 - Force-length relations for all frequencies

These lookup tables accept the contractile element's length as the input. The length of the contractile element is measured by subtracting Body1's position from Body2's position. The range of the force-length relation is as shown in Figure 4.29.
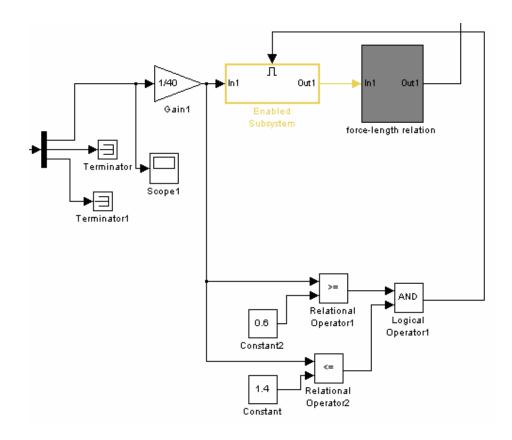

Figure 4. 29 - Force-length relation implementation

The logical operator (AND) defines the range of the force-length relation. Through an enabled subsystem the out of range values for the contractile element length is assured to be zero. Within the enabled subsystem states are held and zero crossing detection is enabled.

During the simulations when the range is violated, the output of the force-length relation is zero. The force-length relation is in product to the force of the contractile element. This means physically no force production at these moments.

The results are in Chapter 5.3.

4.3.2.4. Model 4 - Force-velocity and Force-length Relations Implemented together

This model has both force-length and force-velocity relations implemented. They are both in product with the active force function, so they both affect the muscle motion.

An early guess is that the results would show the characteristics of both of the relations.

The results are in Chapter 5.4.

4.3.2.5. Model 5 - Muscle Spindle Implementation in Isometric Conditions

This model is to investigate the effect of the spindle on the muscle. After finishing the muscle model with inherent relations, the muscle spindle model is now to be implemented. While building Model 5, section 3.1.2 is followed. The muscle spindle model here is not considered as a single spindle or a specific group of spindles but it is rather considered as the one whose response is the cumulative response of all spindles in the muscle. The parameters implemented to the blocks are mainly for bag 2 fiber type though some parameters have the properties for a general spindle. Besides, the intrafusal fibers (spindle fiber) that make up the spindle are assumed to have equivalent properties.

For Model 5 the conditions are isometric. The extrafusal fiber (muscle itself) has no force-length and force-velocity relations. Besides, the intrafusal fiber has no contractile element and feedback (stretch reflex) to the system. The conditions resemble those of Model 1. This way the spindle serves only as a mechanical component parallel to the extrafusal muscle.

In Model 5 one lumped muscle spindle is implemented in parallel to the contractile element of the muscle. The ends of the the spindle are assumed to extend from the beginning to the end of the contractile element so the lengths of it and the spindle are assumed to be the same.

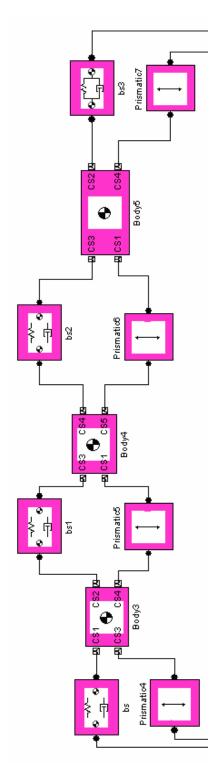In Figure 4.30 the implementation of the spindle model into the model is introduced.

Figure 4. 30 - Spindle model implementation

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In Figure 4.30 there are four different body spring and damper blocks; Bs,Bs1,Bs2,Bs3. Bs and Bs3 which are the leftmost and rightmost bodies represent the very stiff connective tissue by which the spindle attaches to the extrafusal fiber.

The physical parameters are taken as:

Bs&Bs3:

k= 20000 N/m

b= 0 N.s/m

Spring Natural Length = 0.015 m

Bs1:

K_{pe}= 10 N/m [13]

b= 5 N.s/m [13]

Spring Natural Length = 0.008 m

Bs2:

K_{se}= 35 N/m [13]

Spring Natural Length = 0.002 m

Body 3&Body 4&Body 5:

Mass = 0.002 g [17]

Inertia = [0.000002 0 0;0 0.000001 0;0 0 0.000001] g.cm^2

The geometrical locations of the bodies are:

Body3:

[20 0 0] (mm)

Body4:

[28 0 0] (mm)

Body5:

[30 0 0] (mm)

The stiffness value of the connective tissues is assumed as 20000 N/m. As the stiffness being very high, the tissues will have no length change. But except the tissue, the other parts of the spindle will contract or stretch. This way the behavior of motion in the extrafusal fiber is copied onto the spindle. The connective tissue is symmetrically divided. Besides, all body spring and dampers have prismatic joints with axis of translation of motion [1 0 0].

Bs1 & Bs2 represent the pole region and nuclear bag region (sensory region) of the spindle respectively.

The mass of the spindle is 1/10000 [17] while the inertia values are 1/10 of the extrafusal fiber. Mass and inertia values of the spindle makes it like a dummy part.

The contractile element length is 40 mm for the extrafusal fiber and the resting length of the intrafusal fiber active part is 8mm. This depicts that the length ratio is 0.2 for bag 2 fibers [17]. The total length is divided as 1/4 between. (PE length=8 mm, SE length=2 mm) The spring natural lengths defined in the body spring and damper blocks Bs1 and Bs2 are the slack lengths of the springs and they are 1/8 of them (PE slack length=1 mm, SE slack length=0.25 mm). Thus, according to this configuration the springs are already in extension beyond their slack lengths. The tension on the springs can explain the contribution of the muscle spindle to tonus in the resting muscle [42]. Section 2.1.1 defines the geometrical relations mentioned here.

The results are in Chapter 5.5.

4.3.2.6. Model 6 – Muscle with the Muscle Spindle in Non-isometric Conditions

In Model 6 one end of the muscle is left free and then force-velocity and force-length relations of the extrafusal fiber are implemented similiar to what is done in

Model 2 and Model 3. This model is the sum of Model 4 and Model 5, i.e. extrafusal fiber has both relations with one end subject to a constant external force which tries to stretch the muscle and the intrafusal fiber is in parallel to the contractile element of the extrafusal fiber. The spindle still has no contractile element and stretch reflex as the feedback.

The results are in Chapter 5.6.

4.3.2.7. Model 7 - Stretch Reflex Implementation

This model is to investigate the effect of the stretch reflex on the muscle. In addition to Model 6, the Model 7 has the stretch reflex as the feedback. As mentioned in section 3.2.1. stretch reflex is a countereffect to prevent the muscle from stretch.

The primary and secondary endings both contribute to stretch reflex though their functions differ. In this model only the primary endings(Ia) are considered to contribute to the stretch reflex [29] since it responds as the major feedback to stretch. The implementation of the stretch reflex can only be done through a mathematical formula.

The formula found makes the assumption that the firing rate of the spindle is defined as:

$$Iafiringrate = 4.3 * velocity^{0.6} + 2 * displacement + \frac{K}{100} * EMG + meanrate \quad (4.3)[39]$$

where *Iafiringrate* represents the feedback in units of frequency (impulse/s), *velocity* the stretch velocity of the sensory region in mm/s and *displacement* the displacement of the sensory region in mm.

EMG is the normalized, high-pass-filtered, averaged, rectified EMG receptor-bearing of the muscle. $\frac{K}{100}$ is the percentage of maximal EMG recruitment possible

in the muscle [39]. Since the study does not contain any experimental work, the EMG term can be cancelled. Besides, the *meanrate* is 80 impulses/s. This mean rate serves as a base value for the Ia feedback. Since the history of the rate of the firing rate on the spindle before stretch is not known, mean rate term is assumed to be zero for pre-stretch.

The stretch reflex is implemented in addition to the frequency of the model. (i.e. 20, 40, 60, 80, 100 Hz) The feedback changes the frequency at each time step thus affects the force production of the contractile element.

Equation 4.3 is now to be defined in the model. To define the velocity and the displacement of the spindle a new subsystem called Firing is created as shown in Figure 4.31.
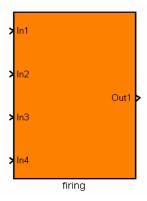

Figure 4. 31 - Firing subsystem

The Firing subsystem has two length inports and two velocity inports. The outport signifies the Ia feedback. The length between Body4 and Body5 defines the serial elastic element of the spindle (Figure 4.32).
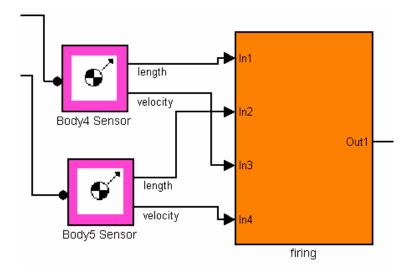
Figure 4. 32 - Firing inports and outports. In1 & 3 from the Body4 sensor and In2 & 4 from Body5 sensor.
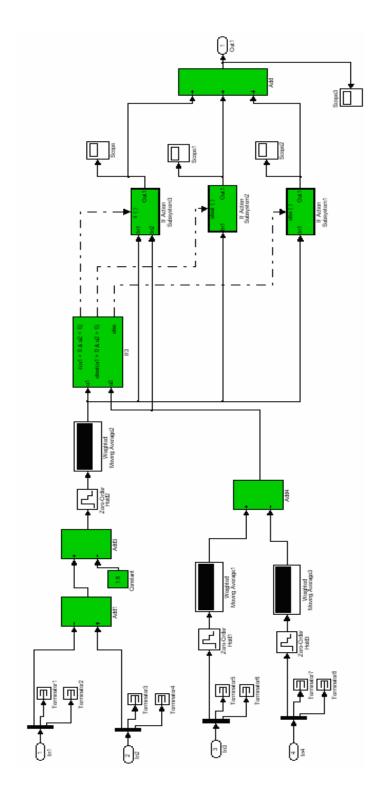
The Firing subsystem's definition is in Figure 4.33.

Figure 4. 33 – Firing Subsystem definition

When the sensory region's length is higher than its initial length (2 mm), it will cause firing. But the firing rate is also dependent on the velocity term in Equation 4.3. According to [42] the velocity term is a stretch velocity. Looking at Figure 4.33, it is seen that the velocity definition of the serial elastic element of the spindle is made according to the stretch velocity in our model which is negative. When the velocity is positive, the velocity term has no contribution to the stretch reflex.

In Model 2, a filtering diagram was used in front of the velocities of the bodies. The same procedure is again followed in Model 7 as the velocity data again has big oscillations. The same filtering diagram is used with the same parameters.

The conditions of firing for both velocity and position are bound together inside the If blocks.

In Figure 4.33 u1 shows the displacement(mm) while u2 the velocity(cm/s) of the serial elastic element of the spindle. The firing algorithm is as follows:
If u1>0 & u2<0 $\rightarrow$ firing
Elseif u1>0 & u2>0 $\rightarrow$ firing
Else$\rightarrow$ no firing

The If Action Subsystem 3 (corresponds to If case) block in Figure 4.33 defines the firing denoted by Equation 4.3. The If Action Subsystem 2 (corresponds to Elseif case) block omits the contraction velocity term and lets the displacement term add to the stretch reflex in the same equation. The other preference If Action Subsystem 1 (corresponds to Else case) has always zero as the output. One important point here is that when the spindle fires and then does not fire for a period of time, the firing rate should be reset to zero. To apply this to the model 'States when execution is resumed' is chosen as 'reset' in all of the action ports.

Now it is time to tell how the frequency of the model is changed by the stretch reflex feedback.

