$$T(n) = \begin{cases} 1 & n=1 \\ T(n) = T(\lfloor\sqrt{n}\rfloor) + 1 & \text{otherwise} \end{cases}$$

$n = 2^k$

$\lg n = k$

Suponer que $n = 2^k$, con k potencia de 2

$$\begin{aligned} T(2^k) &= T(2^{k/2}) + 1 \\ &= (T(2^{k/4}) + 1) + 1 \\ &= T(2^{k/4}) + 2 \\ &\vdots \\ &T(2^{k/2^i}) + i \\ &\vdots \quad \text{con } i = \lg(\lg(n)) \\ &T(2^1) + \lg(\lg n) \\ &T(1) + 1 + \lg(\lg n) \\ T(n) &= \lg(\lg n) + 2 \end{aligned}$$

$$2^k$$
$$2^{k/2}, 2^{k/2}$$
$$2^{k/4}, 2^{k/4}\ 2^{k/4}, 2^{k/4}$$
$$\underbrace{2, 2, \ldots\ldots 2\ 2}_{k \text{ veces}}$$

$k = \lg n = 2^i$

$\lg(\lg n) = i$

Si se sabe que $\forall n \in \mathbb{N}$ $\exists k$ tal que $2^k \leq n < 2^{2k}$, con $n \geq 2$

Entonces:

$$T(2^k) \leq T(n) < T(2^{2k})$$

$$\lg\lg(2^k) + 2 \leq \lg(\lg(n)) + 2 < \lg\lg(2^{2k}) + 2$$

$$\lg\lg(2^k) \leq \lg\lg(n) + 2 < \lg\lg(2^{2k}) + 2$$

$$\lg k \leq \lg(\lg(n)) + 2 < \lg 2k + 2$$

$k = \lg n$

$$\lg\lg n \leq \lg\lg(n) + 2 < \lg(2\lg n) + 2$$

$$1 \cdot \lg\lg n \leq \lg(\lg(n)) + 2 < \underbrace{\lg(2)}_{1} + \lg\lg(n) + 2$$

$$\lg\lg(n) + 2 < \lg\lg(n) + 3 \leq \lg\lg(n) + 3\lg\lg(n) \quad (n \geq 4)$$

Como:

$$1 \cdot \lg\lg n \leq \lg\lg(n) + 2 \leq 4\lg\lg(n), \text{ con } n \geq 4 \quad \blacksquare$$

$$T(n) = \Theta(\lg(\lg n))$$

Ejercicio 2

```
merge sort( A, n) :
    let A = [(1,1), (2,2), ..., (n,n)]
    L = 1, R = n
    while L != R :
        merge( A[L], A[L+1])
        A[L+1] = (A[L][1], A[L+1][2])
        L = L+1
        if R-1 != L
            merge(A[R-1], A[R])
            A[R-1] = (A[R-1][1], A[R][2])
            R = R-1
```

merge recibe dos pares de ~~no~~ indices ~~[illegible]~~ de los subarreglos ordenados