

Sesión 13.1: Dimension Reduction Methods

CS3102 EDA

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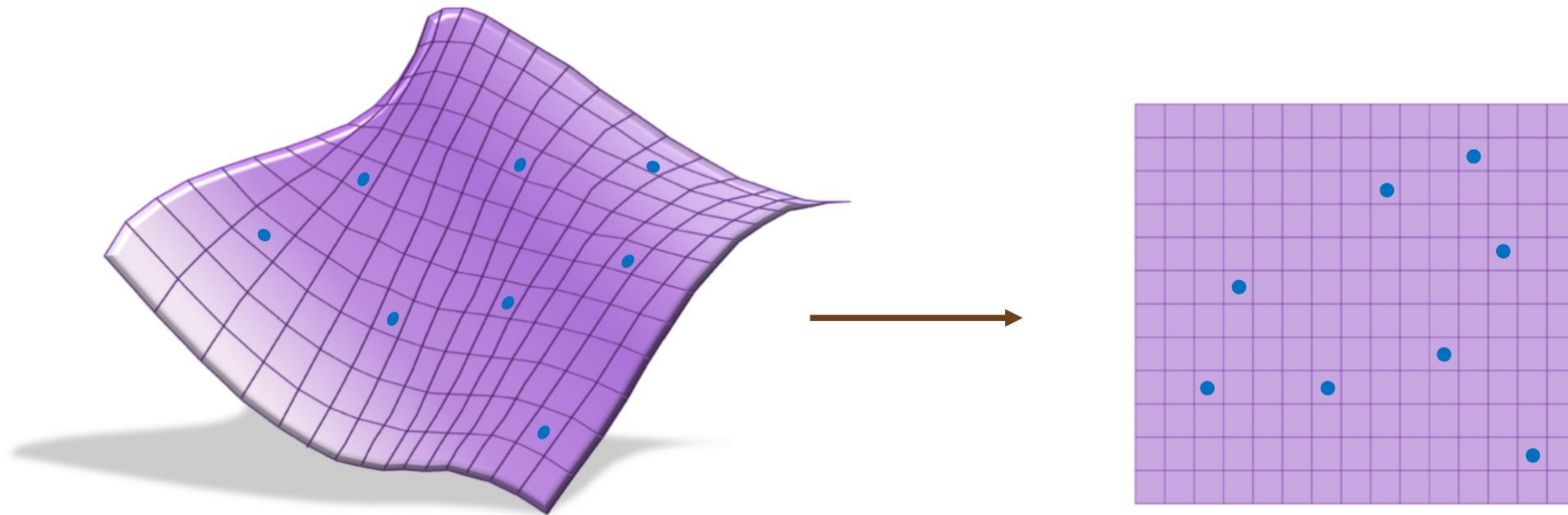


1 Dimension ● Reduction Methods

Dimension Reduction *Methods*

Original Space

Reduced Space



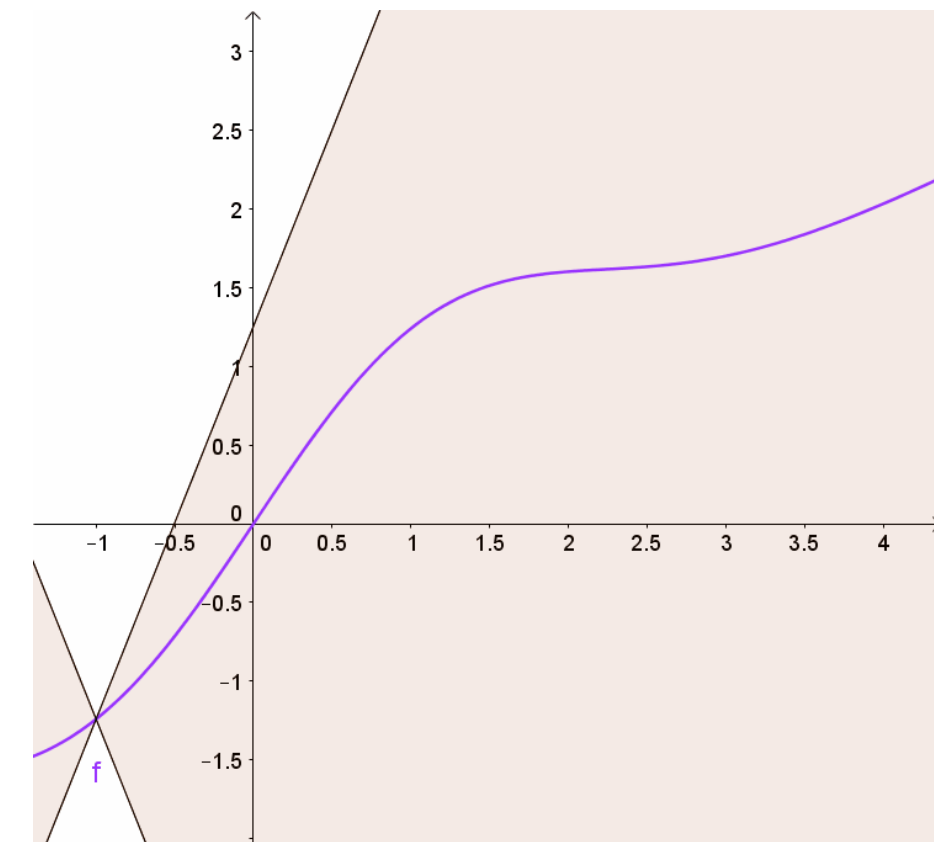
Lipschitz continuity

Sea la función $f: M \rightarrow N$ entre espacios métricos (M, d_M) y (N, d_N) se dice que es Lipschitz continua

