

1. Técnicas de programación

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April 8, 2023

Efficiency

Recursive Complete Search

Recursive complete search

Efficiency

Time Complexity

- How much time does the program will take to run for a given input?
- How many operations does de program make to run for a given input?
- **(big O notation): $O(\dots)$**

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- $a > 1 \rightarrow \log_a(n) \ll n^\alpha \ll a^n$

Time Complexity

$$O(1) \ll O(\log_2(n)) \ll O(\sqrt{n}) \ll O(n) \ll O(n \cdot \log_2(n)) \\ \ll O(n^2) \ll O(n^2 \cdot \log_2(n)) \ll O(n^3) \ll O(2^n) \ll O(n!)$$

Time Complexity

| Input size | Expected time complexity |
|---------------|--------------------------|
| $n \leq 10$ | $O(n!)$ |
| $n \leq 20$ | $O(2^n)$ |
| $n \leq 500$ | $O(n^3)$ |
| $n \leq 5000$ | $O(n^2)$ |
| $n \leq 10^6$ | $O(n \log n)$ or $O(n)$ |
| n is large | $O(1)$ or $O(\log n)$ |

Recursive Complete Search

- Kattis - Closest Sums
- Each query gives you a number and asks to find the closest sum of 2 elements of a vector.
- $O(n^2 \cdot m)$

Complete Search

- We already saw complete search using iteratives(for, while)
- Sometimes is more efficient solve a problem recursively

Recursive Complete Search

Some examples:

- Generate all subsets of n items
- Generate all permutations of n items
- Generate all ways to put n queens on a $n \times n$ chessboard with no queen attacking each other.

- Uva 12455 - Bars
- Given a list I containing $1 \leq n \leq 20$ integers, is there a subset of list I that sums to another given integer X
- We can try all 2^n possible subsets of integers, sum the selected integers for each subset in $O(n)$, and see if the sum of the selected integers equals to X
- Overall time complexity is thus $O(n \cdot 2^n)$
- When $n = 20$, this is just $20 \times 2^{20} \sim 21M$

Recursive complete search

Simple Backtracking

- UVa 750 - 8 Queens Chess Problem
- In chess (with an 8×8 board), it is possible to place eight queens on the board such that no two queens attack each other.
- Determine all such possible arrangements given the position of one of the queen

Gracias