To change the frequency of the model at each time step a subsystem called Frequency Change is created. See Figure 4.34.
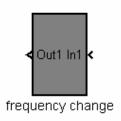

Figure 4. 34 - Frequency Change block. Input to the block is the Ia feedback

As shown previously in Figure 4.18 the signals for the force production are sent from a Signal generator in the form of sinusoidal waves. These waves have the form $Sin(\omega t)$. As known:

$$\omega = 2 * \pi * f \qquad\qquad (4.4)$$

where f denotes the frequency term. The stretch reflex has the output in terms of frequency. The frequency of the model can be changed when the feedback only affects this term. Schematically how the update can be done is shown in Figure 4.35.
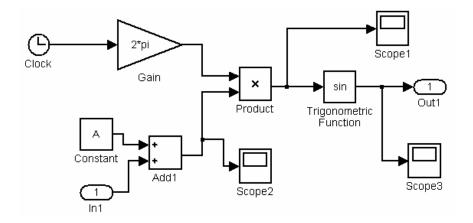
Figure 4. 35 - The Frequency Change block definition

In Figure 4.35 the input to the Frequency Change block is the stretch reflex feedback and the constant A is the masked parameter for frequency which can be 100, 80, 60, 40 or 20 Hz. The feedback and input frequency are in addition and together determine the new frequency of the model at each time step. The Frequency Change block's output is input to the Force block.

The results are in Chapter 5.7.

4.3.2.8. Model 8 - The Serial Elastic Element of the Spindle Slackening Problem Investigation

Slackening is a problem in the serial elastic element of the spindle. As mentioned, without efferent innervation, the muscle spindles would become slack when extrafusal fibers shorten. The spindle afferents would then become silent and the CNS would lose information about muscle length. Thus when the muscle is about to shorten, the CNS must activate the γ-motor neurons by the amount g(t) to maintain tension in the spindle [31]. In this way one can imagine the spindle's serial elastic

element as a rope carrying only tension but slacken under a certin length for compression.

Instead of finding the active force function of the spindle, we deal with the slackening problem with Translational Hard Stop block under SimScape/Foundation Library/Mechanical/Translational Elements. By the help of the block one can specify the range of one dimensional motion between two bodies with lower and upper boundaries. This block will not let the length go down below a slack length but will let tension occur as much as possible. The model is now solved with ode15s variable step solver.

For this purpose one subsystem is created called Serial Elastic Element shown in Figure 4.36 attached to prismatic joint 6 which replaced bs2 in Figure 4.30. The definition of this block is shown in Figure 4.37.
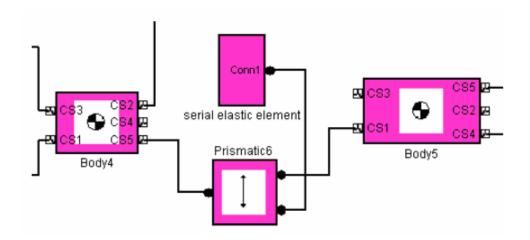

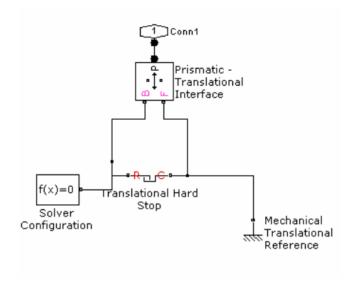Figure 4. 36 – The new serial elastic element block

Figure 4. 37 – The serial elastic element block definition

The parameters of the serial elastic element block is defined as:

Upper bound: 1 m

Lower bound: -1.75 mm

The lower bound is the difference between the initial and the slack lengths of the serial elastic element. So the element is expected to move maximum this amount when under compression.

The results are in Chapter 5.8.

4.3.9. Model 9 - Quick Release Experiment Simulation

As the experiment is observed, the stimulation is allowed to take place at first while one end is held fixed producing a buildup in force production in the muscle. After the maximum buildup force is reached, the muscle is left free. From this point on the external force acting on the muscle (the weight in the basket) tries to stretch it.

While building the model, the muscle has the force-velocity and force-length relations in place. But the spindle and its reflex are not necessary. This is because the muscle is in never stretch condition during the experiment. Besides, the stimulation of the muscle goes on till the end of the experiment.

In the model, the total length of the body can be observed from the external body (Body3) shown in Figure 4.20 as the other one end of the muscle model is fixed with Ground block. This body should be kept for a certain amount of time at its fixed place so that the force buildup can occur. To accomplish this, there is need for a body actuator which supplies Body3 with the reverse sign and the same magnitude of the total net force on it for that amount of time. Figure 4.38 shows the implementation of the quick relase experiment according to what is mentioned up to this point in this section. The joint Prismatic4 is connected to the muscle on the left side.
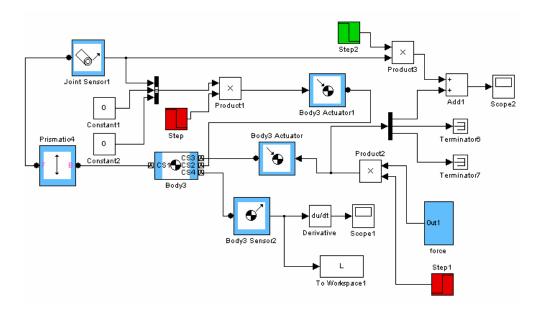

Figure 4.38 – Implementation of the blocks for the quick release experiment

The joint sensor 1 in Figure 4.38 reads the computed force from the serial elastic element of the muscle (not shown here). This computed force is the reverse of the net force acting on Body3. The muscle length is held constant when this force acts on Body3. The simulations last 2 seconds and during the first second the muscle is held fixed then relased. This time is enough for the force buildup. Step functions help to define when these forces act on the model.

The external force is kept smaller than the buildup force for each frequency during the simulations to keep the muscle in contraction as mentioned in section 2.3. At different frequencies (100,80,60,40,20 Hz) the experiment is repeated. Velocities are obtained from the total muscle length scopes with different methods. The results and the methods followed are in Chapter 5.9.

CHAPTER 5

RESULTS

In this chapter the results with different parameters for the models are investigated. In the results "ef" refers to the constant external force in N and "b" to the damping in parallel to the contractile element in N.s/m. The other abbreviations are in the Nomenclature.

5.1. Model 1 Simulation Results

This model investigates how the model works under isometric conditions. The length of the muscle is held constant. Simulation time is 1 second for all tries.

- The model has reached isometric tetanus for all frequencies(100, 80, 60, 40, 20 Hz) as Figures 5.1 and 5.2 reveal when they are compared to Figure 1.7. Figure 5.1 shows the buildup force read from the joint sensors of the serial elastic element on both ends of the muscle model. Figure 5.2 shows the CE length change for all frequencies. As observed in Figure 5.1, the buildup force increases as the frequency increases. Besides, the time to reach the buildup force is about 0.2 s for all frequencies. As seen in Figure 5.2, the ripples of the contractile element are smaller in size and the CE contracts

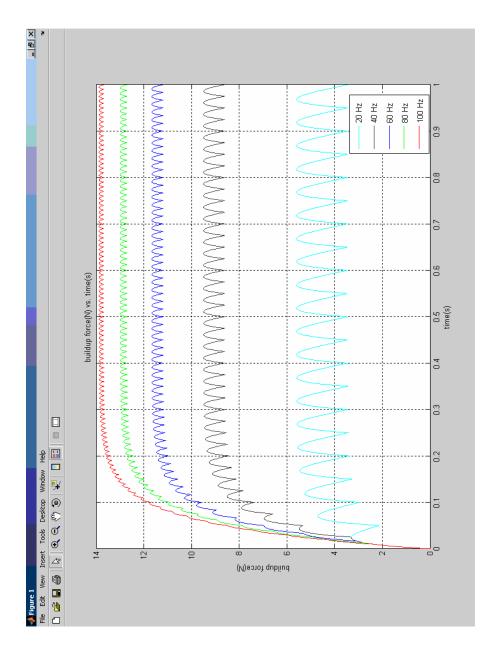more as the frequency increases. The CE does not change length as the tetanus is reached.

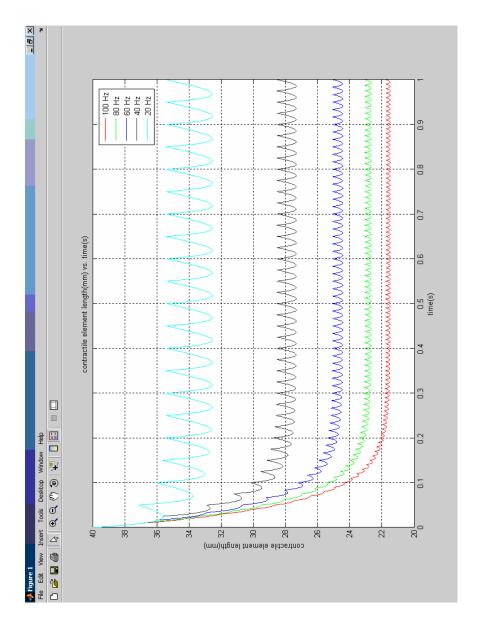
Figure 5.1 - Buildup force(N) for all frequencies

Figure 5.2 - CE length(mm) for all frequencies

- Referring to Figures 5.1 and 5.2, the point beyond which the buildup force does not change is called the saturation point and the point beyond which the contractile element does not change length is the ending point for each frequency. As both figures have ripples, the average of the sum of upper and

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lower points of the ripples is taken as the saturation and ending points. The saturation and ending points are in Tables 5.1 and 5.2 respectively.

Table 5.1 - Saturation points(N) for all frequencies

frequency(Hz)	saturation point(N)
100	13,78
80	12,85
60	11,43
40	9,05
20	4,53

Table 5.2 - Ending points(mm) for all frequencies

frequency(Hz)	ending point(mm)
100	21,62
80	22,86
60	24,75
40	27,93
20	33,96

- If there were no joint spring and damper in the model, it would not be possible to see the buildup force.

- The total muscle mass is 20 g. For a moment this mass is increased to 200 g randomly and divided equally between bodies on both sides of the CE in the model. When the model is simulated at 100 Hz, the buildup force(N) can be seen in Figure 5.3. The ripples are less when compared to the CE length scope for 100 Hz of Figure 5.1.
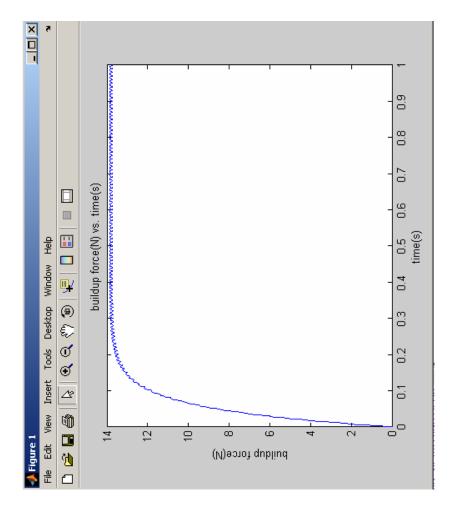
Figure 5. 3 - Buildup force(N) for 100 Hz, mass of muscle=200 g, ripples less when compared to the same frequency scope of Figure 5.1.

