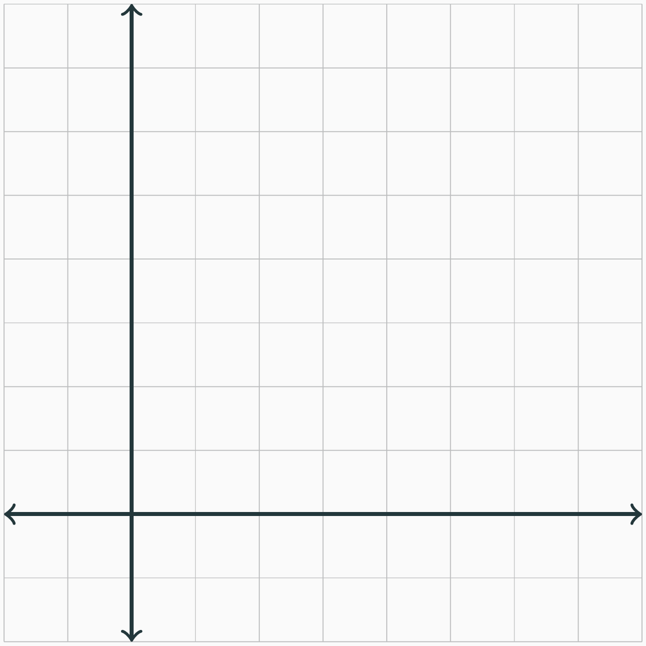
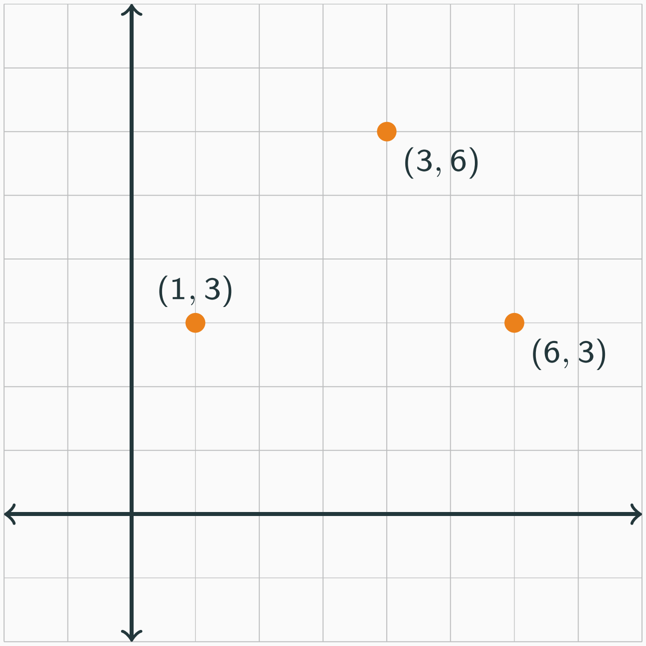
- Geometry
- Computational geometry

Geometry

Points and vectors



Points and vectors



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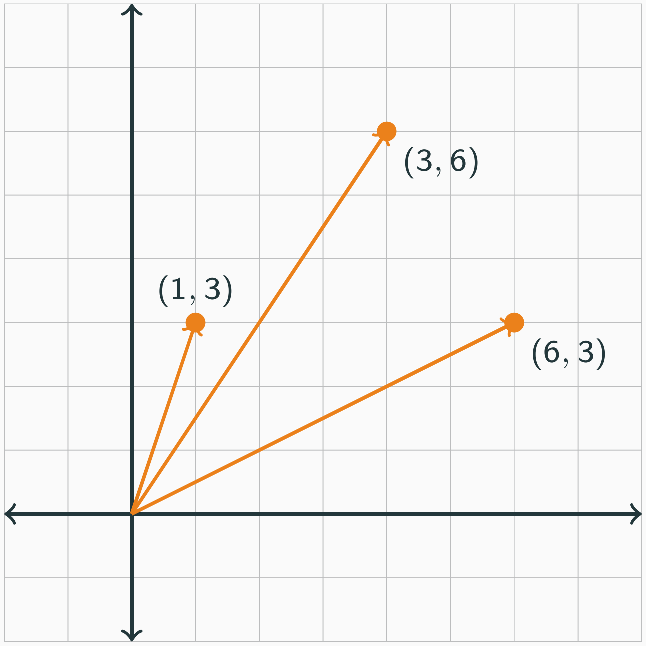
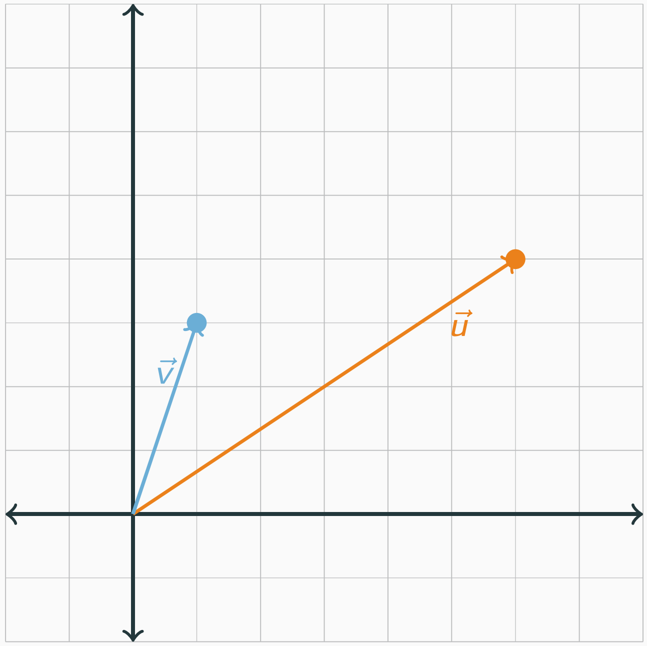
Points and vectors



- Points are represented by a pair of numbers, (x, y).
- Vectors are represented in the same way.
- Thinking of points as vectors allows us to do many things.

Points and vectors



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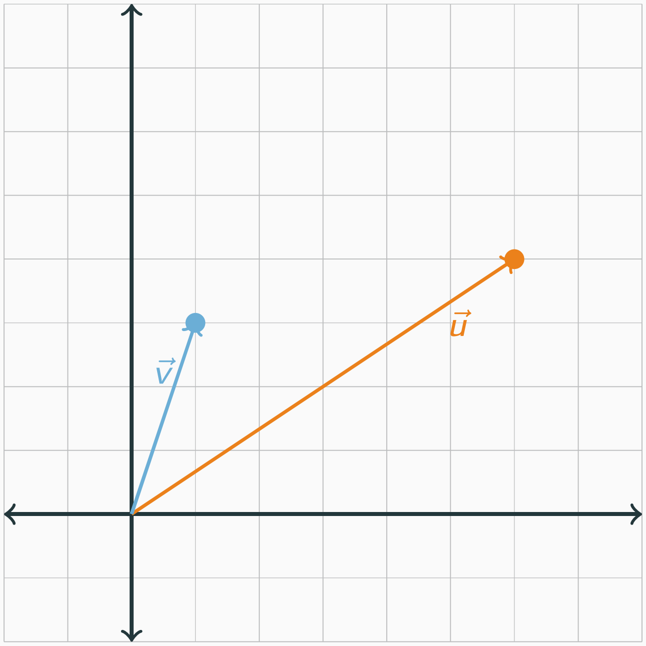

- Simplest operation, addition is defined as

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 + x_1 \\ y_0 + y_1 \end{pmatrix}$$

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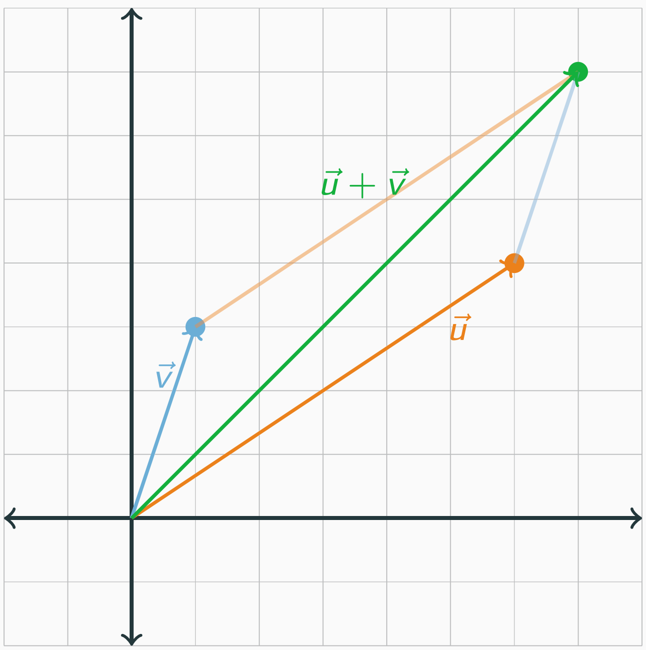

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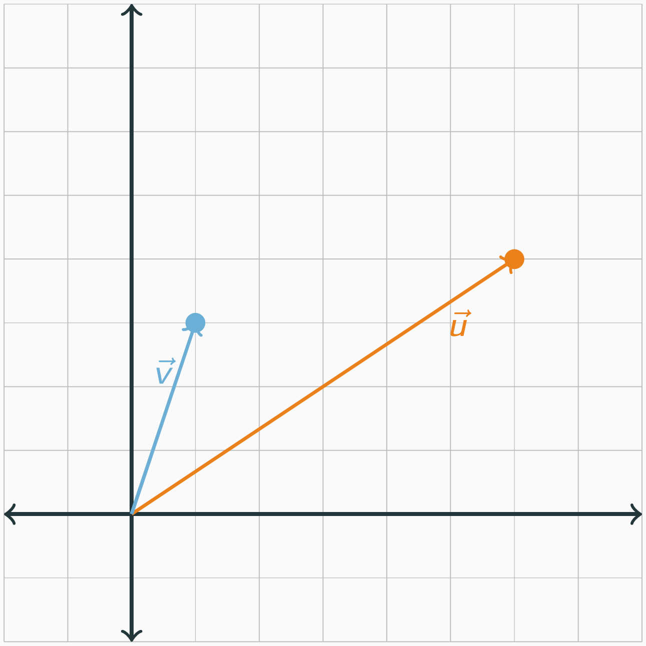

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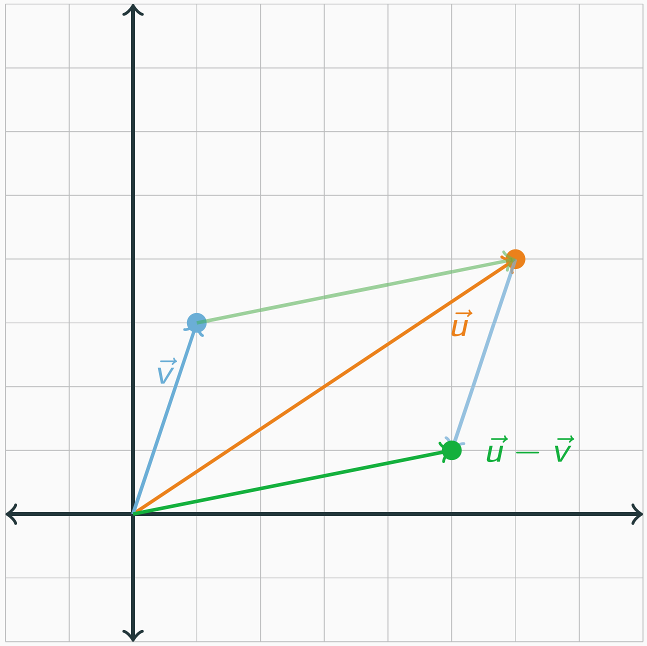

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Points and vectors

```
struct point {
    double x, y;
    point(double _x, double _y) {
        x = _x, y = _y;
    }

    point operator+(const point &oth){
        return point(x + oth.x, y + oth.y);
    }

    point operator-(const point &oth){
        return point(x - oth.x, y - oth.y);
    }
};
```

Points and vectors

. . . or we could use the `complex<double>` class.

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typedef complex<double> point;
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The `complex` class in `C++` and `Java` has methods defined for

- Addition
- Subtraction
- Multiplication by a scalar
- Length
- Trigonometric functions

Points and vectors

Complex numbers have the real part and the imaginary part. Can be thought of as vectors or points on the complex plane.

