UTEC
UNIVERSIDAD DE INGENIERÍA Y TECNOLOGÍA

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# Computación Gráfica

*Class 16. Quadtrees. k-d trees. Voronoi diagram. Delaunay triangulation.*

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TRANSFORMATEC

# Today

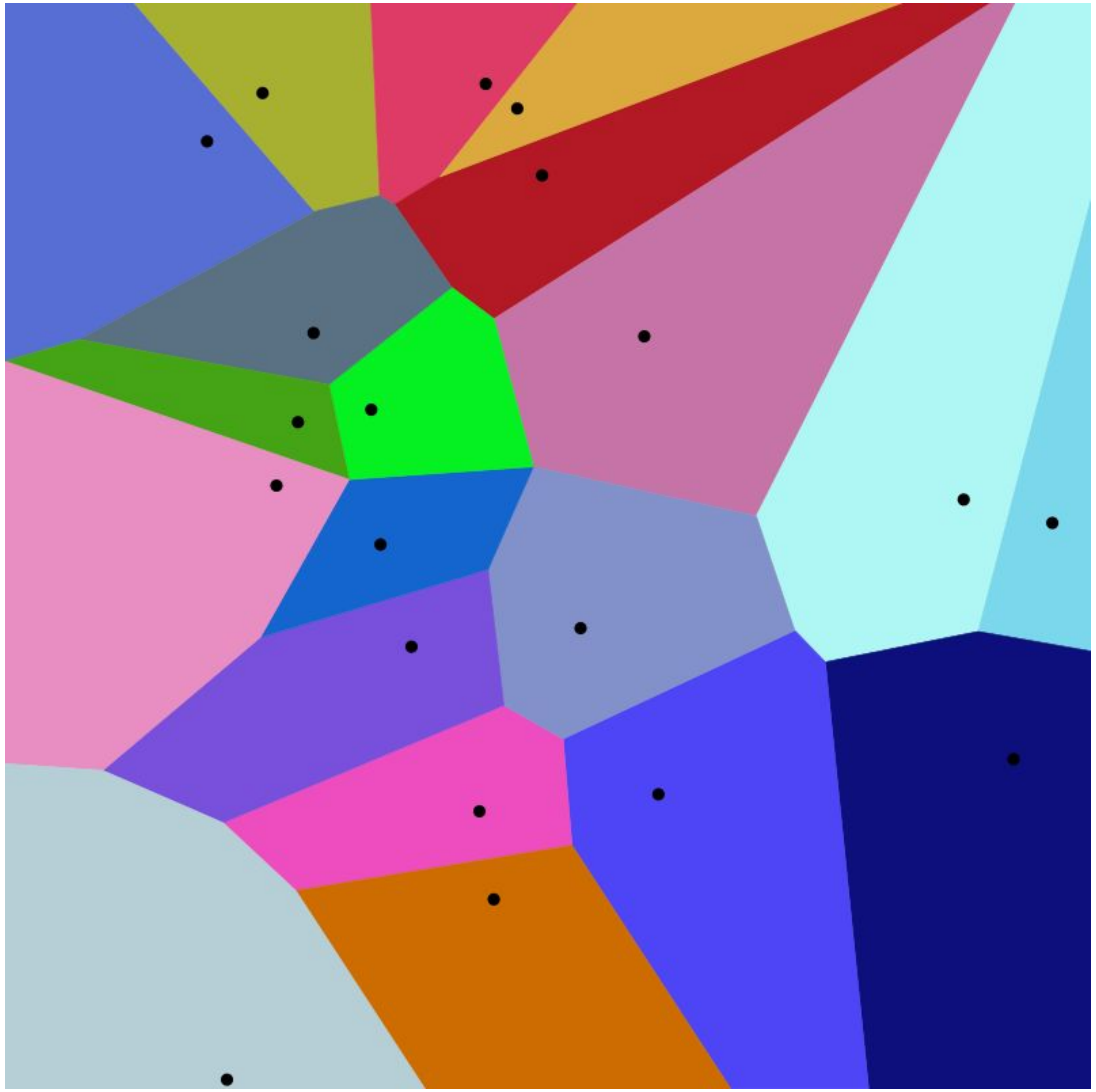
- Quadtrees.
- k-d trees.
- Overall ideas on voronoi diagram and delaunay triangulation.