- If the damping in parallel to the CE is changed, the saturation point will stay the same. But the time to reach that point will change due to damping effect. If there were no damping (b=0 N.s/m) but still the same force were to be produced by the CE, serial elastic elements on both ends of the model would respond quickly and it would not be possible to obtain isometric tetanus as the springs would oscillate back and forth creating an unstable condition. This result can be seen in Figure 5.4.
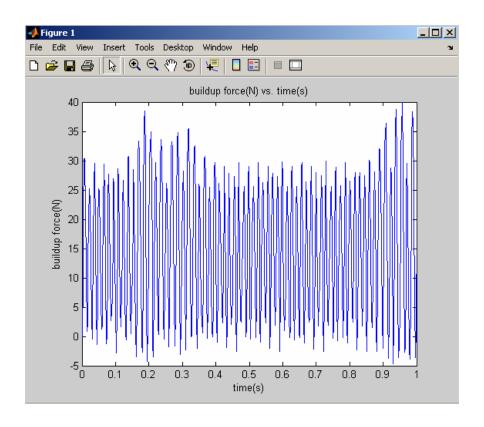
Figure 5. 4 - Buildup force(N) for 100 Hz, no damping

- The length of the PE is 40 mm, the same length with the CE. For a moment the natural spring length of the PE which is defined in its own body spring and damper block is taken as 30 mm. But the positions of the bodies that define the boundaries of the CE is not changed so the CE length is kept at 40 mm. Thus, the PE has some initial tension on it. When the model is simulated at 100 Hz, the buildup force(N) and the CE length(mm) scopes are obtained. As observed in Figure 5.5, the saturation point is 15 N. This value is higher than the saturation point 13.78 N for 100 Hz in Table 5.1 when there is no tension on the PE. When Figure 5.6 is observed, the ending point is 19.4 mm which is lower than 21.62 mm, the ending point for 100 Hz in Table 5.2. This means the muscle has contracted more when there is initial

tension on PE. The initial tension on the springs will become important when we deal with the spindle.
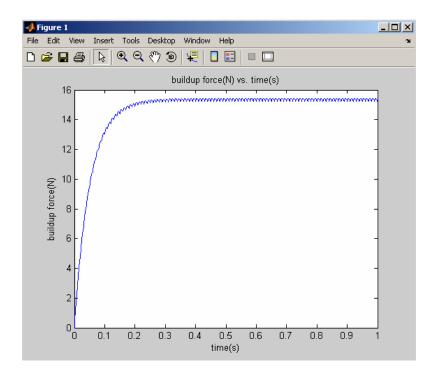
Figure 5. 5 – Buildup force(N) for 100 Hz with initial tension on PE. When there is some initial tension on PE, the saturation point is 15 N. This value is higher than 13.78 N which is the saturation point for 100 Hz.
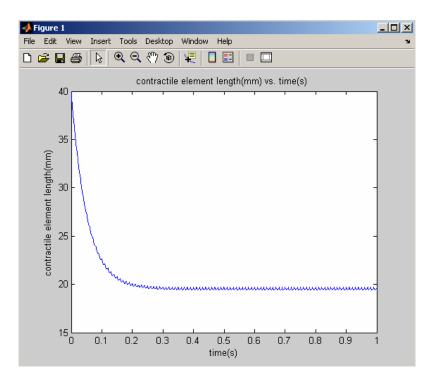
Figure 5. 6 - CE length(mm) for 100 Hz with initial tension on PE. When there is some initial tension on PE, the ending point is 19.4 mm. This value is lower than 21.62 mm which is the ending point for 100 Hz, meaning the muscle has contracted more.

- When a single twitch is applied to Model 1, the CE length (mm) and the force on the SE (N) are shown in Figures 5.7 and 5.8 respectively. In Figure 5.7 it can be observed that the muscle begins to contract immediately. As the single twitch is a small instantaneous force, the CE is expected to come to its initial value of 40 mm. But the result is different since for a short time the CE length is higher than 40 mm. This is the effect of the negative part of the force produced by the CE as shown previously in Figure 4.15. The same countereffect can be seen in Figure 5.8. In this figure, the force on the SE is negative for a short time.
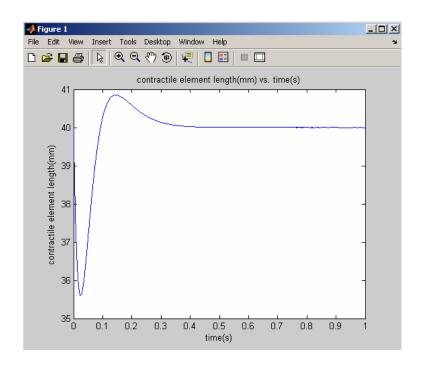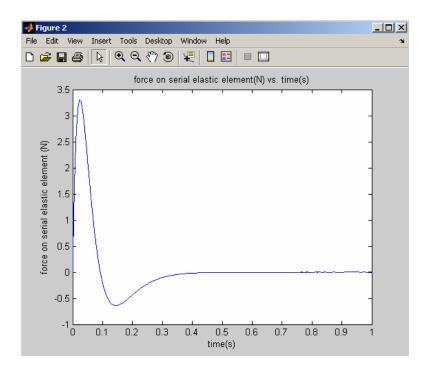
Figure 5. 7 - CE length(mm) for single twitch

Figure 5. 8 - Force on SE(N) for single twitch

To block the countereffect, the value of damping in parallel to the CE can be changed. The higher the damping is, the more we guarantee that the CE will not pass its initial value of 40 mm. But more damping increases the time it takes the CE to come to its initial value. When the damping is changed to b=500 N.s/m from its initial value of b=49 N.s/m for a moment, the new results are in Figures 5.9 and 5.10. As observed in Figure 5.9, the force on the serial elastic element does not go below 0 N as it goes in Figure 5.8. Besides, the length of the contractile element does not pass 40 mm. As expected the time to reach the initial values increase for both scopes.
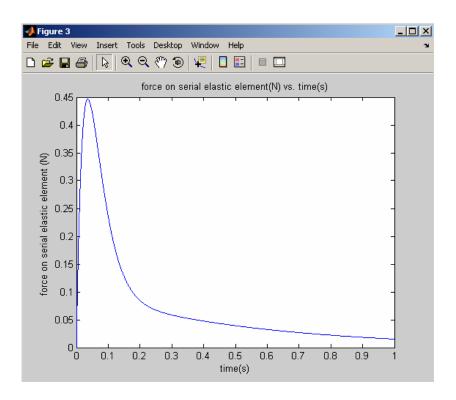
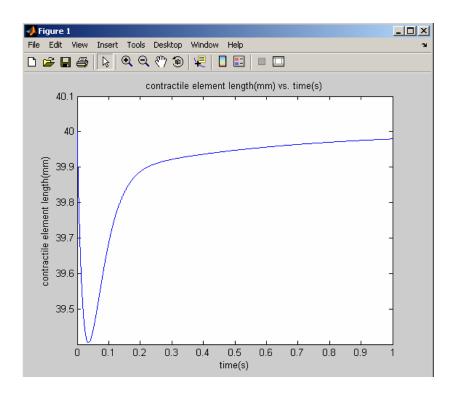Figure 5. 9 – Force on SE(N), b=500 N.s/m, single twitch

Figure 5. 10 - CE length(mm), b=500 N.s/m, single twitch

- As mentioned in section 4.3.2.2, the velocity of the CE is defined as the relative velocity of the bodies on both sides of the CE. A positive value means contraction whereas a negative value means stretch. The CE velocity scopes are obtained for all frequencies. They are shown in Figures 5.11,5.12,5.13,5.14 and 5.15 which correspond to frequencies 100, 80, 60, 40 and 20 Hz respectively. Velocities are much more oscillated than the displacements as expected, being the time derivative of the displacement. The maximum velocities are nearly the same for all cases. The amplitude of oscillations are larger for smaller frequencies.
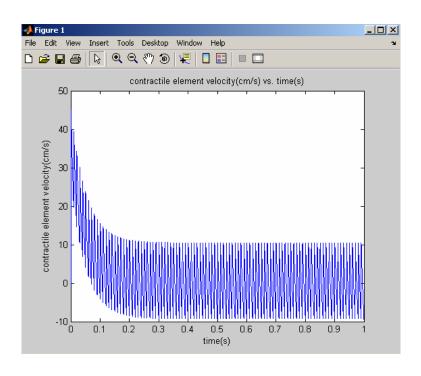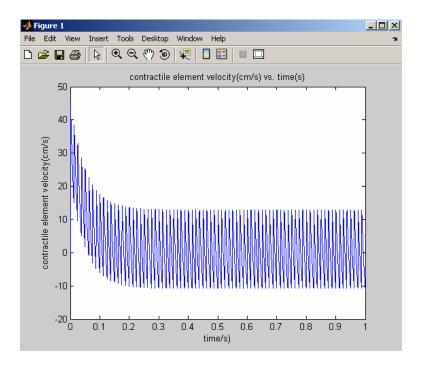
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Figure 5. 11 – CE velocity(cm/s) for 100 Hz
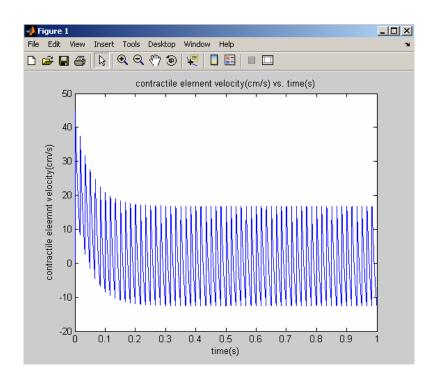

Figure 5. 12 – CE velocity(cm/s) for 80 Hz

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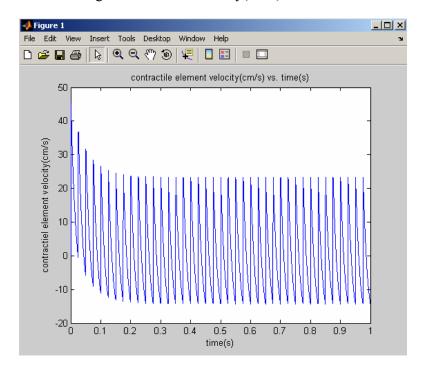Figure 5. 13 – CE velocity(cm/s) for 60 Hz

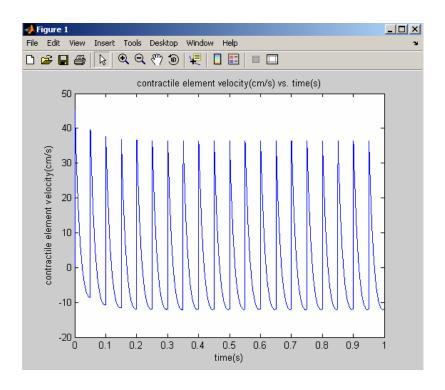
Figure 5. 14 – CE velocity(cm/s) for 40 Hz

Figure 5. 15 – CE velocity(cm/s) for 20 Hz

- The contraction phenomenon of the muscle is directly related to the definition of the base-follower sequence. This sequence can be seen in Figure 4.6. If the places of the Base and Follower are exchanged, there occurs a stretch of the contractile element. Besides, in all prismatic joints, the axis of transmission is [1 0 0]. The direction of motion can also be changed by this axis of transmission. When the axis of transmission is replaced by [-1 0 0], the stretch of the CE can be seen instead of the stretch.

5.2. Model 2 Simulation Results

This model is to investigate the effect of the force-velocity relation on the muscle. As mentioned in section 4.3.2.2 there is no force-length relation here.

- When the model is simulated for 5 seconds at 100 Hz with the external force 0 N, the tension on the muscle can be observed in Figure 5.16. This tension can be read from both sensors of the joint spring and damper as it is done in section 5.1. The Figure 5.16 shows no buildup force. This stems from the external body's motion (Body3 in Figure 4.20) preventing a stabilized buildup. When the simulation is observed, this body seems to oscillate back and forth in an unstable manner. This results in an unstable force occuring on the serial elastic elements. But in Model 1 because both ends are fixed, the tetanus condition is easily reached and a force buildup is observed.
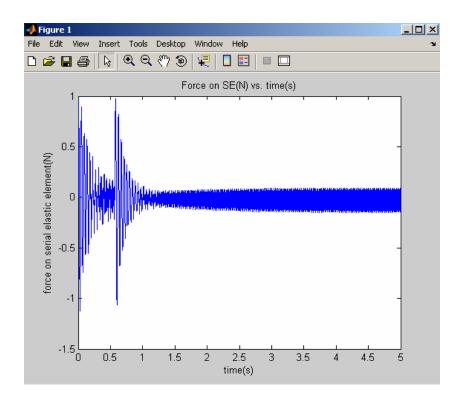
Figure 5. 16 – Force on SE(N) for 100 Hz and ef=0 N. There is no buildup force.

- Model 2 was initially simulated for 1 second at 100 Hz with 0 N external force. The CE length is observed as shown in Figure 5.17.
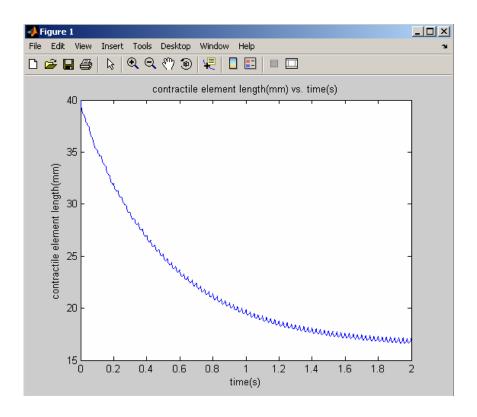
Figure 5. 17 – CE length(mm) for 100 Hz and ef=0 N

Observing Figure 5.17, the ending point of the contractile element length seems not to be reached during the period of 1 s, we extended the time of simulation to 5 seconds. This initially reminded us that some damping like property has existed here so that the time to reach the ending point is longer.

