UTEC
UNIVERSIDAD DE INGENIERÍA Y TECNOLOGÍA

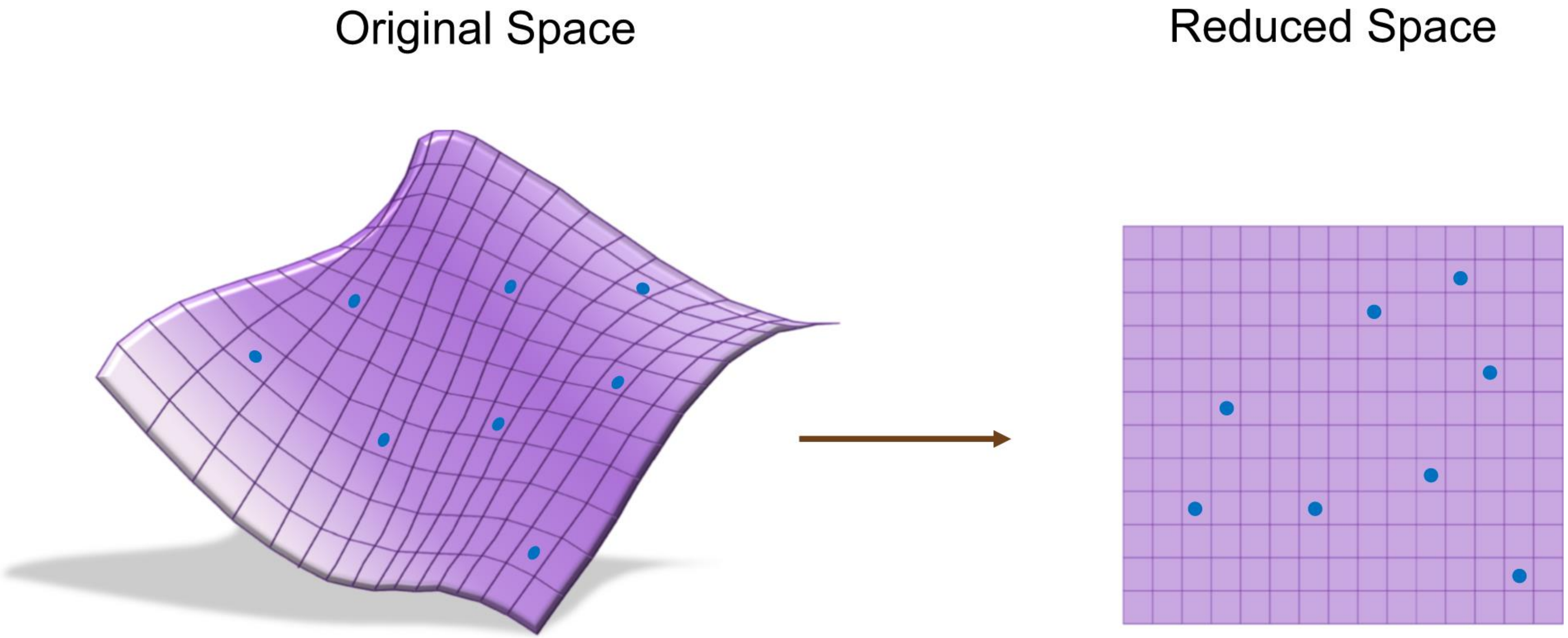
Sesión 13.1: Dimension Reduction Methods

CS3102 EDA

Índice

1. Dimension Reduction Methods

Dimension Reduction *Methods*



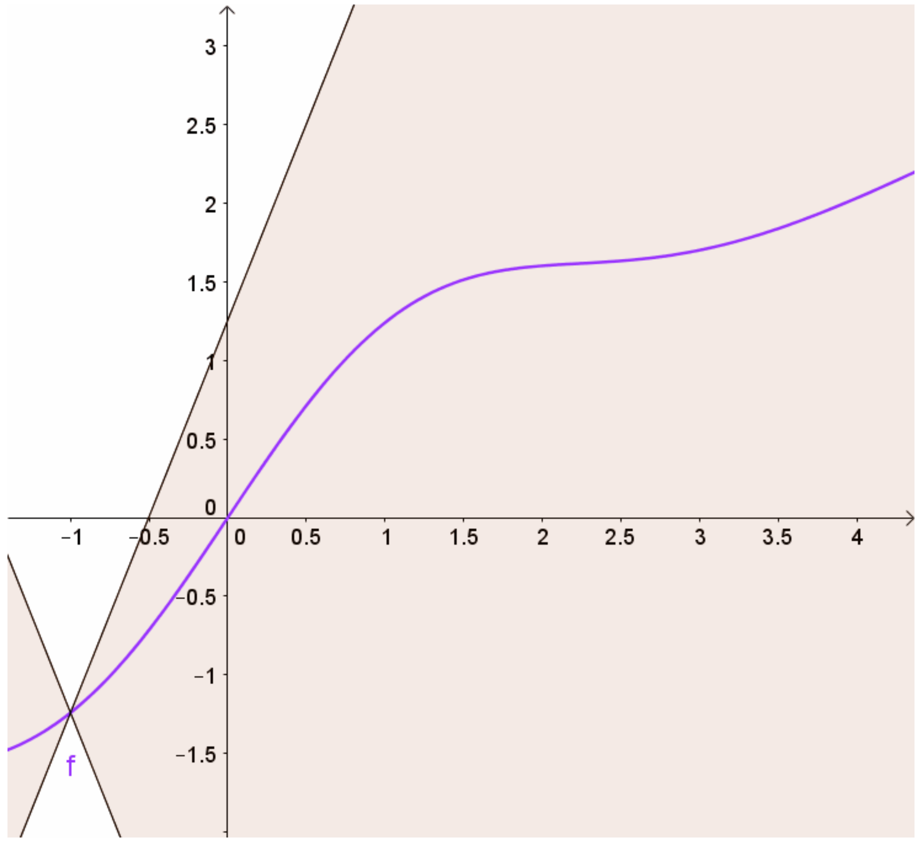
Lipschitz *continuity*

Sea la función $f: M \to N$ entre espacios métricos (M, d_M) y (N, d_N) se dice que es Lipschitz continua