$$d_N(f(x), f(y)) \leq k \cdot d_M(x, y) \quad \forall x, y \in M$$

Metric map: $k = 1$

Contraction mapping: $0 \leq k < 1$



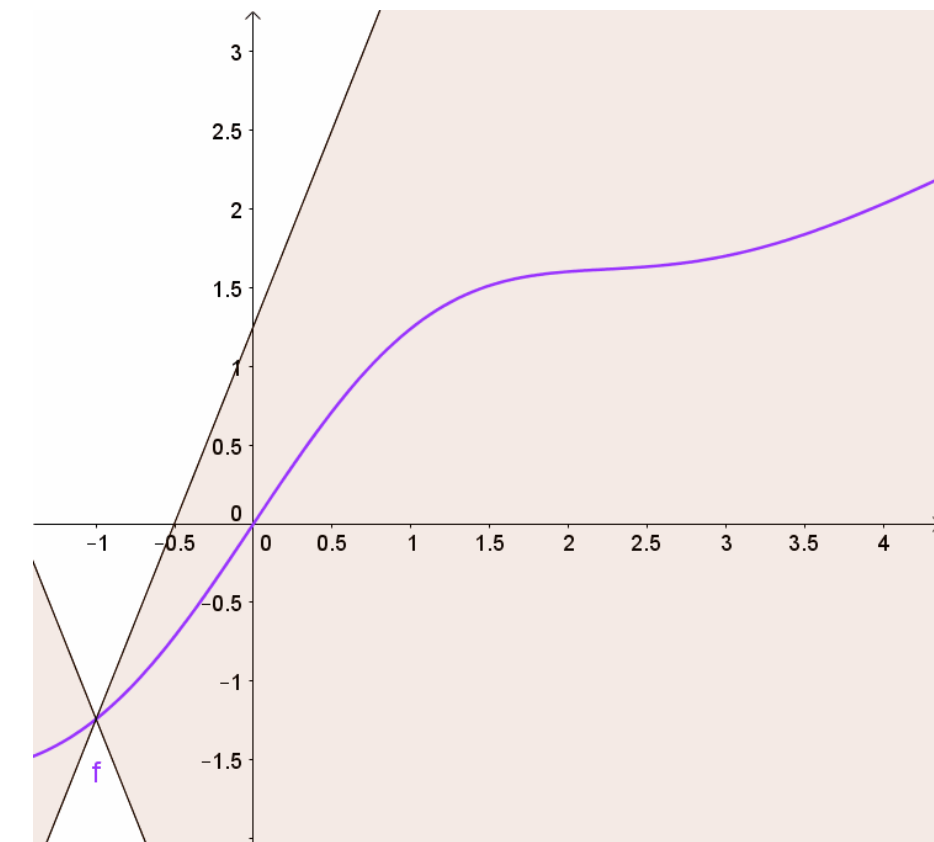
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Pruning Property:

$$d_N(f(x), f(y)) \leq d_M(x, y)$$



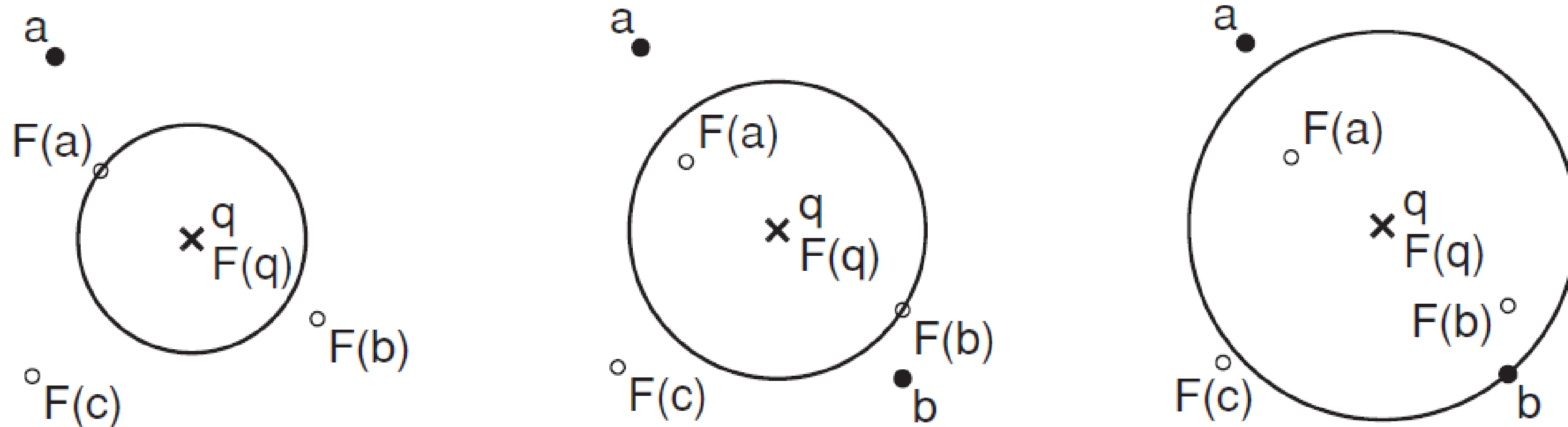
Proximity-Preserving *Property*

$$d_M(a, b) \leq d_M(a, c)$$

$$d_N(f(a), f(b)) \leq d_N(f(a), f(c))$$

Es muy difícil encontrar transformaciones que reduzcan dimensionalidad y cumplan esta propiedad

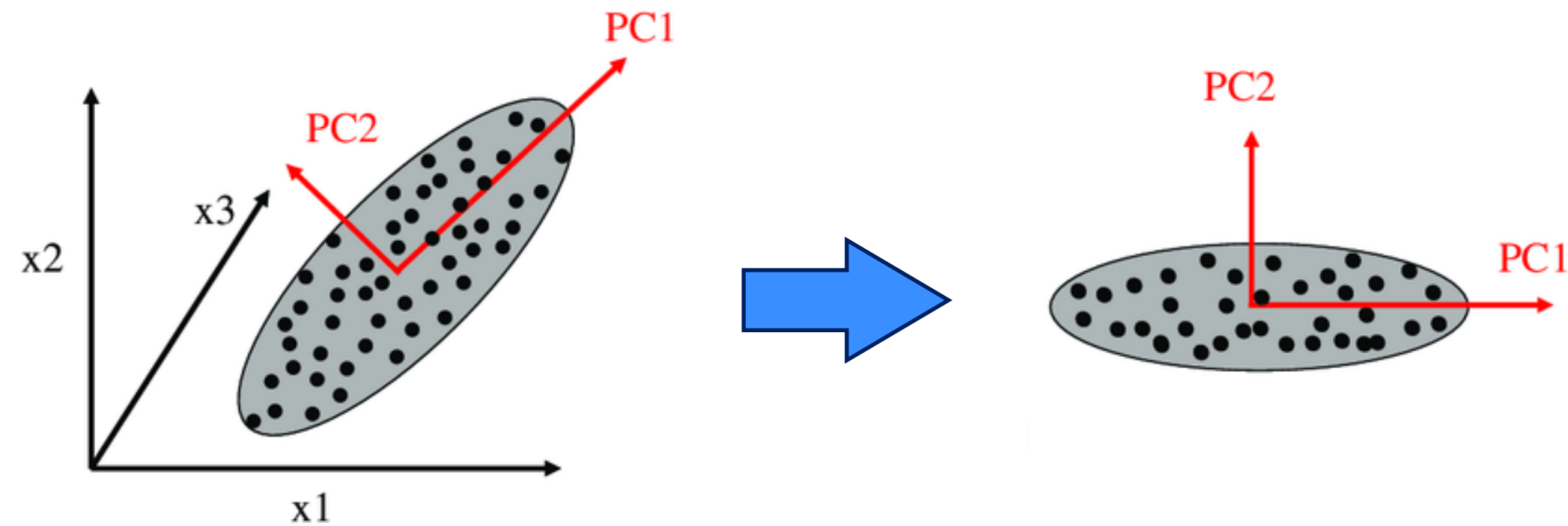
Incremental nearest *neighbor algorithm*