**References for the class of today:** (part of the first partial exam)

- Berg, M. d., Cheong, O., Kreveld, M. v., and Overmars, M. Computational Geometry: Algorithms and Applications, 3rd ed. ed. Springer-Verlag TELOS, Santa Clara, CA, USA, 2008.
    - Sections 7.1 and 7.2 (Voronoi Diagrams)
    - Sections 5.2 (kd-trees), 5.3 (range trees)
    - Section 14.2 (quadtrees)
    - Sections 9.1, 9.2, 9.3 (Delaunay triangulation)

- Other sources:
    - https://www.geeksforgeeks.org/voronoi-diagram/ (Fortune's algorithm)
    - https://www.cs.cornell.edu/info/people/chew/Delaunay.html
    - https://jacquesheunis.com/post/fortunes-algorithm/
    - https://www.youtube.com/watch?v=ysLCuqcyJZA

# Voronoi diagram



# Voronoi diagram

- Divides the space into regions based on the distance to a set of the points called “seeds” or “sites“.
- the region of the seed s is formed by the points whose closest seed is s.
- Voronoi diagrams have many applications in multiple fields. Among them:
  - Geographical Information Systems.
  - Computer Vision.
  - Terrain generation.
  - Texture generation.
  - Collision detection
  - Rendering.
  - Texture maping.