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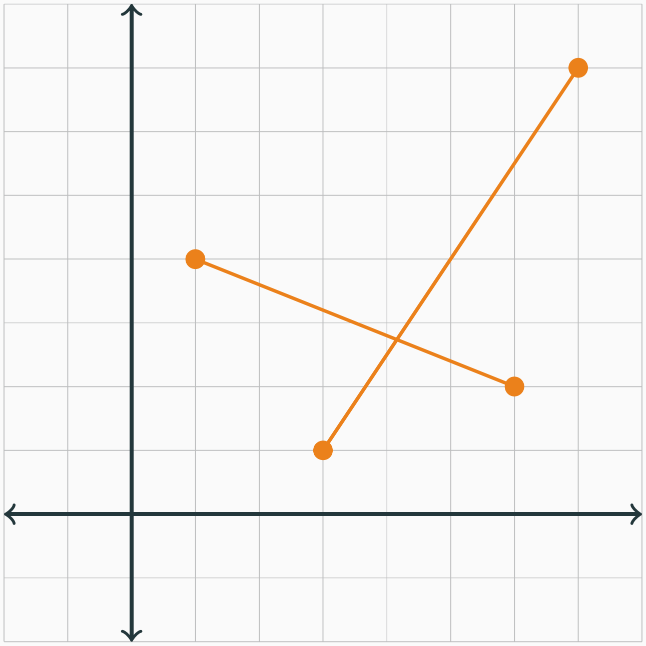
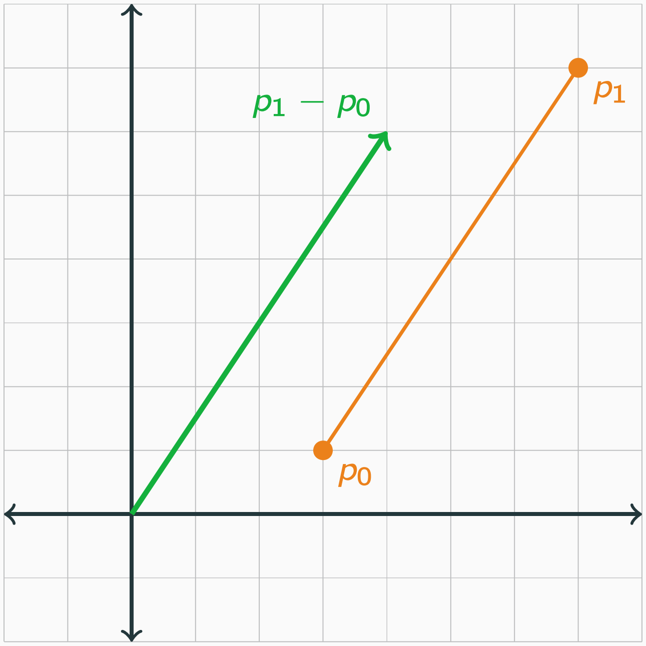
- `double sin(p)`, `double cos(p)`, `double tan(p)`, trigonometric functions.

Lines and line segments



- Line segments are represented by a pair of points, $((x_0, y_0), (x_1, y_1))$.

Lines and line segments



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- Distance between two points is the length of the line segment or vector between the points.

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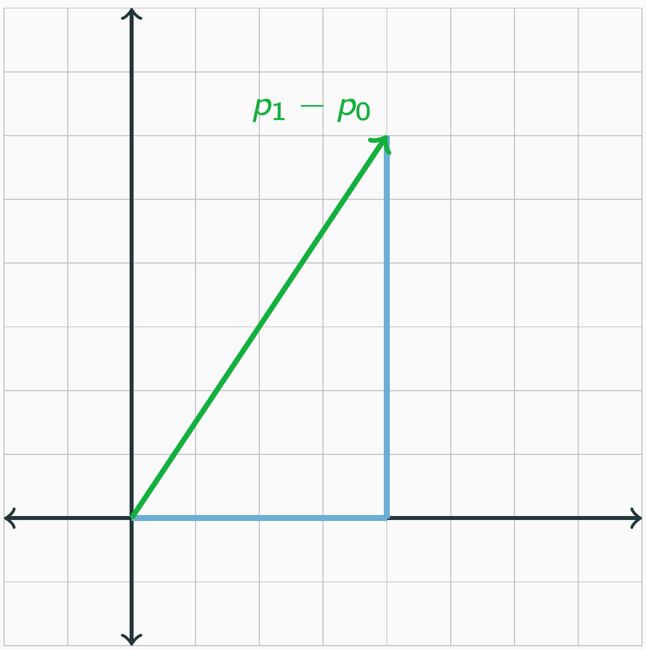

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$$
\begin{aligned}
d((x_0,&y_0),(x_1,y_1)) \\
&= |(x_1 - x_0, y_1 - y_0)| \\
&= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}
\end{aligned}
$$

Lines and line segments

```
struct point {
    ...
    double distance(point oth = point(0,0)) const {
        return sqrt(pow(x - oth.x, 2.0)
                  + pow(y - oth.y, 2.0));
    }
    ...
}
```

Lines and line segments

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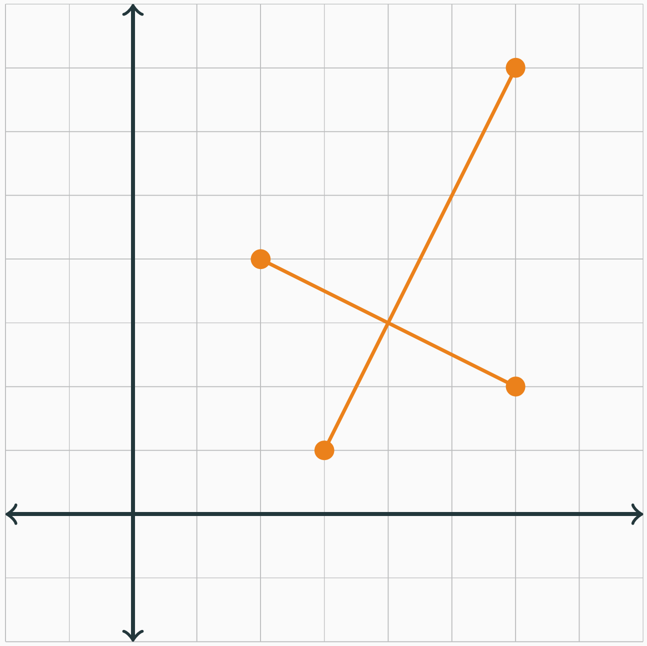
    ...
}
```

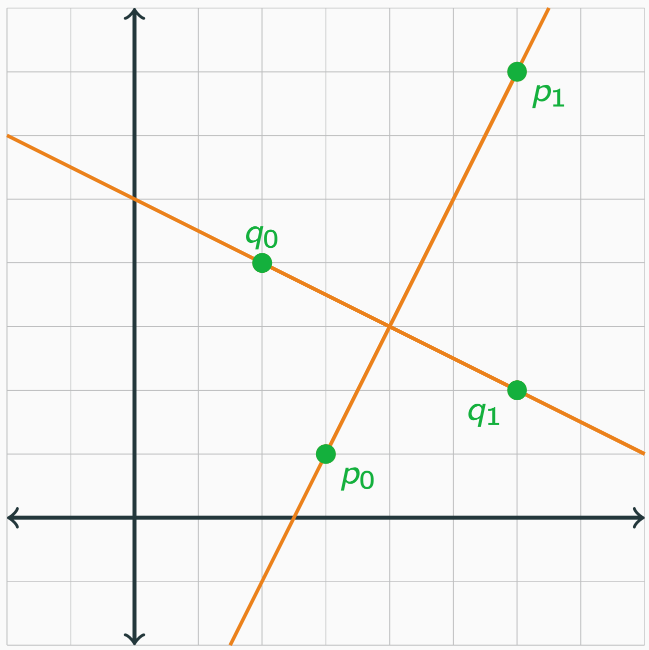
Or use the `abs` function with `complex<double>`.

Lines and line segments



- Line representation same as line segments.

Lines and line segments



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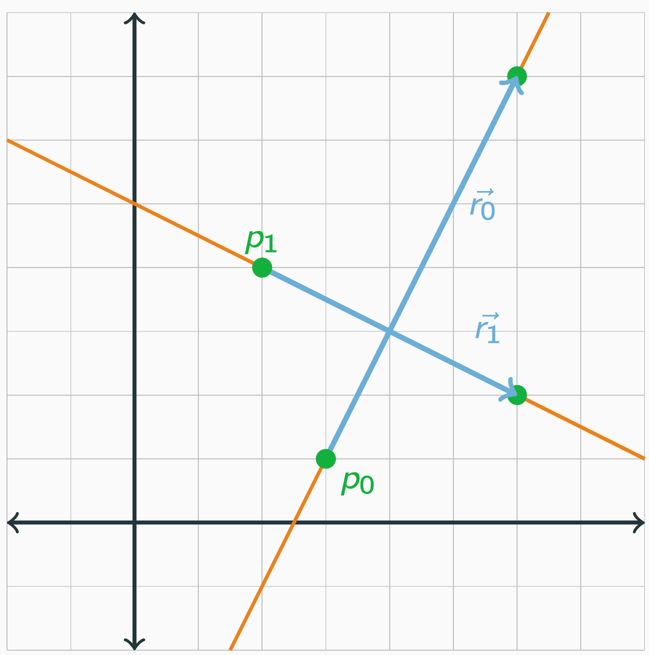

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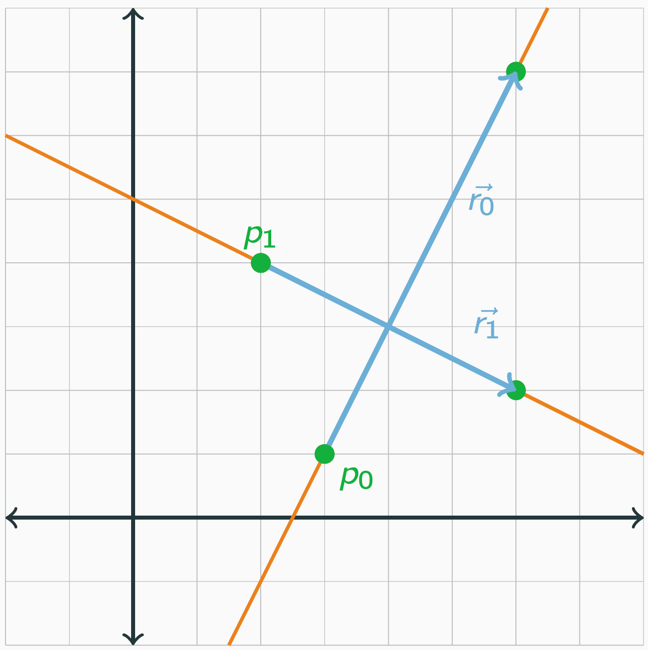
- Or as a point and a direction vector.

$$p + t \cdot \vec{r}$$

Lines and line segments



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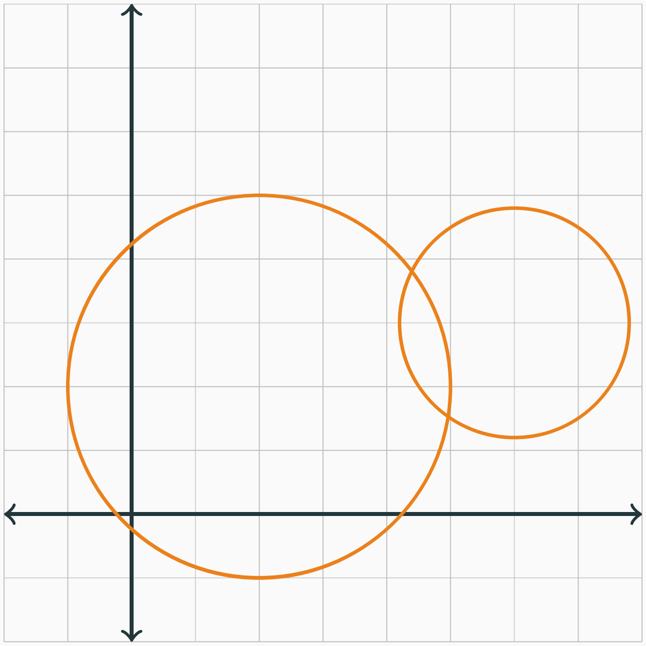
- Either way

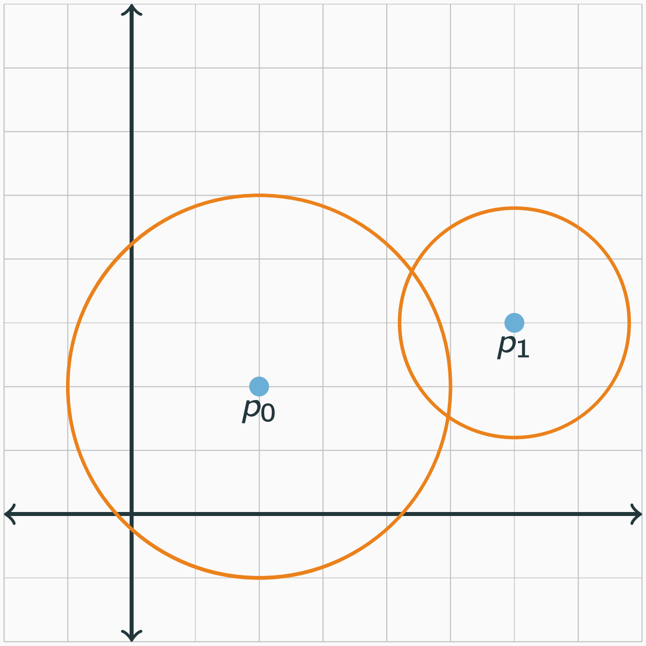
`pair<point,point>`

Circles



- Circles are very easy to represent.

Circles



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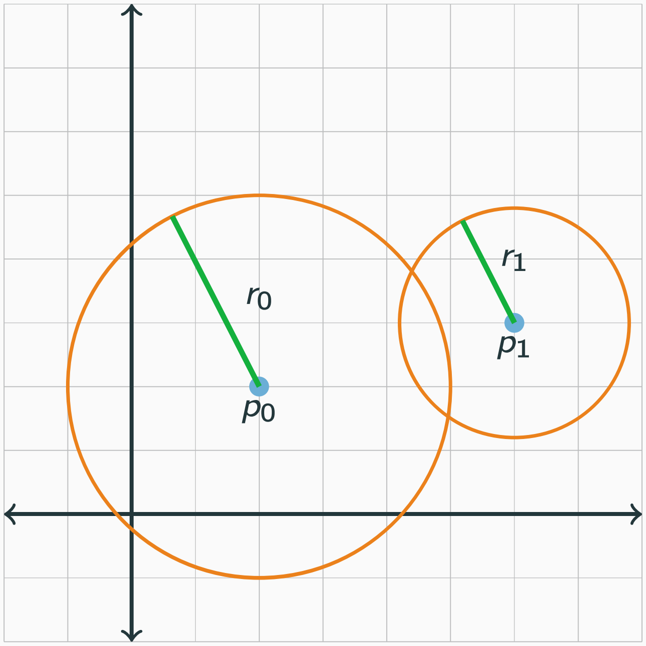

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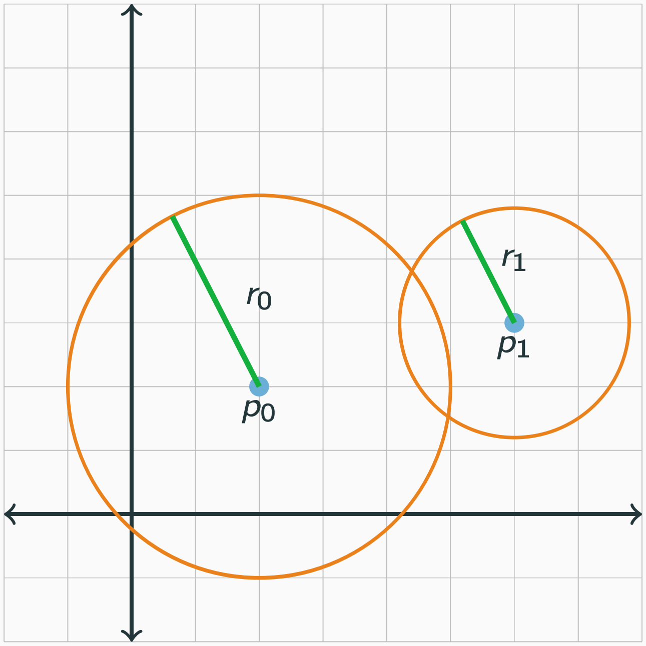
Circles



- Circles are very easy to represent.
- Center point $p = (x, y)$.
- And the radius r.

`pair<point,double>`

Dot product

Given two vectors

$$\vec{u} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

the dot product of $\vec{u}$ and $\vec{v}$ is defined as

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = x_0 \cdot x_1 + y_0 \cdot y_1$$

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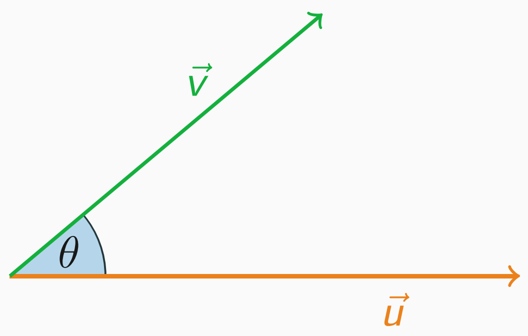
Which in geometric terms is

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos \theta$$

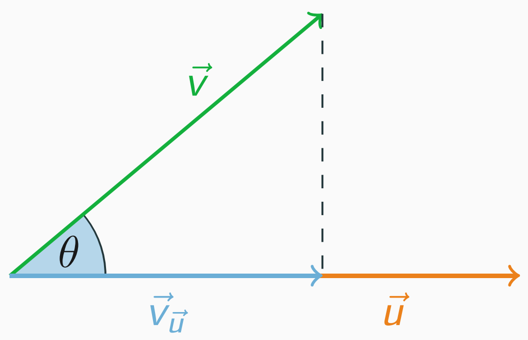
Dot product



- Allows us to calculate the angle between $\vec{u}$ and $\vec{v}$.

$$\theta = \arccos\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right)$$

Dot product



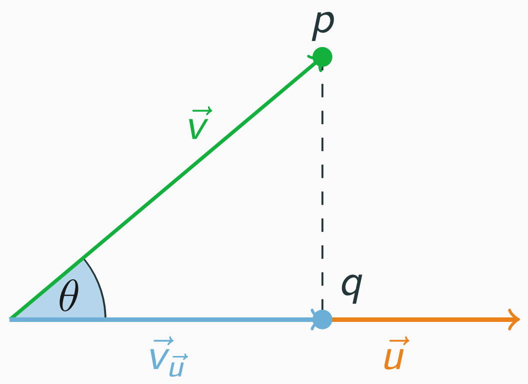
- Allows us to calculate the angle between $\vec{u}$ and $\vec{v}$.

$$\theta = \arccos\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right)$$

- And the projection of $\vec{v}$ onto $\vec{u}$.

$$\vec{v}_{\vec{u}} = \left(\frac{\vec{u}\cdot\vec{v}}{|u|^2}\right)\vec{u}$$

Dot product



- The closest point on $\vec{u}$ to p is q.

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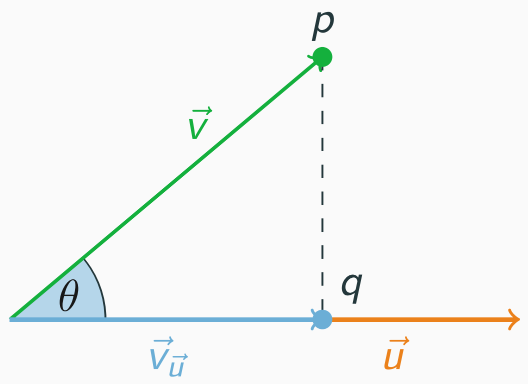

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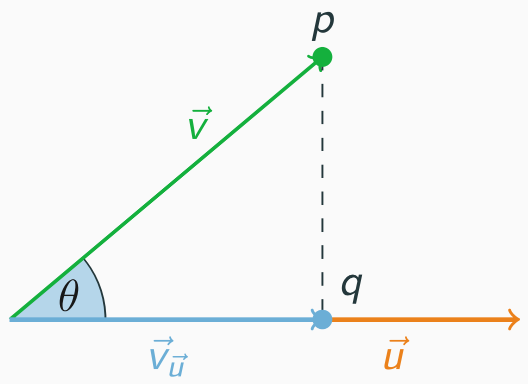

- The closest point on $\vec{u}$ to p is q.
- The distance from p to $\vec{u}$ is the distance from p to q.
- Unless q is outside $\vec{u}$, then the closest point is either of the endpoints.

Dot product

Rest of the code will use the `complex` class.