2. PCA

(Principal component analysis)

Principal component analysis



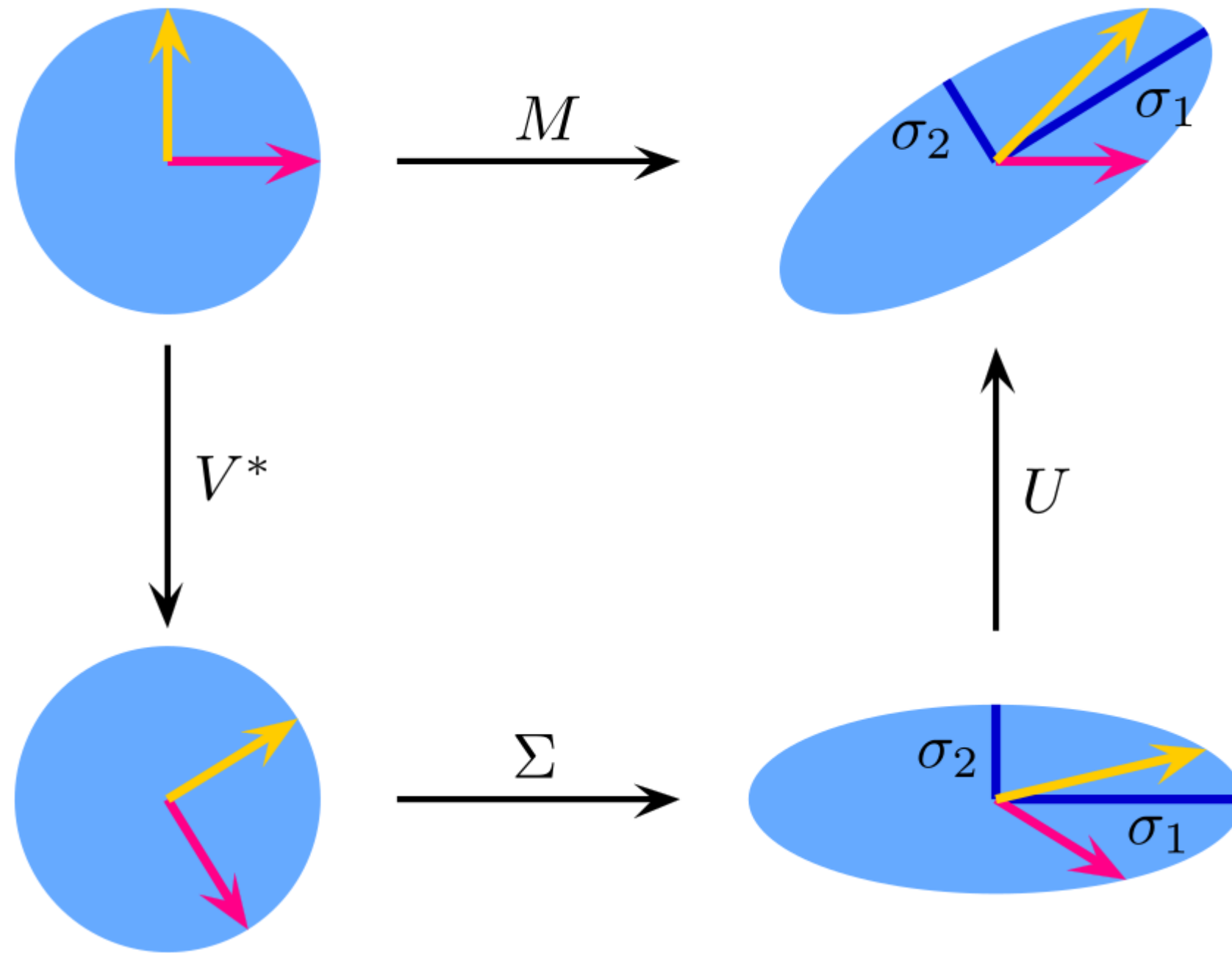
3. *SVG* (Singular value decomposition)