# Voronoi diagrams: how to compute it?

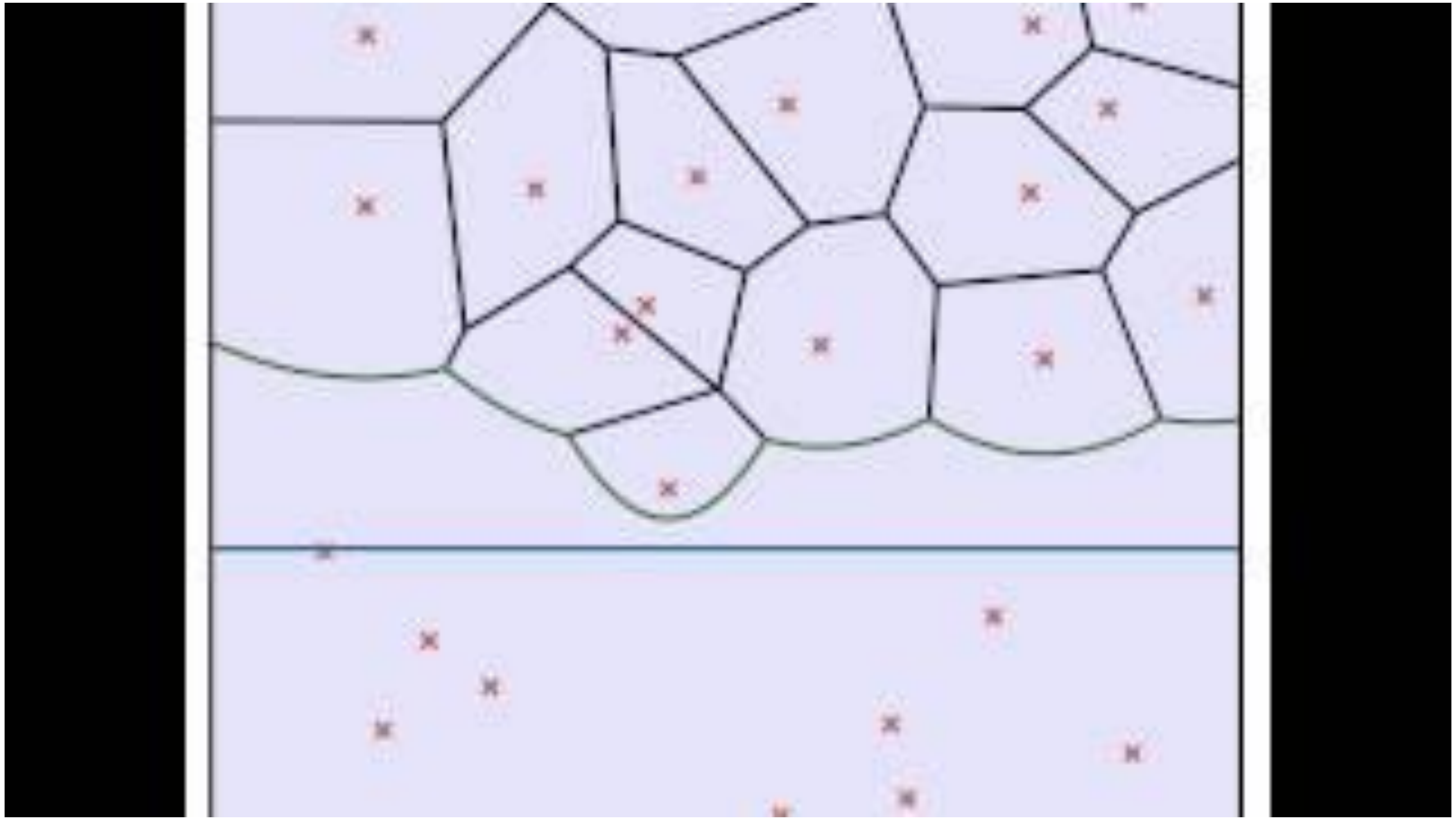
The **Fortune's algorithm**

# Voronoi diagrams: how to compute it?

## The **Fortune's algorithm**

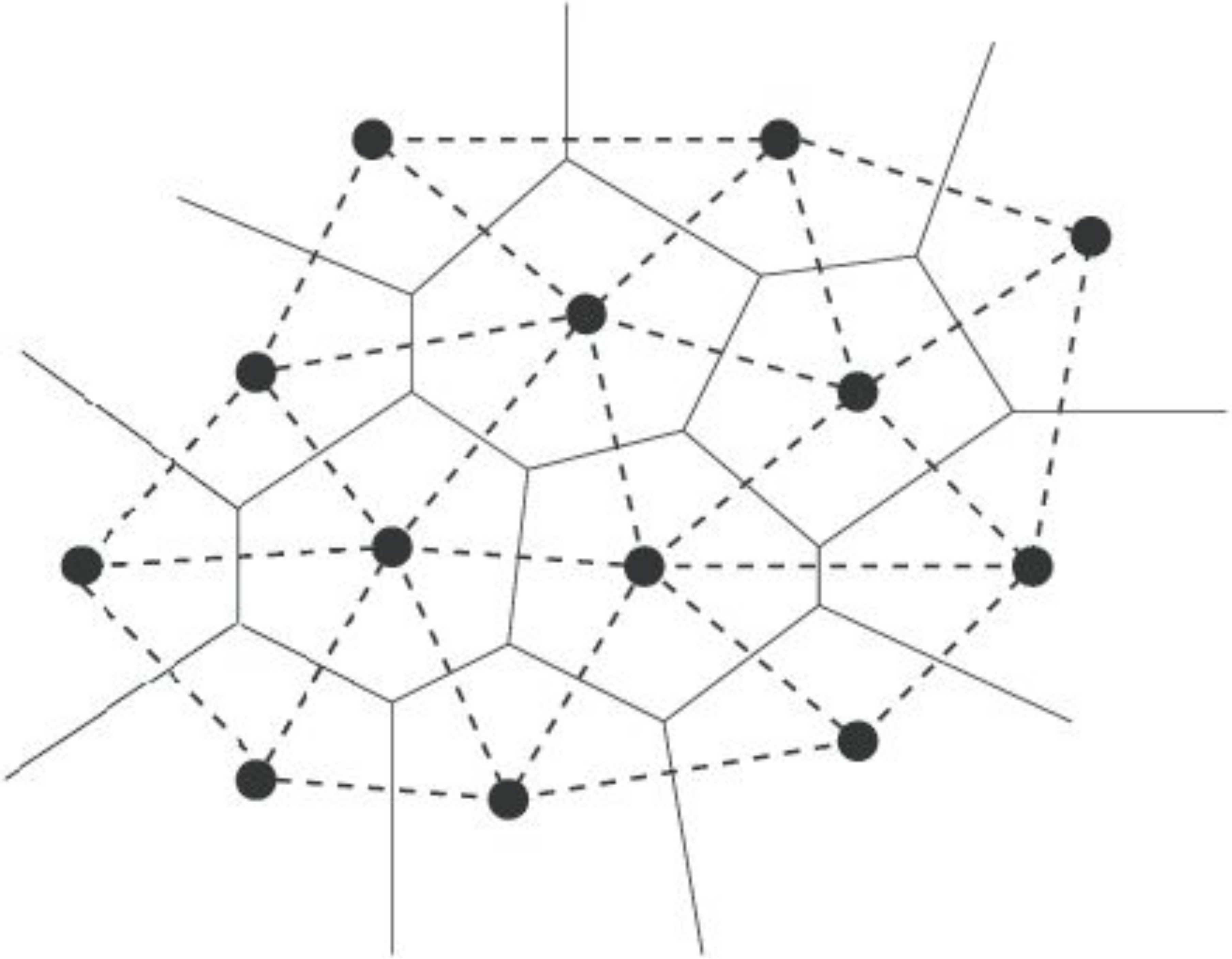
- ➢ The name is due to its inventor: Steven Fortune
- ➢ Invented in 1986 (38 years ago)
- ➢ Sweep line.
- ➢ Runtime O(n log n) and memory O(n), where n is the number of initial seeds.