```
#define P(p) const point &p
#define L(p0, p1) P(p0), P(p1)
double dot(P(a), P(b)) {
    return real(a) * real(b) + imag(a) * imag(b);
}
double angle(P(a), P(b), P(c)) {
    return acos(dot(b - a, c - b) / abs(b - a) / abs(c - b));
}
point closest_point(L(a, b), P(c), bool segment = false) {
    if (segment) {
        if (dot(b - a, c - b) > 0) return b;
        if (dot(a - b, c - a) > 0) return a;
    }
    double t = dot(c - a, b - a) / norm(b - a);
    return a + t * (b - a);
}
```

Cross product

Given two vectors

$$\vec{u} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

the cross product of $\vec{u}$ and $\vec{v}$ is defined as

$$\left| \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \times \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \right| = x_0 \cdot y_1 - y_0 \cdot x_1$$

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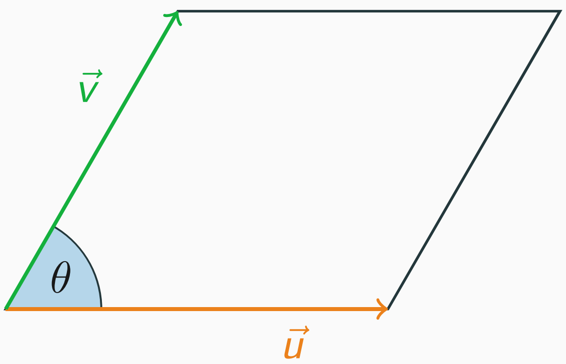
Which in geometric terms is

$$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin \theta$$

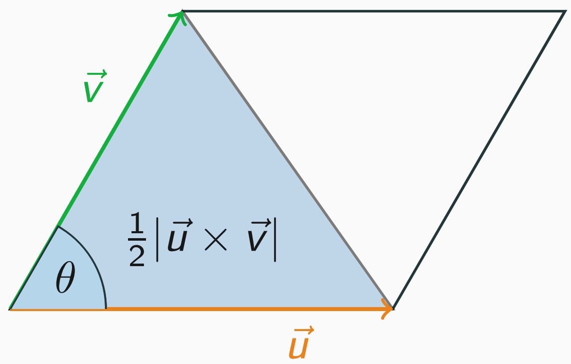
Cross product



- Allows us to calculate the area of the triangle formed by $\vec{u}$ and $\vec{v}$.

$$\frac{|\vec{u} \times \vec{v}|}{2}$$

Cross product



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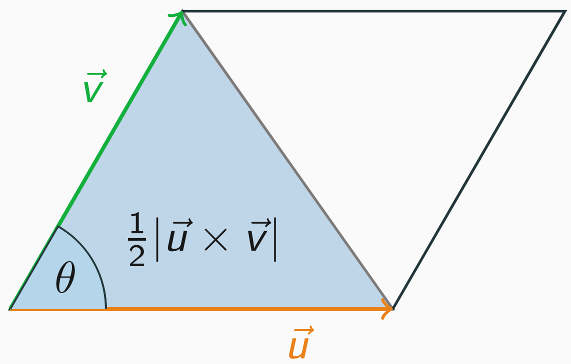


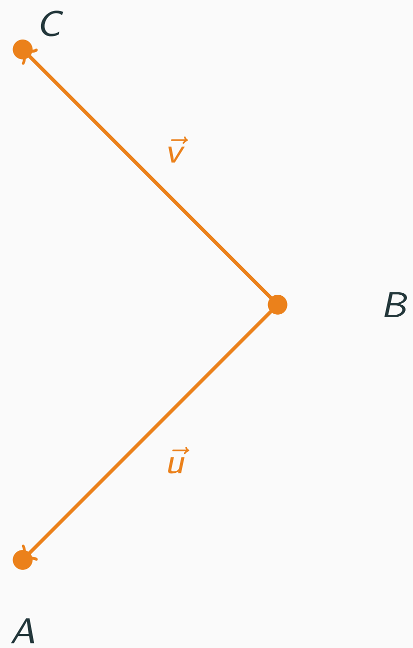
- Allows us to calculate the area of the triangle formed by $\vec{u}$ and $\vec{v}$.

$$\frac{|\vec{u} \times \vec{v}|}{2}$$

- And can tell us if the angle between $\vec{u}$ and $\vec{v}$ is positive or negative.

$$\begin{aligned}
|\vec{u} \times \vec{v}| < 0 \quad &\text{iff} \quad \theta < \pi \\
|\vec{u} \times \vec{v}| = 0 \quad &\text{iff} \quad \theta = \pi \\
|\vec{u} \times \vec{v}| > 0 \quad &\text{iff} \quad \theta > \pi
\end{aligned}$$

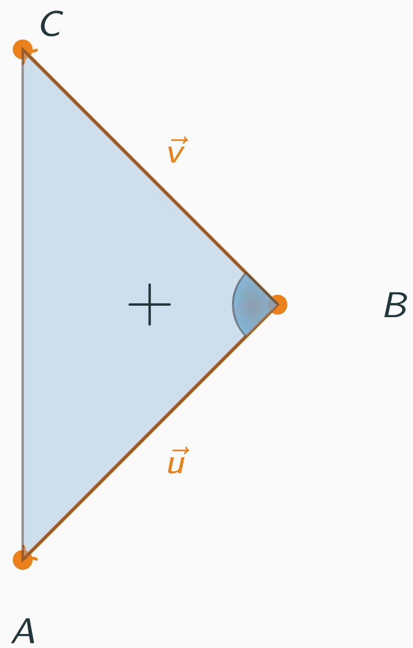
Counterclockwise



- Given three points A, B and C, we want to know if they form a counter-clockwise angle in that order.

$$A \rightarrow B \rightarrow C$$

Counterclockwise



- Given three points A, B and C, we want to know if they form a counter-clockwise angle in that order.

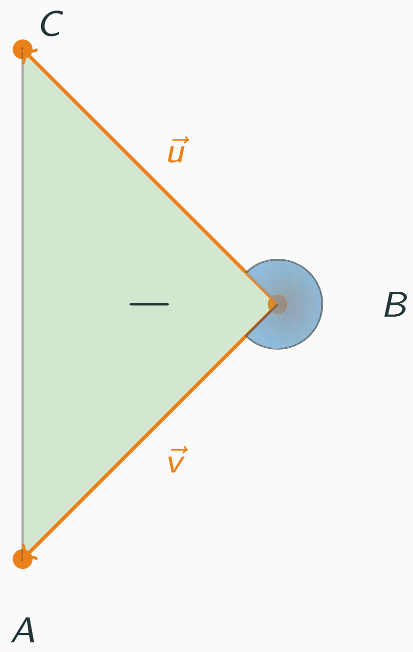
$$A \to B \to C$$

- We can examine the cross product of and the area of the triangle formed by

$$\vec{u} = B - C \quad \vec{v} = B - A$$

$$\vec{u} \times \vec{v} > 0$$

Counterclockwise



- The points in the reverse order do not form a counter clockwise angle.

$$C \to B \to A$$

- In the reverse order the vectors swap places

$$\vec{u} = B - A \quad \vec{v} = B - C$$
$$\vec{u} \times \vec{v} < 0$$

Counterclockwise

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$$C \to B \to A$$

- In the reverse order the vectors swap places

$$\vec{u} = B - A \quad \vec{v} = B - C$$
$$\vec{u} \times \vec{v} < 0$$

- If the points A, B and C are on the same line, then the area will be 0.

Counterclockwise