Singular value *decomposition*

$$\begin{array}{c}
 \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \end{array}
 &
 =
 &
 \begin{array}{|c|c|c|c|} \hline \text{teal} & \text{green} & \text{blue} & \text{green} \\ \hline \text{teal} & \text{green} & \text{blue} & \text{green} \\ \hline \text{teal} & \text{green} & \text{blue} & \text{green} \\ \hline \text{teal} & \text{green} & \text{blue} & \text{green} \\ \hline \end{array}
 &
 \begin{array}{|c|c|c|} \hline \text{orange} & 0 & 0 \\ \hline 0 & \text{yellow} & 0 \\ \hline 0 & 0 & \text{yellow} \\ \hline 0 & 0 & 0 \\ \hline \end{array}
 &
 \begin{array}{|c|c|c|} \hline \text{light blue} & \text{light blue} & \text{light blue} \\ \hline \text{purple} & \text{purple} & \text{purple} \\ \hline \text{pink} & \text{pink} & \text{pink} \\ \hline \end{array}
 \\
 \mathbf{M}_{m \times n} & = & \mathbf{U}_{m \times m} & \mathbf{\Sigma}_{m \times n} & \mathbf{V}^*_{n \times n}
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{|c|c|c|c|} \hline \text{teal} & \text{green} & \text{blue} & \text{green} \\ \hline \text{teal} & \text{green} & \text{blue} & \text{green} \\ \hline \text{teal} & \text{green} & \text{blue} & \text{green} \\ \hline \text{teal} & \text{green} & \text{blue} & \text{green} \\ \hline \end{array}
 &
 \begin{array}{|c|c|c|c|} \hline \text{teal} & \text{teal} & \text{teal} & \text{teal} \\ \hline \text{green} & \text{green} & \text{green} & \text{green} \\ \hline \text{blue} & \text{blue} & \text{blue} & \text{blue} \\ \hline \text{green} & \text{green} & \text{green} & \text{green} \\ \hline \end{array}
 &
 =
 &
 \begin{array}{|c|c|c|c|} \hline 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \\ \hline \end{array}
 \\
 \mathbf{U} & \mathbf{U}^* & = & \mathbf{I}_m
 \end{array}
 \qquad
 \begin{array}{c}
 \begin{array}{|c|c|c|} \hline \text{light blue} & \text{purple} & \text{pink} \\ \hline \text{light blue} & \text{purple} & \text{pink} \\ \hline \text{light blue} & \text{purple} & \text{pink} \\ \hline \end{array}
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 &
 =
 &
 \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ \hline \end{array}
 \\
 \mathbf{V} & \mathbf{V}^* & = & \mathbf{I}_n
 \end{array}$$

