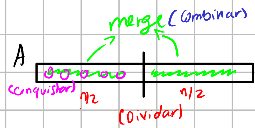
Divide y Conquista

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Merge-Sort (A,p,r)
if p<r
    q = ⌊(p+r)/2⌋
    Merge-Sort (A,p,q)
    Merge-Sort (A, q+1, r)  ← Dividir
    Merge (A, p, q, r)  ← Combinar
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Merge (A, p, q, r) // Recibe un arreglo A[1..n], tre ind p, q, r tales que A[p..q], A[q+1...r] están ordenados. Ordena A[p...r].

$n_1 = q - p + 1$	c	1
$n_2 = r - q$	c	1
for i=1 to n_1	c	n_1
L[i] = A[p+i-1]	c	n_1+1
for j=1 to n_2	c	n_2+1
R[j] = A[q+j]	c	n_2
L[n_1+1] = ∞	c	1
R[n_2+1] = ∞	c	1
i=1	c	1
j=1	c	1
for k=p for r	c	n_1+n_2+1
if L[i] ≤ R[j]	c	n_1+n_2
A[k] = L[i]	c	n_1+n_2
i=i+1	c	n_1+n_2
else A[k] = R[j]	c	n_1+n_2
j=j+1	c	n_1+n_2

Analisis Tiempo de ejecución

$$\boxed{T(n) = a\,T(n/b) + D(n) + C(n)}$$

a: cantidad de sub problemas
b: División de sub problemas

T(n) del MergeSort: $T(n) = 2T(n/2) + C_1 + C_2 n$

$$T(n) = \begin{cases} 2F(\lfloor n/2 \rfloor) + n & : n \geq 1 \\ 1 & : n = 1 \end{cases}$$

Supongamos que $n = 2^k$ para algún $k \in \mathbb{N}$.

$$F(2^k) = 2F(2^{k-1}) + 2^k$$
$$= 2(2F(2^{k-2}) + 2^{k-1}) + 2^k$$
$$= 2^2 F(2^{k-2}) + 2^k + 2^k$$
$$= 2^2(2F(2^{k-3}) + 2^{k-2}) + 2^k + 2^k + 2^k$$
$$\vdots$$
$$= 2^i \cdot F(2^{k-i}) + i \cdot 2^k$$
$$\vdots$$

reemplazamos $n=2^k$

$$= 2^k \cdot F(2^0) + k \cdot 2^k = 2^k + k\,2^k = 2^k(k+1) = n(\lg n + 1)$$

Cuando n es potencia de 2 $\Rightarrow F(n) = n(\lg n + 1)$

Ahora, suponga que $n \in \mathbb{N}$. Sea $k \in \mathbb{N}$ tal que $2^k \leq n < 2^{k+1}$

Como F es creciente (probaremos después), $F(2^k) \leq F(n) < F(2^{k+1})$

Luego, $F(n) < F(2^{k+1}) = 2^{k+1} \cdot (\lg(2^{k+1}) + 1)$
$$= 2^{k+1} \cdot (k+2)$$
$$\leq 2^{k+1} \cdot 3k$$
$$= 2 \cdot 2^k \cdot 3k$$
$$= 6k\,2^k \leq 6n\lg n$$

$$F(n) \geq F(2^k)$$
$$= 2^k(\lg(2^k) + 1)$$
$$= 2^k(k+1)$$
$$= \frac{2^{k+1}}{2}(k+1) > \frac{n\lg n}{2}$$

porque sabemos que $2^k \leq n < 2^{k+1}$

Concluimos que

$$\frac{1}{2} n \lg n < F(n) \leq 6n\lg n \quad \blacksquare$$

Falta demostrar que F(n) es creciente.

Recordar

$$F(n) = \begin{cases} 1 & : n = 1 \\ 2F(\lfloor n/2 \rfloor) + n & : n > 1 \end{cases}$$

Basta probar que $F(n) < F(n+1)$ para todo $n \geq 1$

Por Ind en n. Si $n=1$, tenemos que $F(n) = F(1) = 1 < 4 = F(2) = F(n+1)$

Si $n > 1$

Caso 1: n es par. $F(n) < F(n) + 1 = 2F(n/2) + n + 1 = F(n+1)$

Caso 2: n es impar. $F(n) = 2F\left(\frac{n-1}{2}\right) + n$

(H.I)

$$< 2F\left(\frac{n-1}{2} + 1\right) + n$$
$$= 2F\left(\frac{n+1}{2}\right) + n$$
$$< 2F\left(\frac{n+1}{2}\right) + n + 1 < 2F\left(\left\lfloor\frac{n+1}{2}\right\rfloor\right) + n + 1 = F(n+1)$$