The damping like property of the force-velocity relation can not be deducted from this single scope. Indeed, we need to compare the results of Model 3 and Model 4 as Model 3 has the force-length relation only and Model 4 has the force-velocity and the force-length relations together. The difference between the results will show us the true effect of the force-velocity relation, of course keeping the parameters like the frequency and the external force the same for all simulations.

- As mentioned in section 4.3.2.2 a filtering diagram is applied to the velocities as shown in Figure 4.22. The reason of the filtering and the parameters of the filtering diagram is mentioned in Appendix. Mainly, the filtering is applied to smoothen the oscillations in the velocity scopes. The oscillations in the velocity scopes have already been shown in Figures 4.11, 4.12, 4.13 4.14 and 4.15. When the Model 2 is simulated at 100 Hz with 1 N external force, we checked out the velocity scopes of the CE. The velocities have changed after and before the filtering diagram as presented in Figure 5.18. In this figure, Y axis depicts contractile element velocity in cm/s and X axis shows the time in seconds.
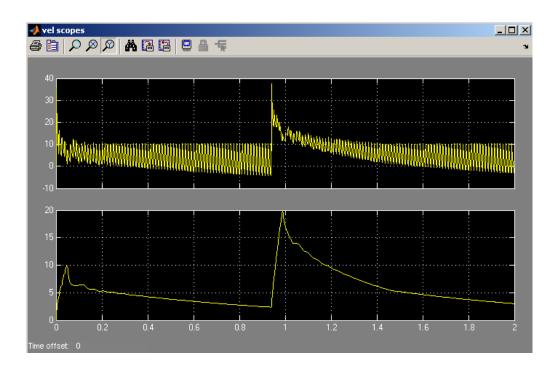

Figure 5. 18 – CE velocity(cm/s) vs. time(s) scopes at 100 Hz with ef=1 N. The upper scope shows the velocity obtained from Model 2 before the filtering diagram and the lower scope shows the velocity after the filtering diagram.

As observed in Figure 5.18, the maximum velocity occured at some time close to t=0 s before the filtering. But we can not see the maximum velocity at the initial times of the lower velocity scope in Figure 5.18. The rest of the velocity profile is preserved except the initial time though.

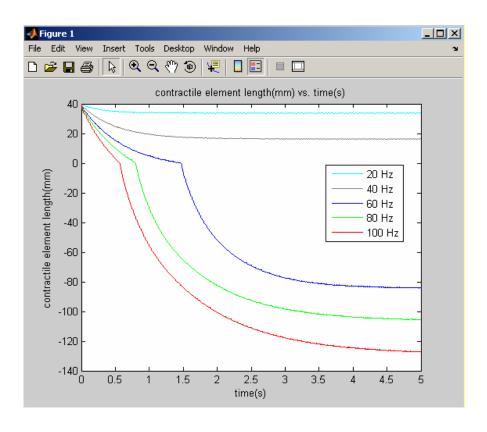- CE lengths for all frequencies with 0 N external force are shown in Figure 5.19.

Figure 5. 19 - CE length(mm) for all frequencies and ef=0 N

As the results of Figure 5.19 show, the contractile element length of Model 2 goes under 0 mm for 60, 80 and 100 Hz. As Model 2 has the force-velocity relation, it is clear that this relation can not prevent the contractile element from shortening much,

even down to zero. The contractile element length can go below 0 mm without a warning. This happening is due to the transparent bodies in SimMechanics. The Body 1 and Body 2 pass through each other during the simulation.

As known the muscle can produce force in a length range shown by force-length relation mentioned in section 4.3.2.3. The Figure 5.19 also shows that the force-velocity relation can not keep the motion in such a range. Indeed, this is the function of force-length relation.

5.3. Model 3 Simulation Results

This model is buit in order to see the effect of the force-length relation separately. Simulation time is 2 seconds for all tries.

- The force-length relation is the relation which determines the contractile element length. The results do change when compared to the results of Model 1 & Model 2. The contractile element lengths of Model 1,Model 2 and Model 3 for 100 Hz with external force 0 N can be seen in Figure 5.20.
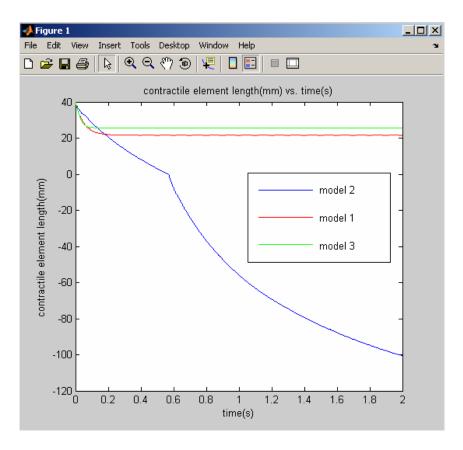
Figure 5. 20 - CE length(mm) of Model 1, Model 2, Model 3 for 100 Hz and ef=0 N

Observing the Figure 5.20, when the CE length of Model 3 is compared to the CE length of Model 1 and Model 2, it is seen that the force-length relation prevents the muscle from the shortening mentioned on Figure 5.19 in the last case of section 5.3 and thus keeps the motion in the range determined by the force-length relation.

- The contractile element lengths of Model 3 for all frequencies with external force of 1 N are given in Figure 5.21. As seen the CE length again reaches the tetanus. These scopes resemble Figure 5.2. For higher frequencies, more contraction is obtained.
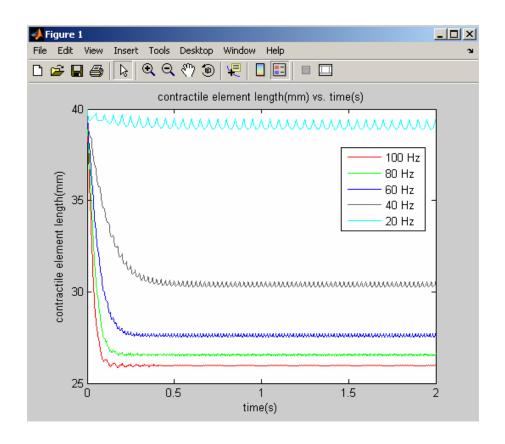
Figure 5. 21 - CE length(mm) for all frequencies and ef=1 N

5.4. Model 4 Simulation Results

This model investigates the mechanical behavior of the muscle when both of the force-length and force-velocity relations are implemented to the model. Simulation time is 2 seconds for all tries.

As mentioned in section 5.3, because this model has both relations, the results of it can be compared to the results of the Models 2 & 3. This model's output will show the characteristics of the Models 2 & 3. When the models are simulated at 100 Hz with external force of 5 N, the CE length can be seen in Figure 5.22. The results of Models 3 & 4 show that the muscle contracts the same amount for these models. But

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there is some time delay in reaching the ending point. As known the difference between Models 2 & 3 is the force-velocity relation. This shows that the force-velocity relation works as an inherent damper causing time delay in the scopes but not changing the ending point.

When the results of Model 2 & 4 are compared, it is seen that again the force-length relation, which is the difference between the models, keeps the muscle contraction in a range as previosly shown in the results of Model 3.
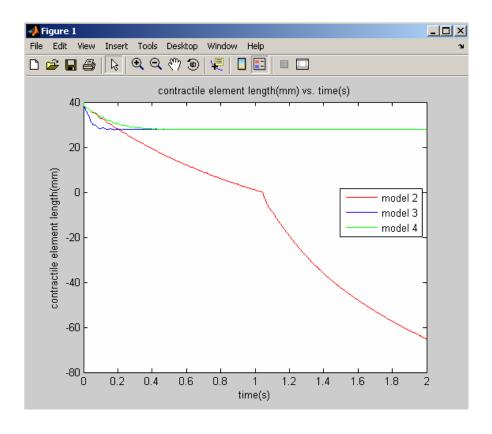

Figure 5. 22 - CE length of Model 2, Model 3 and Model 4 for 100 Hz and ef= 5 N. Indeed these results led us to the conclusion that the force-velocity relation works as an inherent damper.

5.5. Model 5 Simulation Results

This model investigates the effect of the muscle spindle on the mechanical behavior of the muscle added in parallel to the muscle fiber. The spindle is added solely as a mechanical element with no feedback. The length of the muscle is held constant in the Model 5. Simulation time is 1 second for all tries.

- The model 5 is simulated for all frequencies. As this model is under isometric conditions, the isometric tetanus is reached for all frequencies similiar to the Model 1. The buildup forces are obtained from the sensors of the serial elastic elements of the extrafusal fiber. It is expected that the spindle carries some tension on it so the buildup forces decrease. The saturation points and ending points are shown in Tables 5.3 and 5.4. These points are a little different for Model 5 when compared to the corresponding points of Model 1 shown in Tables 5.1 and 5.2. As expected the saturation points and the contractions are less for Model 5 compared to Model 1. In Figure 5.24 the decrease in the saturation point is shown when Model 1 & 5 are simulated at 100 Hz with an external force of 1 N. The difference in tension between is the tension carried by the muscle spindle.

- The stiffness of the Model 5 is different from that of Model 1. Model 5 is a little stiffer since the existence of the springs in the muscle spindle model in parallel to the CE.
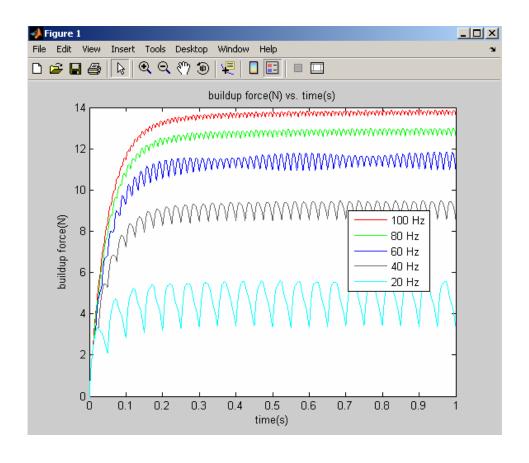
Figure 5. 23 - Buildup force(N) for all frequencies

Table 5.3 - Saturation point(N) for all frequencies

frequency(Hz)	buildup force(N)
100	13,73
80	12,79
60	11,38
40	8,99
20	4,50

Table 5.4 - Ending point(mm) for all frequencies

frequency(Hz)	ending point(mm)
100	21,675
80	22,09
60	24,91
40	27,96
20	34,01

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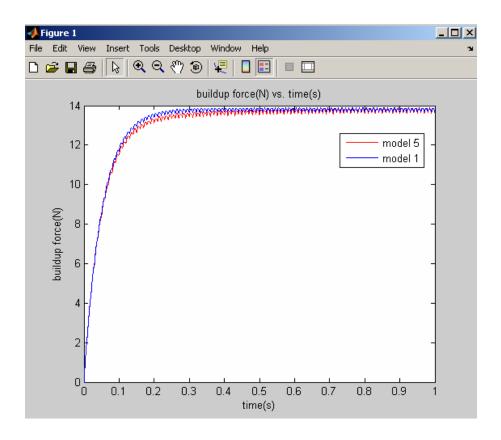
Figure 5. 24 - Buildup force(N) of Model 1 and Model 5 for 100 Hz

- To see the damper effect on Model 5, the model is simulated at 100 Hz with and without the damping (b=0 N.s/m) of the spindle. The resulting buildup force can be seen in Figure 5.25. The saturation point is higher when there is no damping as some of the tension is absorbed by the CE.
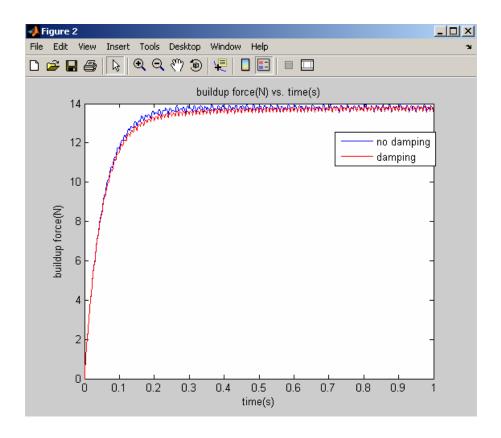
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Figure 5. 25 - Buildup force(N) of Model 5 for 100 Hz with and without damping on the spindle

- The connective tissue does not change length as it is a very stiff tissue when Model 5 is simulated.