Singular value *decomposition*

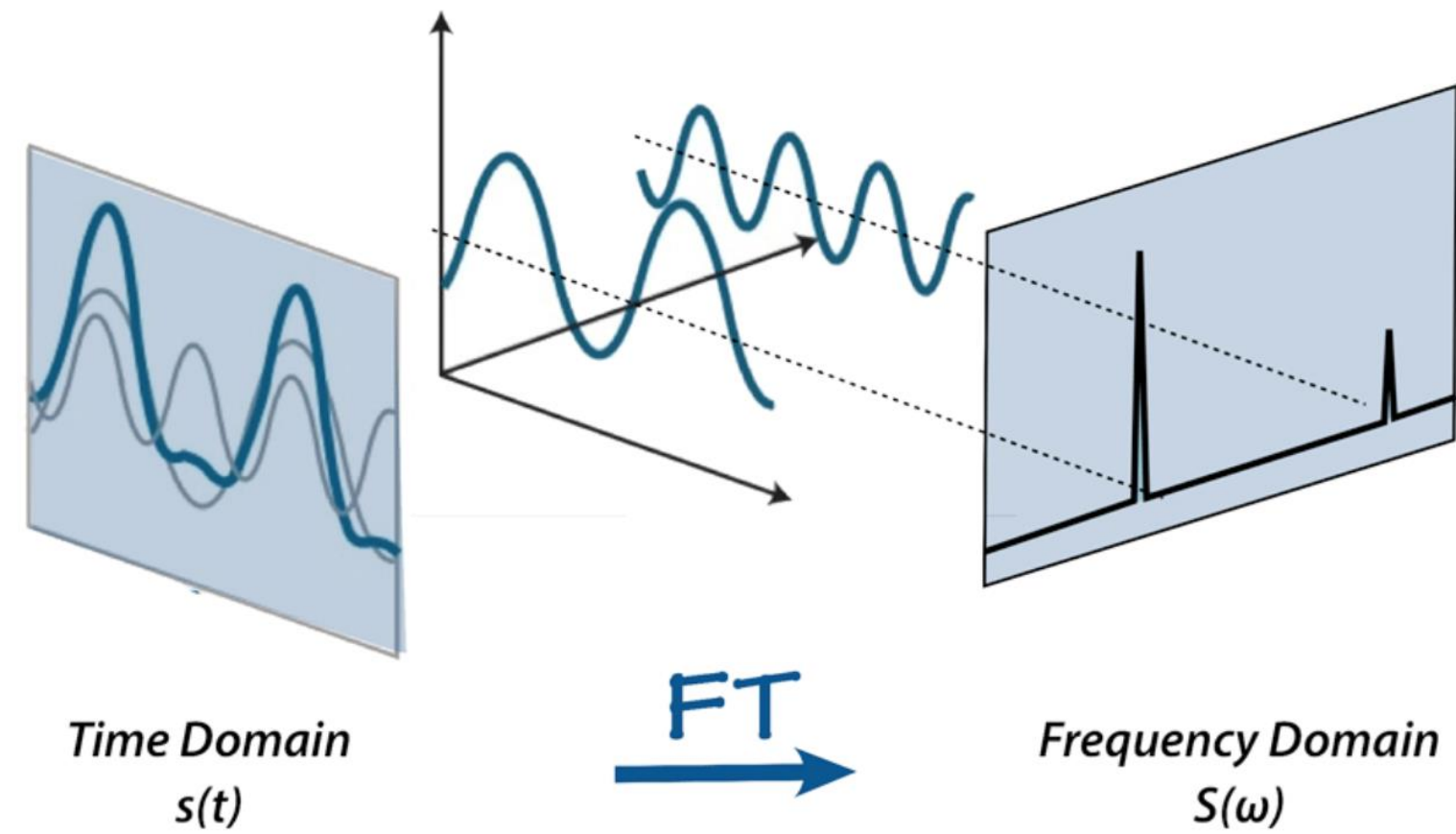


$$M = U \cdot \Sigma \cdot V^*$$

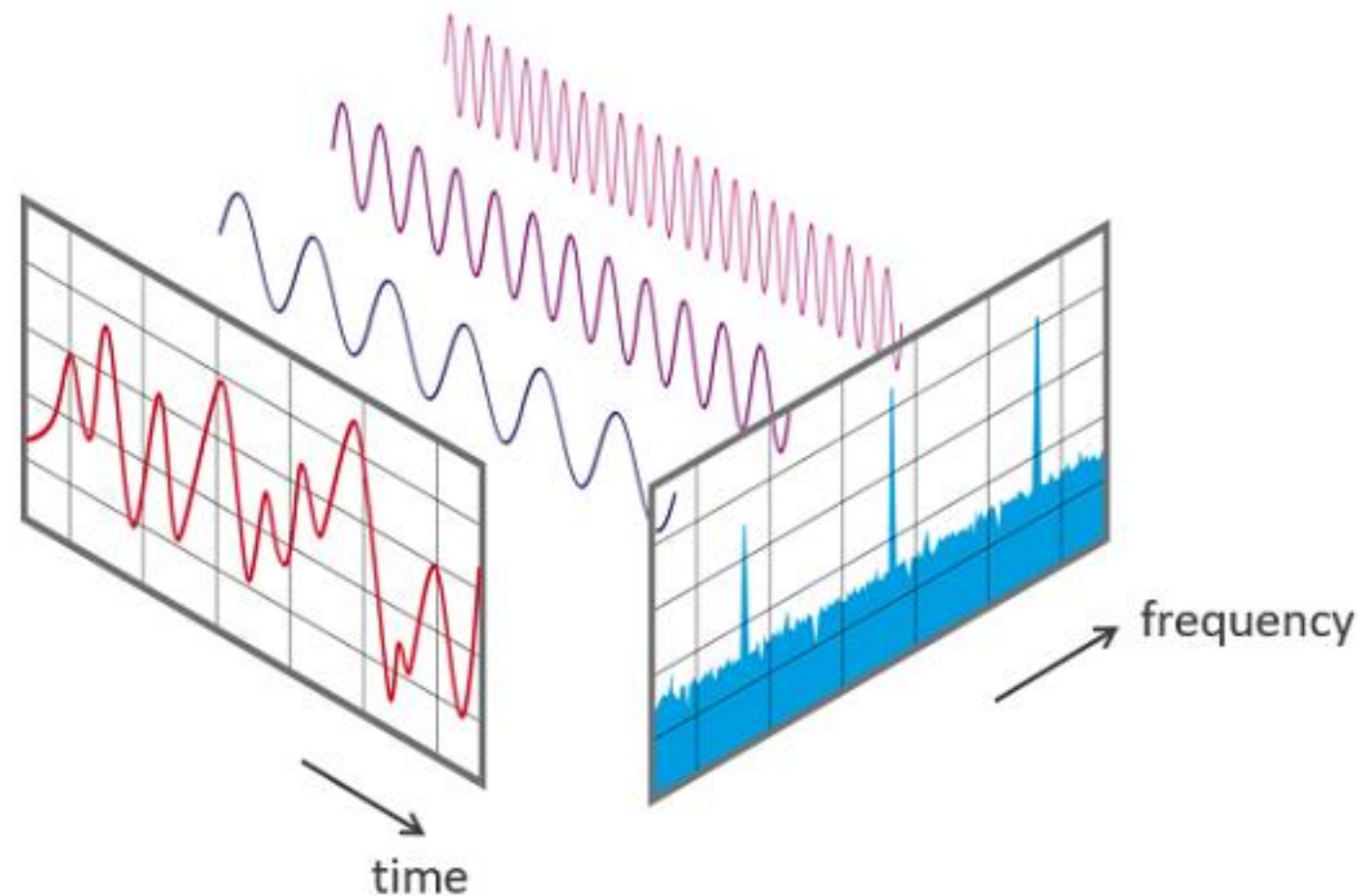
4 ● DFT

(Discrete Fourier Transform)

Dominio de *Frecuencia*



Dominio de *Frecuencia*



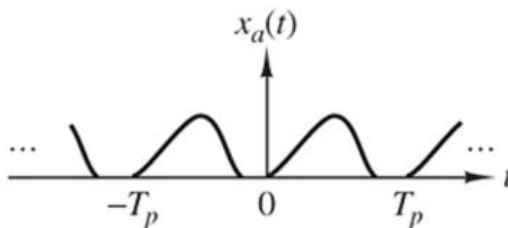
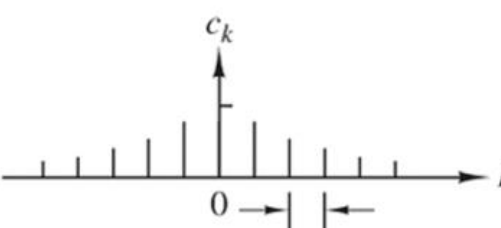
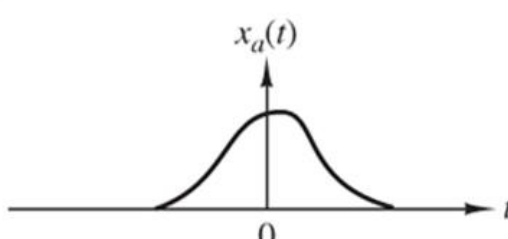
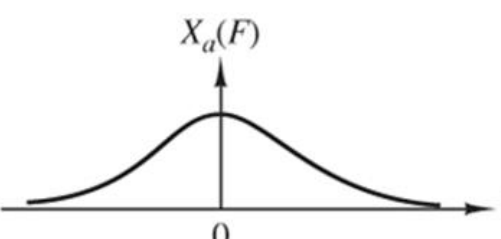
Serie de Fourier

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$$

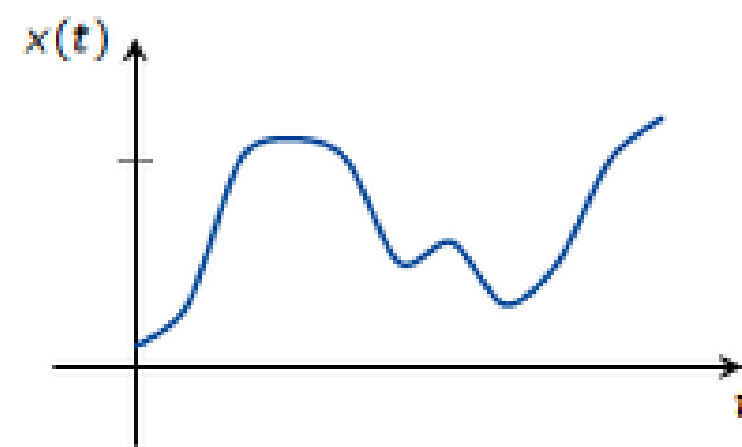
Transformada de Fourier

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i \omega t} dt$$

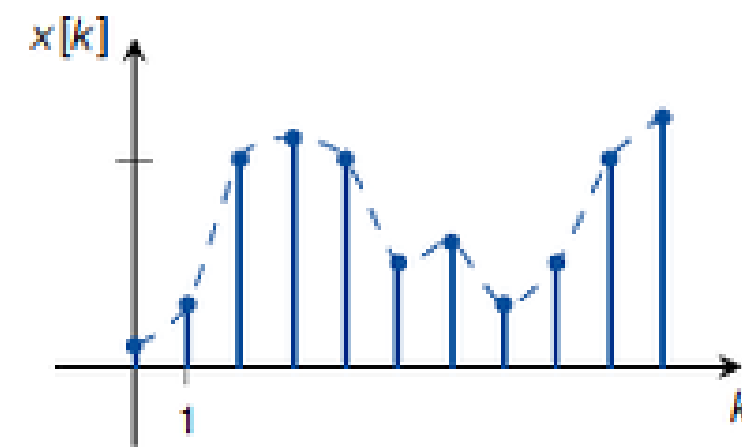
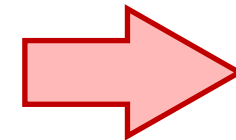
Fourier Transform