# Voronoi diagrams: how to compute it?



https://www.youtube.com/watch?v=k2P9yWSMaXE

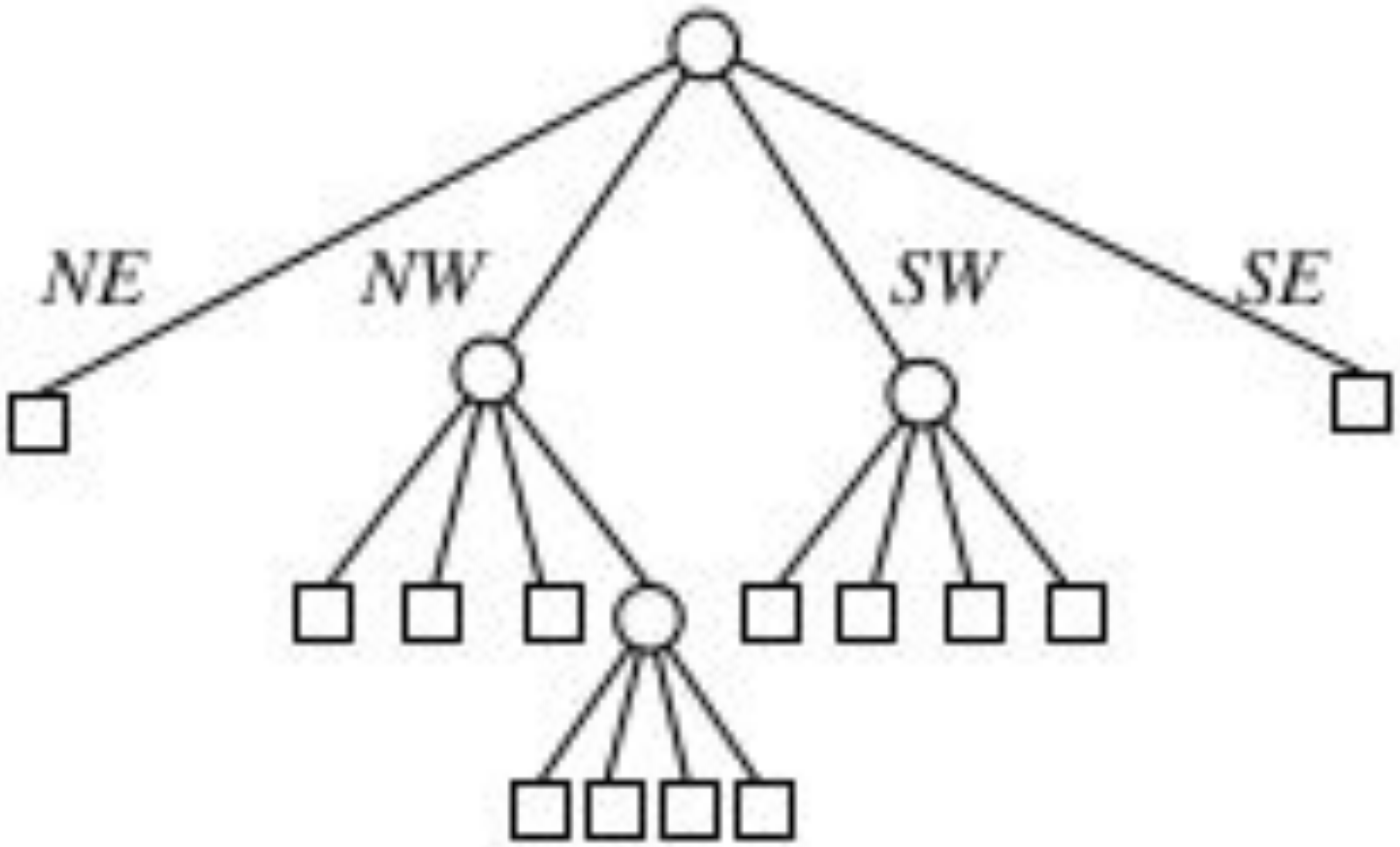
# Delaunay triangulation

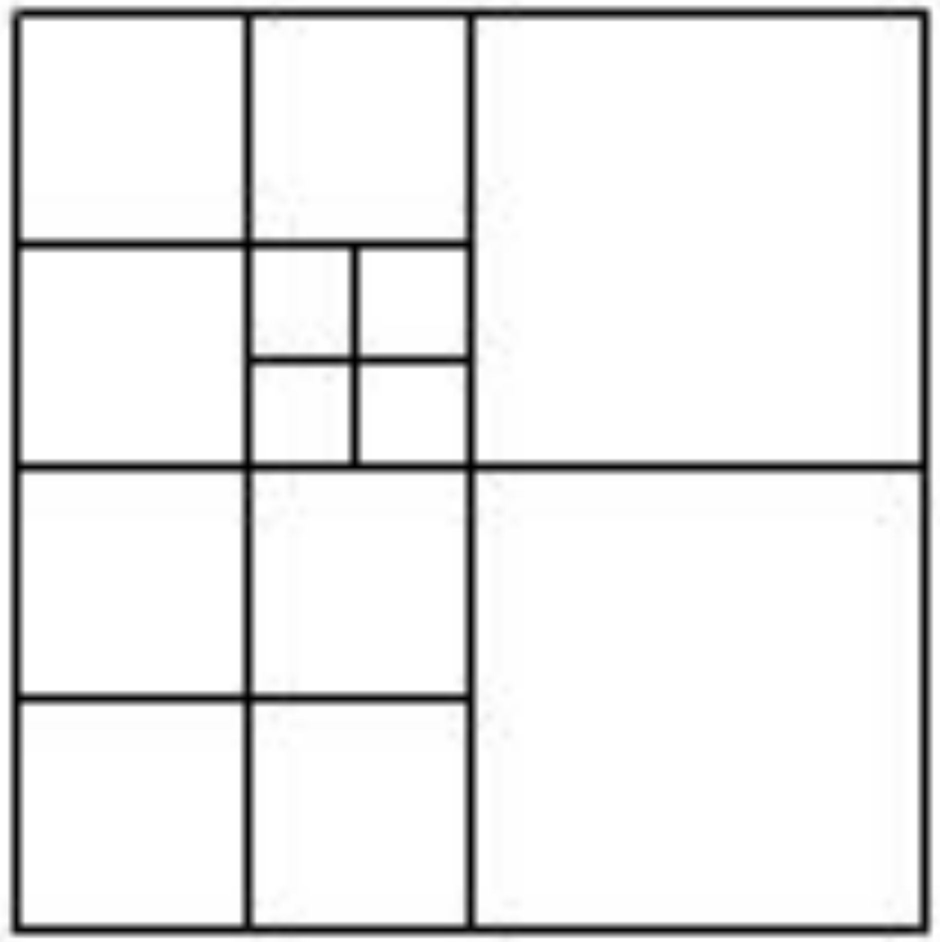