```
double cross(P(a), P(b)) {
    return real(a)*imag(b) - imag(a)*real(x);
}
double ccw(P(a), P(b), P(c)) {
    return cross(b - a, c - b);
}
bool collinear(P(a), P(b), P(c)) {
    return abs(ccw(a, b, c)) < EPS;
}
```

Intersections

Very common task is to find the intersection of two lines or line segments.

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Very common task is to find the intersection of two lines or line segments.

- Given a pair of points (x_0, y_0), (x_1, y_1), representing a line we want to start by obtaining the form $Ax + By = C$.
- We can do so by setting

$$A = y_1 - y_0$$
$$B = x_0 - x_1$$
$$C = A \cdot x_0 + B \cdot y_1$$

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$$
\begin{aligned}
A &= y_1 - y_0 \\
B &= x_0 - x_1 \\
C &= A \cdot x_0 + B \cdot y_1
\end{aligned}
$$

- If we have two lines given by such equations, we simply need to solve for the two unknowns, x and y.

Intersections

For two lines

$$A_0x + B_0y = C_0$$
$$A_1x + B_1y = C_1$$

The intersection point is

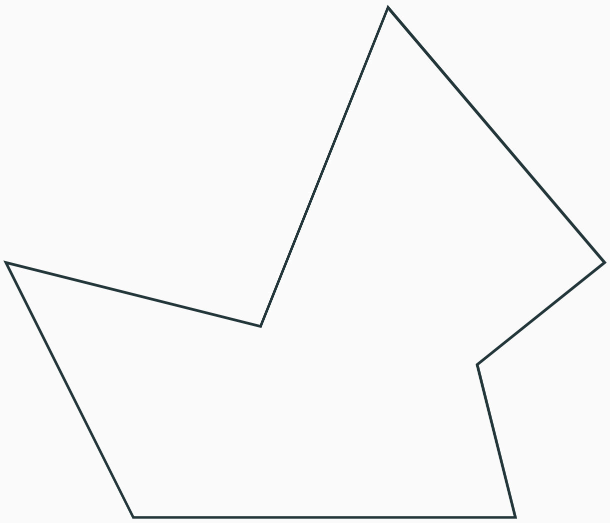
$$x = \frac{(B_1 \cdot C_0 - B_0 \cdot C_1)}{D}$$
$$y = \frac{(A_0 \cdot C_1 - A_1 \cdot C_0)}{D}$$

Where

$$D = A_0 \cdot B_1 - A_1 \cdot B_0$$

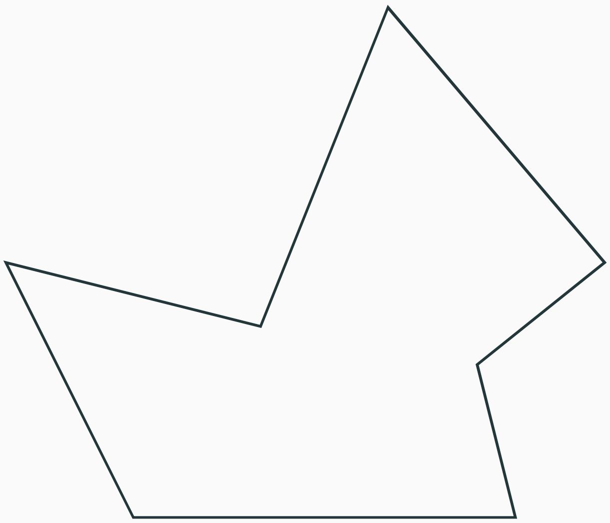
Computational Geometry

Polygons



- Polygons are represented by a list of points in the order representing the edges.

Polygons