$$d_N\big(f(x), f(y)\big) \leq k \cdot d_M(x, y) \qquad \forall x, y \in M$$

Metric map: $k = 1$

Contraction mapping: $0 \leq k < 1$



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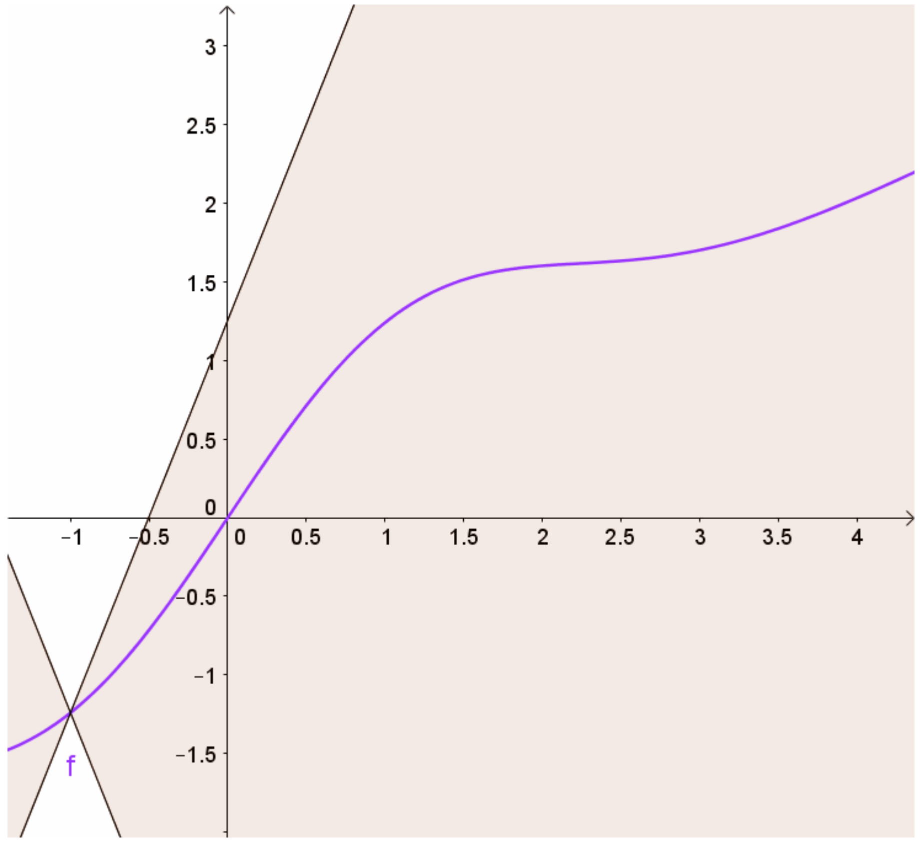


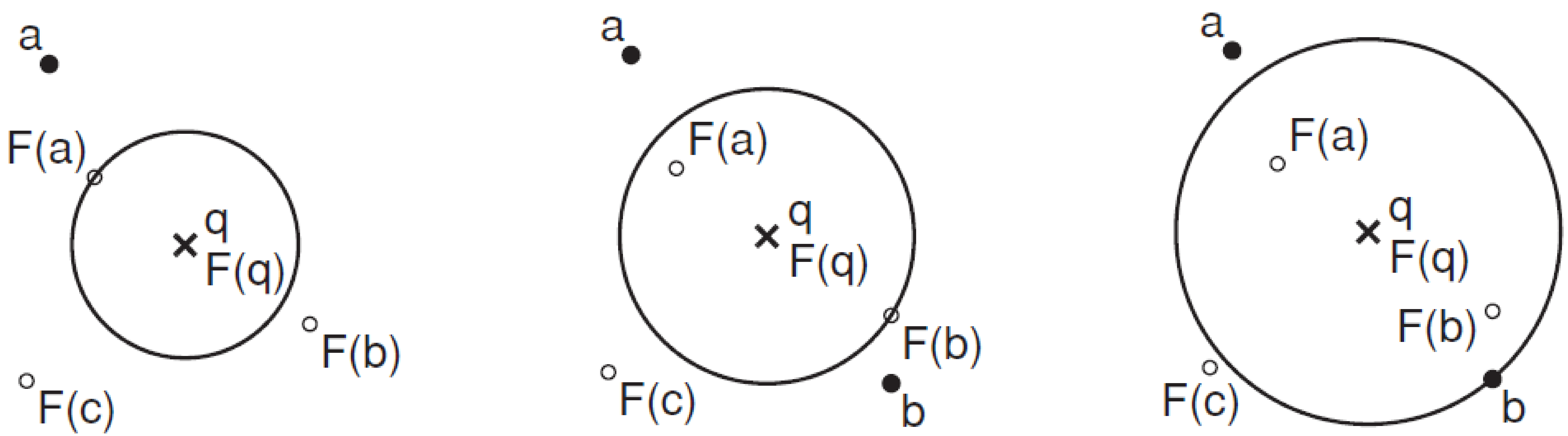
Pruning Property: $$d_N\big(f(x), f(y)\big) \leq d_M(x, y)$$

Proximity-Preserving *Property*

$d_M(a,b) \leq d_M(a,c)$ $\qquad$ $d_N\big(f(a), f(b)\big) \leq d_N\big(f(a), f(c)\big)$

Es muy difícil encontrar transformaciones que reduzcan dimensionalidad y cumplan esta propiedad