- There is some initial tension on the serial elastic elements and the initial tensions are 0.028 N (on the PEE) and 0.049 N (on the SEE). Without the initial tension on the springs the tension on the extrafusal fiber for all frequencies do not change much.

- The serial elastic element length of the spindle goes below 0 mm for a contraction more than 10 mm which is the spindle length. This is due to the transparent bodies which pass through each other. Besides, all the

contraction on the extrafusal fiber can be observed on the spindle, i.e. the same amount of contraction is seen on the intrafusal fiber. This is due to the serial elastic elements of the intrafusal fiber which do not change length. The contractile element's length change is copied onto the total length of the spindle.

- The force on the spindle can be read from the sensor of the serial elastic element of the spindle. The force on the SE of the intrafusal fiber at 100 Hz can be seen in Figure 5.26. As observed the force on the spindle is small.
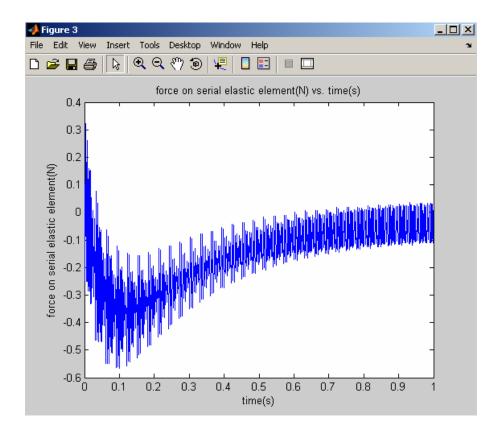

Figure 5. 26 – Force on the serial elastic element(N) of the spindle at 100 Hz

5.6. Model 6 Simulation Results

This model investigates the effect of the muscle spindle on the mechanical behavior of the muscle when one end is free.

- When the model is simulated at 100 Hz with an external force of 0 N, the force on the the spindle shows no buildup as shown in Figure 5.27. As observed the force on the spindle is unstable.
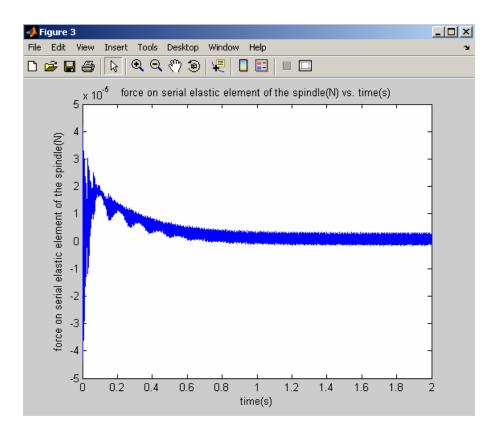

Figure 5. 27 - Force read from SE of the spindle(N) for 100 Hz and ef=0 N

5.7. Model 7 Simulation Results

This model investigates the stretch reflex effect on the mechanical behavior of the muscle.

- Without the feedback (stretch reflex) the muscle is not able to support high external forces stretching the muscle. The Model 4 & 7 are simulated at 100 Hz with an external force of 20 N which is high enough to stretch the muscle. The CE length of the two models can be seen in Figure 5.28. As observed, the contractile element does not change much length for Model 7 but the CE of Model 4 stretches about 100 mm. So the reflex is helpful for preventing the muscle from more stretch.
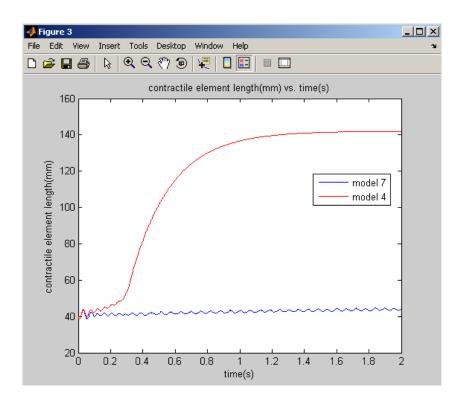

Figure 5. 28 - CE length(mm) of Model 4 and Model 7 for 100 Hz and ef=20 N

- The feedback results for all frequencies and an external force of 5 N are as shown in Figures 5.29,5.30,5.31,5.32 and 5.33. In all scopes, X axis depicts time in seconds while Y axis shows feedback in Hz(1/s). As seen in Figure 5.29, 5.30 and 5.31, the feedbacks are zero meaning that the external force 5 N is not enough to cause a stretch in the muscle and thus fire the spindle.
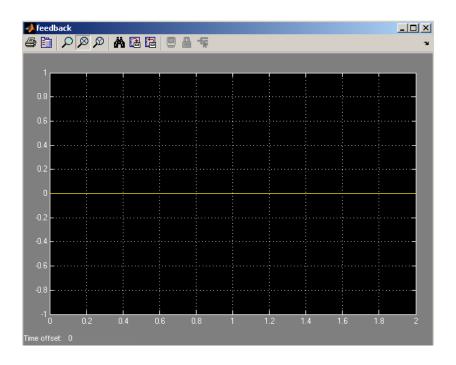

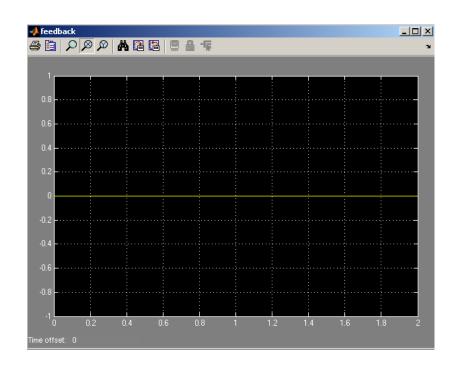Figure 5. 29 - Firing rate(1/s) vs. time(s) for 100 Hz and ef=5 N

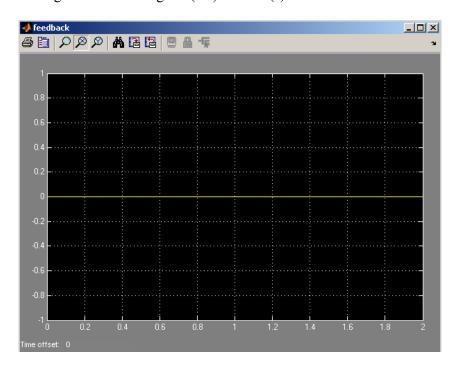Figure 5. 30 - Firing rate(1/s) vs. time(s) for 80 Hz and ef=5 N

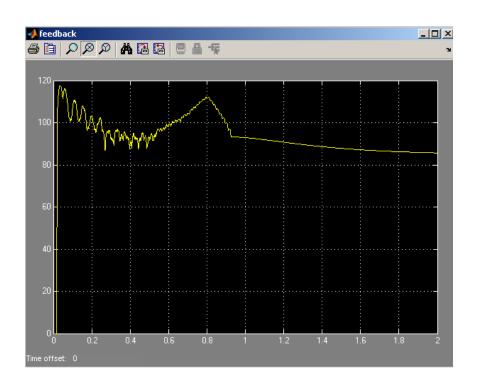Figure 5.31 - Firing rate(1/s) vs. time(s) for 60 Hz and ef=5 N

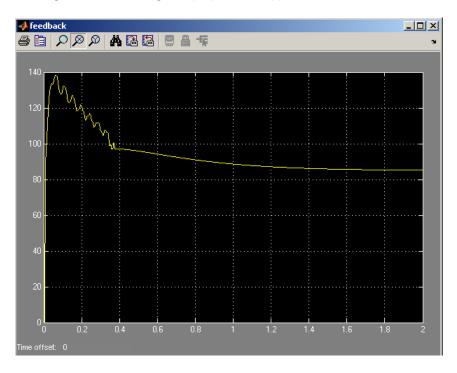Figure 5. 32 - Firing rate(1/s) vs. time(s) for 40 Hz and ef=5 N

Figure 5. 33 - Firing rate(1/s) vs. time(s) for 20 Hz and ef=5 N

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If the external force is not large enough to stretch the muscle up against the contraction force, the feedback comes out as zero. As the frequency is decreased keeping the external force constant, the feedback starts to increase as observed in Figures 5.32 & 5.33.

- The main characteristics of the behavior of the Ia feedback shown in Figure 3.5 is obtained. When the model is simulated at 40 Hz with an external force of 10 N, the feedback can be seen in Figure 5.34. Comparing it to Figure 3.5, the results are similiar. Firstly, the shape in Figure 5.34 consists of the dynamic response and the static response of Ia feedback. The dynamic response can be seen from the initial time to nearly t=0.4 s and the static part is the rest from that point on. The oscillatory data is in the dynamic response part. Secondly, there is a sharp increase in the feedback data. Finally, the same shape comes to the final value and shows slow-phase fall before reaching this value.

Figure 5. 34 - Firing rate(1/s) vs. time(s) for 40 Hz and ef=10 N

- For each frequency as the external force begins to increase, the feedback also begins to increase. This result is shown in Figures 5.35 and 5.36 when the model is simulated at 60 Hz with external forces 15 and 25 N respectively.
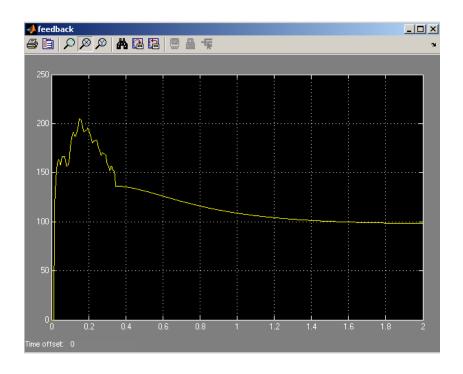
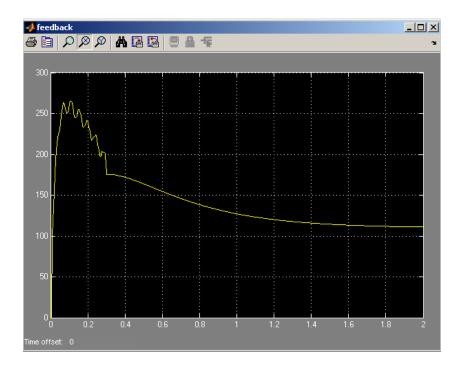Figure 5. 35 - Firing rate(1/s) vs. time(s) for 60 Hz and ef=15 N

Figure 5. 36 - Firing rate(1/s) vs. time(s) for 60 Hz and ef=25 N

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- As Figures 5.37,5.38 and 5.39 reveal, for large external forces such as 25 N the static equilibrium point for the Ia feedback is the same for all frequencies. These scopes reveal that the large external force is the factor which determines the behavior of the Ia feedback.
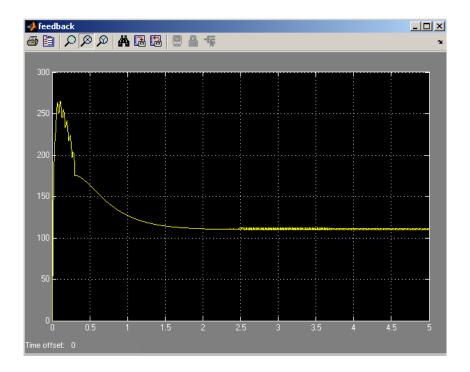