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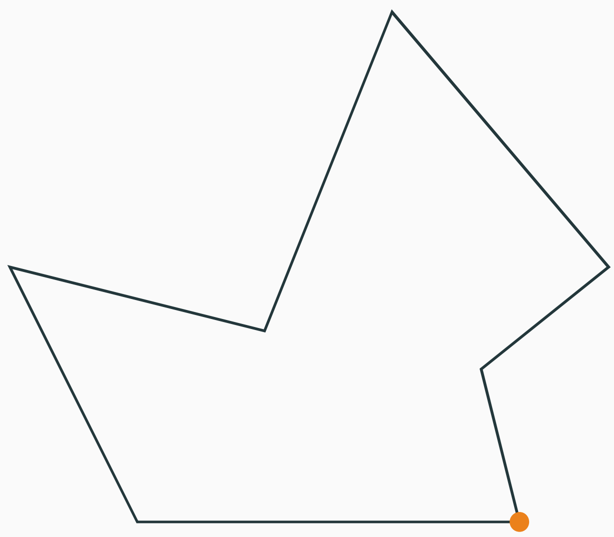
Polygons



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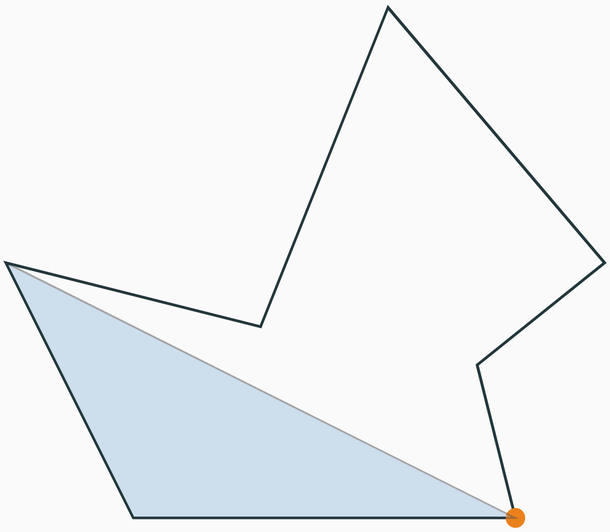
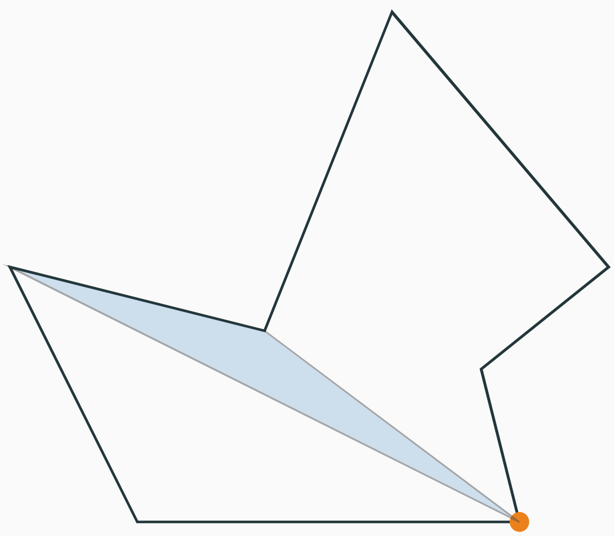
Polygons



- Polygons are represented by a list of points in the order representing the edges.
- To calculate the area
  - We pick one starting point.
  - Go through all the other adjacent pair of points and sum the area of the triangulation.

Polygons



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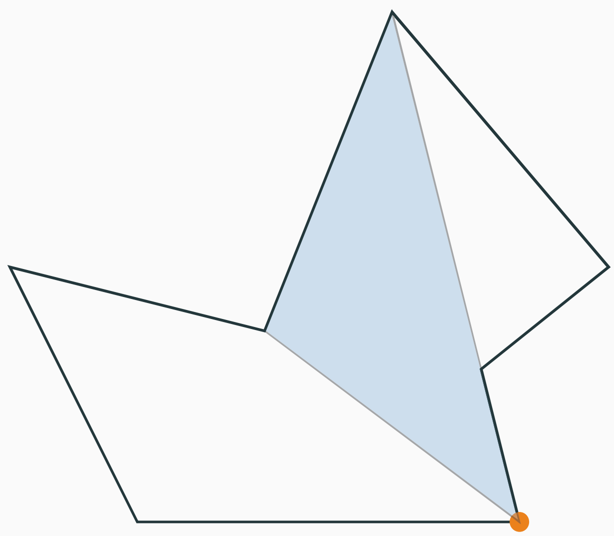

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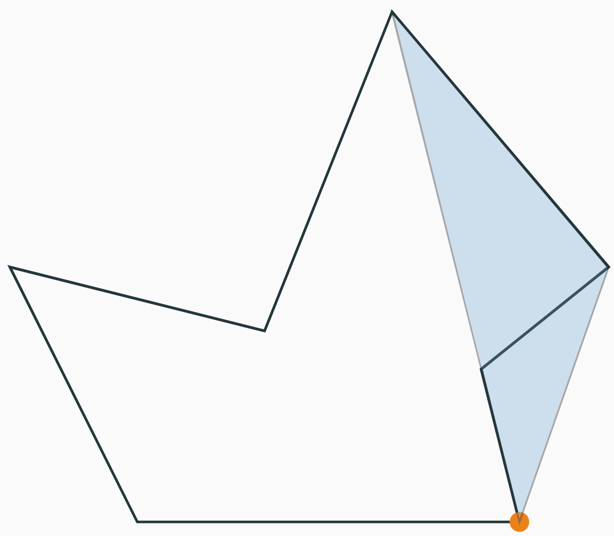

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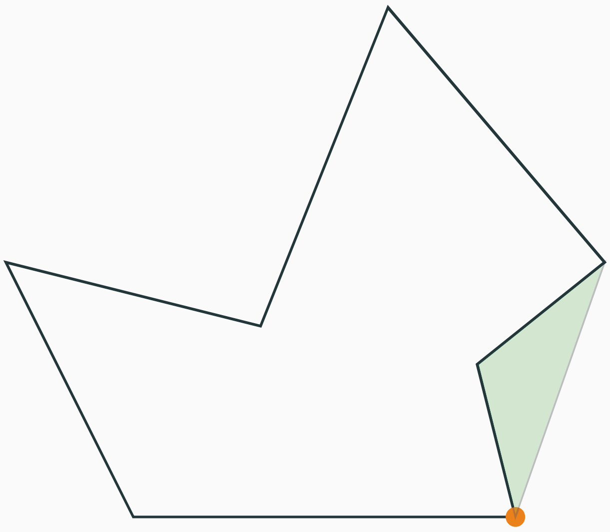

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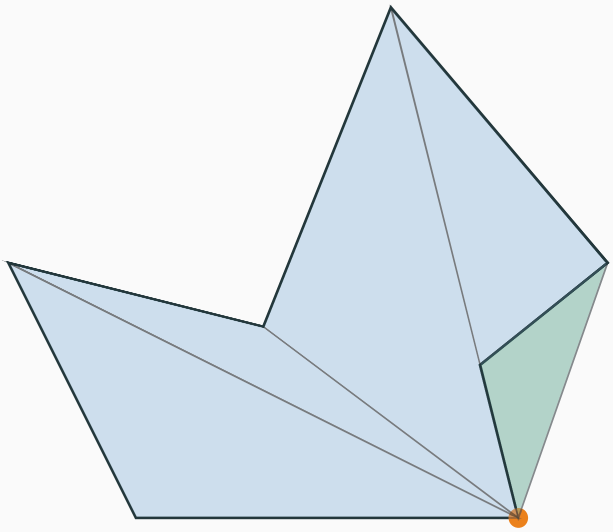

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Convex hull

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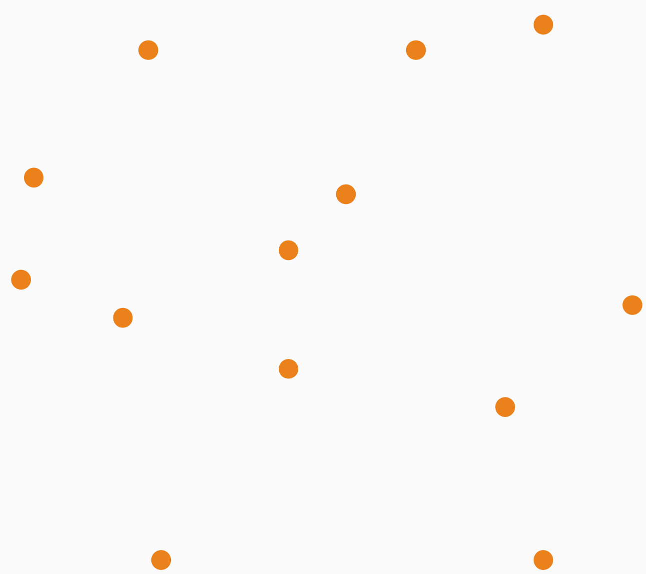

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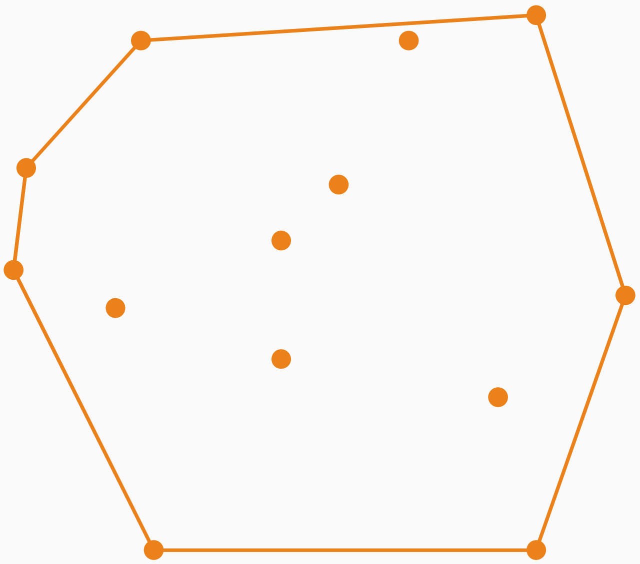

Convex hull

Graham scan:

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Time complexity $O(N \log N)$.

Convex hull

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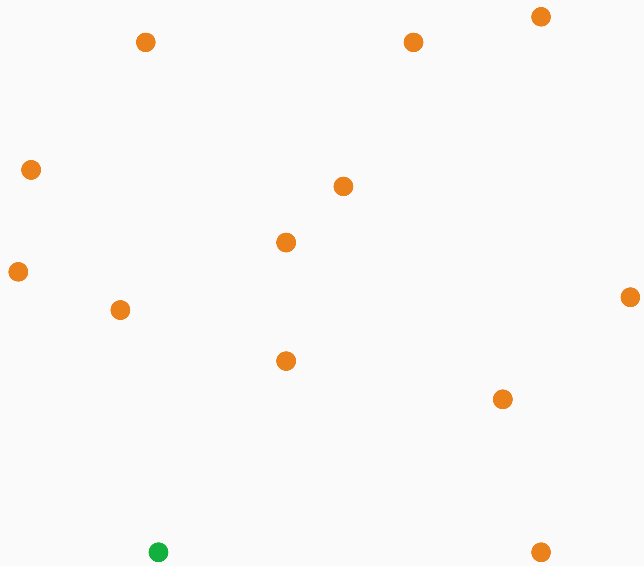