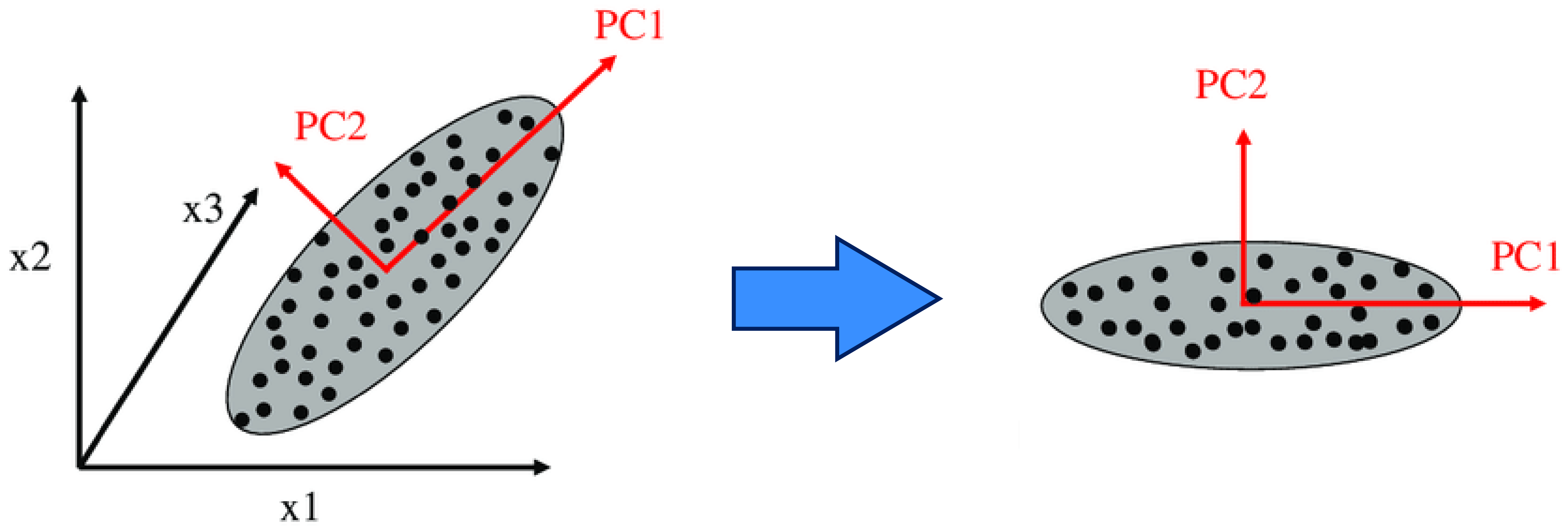
Incremental nearest *neighbor algorithm*



2. PCA

(Principal component analysis)

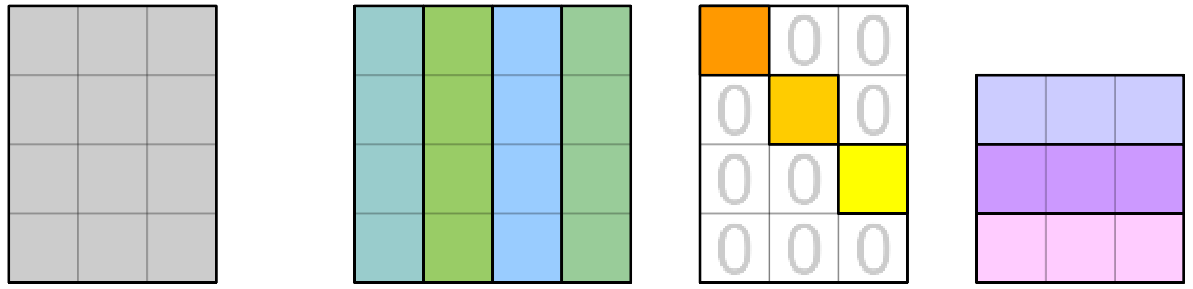
Principal *component analysis*



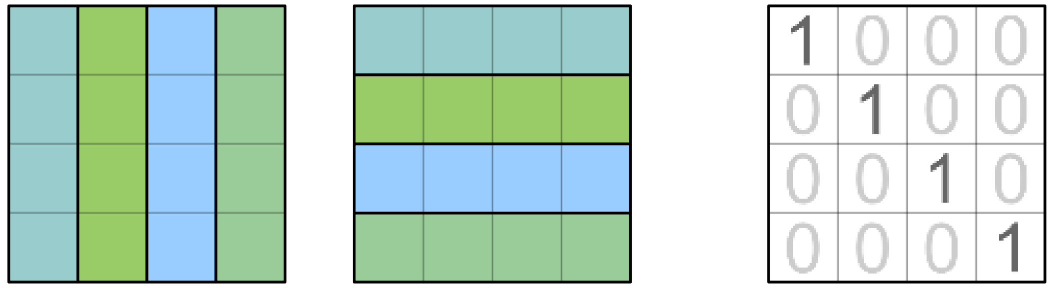
3. SVG

(Singular value decomposition)

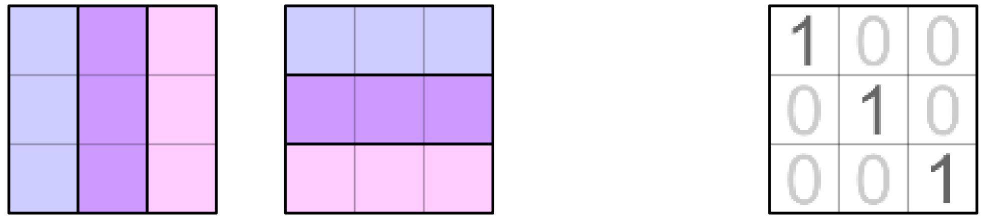
Singular value *decomposition*



$$\underset{m\times n}{\mathbf{M}} = \underset{m\times m}{\mathbf{U}}\ \underset{m\times n}{\mathbf{\Sigma}}\ \underset{n\times n}{\mathbf{V}^*}$$

