Quick Sensitivity Curves for Pulsar Timing Arrays

Making unrealistic sensitivity curves more realistic

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Outline

Sensitivity Curves

In general, sensitivity curves are a way to characterize the sensitivity of a detector to a given signal.

- Encode the noise properties of the detector.
- Can be used to assess to the "detectability" of something.

Sensitivity Curves

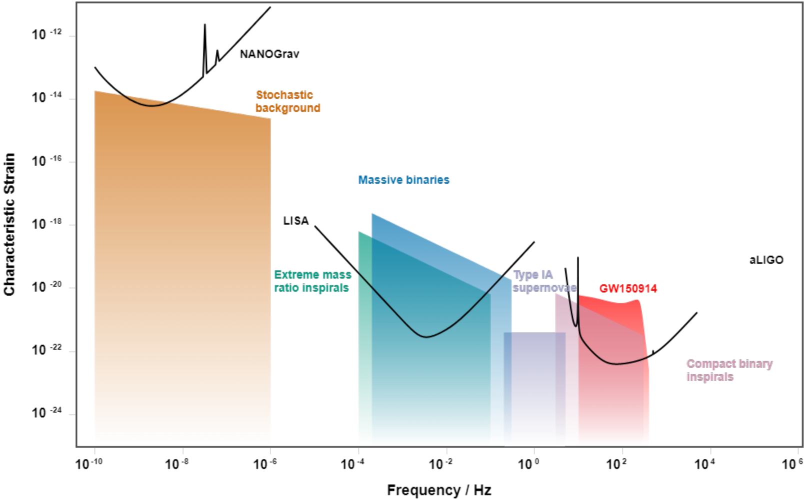

Figure: Sensitivity Curve Example

Quick Sensitivity Curve Derivation

We will start from the realistic sensitivity curves presented in eq. 92 of Hazboun et al. (2019),

$$S_{\rm eff}(f) = \left(\sum_I \sum_{J>I} \frac{T_{IJ}}{T_{\rm obs}} \frac{\chi_{IJ}^2}{S_I(f) S_J(f)} \right)^{-1/2}$$

- $S_{\rm eff}$ is the effective background sensitivity
- $T_{\rm obs}$: Total observational time span
- T_{IJ}: Overlapping observational time span between pulsars I and J
- χ_{IJ}: correlation coefficients
- $S_I(f)$: Individual pulsar sensitivity I

Derivation of Quick Sensitivity Curves

Simplifying assumptions:

- Uniform distribution of pulsars across the sky
- Identical noise properties for all pulsars
- $T_I = T_{\mathrm{obs}}$ (all pulsars observed for full timespan)
- Same observational cadence for all pulsars

Simplified Sensitivity Equation

Assuming uniform pulsar distribution across the sky,

$$\chi_{IJ} \approx 1/48$$

and identical noise properties:

$$S_{\text{eff}}(f) = \left(\frac{N_{\text{psr}} \left(N_{\text{psr}} - 1 \right)}{2 \times 48 \times S_I(f)^2} \right)^{-1/2}$$

Final Sensitivity Equation

From Equation 2, it follows that:

$$S_{\text{eff}}(f) = \left(\frac{96}{N_{\mathrm{psr}}(N_{\mathrm{psr}} - 1)}\right)^{1/2} \frac{12\pi^2 f^2}{\mathcal{N}_I^{-1}(f)}$$

Derivation of Quick Sensitivity Curves

$$S_I(f) = \frac{1}{\mathcal{N}_I^{-1}(f)\mathcal{R}(f)} = \frac{12\pi^2 f^2}{\mathcal{N}_I^{-1}(f)}$$

where $\mathcal{N}_I^{-1}(f)$ is the noise-weighted inverse transmission function for pulsar I.

Noise-Weighted Inverse Transmission

$$\mathcal{N}_I^{-1}(f) \approx \frac{\mathcal{T}_I(f)}{P_{\mathrm{N}}(f)}$$

where $\mathcal{T}_I(f) \approx \left(1 + \frac{1}{T_{\mathrm{obs}} f}\right)^{-6}$

Noise Model

The power in the noise is given by:

$$P_N = P_{\mathrm{WN}} + P_{\mathrm{RN}} = 2\Delta t \sigma^2 + A_{\mathrm{RN}} f^{-\gamma_{\mathrm{RN}}}$$

where σ is the time of arrival uncertainty, and A_{RN} models red noise.

Conclusion

Accurate PTA sensitivity is expensive to compute.
But with a few oversimplifications, we can get a quick estimate to characterize our detector.

References

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