		Continuous-time signals	
		Time-domain	Frequency-domain
Periodic signals	Fourier series	 $c_k = \frac{1}{T_p} \int_{T_p} x_a(t) e^{-j2\pi k F_0 t} dt$	 $F_0 = \frac{1}{T_p}$ $x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0 t}$
		Continuous and periodic	Discrete and aperiodic
Aperiodic signals	Fourier transforms	 $X_a(F) = \int_{-\infty}^{\infty} x_a(t) e^{-j2\pi F t} dt$	 $x_a(t) = \int_{-\infty}^{\infty} X_a(F) e^{j2\pi F t} dF$
		Continuous and aperiodic	Continuous and aperiodic

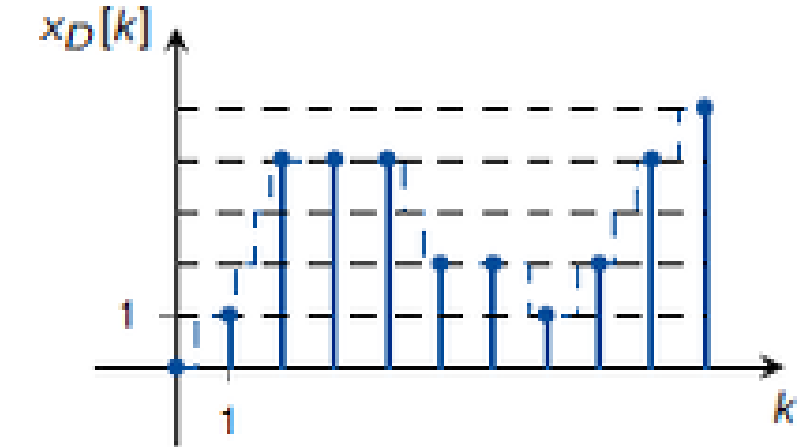
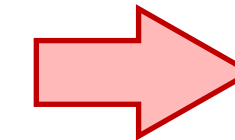
Digitalización



Señal

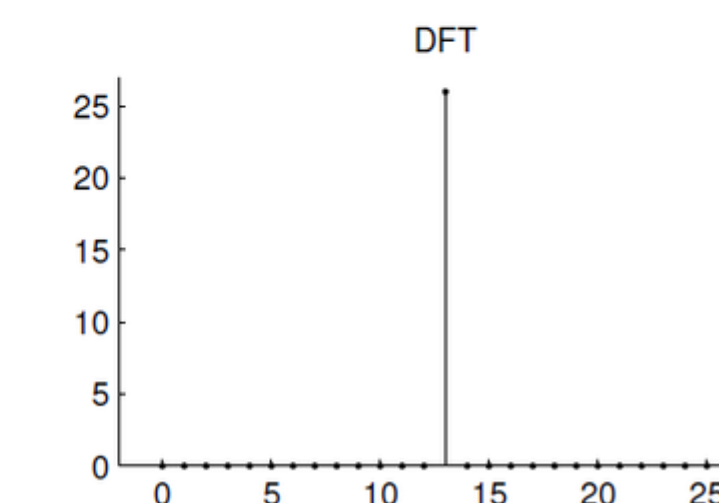
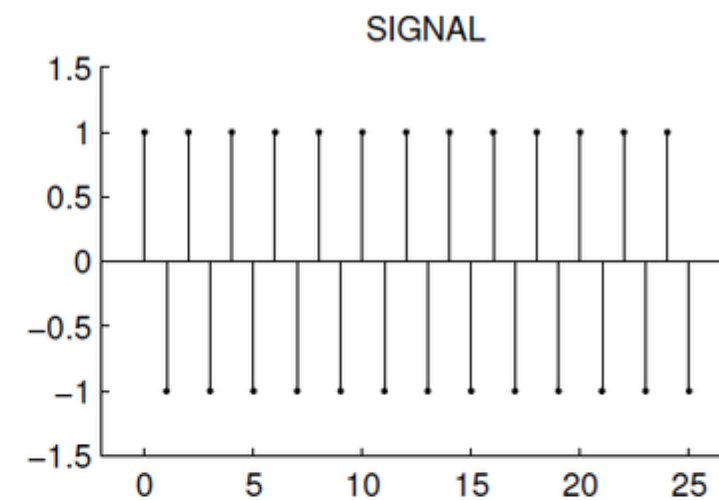
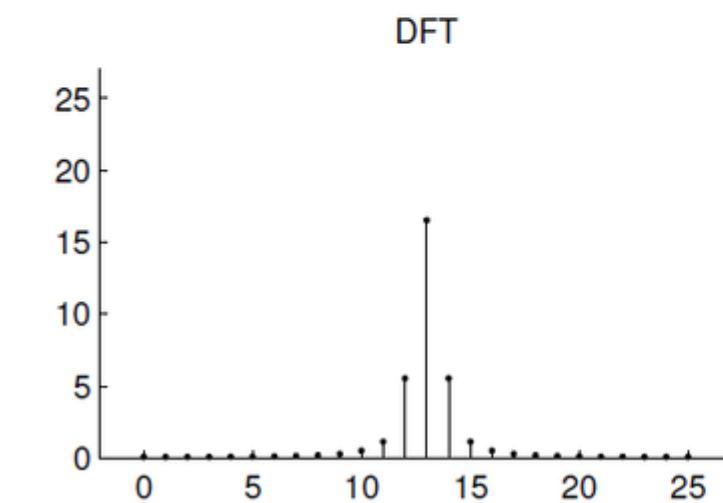
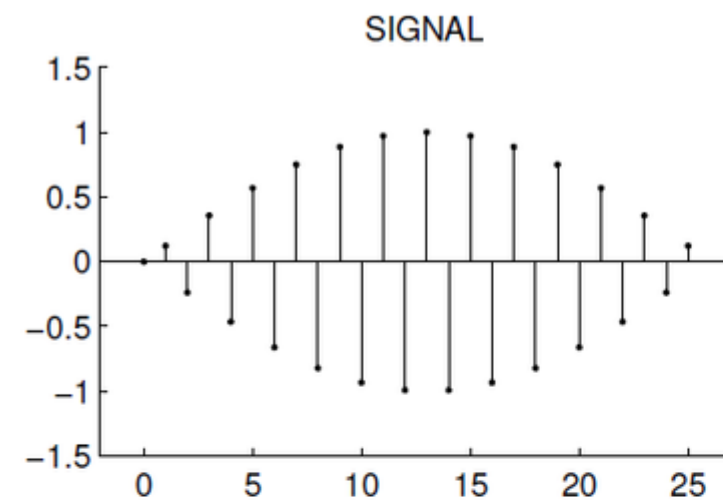