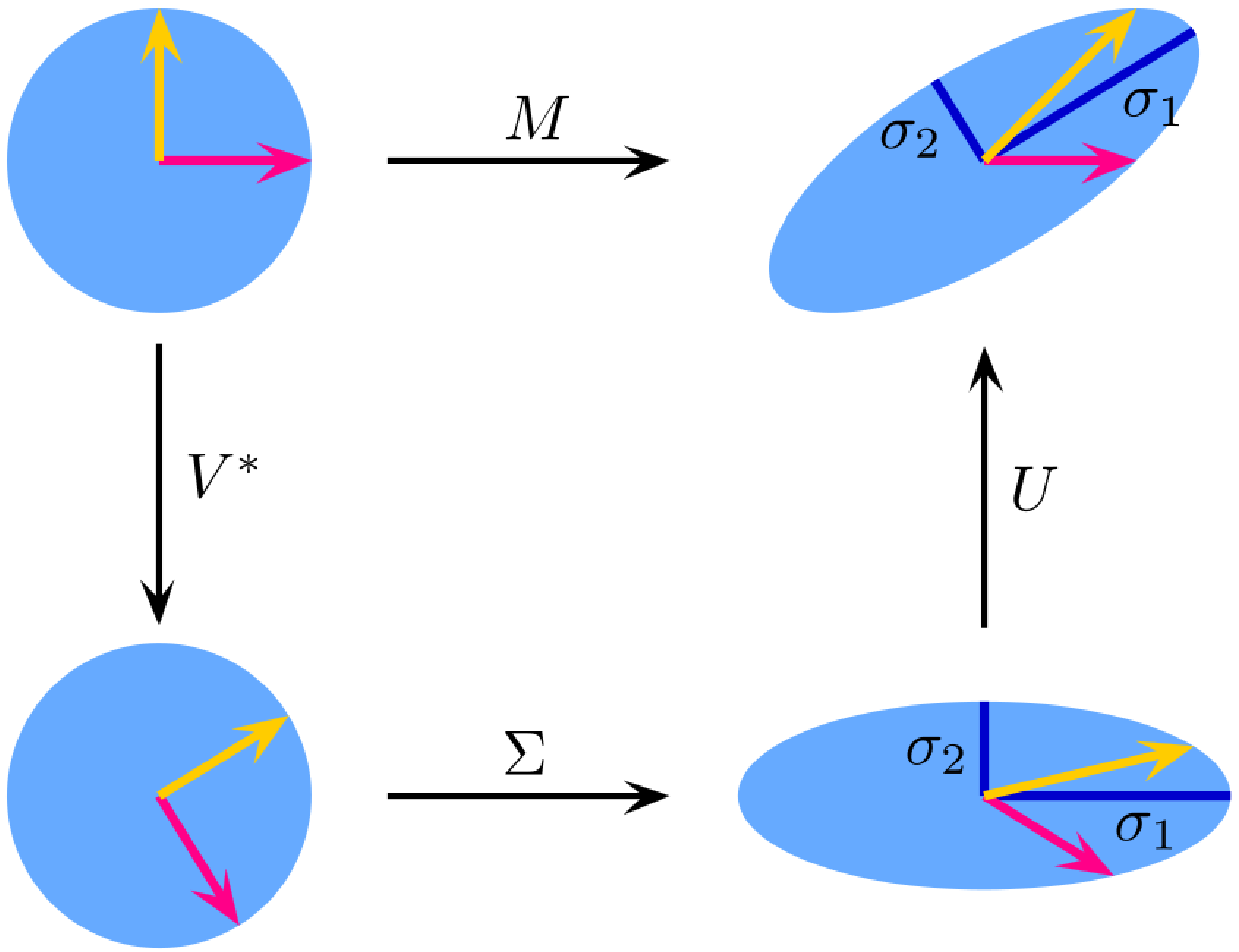


$$\mathbf{U}\ \mathbf{U}^* = \mathbf{I}_m$$



$$\mathbf{V}\ \mathbf{V}^* = \mathbf{I}_n$$

Singular value *decomposition*