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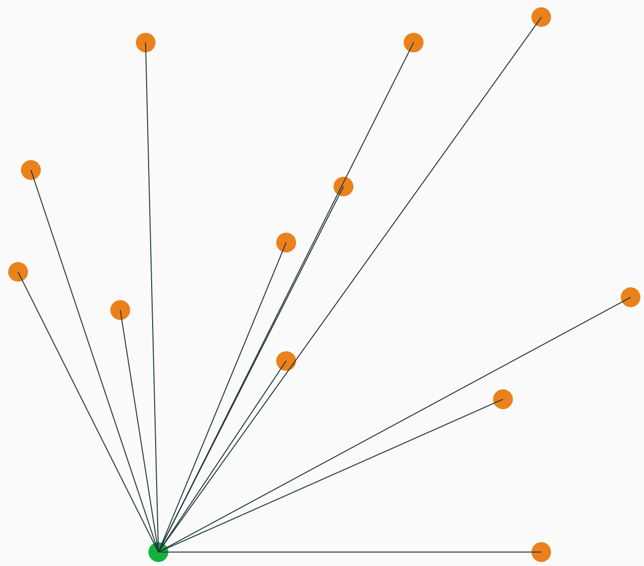

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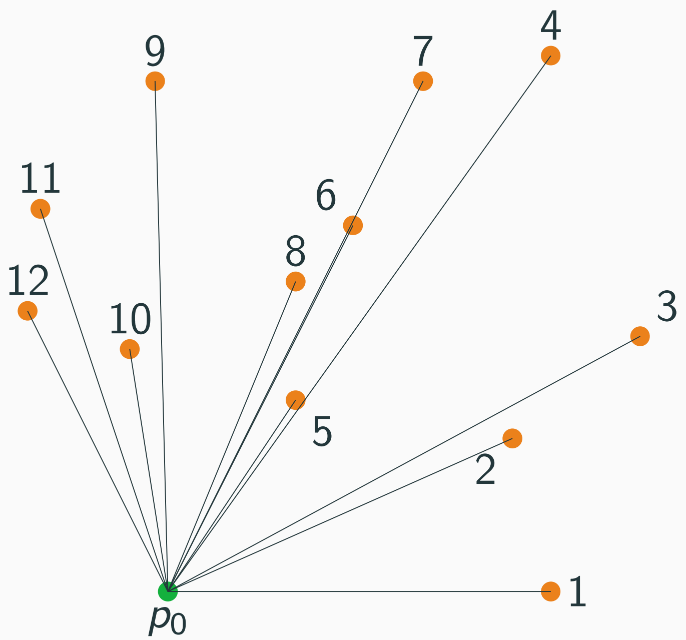

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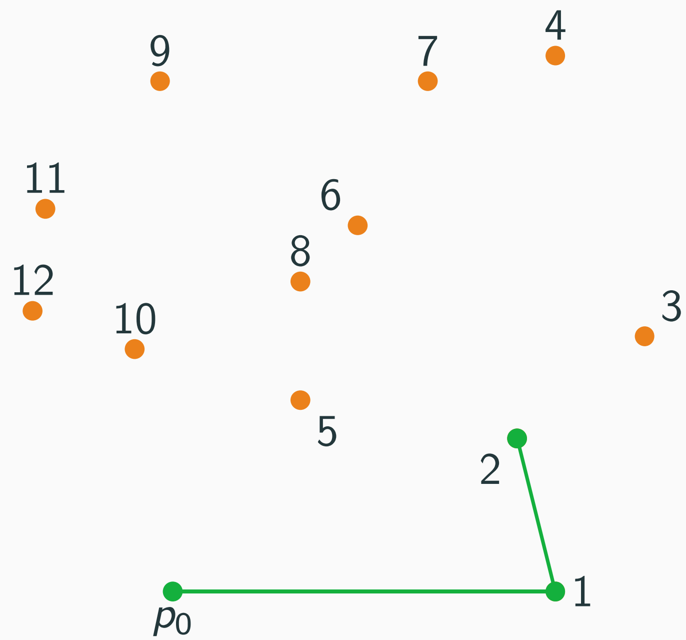

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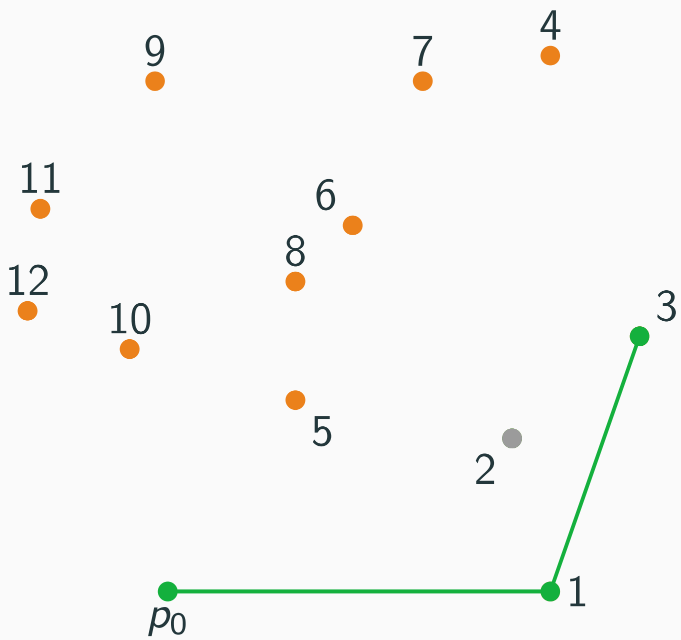

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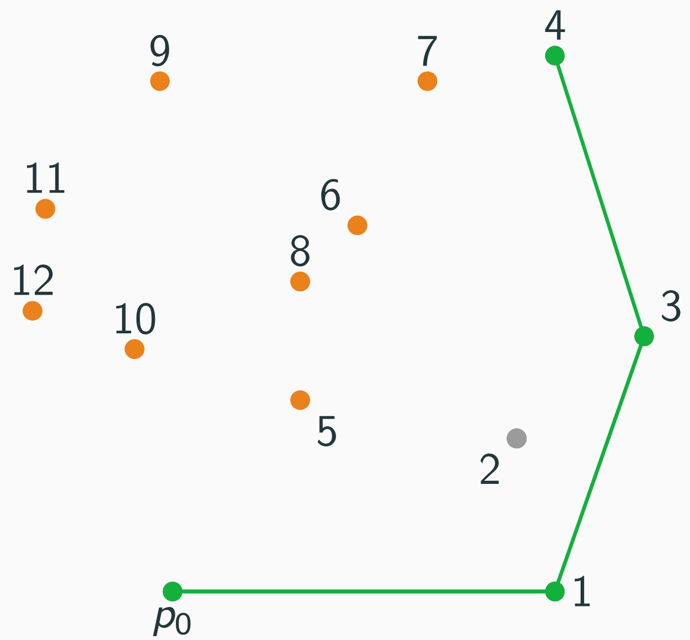

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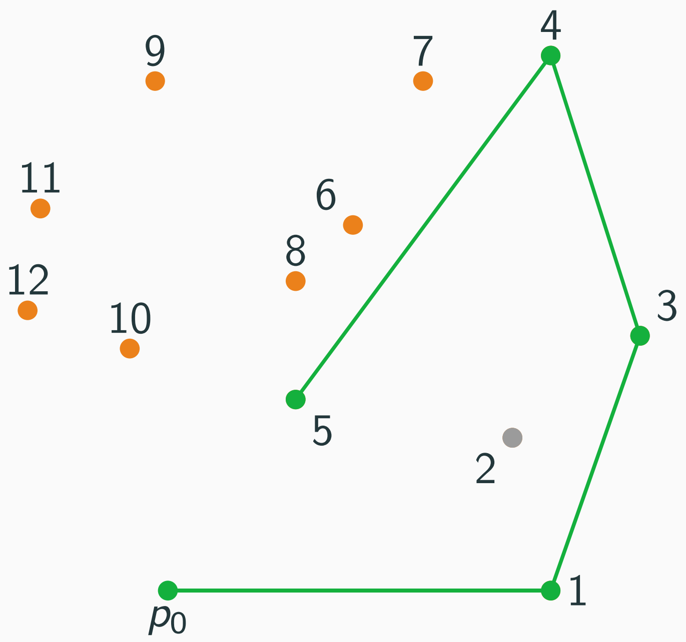

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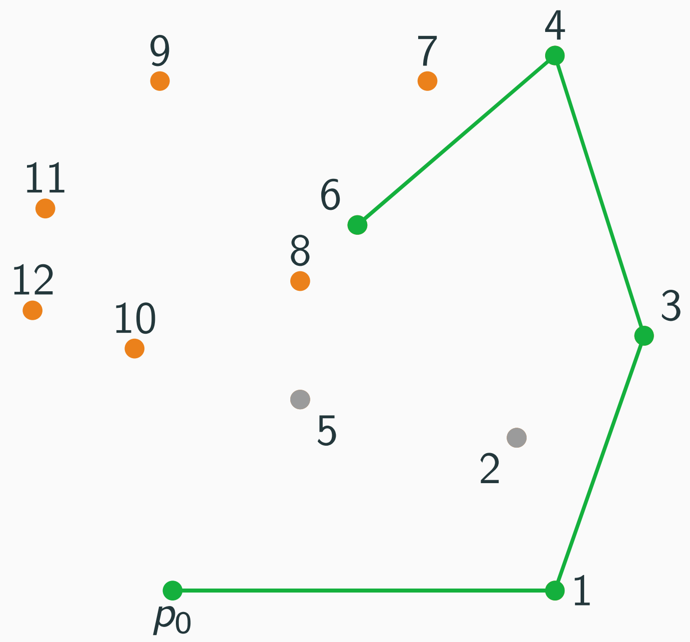

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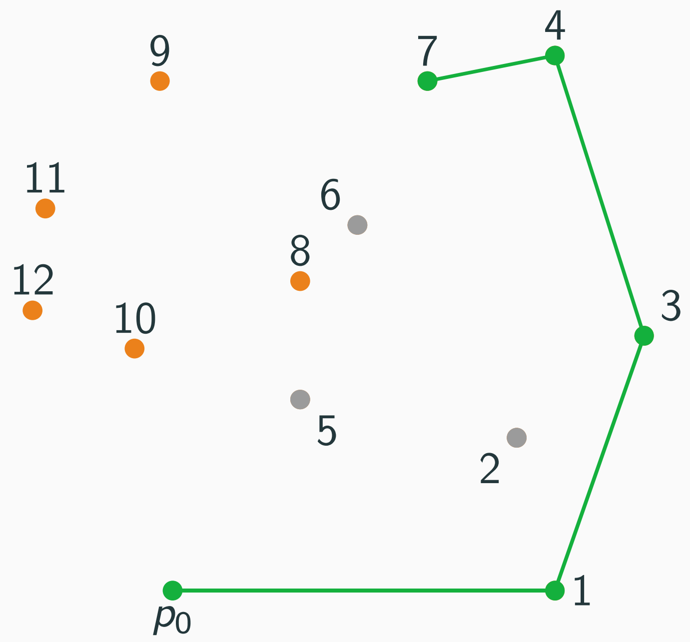

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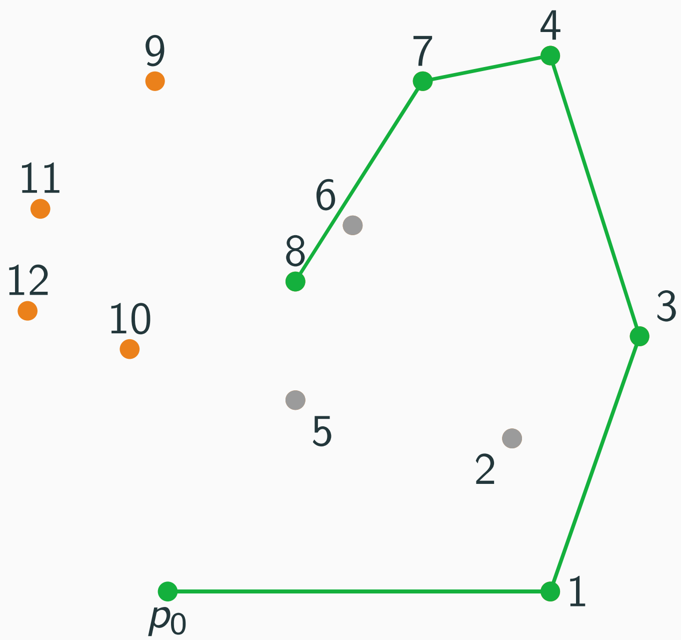

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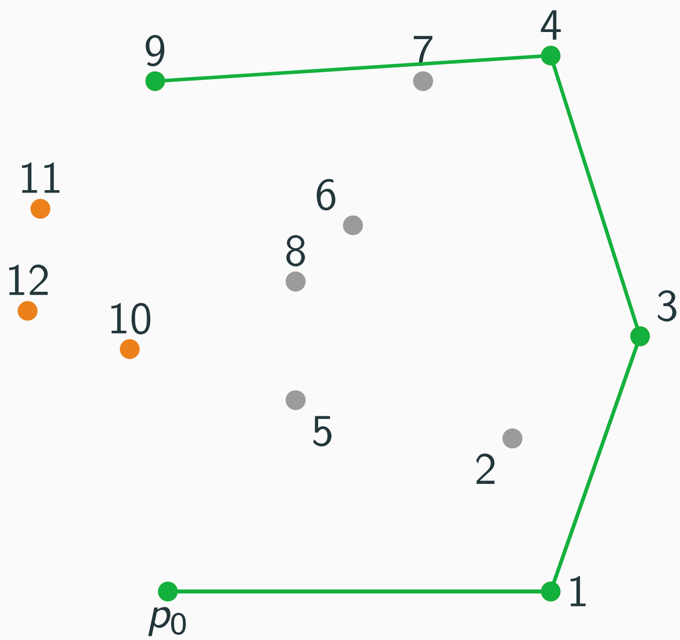

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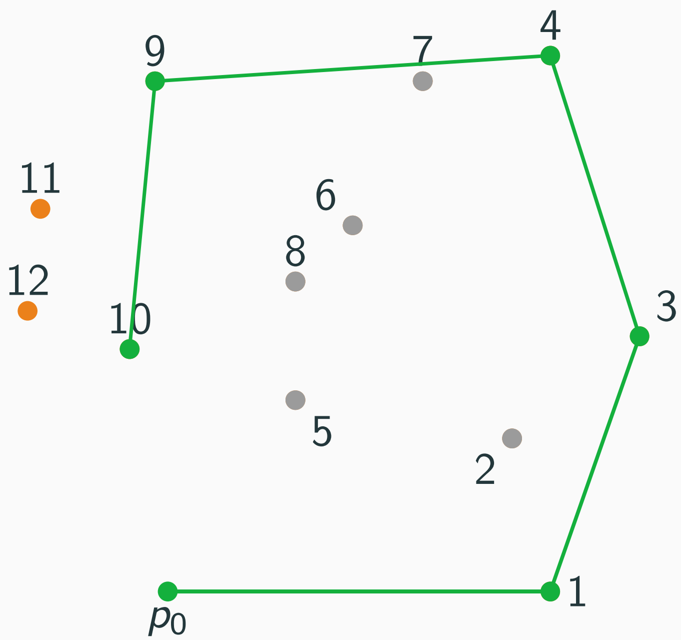

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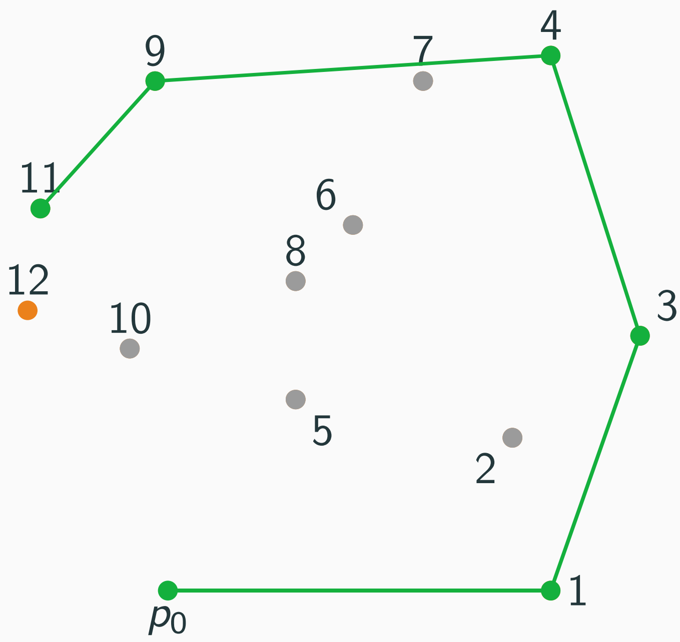

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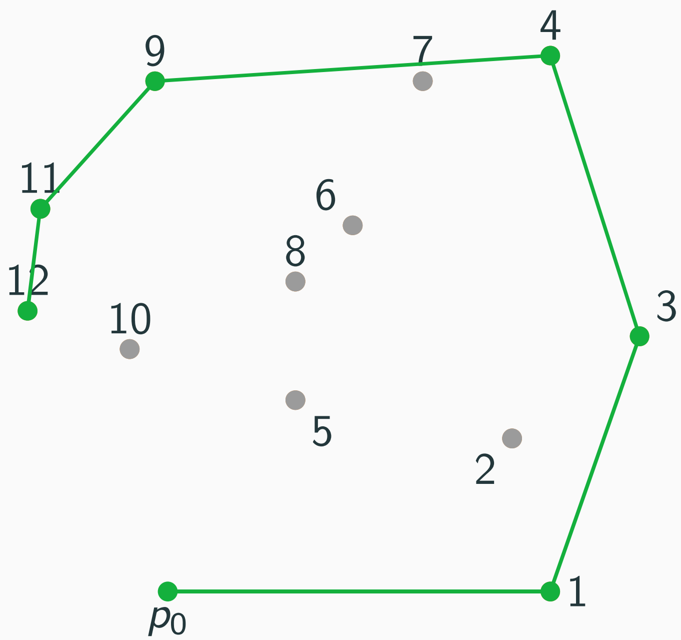

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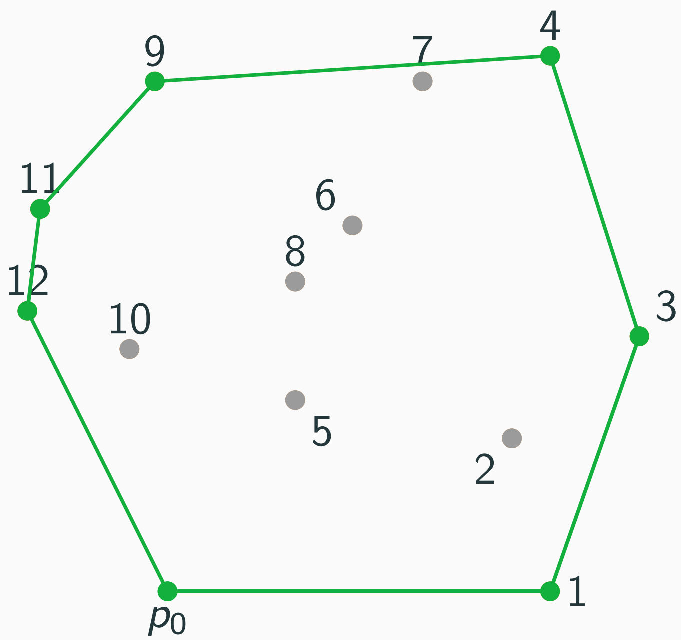

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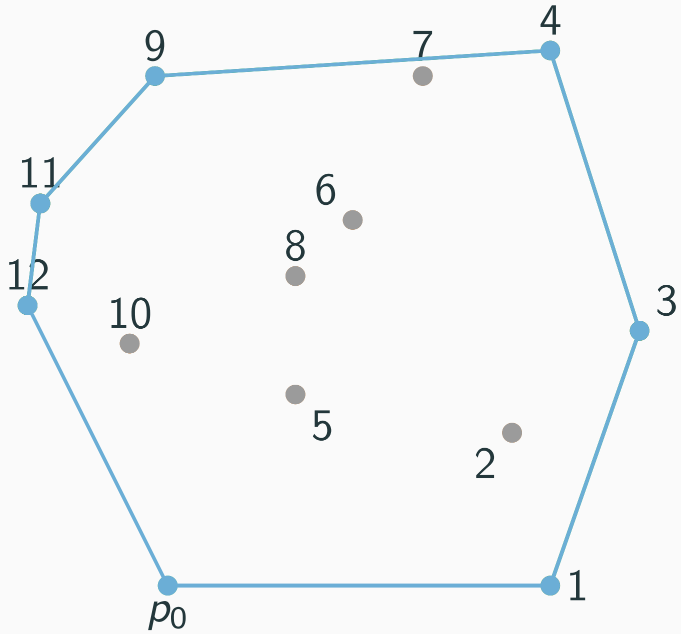

Convex hull