# Delaunay triangulation

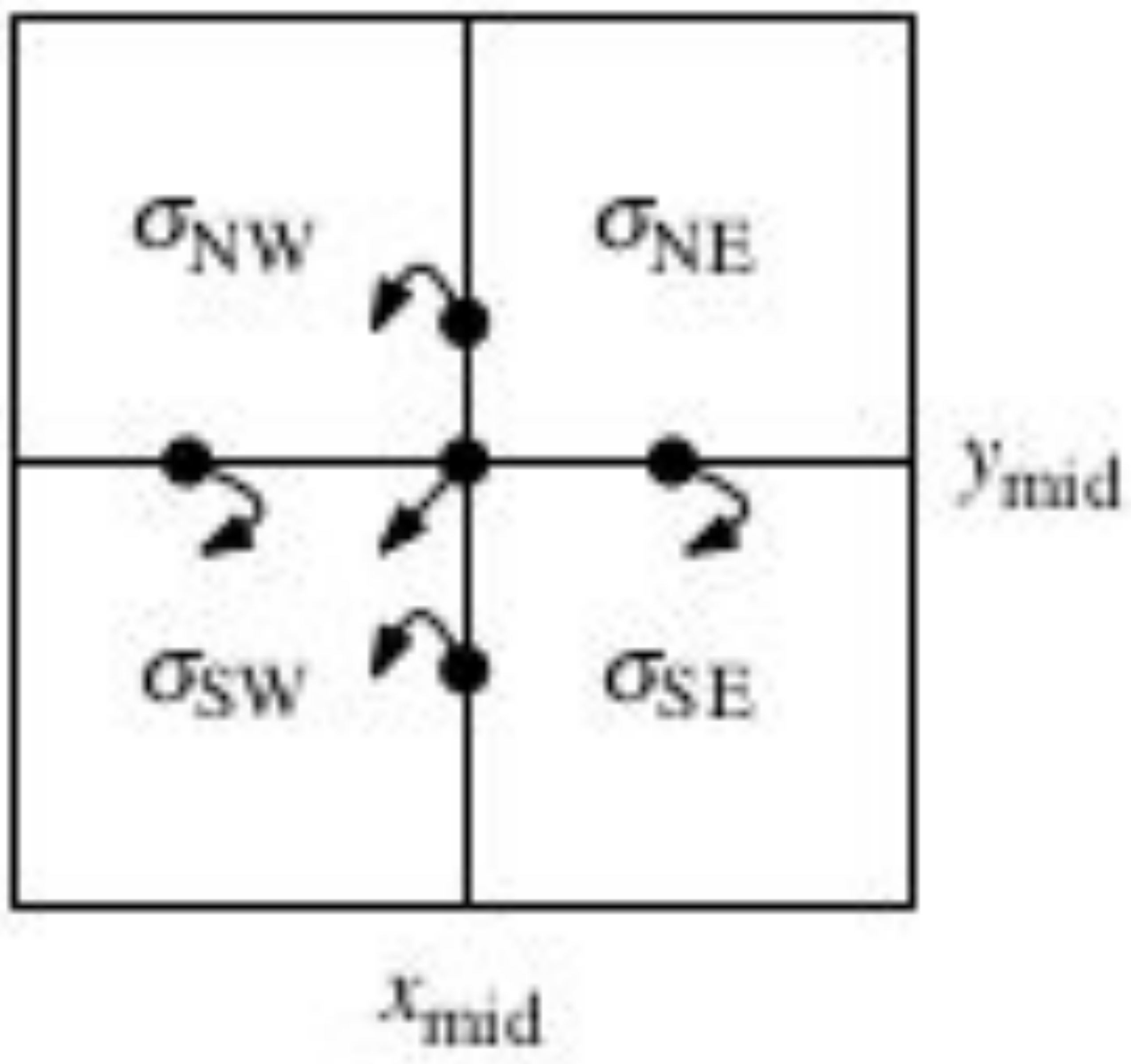
- It is (almost) the dual graph of the Voronoi diagram.
- Can be computed on its own and compute, from it, the Voronoi diagram.
- Can be computed from the Voronoi diagram.