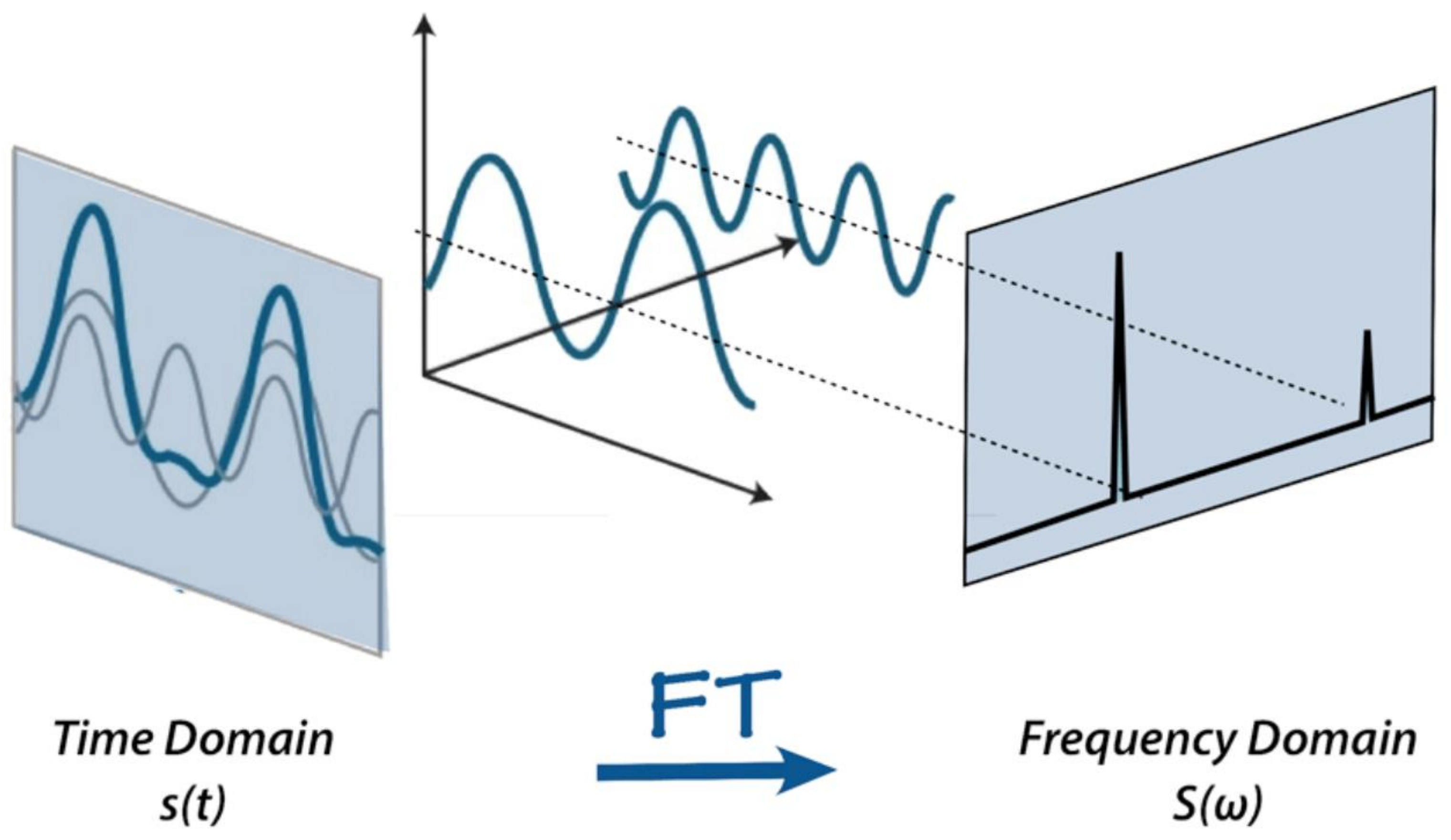


$$M = U \cdot \Sigma \cdot V^*$$

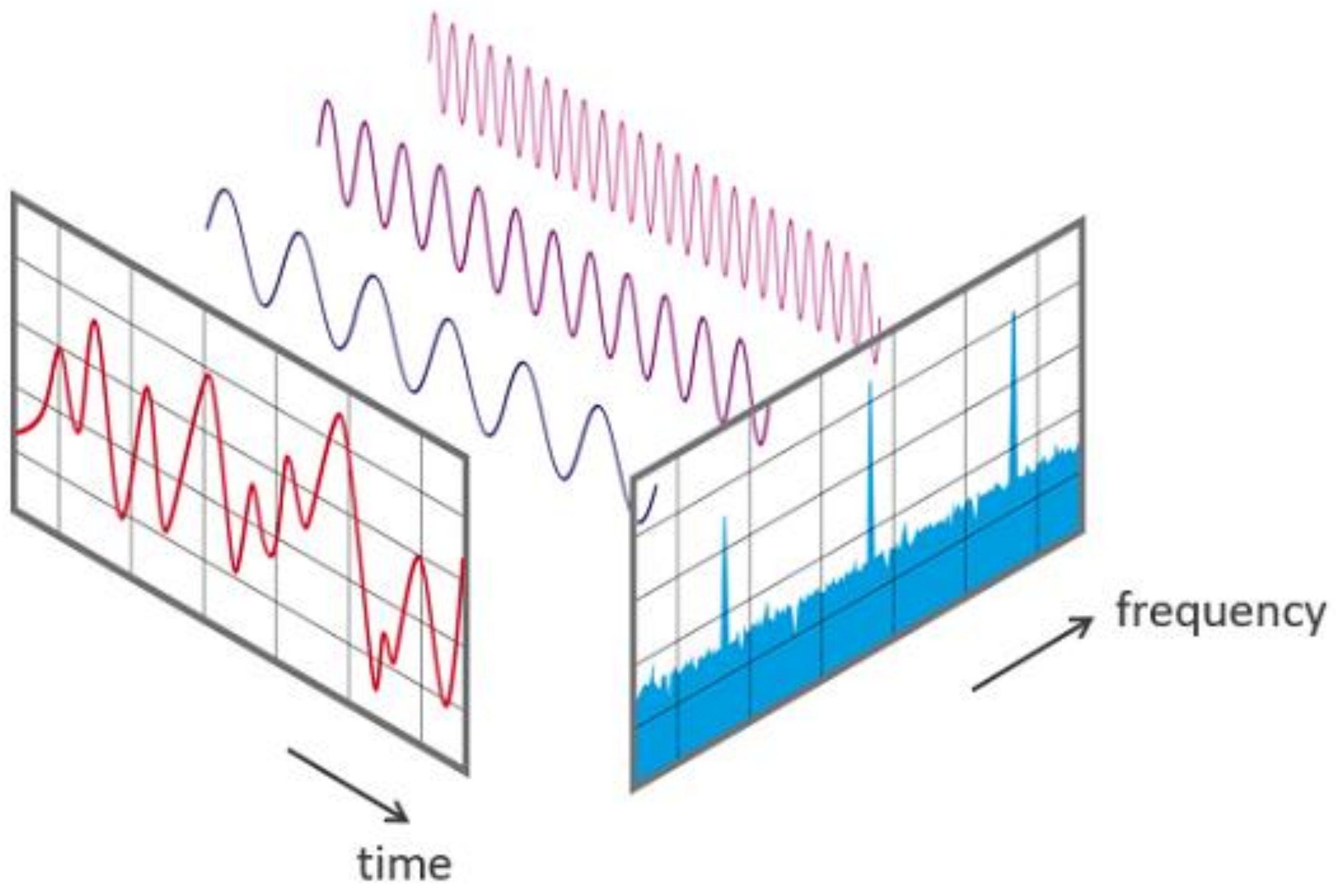
4. DFT

(Discrete Fourier Transform)