```
point hull[MAXN];
int convex_hull(vector<point> p) {
    int n = size(p), l = 0;
    sort(p.begin(), p.end(), cmp);
    for (int i = 0; i < n; i++) {
        if (i > 0 && p[i] == p[i - 1])
            continue;
        while (l >= 2 && ccw(hull[l - 2], hull[l - 1], p[i]) >= 0)
            l--;
        hull[l++] = p[i];
    }
    int r = l;
    for (int i = n - 2; i >= 0; i--) {
        if (p[i] == p[i + 1])
            continue;
        while (r - l >= 1 && ccw(hull[r - 2], hull[r - 1], p[i]) >= 0)
            r--;
        hull[r++] = p[i];
    }
    return l == 1 ? 1 : r - 1;
}
```

Convex hull

Many other algorithms exist

Convex hull

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- Gift wrapping aka Jarvis march.

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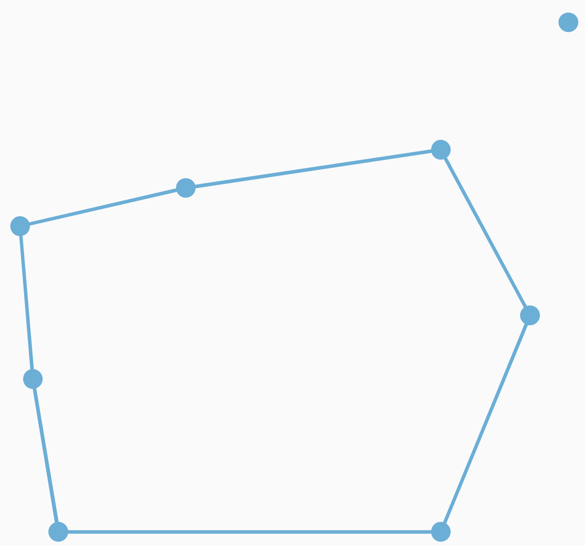
Some can be extended to three dimensions, or higher.

Point in convex polygon

Simple algorithm to check if a point is in a convex polygon.



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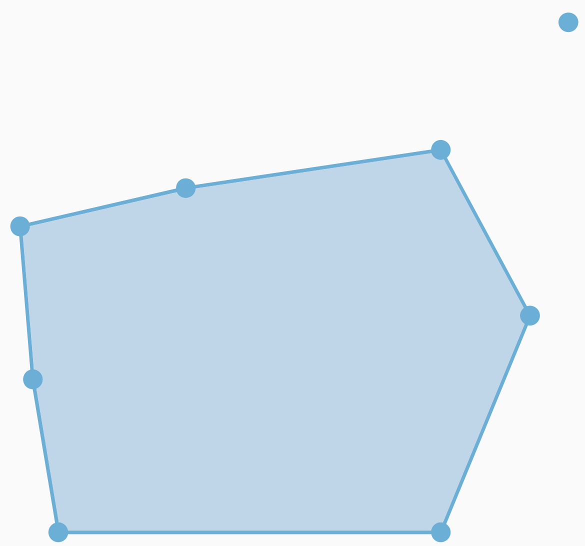

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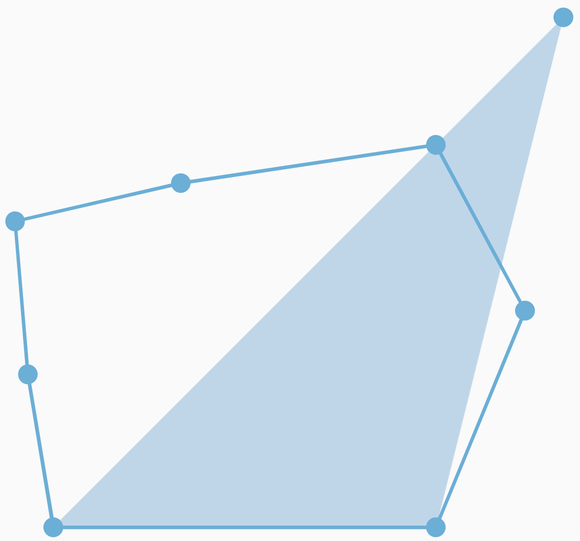

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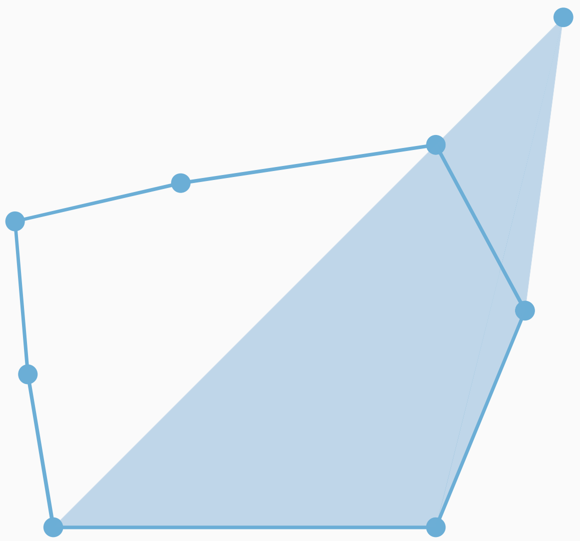

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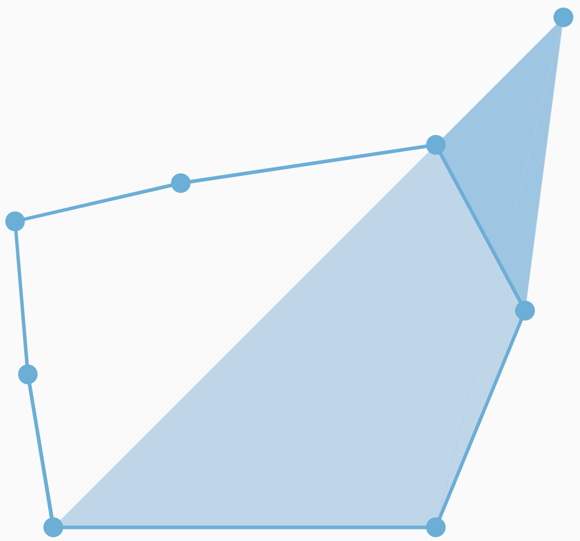

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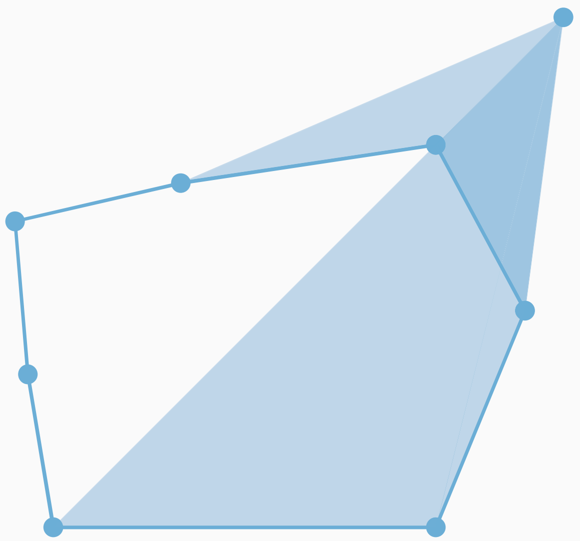

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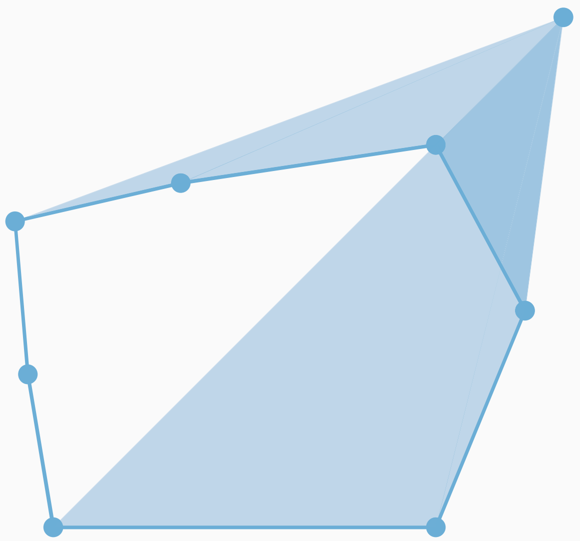

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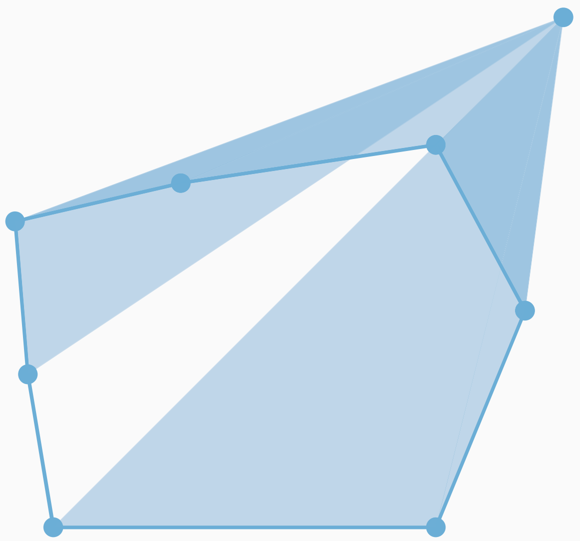

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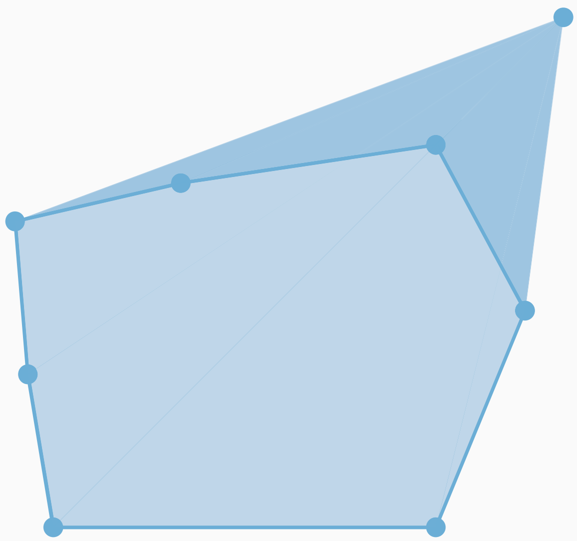

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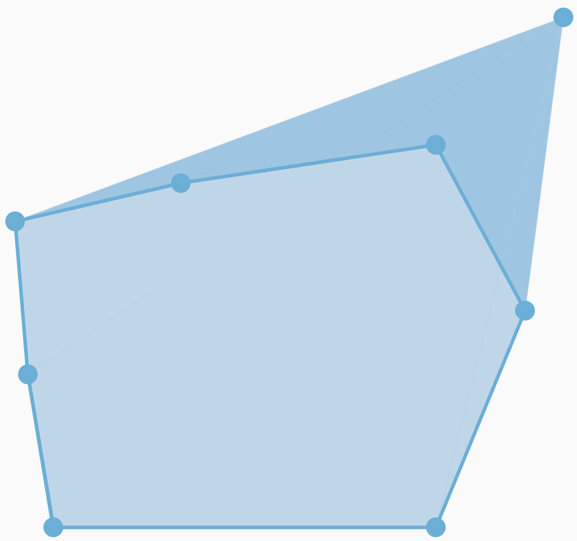

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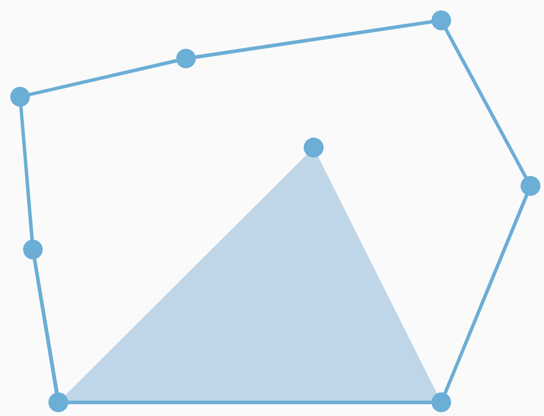

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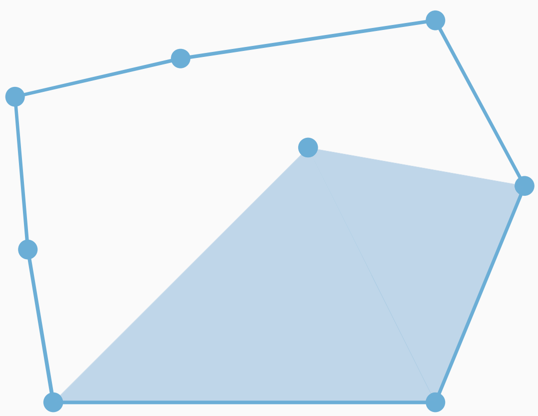

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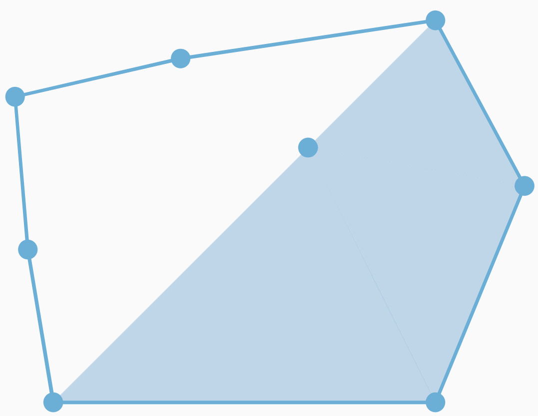

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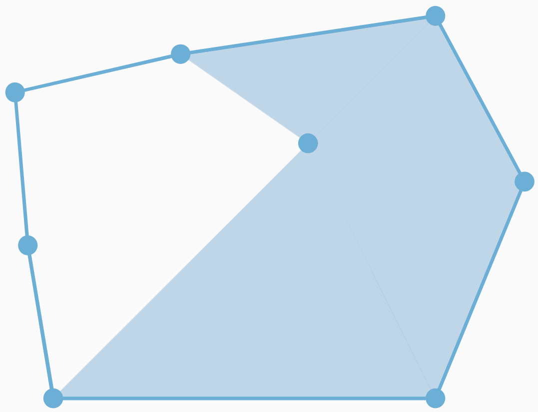

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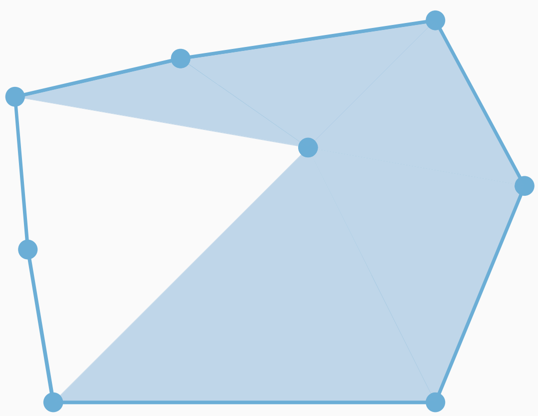