Sea $F: \mathbb{N} \to \mathbb{R}^{>}$ definido por:

$$F(n) = \begin{cases} 1 & n = 1 \\ 2F(n-1) + 1 & \text{caso contrario} \end{cases}$$

¿Cuanto vale F(n)?

$$F(n) = 2F(n-1) + 1$$
$$= 2(2F(n-2) + 1) + 1$$
$$= 2^2 F(n-2) + 2 + 1$$
$$= 2^2(2F(n-3) + 1) + 2 + 1$$
$$= 2^3 F(n-3) + 2^2 + 2 + 1$$
$$= 2^3(2F(n-4) + 1) + 2^2 + 2 + 1$$
$$= 2^4 F(n-4) + 2^3 + 2^2 + 2 + 1$$
$$\vdots$$
$$= 2^i F(n-i) + \sum_{k=0}^{i-1} 2^k$$
$$= 2^i F(n-i) + 2^i - 1$$

$i = n-1$

$$= 2^{n-1} F(1) + 2^{n-1} - 1$$
$$F(n) = 2^n - 1$$

Sea $F: \mathbb{N} \to \mathbb{R}^{>}$ definido por:

$$F(n) = \begin{cases} 1 & n = 1 \\ F(n-1) + n & \text{Caso contrario} \end{cases}$$

¿Cuanto vale F(n)?

$$F(n) = F(n-1) + n$$
$$= (F(n-2) + n - 1) + n$$
$$= F(n-2) + 2n - 1$$
$$= (F(n-3) + n - 2) - 1$$
$$= F(n-3) + 3n - (1+2)$$
$$= (F(n-4) + n - 3) + 3n - (1+2)$$
$$= F(n-4) + 4n - (1+2+3)$$
$$\vdots$$
$$= F(n-i) + in - (1+2+3+\ldots(i-1))$$

$= F(n-i) + in - (1+2+3+\dots(i-1))$

$i = n-1$

$= F(1) + n(n-1) - (1+2+3+\dots(n-2))$

$= 1 + n(n-1) - \frac{(n-2(n-1))}{2}$

$= 1 + (n-1)\left(n - \frac{(n-2)}{2}\right)$

$= 1 + (n-1)\left(\frac{n+2}{2}\right)$

$= 1 + \frac{n^2+n-2}{2}$

$= \frac{n^2+n}{2} = \frac{n(n+1)}{2}$

$T(1) = 1$

$T(n) = 2T(\lfloor n/2 \rfloor) + n^2$

Para n potencia de 2 $\quad n = 2^k$

$= 2T(n/2) + n^2$

$= 2\left(2T(n/4) + \left(\frac{n}{2}\right)^2\right) + n^2$

$= 2^2 T(n/4) + \frac{n^2}{2} + n^2$

$= 2^2\left(2T(n/8) + \left(\frac{n}{4}\right)^2\right) + \frac{n^2}{2} + n^2$

$= 2^3 T(n/8) + 2^2\left(\frac{n}{4}\right)^2 + \frac{n^2}{2} + n^2$

$= 2^3 T(n/8) + \frac{n^2}{4} + \frac{n^2}{2} + n^2$

$\vdots$

$= 2^i T(n/2^i) + n^2\left(\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \dots + \frac{1}{2^{i-1}}\right)$

$= 2^i T(n/2^i) + n^2\left(1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots + \left(\frac{1}{2}\right)^{i-1}\right)$

$= 2^i T(n/2^i) + n^2\left(\frac{(1/2)^{i-1+1} - 1}{1/2 - 1}\right)$

$i = \lg n$

$= 2^{\lg n} T(n/2^{\lg n}) + n^2\left(\frac{(1/2)^{\lg n} - 1}{1/2 - 1}\right)$

$= n\,T(1) + n^2\left(\frac{\frac{1}{n} - 1}{-1/2}\right)$

$= n - 2n + n^2 = n(2n-1)$

Demostrar por inducción

$$T(n) = \begin{cases} 1 & n=1 \\ 2T(\lfloor \frac{n}{2} \rfloor) + n & \text{caso contrario.} \end{cases}$$

Probar $T(n) = \Omega(n \lg n)$

Prop $\quad T(n) \geq \frac{1}{4} n \lg n$ para todo $n \geq 4$.