Dominio de *Frecuencia*



Dominio de *Frecuencia*



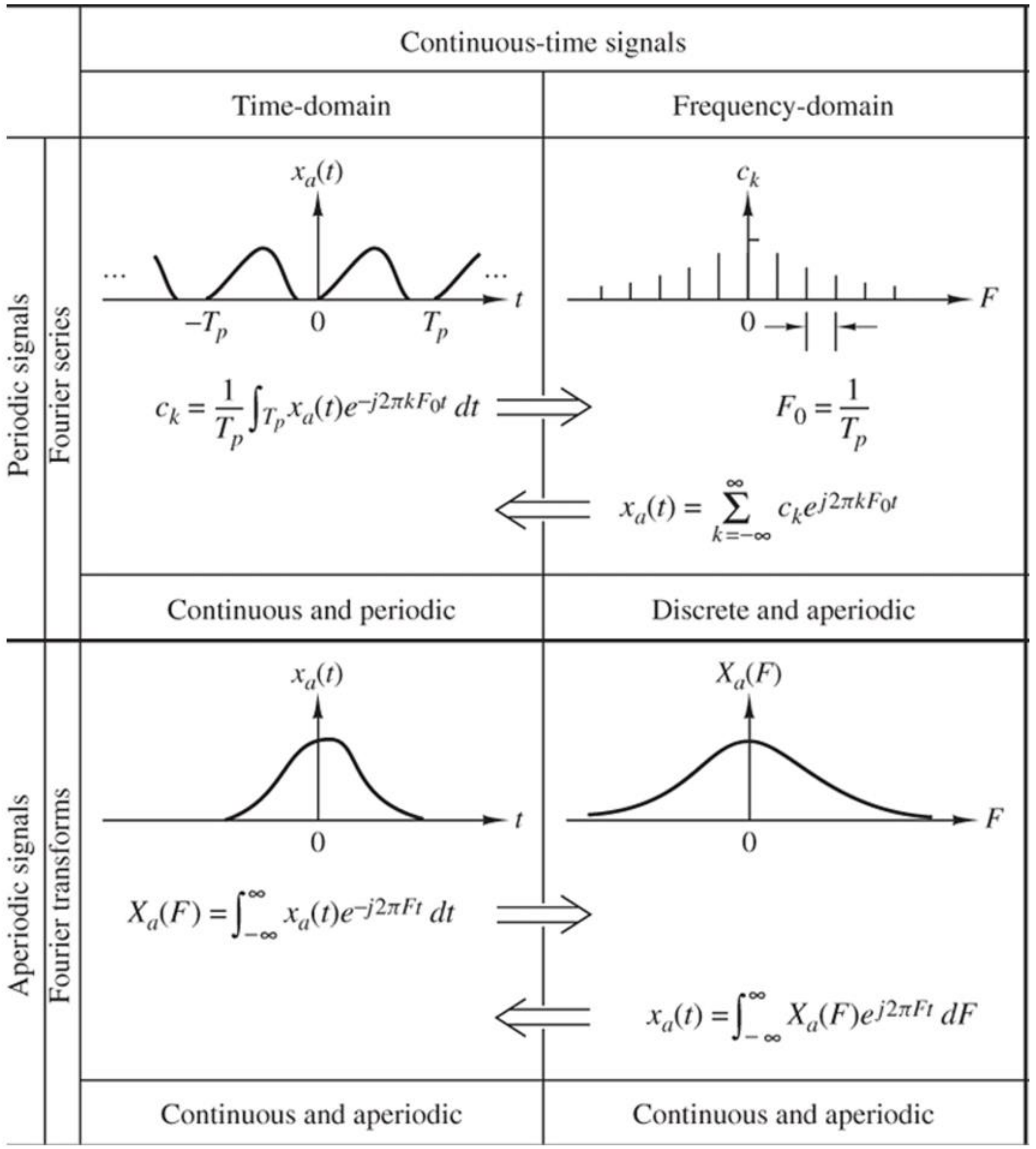
Serie de Fourier

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right)$$

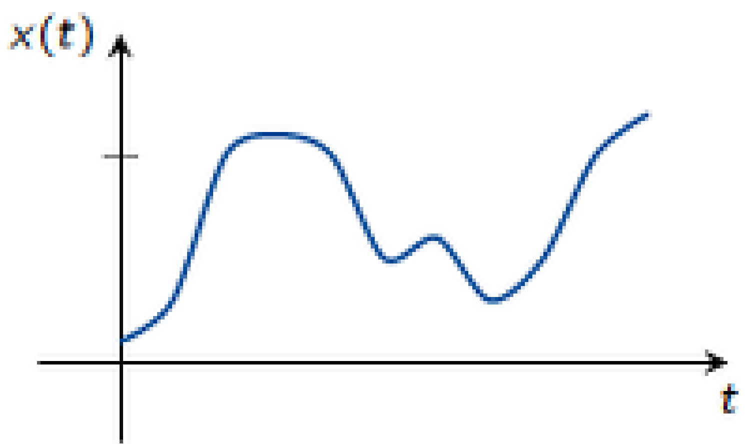
Transformada de Fourier

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-2\pi i\omega t}\partial t$$