Muestreo



Cuantización

Discrete Fourier Transform

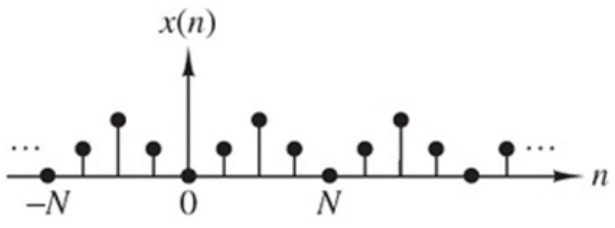
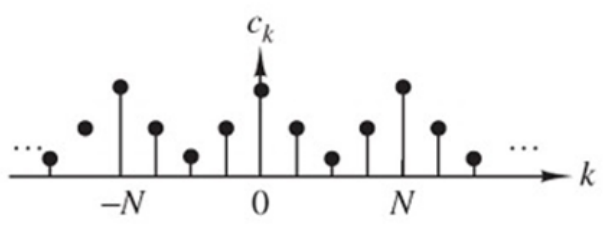
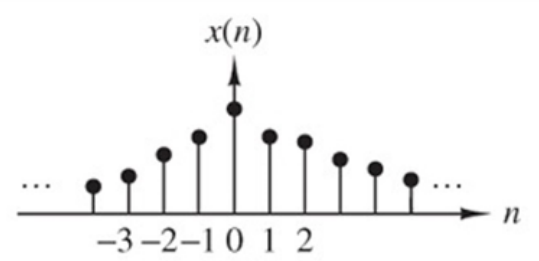
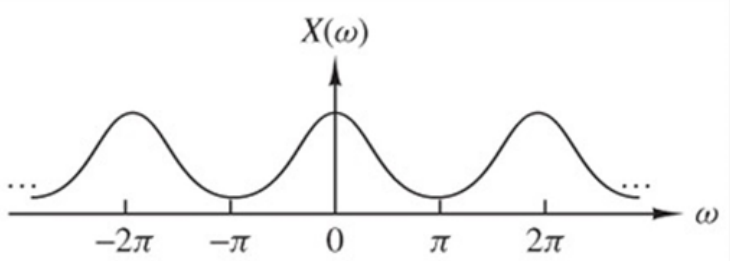


Transformada

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i \frac{2\pi}{N} kn}$$

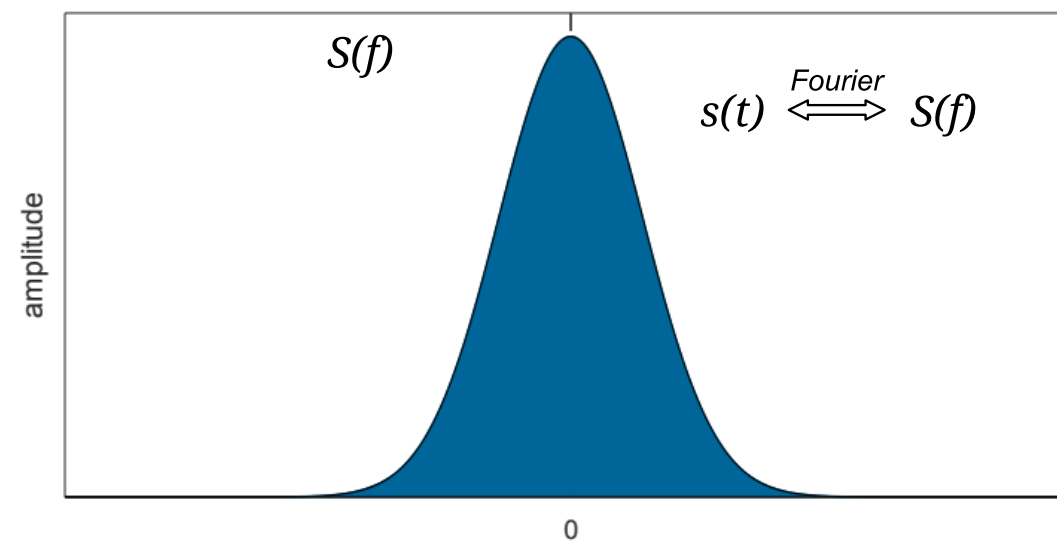
$$X_k = \sum_{n=0}^{N-1} x_n \cdot \left[\cos\left(\frac{2\pi}{N} kn\right) - i \cdot \sin\left(\frac{2\pi}{N} kn\right) \right]$$