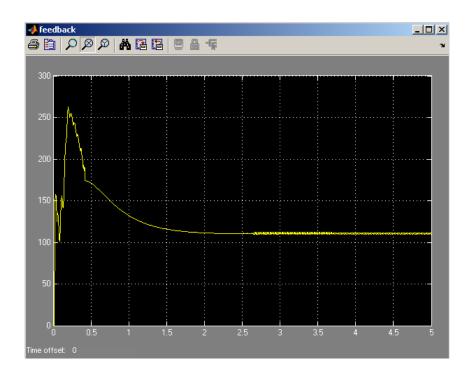Figure 5. 37 - Firing rate(1/s) vs. time(s) for 60 Hz and ef=25 N

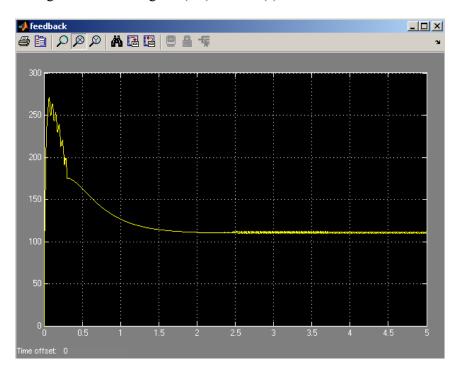Figure 5. 38 - Firing rate(1/s) vs. time(s) for 100 Hz and ef=25 N

Figure 5. 39 - Firing rate(1/s) vs. time(s) for 20 Hz and ef=25 N

- There is still slackening problem in the serial elastic element in Model 7. As seen in Figure 5.40, when the model is simulated at 100 Hz with an external force of 0 N, the contraction occurs immediately on the serial elastic element of the intrafusal fiber. Later, because of the damping the pole region of the intrafusal fiber responds to this contraction and the serial elastic element elongates but the tendon length stays at a length less than 0 mm. X axis depicts time in seconds while Y axis shows the SE length in mm. The slackening issue is investigated in the Model 8.
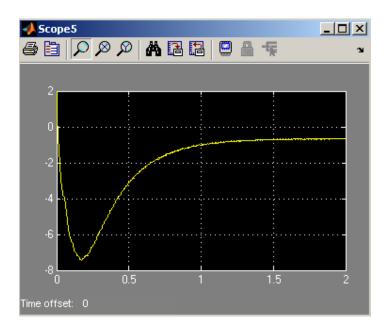
Figure 5. 40 - SE length of the spindle(mm) vs. time(s) for 100 Hz and ef=0 N

- The filtering diagram used in Model 7 in front of the velocities of the bodies on both sides of the SE of the intrafusal fiber helps to smoothen the feedback data. The feedback result of the model without the filter when the model is simulated at 100 Hz with an external force of 20 N is shown in Figure 5.41. The feedback data does not resemble Figure 3.5.
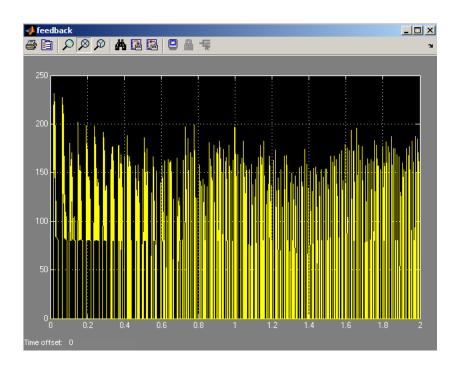
Figure 5. 41 – The feedback(1/s) vs. time(s) when the feedback Ia is not filtered for 100 Hz and ef=20 N.

5.8. Model 8 Simulation Results

This model investigates the slackening problem of the muscle spindle with stretch reflex. The X axes in all of the scopes indicate the time in seconds.

In Figures 5.42,5.43 and 5.44, the results for Model 8 at 100 Hz with an external force of 0 N are shown.
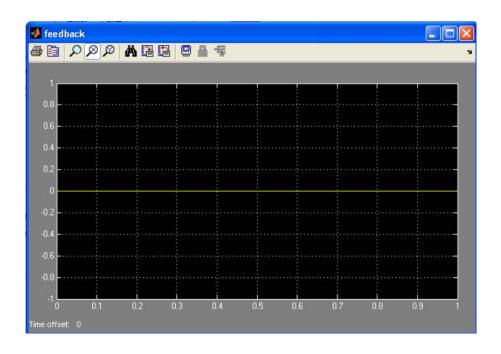
Figure 5. 42 - Firing rate(1/s) vs. time(s) for 100 Hz and ef=0 N
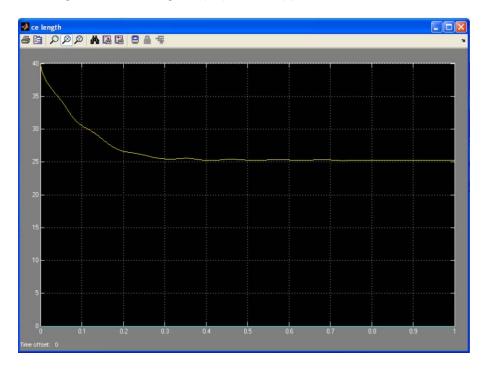
Figure 5. 43 – CE length(mm) vs. time(s) for 100 Hz and ef=0 N
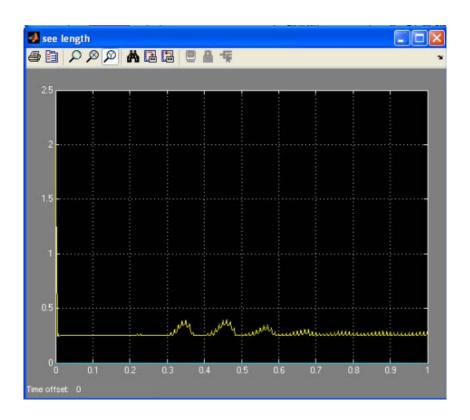
Figure 5. 44 - SE length of the spindle(mm) vs. time(s) for 100 Hz and ef=0 N

As expected in Figure 5.42 for 100 Hz and 0 N external force, the Ia feedback is zero. The CE contracts as shown in Figure 5.43. As the Figure 5.44 shows there is a sharp decrease in the length of the SE of the intrafusal fiber but it does not go below the slackening length (=0.25 mm) though the extrafusal fiber goes on contracting. So the slackening problem is prevented.

The results when the model is simulated at 40 Hz with an external force of 10 N are shown in Figures 5.45,5.46 and 5.47.
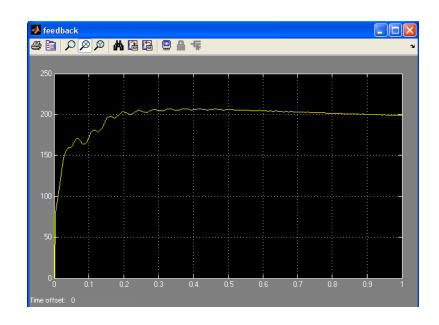
Figure 5. 45 - Firing rate(1/s) vs. time(s) for 40 Hz and ef=10 N
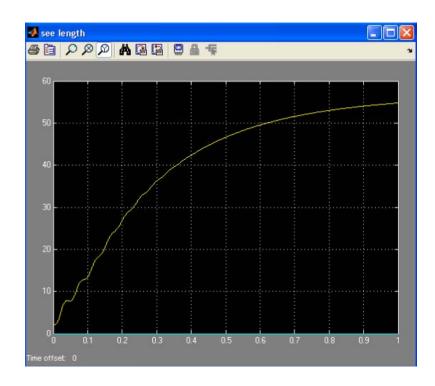
Figure 5. 46 - SE length of the spindle(mm) vs. time(s) for 40 Hz and ef=10 N. The length begins at 2 mm.
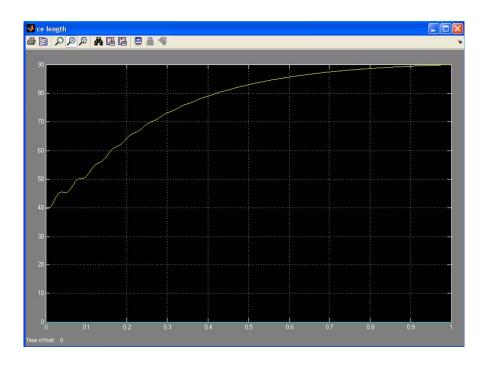
Figure 5. 47 - CE length(mm) vs. time(s) for 40 Hz and ef=10 N. The length begins at 40 mm.

As the results of Figures 5.45,5.46 and 5.47 show, under these conditions the model is in continuous stretch.

5.9. Model 9 Simulation Results

This model is built up to simulate the quick release experiment. At the end of the simulation some methods are applied to the total muscle length(mm) vs. time(s) scopes to obtain the force-velocity relation of the contractile element. Simulation time is 2 seconds for all tries.

A scope showing total muscle length vs. time for 100 Hz frequency and 1 N external force is shown in Figure 5.48. As observed, the muscle is held constant for 1 second, i.e. the total muscle length is held at 50 mm. When left free at t=1 s, the length data

of the muscle is oscillatory. The oscillation occurs at the damped natural frequency of the system, $2\pi/0.044=142.8$ rad/s for this case. This figure is in accordance with Figure 2.11.
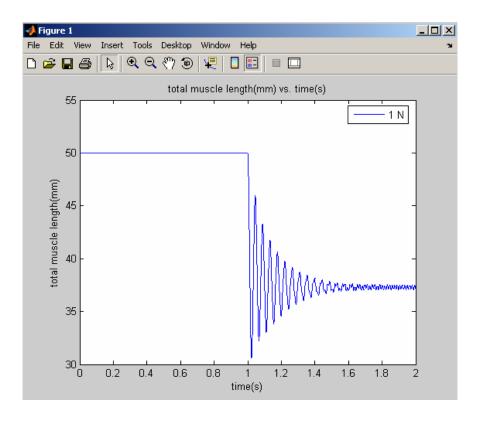
Figure 5. 48 –Total muscle length(mm) for 100 Hz and ef=1 N for quick release simulation.

When we zoom at the onset of the release in Figure 5.48, we see in Figure 5.49 that there is not a sharp decrease when the muscle is released as depicted in Figure 2.3. Actually Figure 2.3 is a sketch, not an experimental data. The data in this figure is impossible to obtain as the velocity of contraction (the slope) goes to infinity because the muscle shortens in zero time when released. If the mass of the external body were decresed, the slope (velocity of contraction) would be sharper than the

slope shown in Figure 5.49 as it becomes easier to move this external body so to contract the muscle.
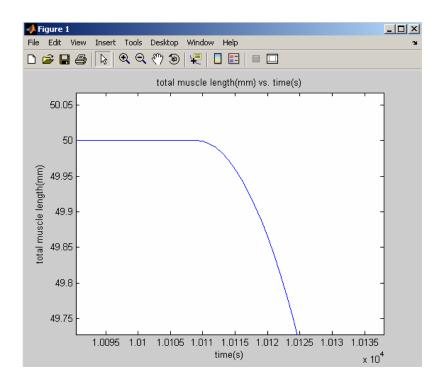
Figure 5. 49 - Closer look at the onset of release in Figure 5.48

The force acting on the model for the same conditions of Figure 5.48 is shown in Figure 5.50 which is also similiar to the force scope in Figure 2.3.
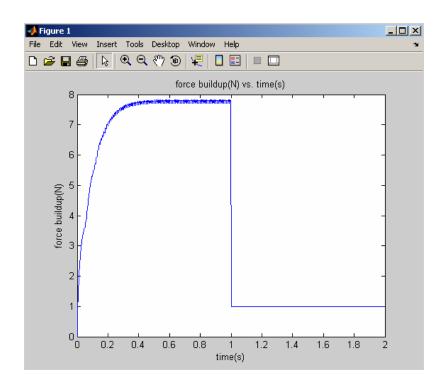
Figure 5. 50 - Force on the model(N) for 100 Hz and ef=1 N

For the first second the buildup in force occurs and for the other second external force(1 N) is acting on the model as Figure 5.50 depicts. The buildup force is obtained for all frequencies as the muscle is held in isometric conditions.