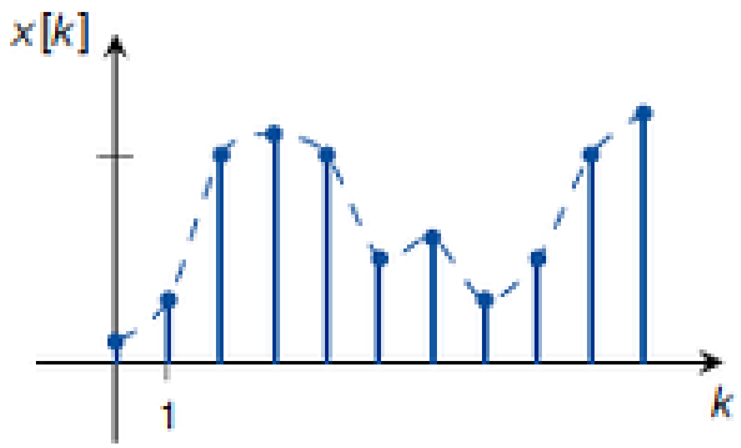
Fourier *Transform*



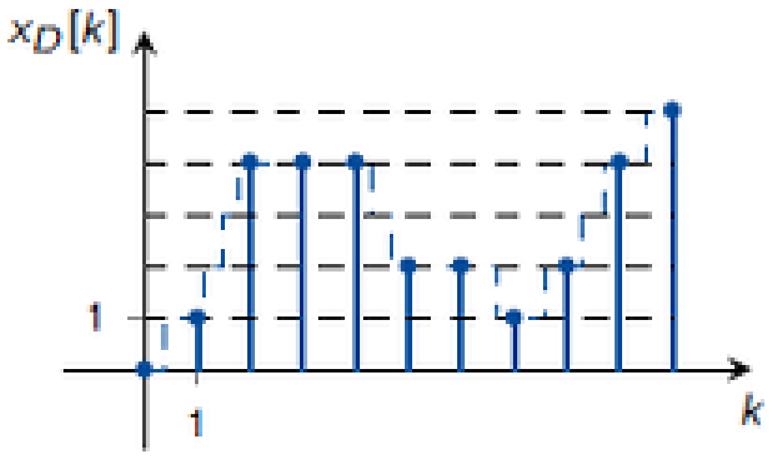
Digital*ización*



Señal

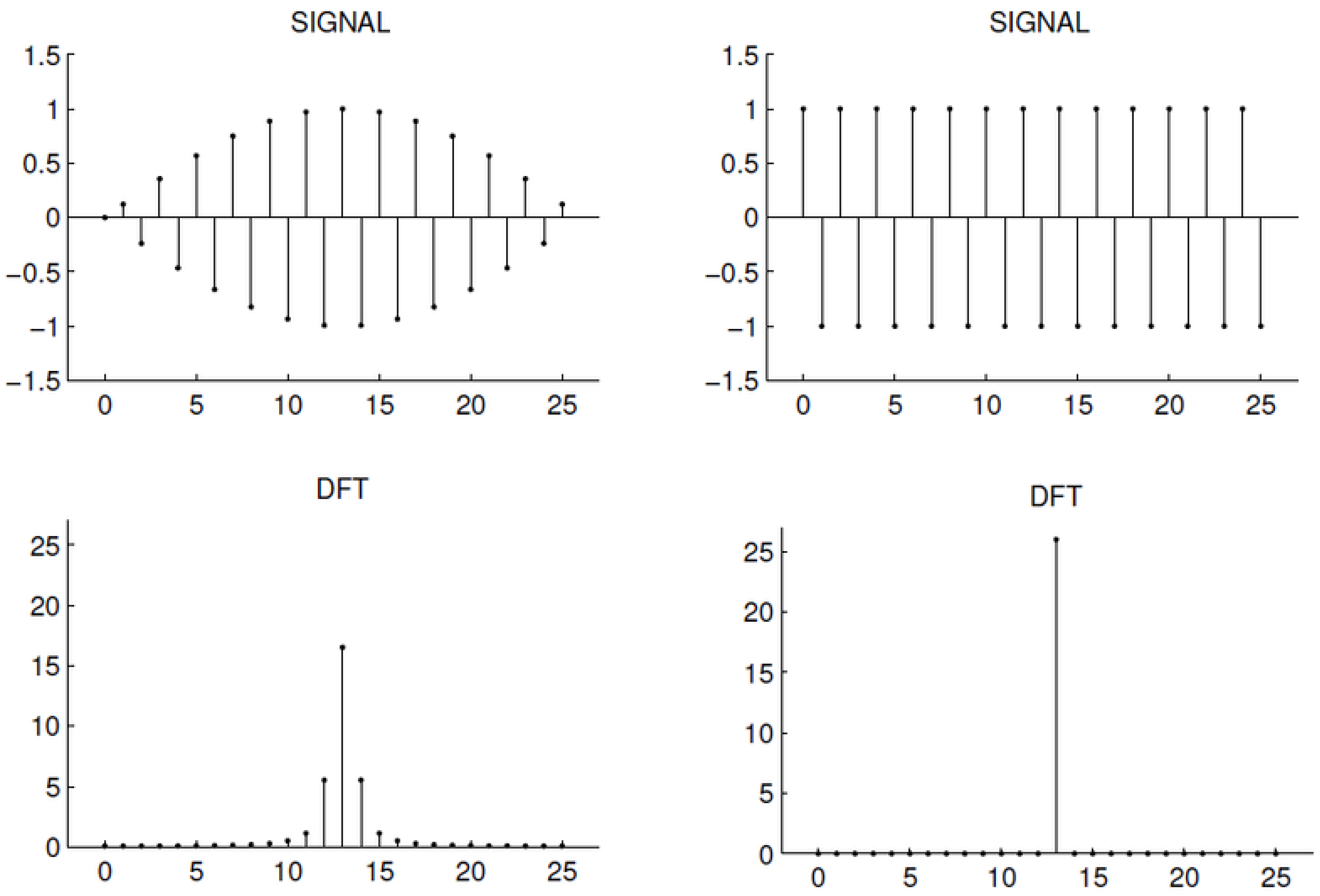


Muestreo



Cuantización

Discrete Fourier *Transform*

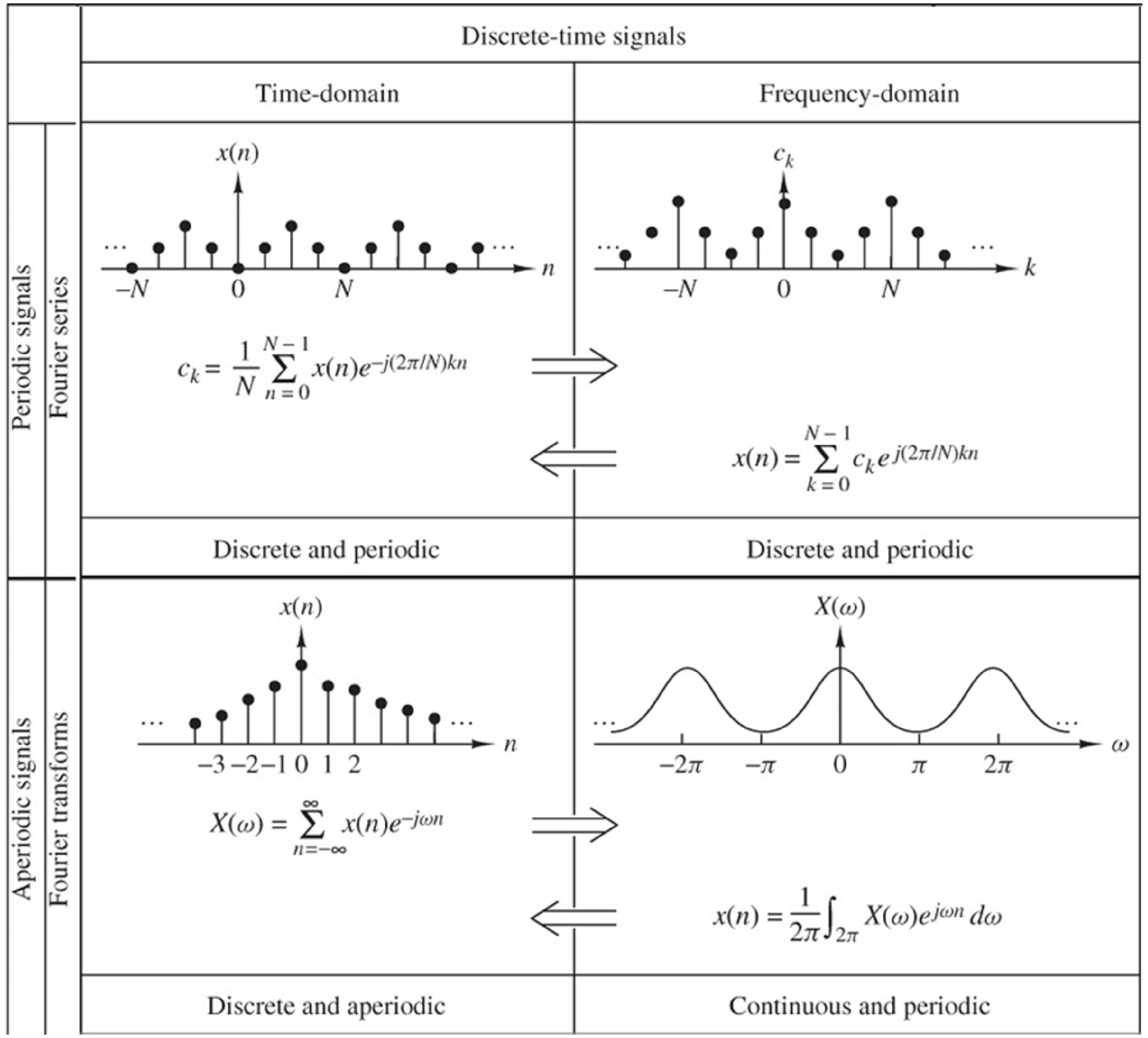


Transformada

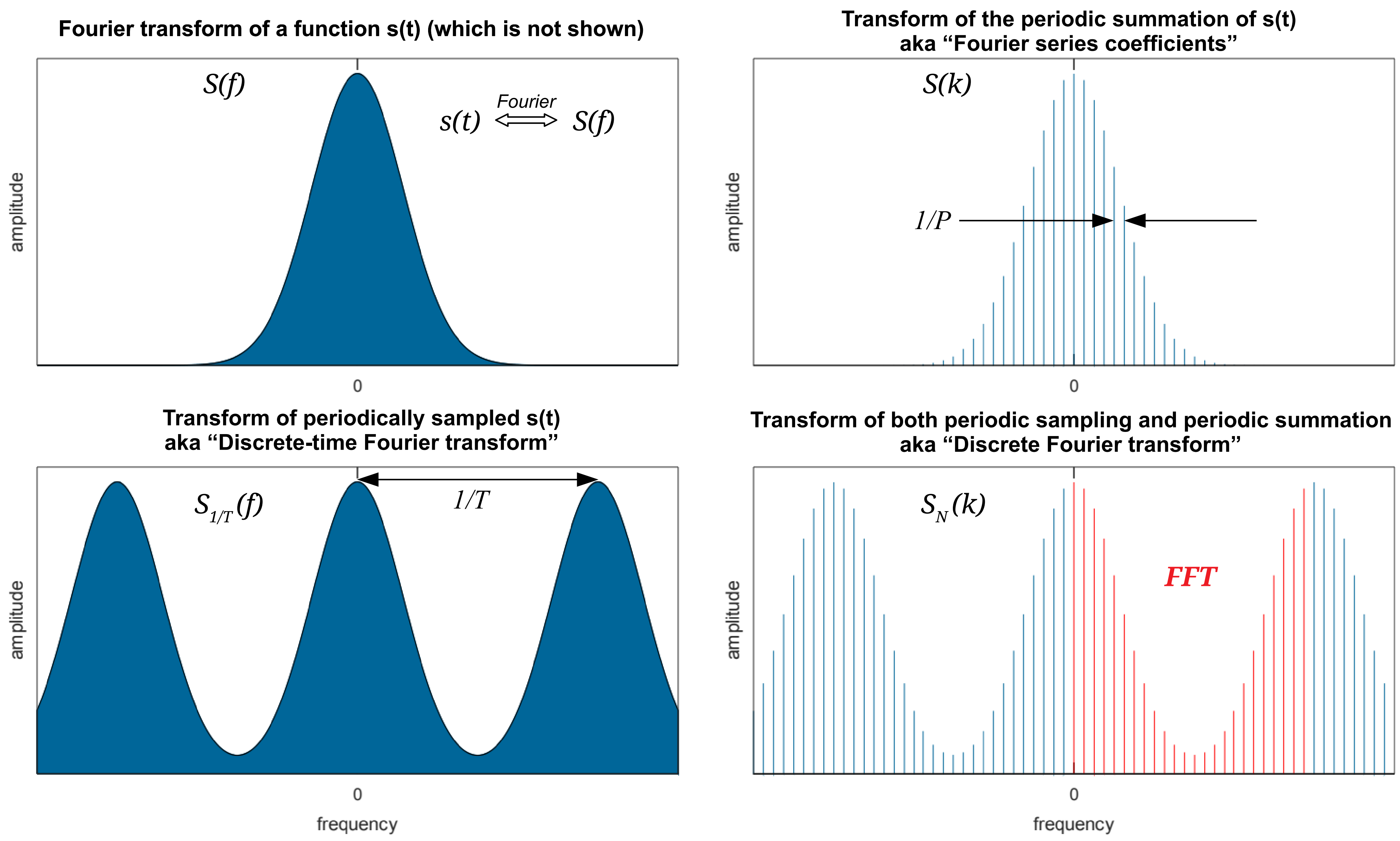
$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i\frac{2\pi}{N}kn}$$

$$X_k = \sum_{n=0}^{N-1} x_n \cdot \left[\cos\left(\frac{2\pi}{N}kn\right) - i \cdot \sin\left(\frac{2\pi}{N}kn\right)\right]$$

Fourier *Transform*



Fourier *Transform*



Discrete Fourier *Transform*

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-i\frac{2\pi}{N}kn}$$

Parseval's *theorem*

$$\int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, \mathrm{d}\omega = \int_{-\infty}^{\infty} |X(2\pi f)|^2 \, \mathrm{d}f$$

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$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