

Lema (Elección noz): Existe una solución óptima $X = (x_1, x_2, \dots, x_n)$ al problema tal que $x_n = \min_{\{1, w/w_n\}}$

Prueba:

Sea X una sol. óptima al problema.

Si X contiene a la elección noz: $x_n = \min \{1, w/w_n\}$ entonces no hay más que probar

Suponga que X no contiene a la elección noz.

Sea Y la misma solución al problema que no contiene la elección noz, entonces:

$$Y = (y_1, y_2, \dots, y_i, \dots, y_n)$$

Sea: $X = (x_1, x_2, \dots, x_i, \dots, x_n)$ la misma solución.

Donde β y d son valores de intercambio.

Luego, se busca mantener la propiedad de los precios

$$w \cdot Y = w \cdot X \leq W$$

$$w \cdot Y = w \cdot X$$

$$w \cdot Y - w \cdot X = 0$$

$$y_i \cdot w_i + y_n \cdot w_n - x_i \cdot w_i - x_n \cdot w_n$$

$$(y_i - x_i) w_i + (y_n - x_n) w_n = 0$$

$$d w_i + -\beta w_n = 0$$

$$\boxed{\frac{d w_i}{w_n} = \beta}$$

$$vX - vY$$

$$x_i v_i + x_n v_n - y_i v_i - y_n v_n$$

$$(x_i - y_i) v_i + (x_n - y_n) v_n$$

$$-d v_i + \beta v_n$$

$$(-d v_i + \frac{d w_i}{w_n} v_n)$$

$$d \left(\frac{w_i}{w_n} v_n - v_i \right) \cdot \frac{1}{w_i}$$

$$\boxed{\frac{d}{w_i} \left(\frac{v_n}{w_n} - \frac{v_i}{w_i} \right) \geq 0 \quad \frac{v_i}{w_i} \leq \frac{v_n}{w_n}}$$

Se puede ver que no es posible construir una solución óptima, sin la elección noz porque $x_n > y_n$. Contradicción.

Ejercicio 2

$$A = [6, 0, 4, 1, 1, 0, 4, 3, 1, 3, 6] \quad k=6$$

$$C = [0, 0, 0, 0, 0, 0, 0]$$

$$C = [0, 0, 0, 0, 0, 0, 1]$$

$$C = [1, 0, 0, 0, 0, 0, 1]$$

$$C = [1, 0, 0, 0, 1, 0, 1]$$

$$C = [1, 1, 0, 0, 1, 0, 1]$$

$$C = [1, 2, 0, 0, 1, 0, 1]$$

$$C = [2, 2, 0, 0, 1, 0, 1]$$

$$C = [2, 2, 0, 0, 2, 0, 1]$$

$$C = [2, 2, 0, 1, 2, 0, 1]$$

$$C = [2, 3, 0, 1, 2, 0, 1]$$

$$C = [2, 3, 0, 2, 2, 0, 1]$$

$$C = [2, 3, 0, 2, 2, 0, 2]$$

$$C = [2, 3, 0, 2, 2, 0, 2]$$

$$C = [2, 5, 0, 2, 2, 0, 2]$$

$$C = [2, 5, 5, 2, 2, 0, 2]$$

$$C = [2, 5, 5, 7, 2, 0, 2]$$

$$C = [2, 5, 5, 7, 9, 0, 2]$$

$$C = [2, 5, 5, 7, 9, 9, 2]$$

$$C = [2, 5, 5, 7, 9, 9, 11]$$

$$C = [2, 5, 5, 7, 9, 9, 11]$$

$$B = [, , , , , , , , ,]$$

$$C = [2, 5, 5, 7, 9, 9, 10]$$

$$B = [, , , , , , , , 6]$$

$$C = [2, 5, 5, 6, 9, 9, 10]$$

$$B = [, , , , , 3, , , 6]$$

$$C = [2, 4, 5, 6, 9, 9, 10]$$

$$B = [, , , , 1, , 3, , , 6]$$

$$C = [2, 4, 5, 5, 9, 9, 10]$$

$$B = [, , , , 1, 3, 3, , , 6]$$

$$C = [2, 4, 5, 5, 8, 9, 10]$$

$$B = [, , , , 1, 3, 3, , 4, , 6]$$

$$C = [1, 4, 5, 5, 8, 9, 10]$$

$$B = [, 0, , , 1, 3, 3, , 4, , 6]$$

$$C = [1, 3, 5, 5, 8, 9, 10]$$

$$B = [, 0, , 1, 1, 3, 3, , 4, , 6]$$

$$C = [1, 2, 5, 5, 8, 9, 10]$$

$$B = [, 0, 1, 1, 1, 3, 3, , 4, , 6]$$

$$C = [1, 2, 5, 5, 7, 9, 10]$$

$$B = [, 0, 1, 1, 1, 3, 3, 4, 4, , 6]$$

$$C = [0, 2, 5, 5, 7, 9, 10]$$

$$B = [0, 0, 1, 1, 1, 3, 3, 4, 4, , 6]$$

$$C = [0, 2, 5, 5, 7, 9, 9]$$

$$B = [0, 0, 1, 1, 1, 3, 3, 4, 4, 6, 6]$$

Ejercicio 3

A = [cow, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, TEA, Now, FOX].

A

cow ~~x~~
DOG ~~x~~
SEA ~~x~~
RUG ~~x~~
ROW ~~x~~
MOB ~~x~~
BOX ~~x~~
TAB ~~x~~
BAR ~~x~~
EAR ~~x~~
TAR ~~x~~
DIG ~~x~~
TEA ~~x~~
Now ~~x~~
FOX ~~15~~

COUNTING-SORT(A[3])
→

SEA ~~x~~
TEA ~~x~~
MOB ~~x~~
TAB ~~x~~
DOG ~~x~~
RUG ~~15~~
DIG ~~x~~
BAR ~~x~~
EAR ~~x~~
TAR ~~x~~
cow ~~x~~
ROW ~~x~~
Now ~~x~~
BOX ~~x~~
FOX ~~x~~

COUNTING-SORT(A[2])
→

TAB ~~x~~
BAR ~~x~~
EAR ~~x~~
TAR ~~x~~
SEA ~~x~~
TEA ~~x~~
DIG ~~x~~
MOB ~~x~~
DOG ~~x~~
cow ~~x~~
ROW ~~x~~
Now ~~x~~
BOX ~~x~~
FOX ~~x~~
RUG ~~x~~

COUNTING-SORT(A[1])
→

BAR
BOX
cow
DIG
DOG
EAR
FOX
MOB
Now
Row
RUG
SEA
TAB
TAR
TEA