Fourier Transform

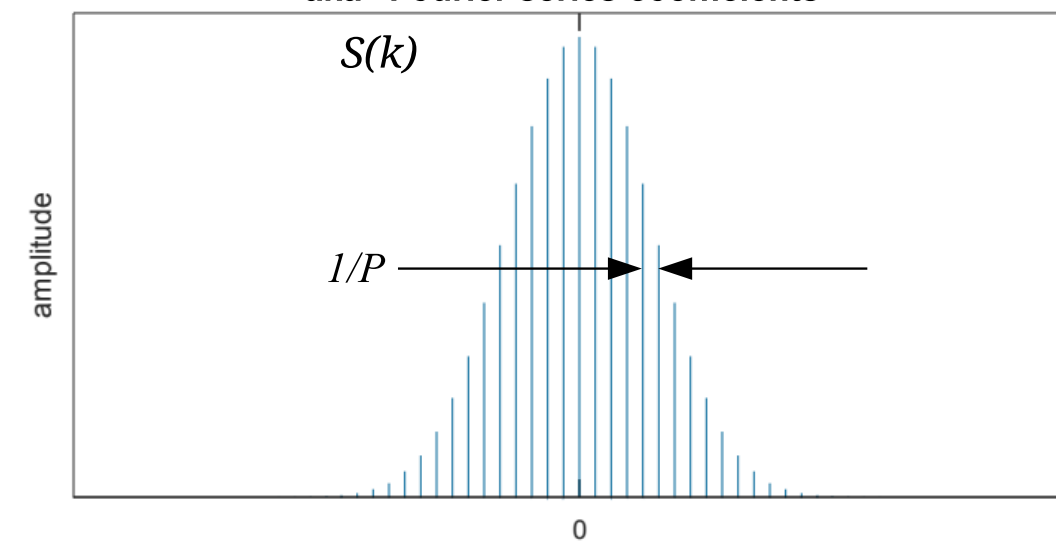
		Discrete-time signals	
		Time-domain	Frequency-domain
Periodic signals	Fourier series	 $c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j(2\pi/N)kn}$	 $x(n) = \sum_{k=0}^{N-1} c_k e^{j(2\pi/N)kn}$
		Discrete and periodic	Discrete and periodic
Aperiodic signals	Fourier transforms	 $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$	 $x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} d\omega$
		Discrete and aperiodic	Continuous and periodic

Fourier Transform

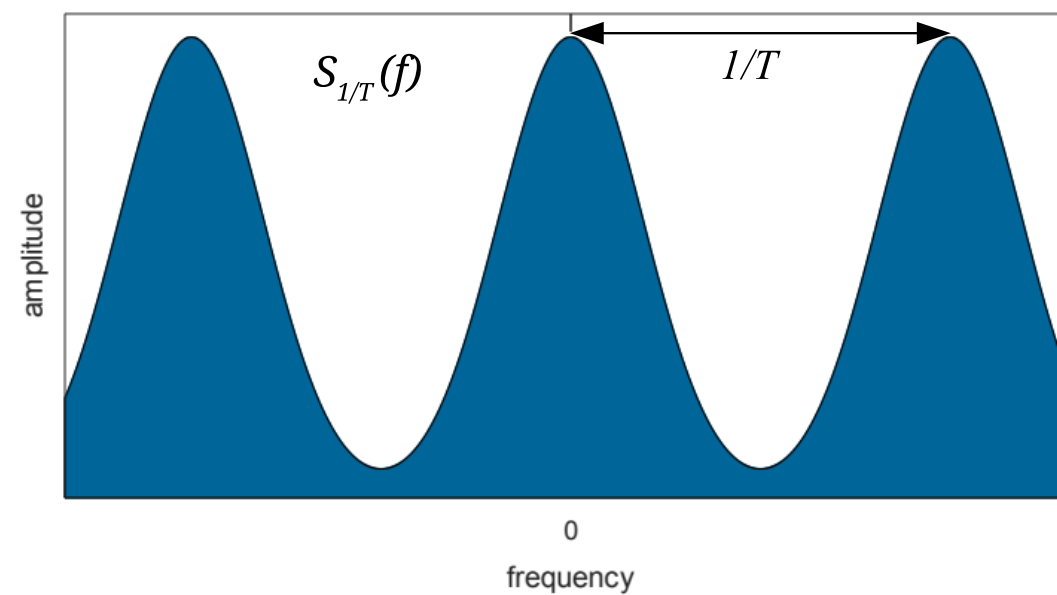
Fourier transform of a function $s(t)$ (which is not shown)



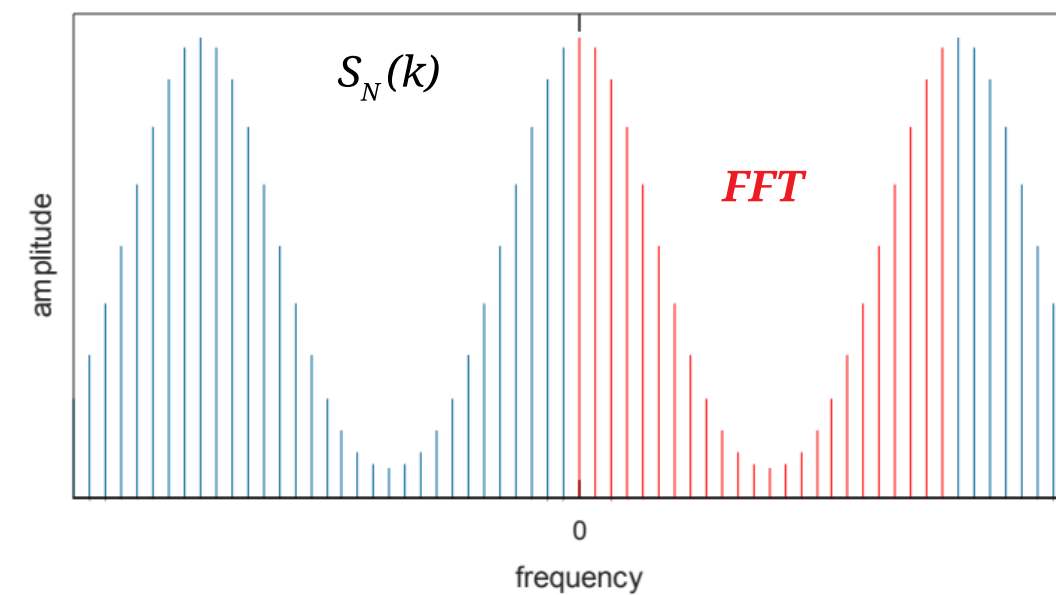
Transform of the periodic summation of $s(t)$
aka "Fourier series coefficients"



Transform of periodically sampled $s(t)$
aka "Discrete-time Fourier transform"



Transform of both periodic sampling and periodic summation
aka "Discrete Fourier transform"



Discrete Fourier Transform

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-i\frac{2\pi}{N}kn}$$

Parseval's *theorem*

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \int_{-\infty}^{\infty} |X(2\pi f)|^2 df$$

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$



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