The saturation points are listed for frequencies as in table 5.5:

Table 5.5 - Saturation points(N) for all frequencies. The saturation points go down compared to Model 1 as the force-length and force-velocity relations are in Model 9.

frequency(Hz)	saturation point(N)
100	7,77
80	6,08
60	3,69
40	1,38
20	0,18

Now it is time to show the total muscle length(mm) vs. time(s) results of quick release experiment for all frequencies. The resulting scopes are in Figures 5.51,5.52,5.53,5.54 and 5.55. As mentioned in section 4.3.9, the external force applied to the model is increased up to the saturation point for all frequencies.
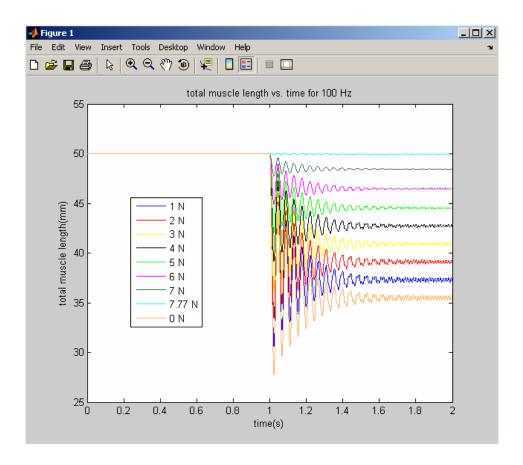
Figure 5. 51 – Total muscle length(mm) for 100 Hz and external forces up to the saturation point
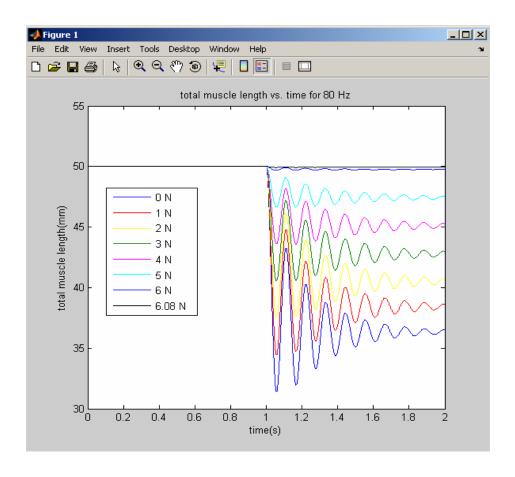
Figure 5. 52 - Total muscle length(mm) for 80 Hz and external forces up to the saturation point
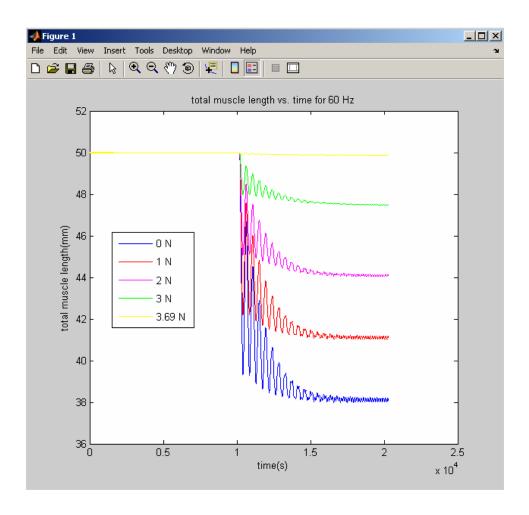
Figure 5. 53 - Total muscle length(mm) for 60 Hz and external forces up to the saturation point
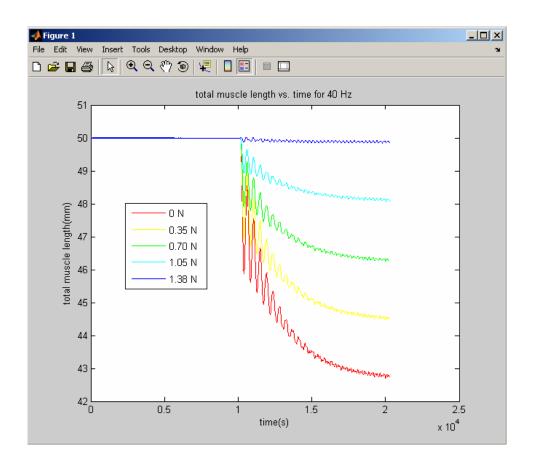
Figure 5. 54 - Total muscle length(mm) for 40 Hz and external forces up to the saturation point
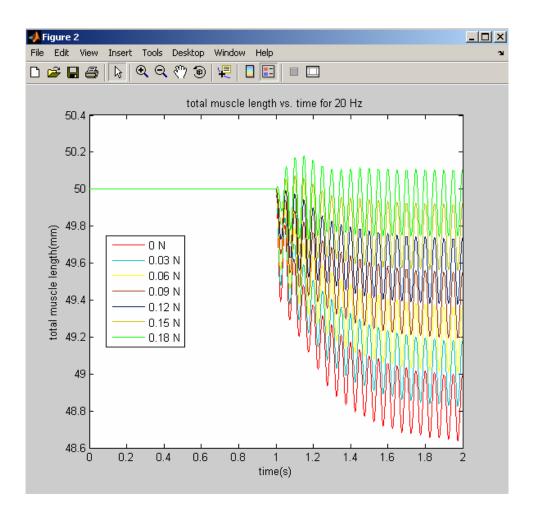
Figure 5. 55 - Total muscle length(mm) for 20 Hz and external forces up to the saturation point

Some work on these scopes is done to find out the force-velocity relation. For this purpose different methods are applied.

One method followed is to fit the position data in Figures 5.51 to 5.54 with different curve fitting methods. Then the slopes (contraction velocities) at the onset of release are taken as mentioned in section 2.3. For this purpose the curve fitting tool in MATLAB is used. Here two different curve fitting methods are applied:

1) Passing the curve through the upper points of each oscillation in the scopes beginning from 50 mm and taking the slope at this point.

2) Passing the curve through the middle points of each oscillation beginning from first middle point and taking the slope at this point. Indeed, the curve begins from 50 mm, goes down rapidly till it reaches the first middle point. The mentioned slope is the one at this middle point where the curve passes through the middle points.

Between these two methods the second methods' results has resembled the force-velocity relation implemented in the model more.

These two different methods are applied to the first four point pairs on the scopes. The points read from the curve to fit the data are the total muscle length(mm) on the y axis and time(s) on the x axis. These four point pairs are curve fitted with exponential, power and polynomial curve fitting. For every curve fitting the initial slopes are calculated for the two different methods. Then the external force(N) is drawn on y axis vs. corresponding contraction velocity(cm/s) for that external force on the x axis. The force-velocity relations are all normalized to their maximum values (F/F_{max} vs. V/V_{max}). These maximum values are the maximum external force (equal to the saturation point) for F_{max} and the maximum initial slope for $V_{max.}$ Among different curve fitting types, the best results are obtained with double exponential fit and cubic fit. The resulting relations are compared to the original relations implemented in the models.

The results of the force-velocity relations for all frequencies are in Figures 5.56,5.57,5.58,5.59 and 5.60. In these figures upper refers to method 1 and middle refers to method 2.
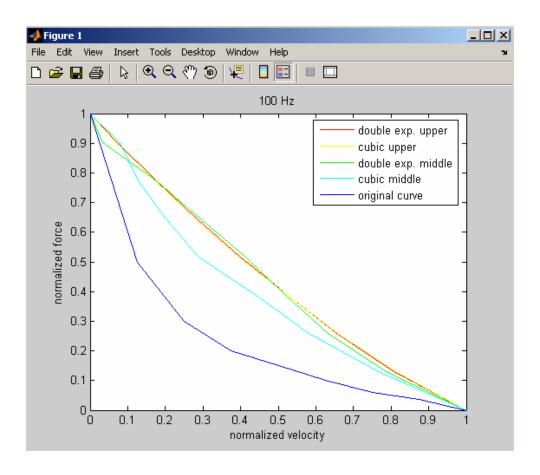
Figure 5. 56 – Quick release results after curve fitting for 100 Hz
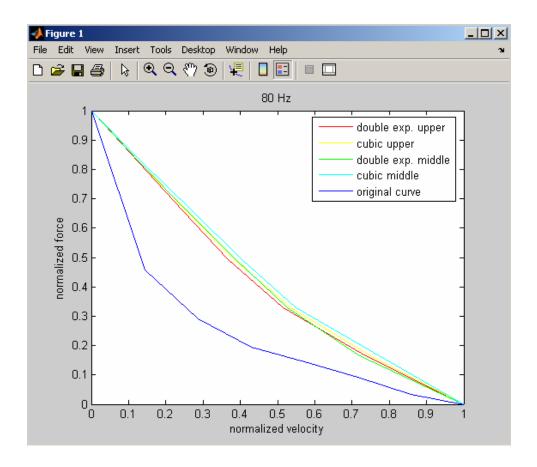
Figure 5. 57 – Quick release results after curve fitting for 80 Hz
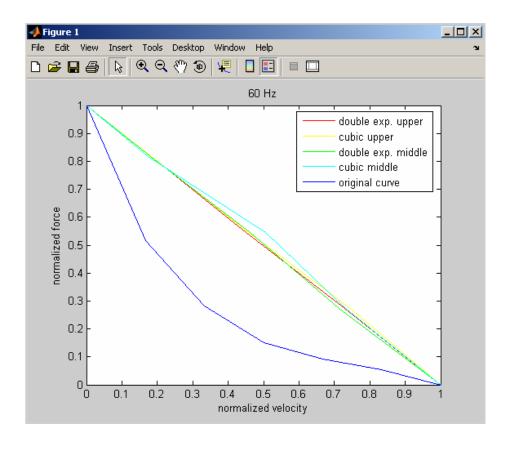
Figure 5. 58 – Quick release results after curve fitting for 60 Hz
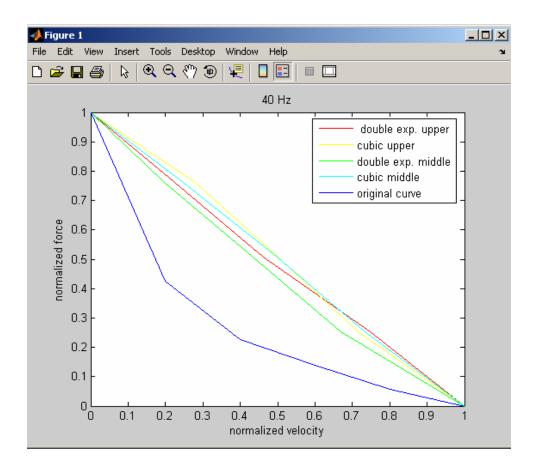
Figure 5. 59 – Quick release results after curve fitting for 40 Hz
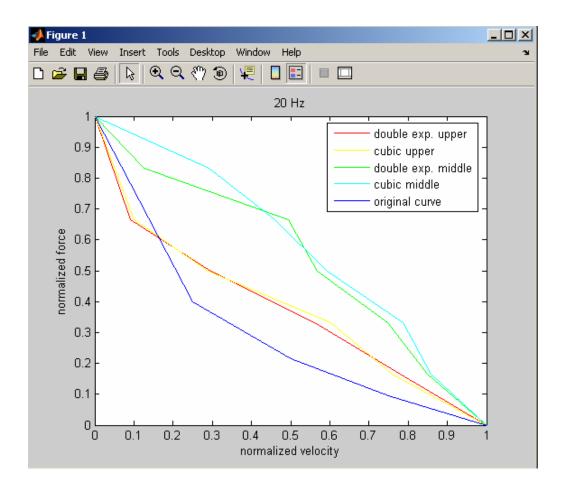
Figure 5. 60 – Quick release results after curve fitting for 20 Hz

One other method applied to the data except curve fitting was to take the derivative of the position data of the muscle (total muscle length) with the derivative block and take the maximum negative point since the slopes were negative.