Caso base

$T(4) = 2T(2) + 4 = 12 \geq \frac{1}{4} 4 \lg 4 = \frac{1}{4} n \lg n$

Paso inductivo

$T(n) = 2T(\lfloor n/2 \rfloor) + n$

$\geq 2 \frac{1}{4} \lfloor \frac{n}{2} \rfloor \lg \lfloor n/2 \rfloor + n$

$\geq \frac{1}{2}\left(\frac{n}{2} - 1\right) \lg(n/4) + n$

$\geq \frac{1}{2}\left(\frac{n}{2} - 1\right)(\lg n - 2) + n$

$= \frac{1}{2}\left(\frac{n}{2} - 1\right)(\lg n - 2) + n$

$= \frac{1}{4} n \lg n - \frac{1}{2} \lg n - \frac{n}{2} + 1 + n$

$= \frac{1}{4} n \lg n + \frac{1}{2}(n - \lg n) + 1 \geq \frac{1}{4} n \lg n \quad \blacksquare$

Dado,

$$T(n) = \begin{cases} 1 & n=1 \\ 2T(\lfloor \frac{n}{2} \rfloor) + 1 & \text{caso contrario} \end{cases}$$

$\lfloor n \rfloor \leq n \leq \lceil n \rceil$

Probar por inducción que $T(n) = O(n)$.

Prop $\quad T(n) \leq 2n-1$ para todo $n \geq 1$

Caso base

$T(1) = 1 \leq 2.1 - 1$

Paso inductivo

$T(n) = 2T(\lfloor \frac{n}{2} \rfloor) + 1$

$\leq 2c\lfloor \frac{n}{2} \rfloor + 1$

$\leq 2c\left(2\frac{n}{2} - 1\right) + 1$

$\leq 2n - 2 + 1$

$= 2n - 1$

Corolario

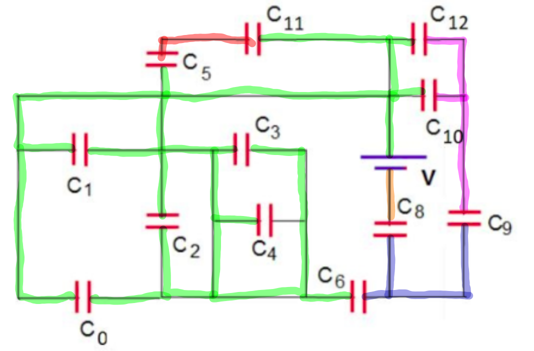
$T(n) \leq 2n \; \forall \; n > 1$

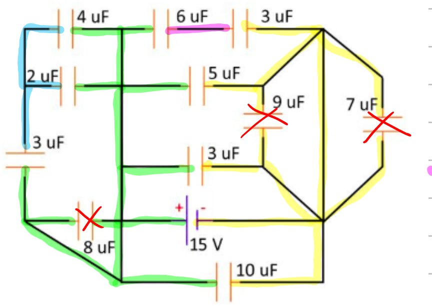
Entonces $T(n) = O(n)$

$T(n) = n\left(\frac{3n}{2} - \frac{1}{2}\right)$

$T(n) = 1 + n^2\left(\frac{3n-3}{2}\right)$

Dado el siguiente circuito, indique a continuación los capacitores





$Ce = 4+2 = 6\mu F$

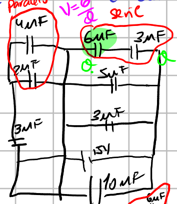
Paralelo

$V = \frac{6}{Q}$

$Ce = 2\mu F$

serie

$V = \frac{6}{1{,}50}$



$C = QV$

Paralelo

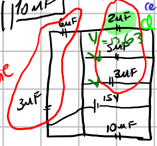
$Ce = 2+5+3 = 10\mu F$

$Q = \frac{2}{1{,}33}$

$Q = 1{,}50$

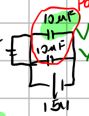
$Ce = 2\mu F$

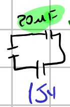
serie



Paralelo

$Ce = 20\mu F$





$V = 1{,}33$

$\frac{1}{ce} = \frac{1}{2} + \frac{1}{20}$

$\frac{1}{ce} = \frac{22}{40}$

$ce = \frac{20}{11}$

$C = QV$

$\frac{20}{13} = V$