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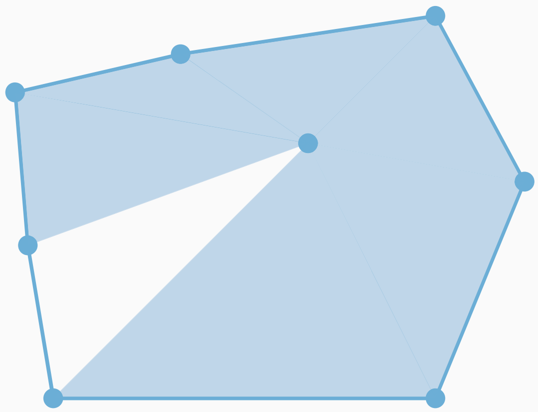

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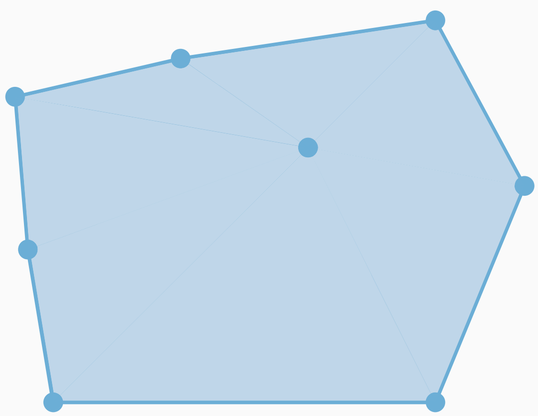

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Point in concave polygon

How about non convex polygon?

Point in concave polygon

How about non convex polygon?

- The *even-odd rule* algorithm.

Point in concave polygon

How about non convex polygon?

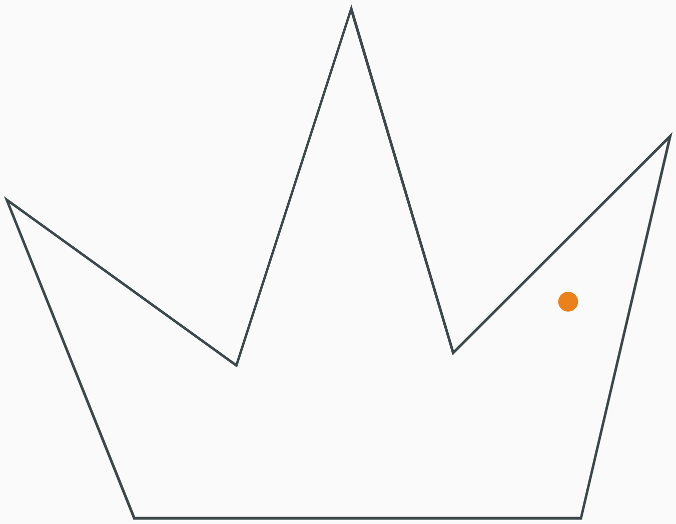
- The *even-odd rule* algorithm.
- We examine a ray passing through the polygon to the point.

Point in concave polygon

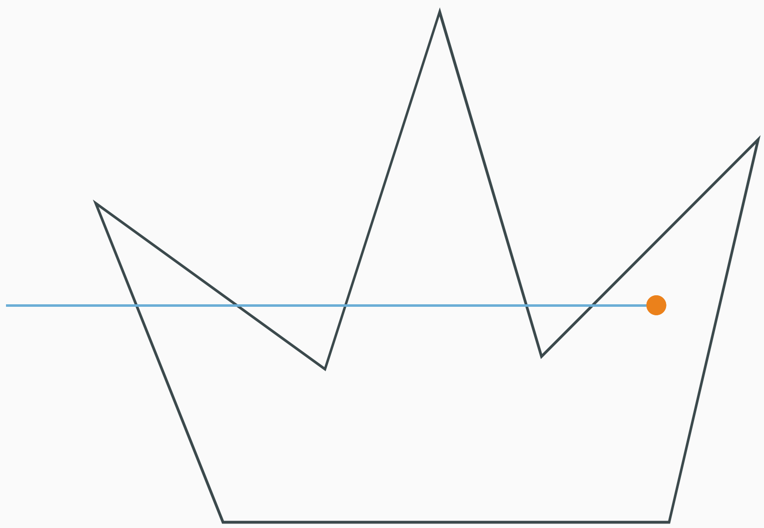
How about non convex polygon?

- The *even-odd rule* algorithm.
- We examine a ray passing through the polygon to the point.
- If the ray crosses the boundary of the polygon, then it alternately goes from outside to inside, and outside to inside.

Point in concave polygon

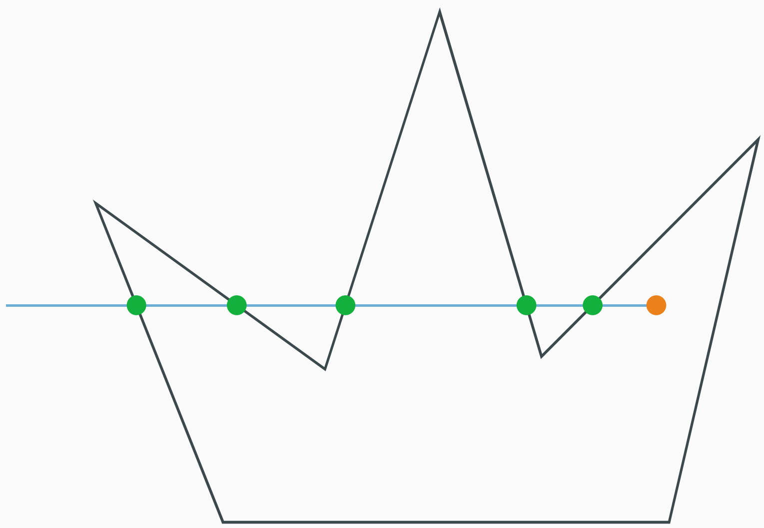


Point in concave polygon



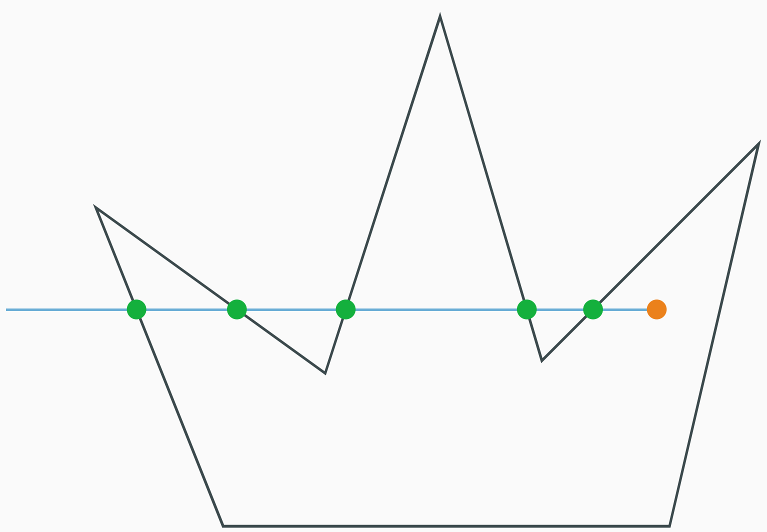
- Ray from the outside of the polygon to the point.

Point in concave polygon



- Ray from the outside of the polygon to the point.
- Count the number of intersection points.

Point in concave polygon



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- Count the number of intersection points.
- If odd, then the point is inside the polygon.
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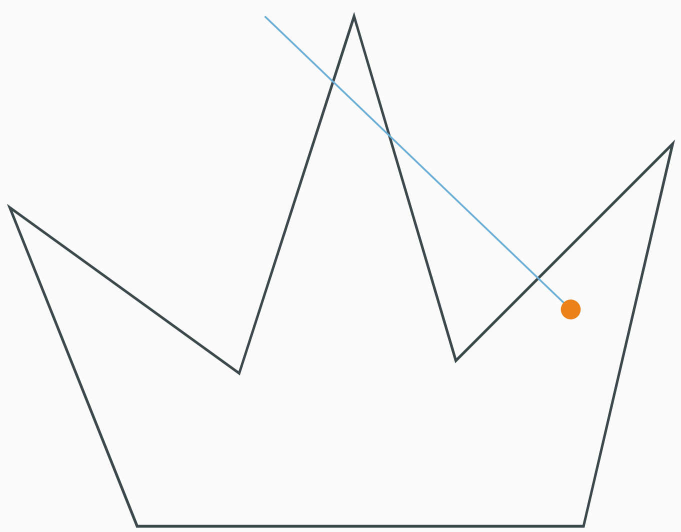
Point in concave polygon



- Ray from the outside of the polygon to the point.
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- If odd, then the point is inside the polygon.
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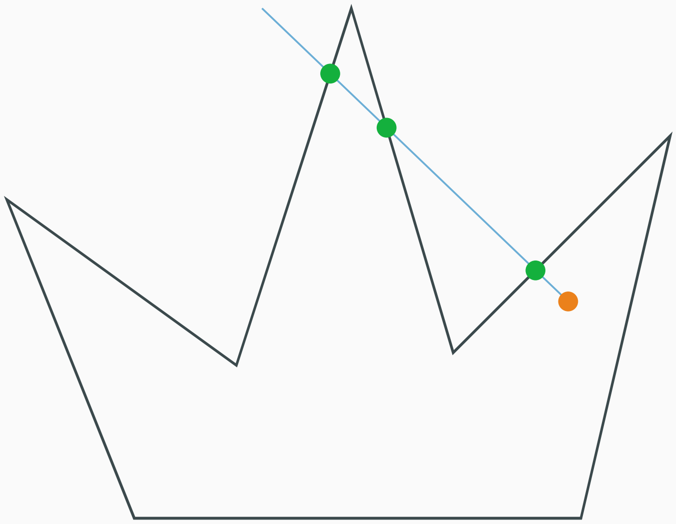
- Does not matter which ray we pick.

Point in concave polygon



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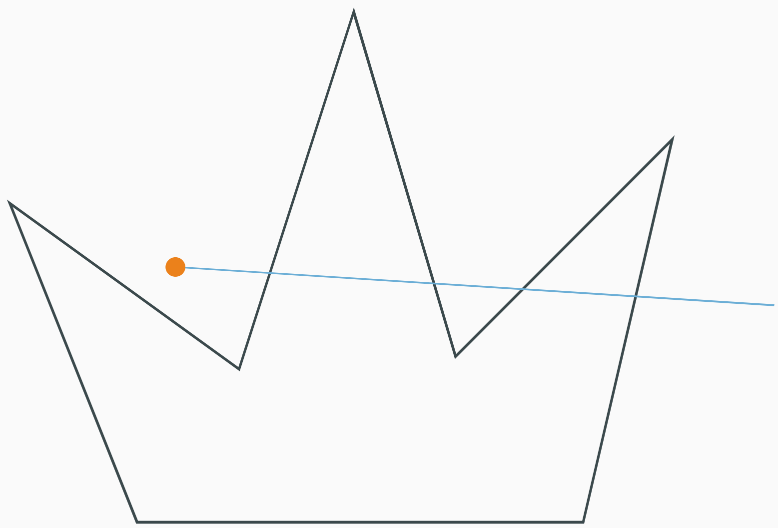

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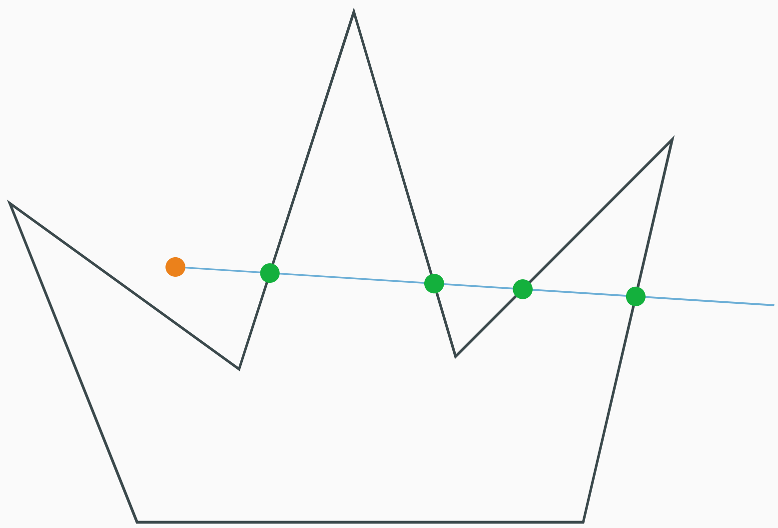

- Ray from the outside of the polygon to the point.
- Count the number of intersection points.
- If odd, then the point is inside the polygon.
- If even, then the point is outside the polygon.
- Does not matter which ray we pick.

Closest pair of points

Given a set of points, we want to find the pair of points with the smallest distance between them.

Divide and conquer algorithm;