The derivative block is put after the body sensor of the external body located at [50 0 0]. The sensor reads the position of this body in mm. That body's motion gives the total muscle's length change as mentioned. Inside of the derivative block Linearization Time Constant is left 'inf' which is default. Implementation of the derivative block is in Figure 5.61.
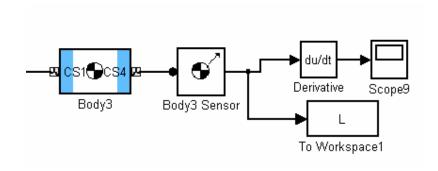
Figure 5. 61 – Implementation of the derivative block

The resulting scope for all frequencies is shown in Figure 5.62. The resulting force-velocity relations are all lines though scattering for the frequencies are obtained.
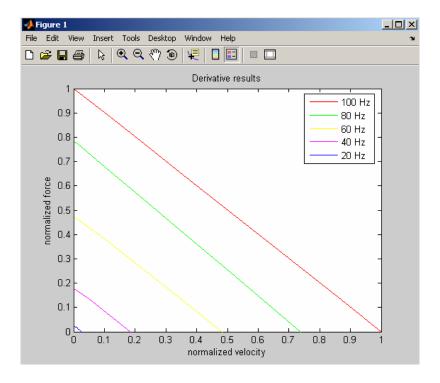
Figure 5. 62 – Force-velocity relations of the quick release experiment with the derivative method

As seen the results of the quick release experiment are not the the same with the force-velocity relations, but still the curve fits reflect the inverse relation between the external force and the contraction velocity especially when the frequency is high. We have no source on the right method of fitting the data and the methods applied for curve fitting may be wrong. Besides, the data for curve fitting is chosen arbitrarily. We are not sure whether they are exactly the needed points in the constitution of the curve. Moreover, there is a filtering applied to the velocity of the CE which smoothens the velocity profile but makes a change in the initial part of it. The effect of parameters of the filtering diagram blocks is not known...

CHAPTER 6

CONCLUSION

In this study, the muscle mechanics has been investigated. The muscle mechanics itself is known to be a very broad subject. Through model development stages, the subject is kept as simple as possible but the most important details of muscle mechanics are included. The modeling path follows the literature and the data available is embedded inside the blocks. Since some of the data needed for use is not available, practical assumptions are made.

All models are based on the Hill muscle model structure. In the first model which is the simplest one, it is possible to obtain the isometric tetanus. The tetanus is one of the most important concepts when dealing with muscle mechanics. In the second model the force-velocity relation and in the third the force-length relation of the contractile element are implemented. Both models' results are compared to the results of Model 4 where they both exist. By using this method the relations' effects on the behavior of motion can be observed when one of the relations is missing. The Figure 5.23 reveals that the force-velocity relation works as an inherent damper. This is true as this relation indicates the magnitude of the force dissipated while the muscle is in motion. The force-length relation works as the relation that determines the force production in the range of motion. Again Figure 5.23 shows that this relation regulates the motion and keeps it in the range.

The muscle spindle is initially implemented only as a mechanical component with no stretch reflex in the Models 5 and 6. In Model 7 the stretch reflex subject is investigated. The stretch reflex is succesful at making change in the frequency of the signals from the CNS as it is the sensory feedback to the muscle. Besides, the shape of this feedback for various results shows the response characteristics of the spindle in Figure 3.5. When the slackening problem of the serial elastic element of the spindle is investigated in Model 8. The new block (hard stop) is implemented so that the element allows the contraction up to the slack length.

In the last model the quick release experiment is implemented where the muscle spindle is not included owing to the fact that the experiment deals with the contraction of the muscle. Firstly, the shape of the force read from the serial elastic element of the muscle in Figure 5.51 is similiar to the shape of figure 2.3. Secondly, the oscillatory shape of figure 2.9 is observed when the muscle is released. But it is not possible to obtain the force-velocity relation of the contractile element of the muscle using the selected experimental procedure is followed. The experimental results are not successful to follow the expected hyperbolic relation though the results have the inverse relation between the applied external force and contraction velocity. This result may be due to the method of curve fitting and filtering method applied to the contractile element's velocity. Some further study on the selection of the convenient curve fitting method and the effect of the parameters in the filtering method for the quick release experiment would make the resulting force-velocity much nearer to the expected results.

As briefly mentioned in this thesis, the muscle's springs in the muscle models have actually all nonlinear behavior according to the load on it. This nonlinearity is more obvious in the tendons. This issue is more important if a single fiber is considered, as the fiber can show different properties from the rest of the fibers. While we are modeling a lumped muscle, these stiffness values are meant to be mean values acting as the total of these fibers. Moreover, the extrafusal muscle's tendons have

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the force-length relation but we could not reach enough experimental results on this subject. Another point is that while the models are all one dimensional, the program allows the three dimensional definiton of the muscle as mentioned in Chapter 4 so some parameters like the pennation angle can easily be added to the model.

Besides the details mentioned above, some factors such as the fatigue in the muscle. can be included in the model. Finally the model would be better if such specific experimental data for a specific muscle can be found.

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APPENDIX

A. Determining the optimum values for weighted moving average and zero-order-hold blocks

Velocity is known to be the differential of the displacement with respect to time. When the velocity graphs are obtained from scopes in the models, the scopes seem to have big oscillations. One example of that kind is below in Figure A.1. This may lead to errors in the movement of the muscle which we call as ripple effect. To smoothen the graphs a filter is needed. Since filter terminology and its usage is cumbersome to a degree for the content of the thesis, we use the simplest one, weighted moving average block.
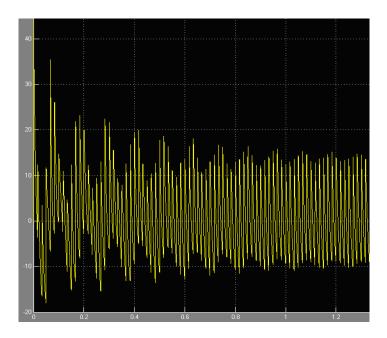

Figure A. 8 - Sample velocity graph at 100Hz and ef=0 N (Y axis velocity in cm/s,X axis time in seconds)

Weighted Moving Average block samples and holds the N most recent inputs, multiplies each input by a specified value (given by the Weights parameter), and stacks them in a vector. This block supports both single-input/single-output (SISO) and single-input/multi-output (SIMO) modes [40].

In the model the inputs have the same weight one. For instance plotting
1/1000*ones(1,1000)
in Weights tab means taking average of 1000 data each with the same weight one. The process can be imagined as a window moving with N data in it, averaging them and giving the most appropriate value for the next data (N+1).

This averaging block is under library Discrete while the model works as a continuous system. For a transition from continuous to discrete, a zero-order-hold block is used.

The Zero-Order Hold block samples and holds its input for the specified sample period. The block accepts one input and generates one output, both of which can be scalar or vector. If the input is a vector, all elements of the vector are held for the same sample period. This block provides a mechanism for discretizing one or more signals in time, or resampling the signal at a different rate.

The time between samples is specified with the Sample time parameter. A setting of -1 means the Sample time is inherited.

Two problems arise at this point:
—What should the optimum number of N for weighted moving average block be?
—What should the sample time for zero-order-hold be?

The optimum number for N should be such that the model does not let ripple effect occur and the sample time should be such that the transition gives the least error.

Since details of no such specific work can be found in literature, only solution to the problems is the criterion we should seek in the results of the model and find the best results by trial and error of the numbers.

B. Optimum Averaging Number:

We know that the force-velocity relation is an inherent damper in the model and delays the movement, but has no effect on the ending point especially scopes such as the contractile element length vs. time is considered in Chapter 5.2. If the ending point is called the equilibrium point, we see no change in the equilibrium point for different averaging numbers. For the purpose of an optimum number, we compare the results of the model with force-velocity and force-length relation and the model with force-length relation only, the difference being the force-velocity relation. If the right number is chosen, we will see a "no change" in the equilibrium point.

Beginning from 0 we go up to a number of 1500 with an increment 100 at each step. The resulting scopes are interesting in the fact that the relation shows that increasing the number at equal steps does not give expected results. Rather the results change widely in each change of the number. The deduction from the scopes is that the number used should be higher than or equal to 500 at least as below that number we can not see the equilibrium point coincide for the two models.

The results tell us that the optimum number is 500.

C. Optimum Sampling Time

The sampling time determines the holding time of each single data. As mentioned, the model is solved with a variable step solver (ode 45 - Dormand-Prince). Because of discretization of the time signal in the model, we obtain a mixing of continuous

and discrete data. Here we recall the multiplication of the force-velocity relation and the force input repeating itself at a frequency. Because the force-velocity relation is of discrete nature and the others are of variable nature, there occurs an error through multiplication. The error reason is that the datas are not one-on-one, i.e. nth data of one input to the product does not correspond to the nth data of the other. The optimum sampling time helps to minimize this error. It should be chosen as small as possible to minimize the error but not very much so not to make the simulation last very long.

The criterion here is that when the model is solved with variable step and fixed step solvers respectively, the resulting scope (s) should be very similiar to each other meaning that the step size difference does not change the results much. Besides, it is a must that the fixed step solver use the same sample time with the zero-order-hold block. For testing the model with force-velocity and force-length relations the model is simulated for 2 seconds with an external force of 10 N, input force with 60 Hz, weighted moving average with 500 data. The contractile element length vs. time is observed from the scope. The variable step solver is ode45 while the fixed step solver is ode5.

Let the sample time be 0.0001 s. This is the step size both defined in configuration parameters tab of the model and the zero-order-hold block itself. This means an average of 20000 data for 2 seconds of simulation. The contractile element length with different solvers and same conditions is in Figure A.2.
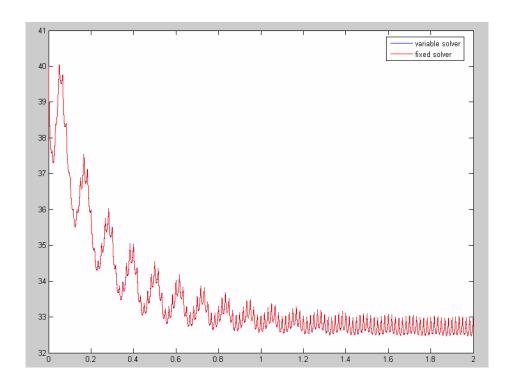
Figure A. 9 - Contractile element length (cm) on Y axis vs. time(s) on X axis with different solvers and same conditions

Here we see approximately no difference between solver types except when the view is zoomed so closely. It comes out that the chosen sample time is a good assumption. So the sample time is 0.0001 s.

We need to search for how much data is passed throughout the simulation. To see the number of points we utilize the block To Workspace from the Sink library as shown in Figure A.3.

Figure A. 10 - To Workspace block

Model 2 is for a moment donated with three workspaces: fv_coefficient keeps the number of data for the force-velocity relation, force for the input force and product for the product of the former two. The implementation is shown in Figure A.4. All workspaces are in Array formats as shown in Figure A.5.
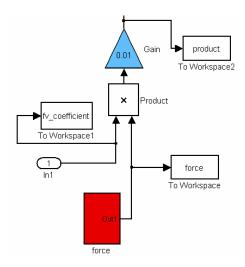

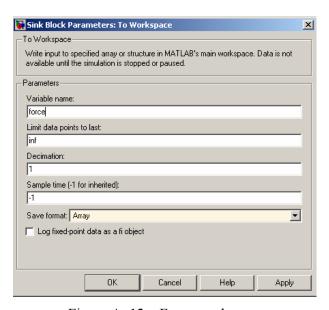Figure A. 11 - Workspaces implementation

Figure A. 12 – Force workspace

The model is simulated now with two different solvers. Keeping the simulation 2 seconds the number of data is 20000, 20312 and 20312 for fv, force and product respectively with variable step solver. The workspaces all contain 20000 data as expected with fixed step solver. The difference 312 data is nearly %1,5 of total data. It should be kept in mind that this difference is the total error summing up at the end of the simulation. This shows a delay of 30 ms when the simulation stops (at the end of 2 s here) which is not considered as an error regarding the whole motion. If simulation time is increased, this difference is also expected to increase. However, the simulation time is good enough as when the scope of the contractile element length is observed, the equilibrium point is reached.

As a result 0.0001 s is our assumption for sample time in zero-order-hold block. The rest of the models contain "0.0001" for zero-order-hold block and "1/500*ones(1,500)" for weighted moving average block.