# Quadtrees

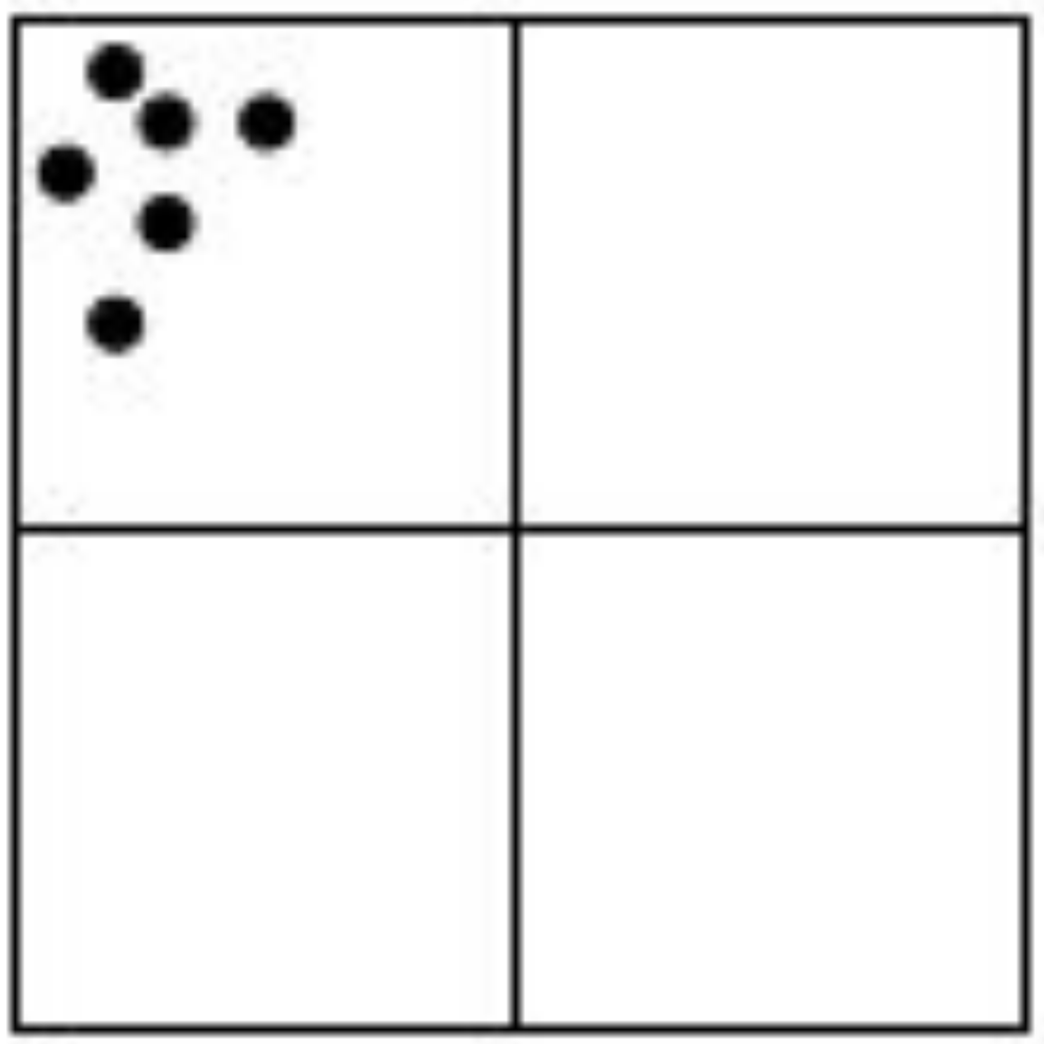


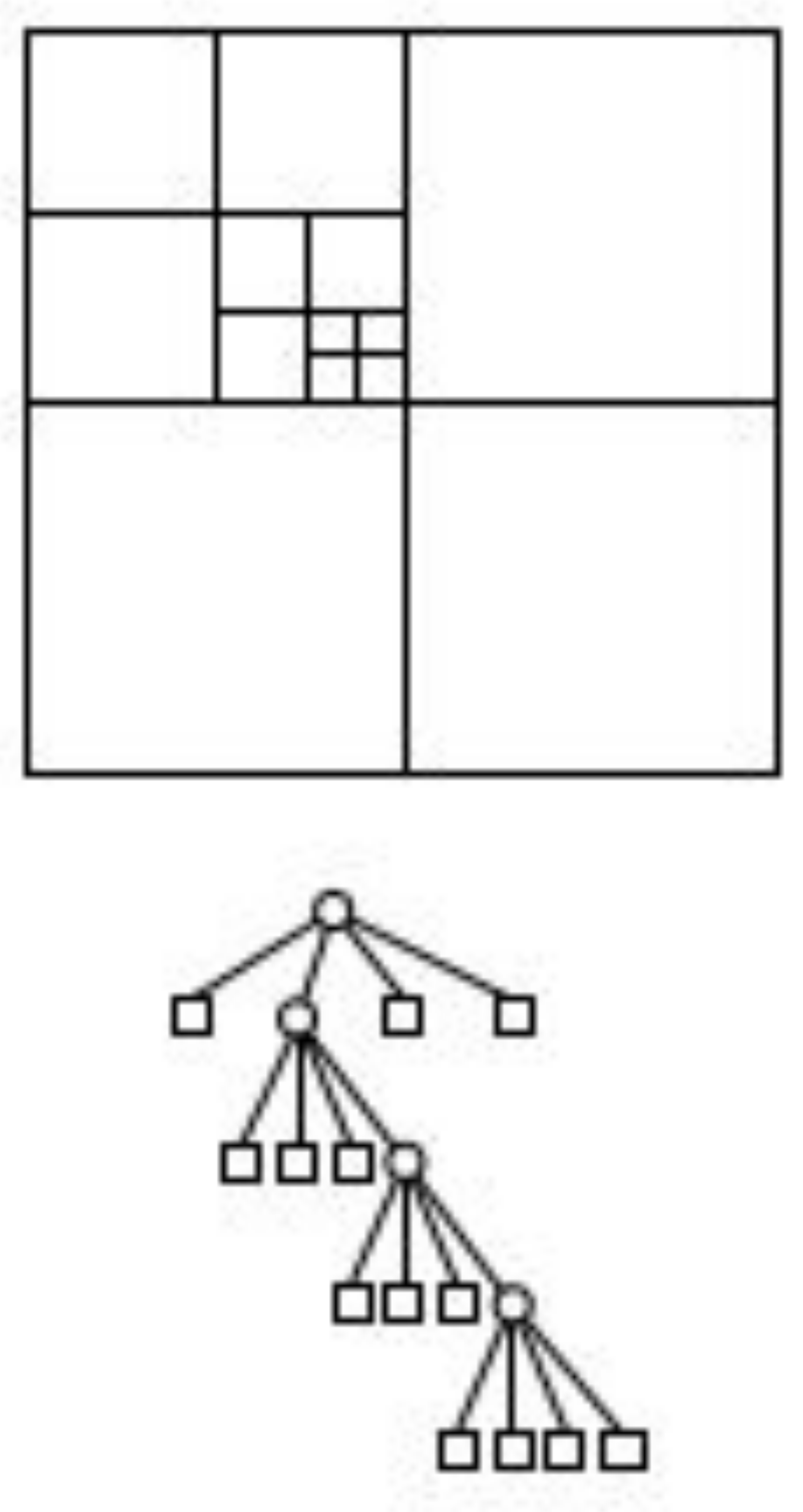
# Quadtrees



# Quadtrees



# Quadtrees



# Quadtrees

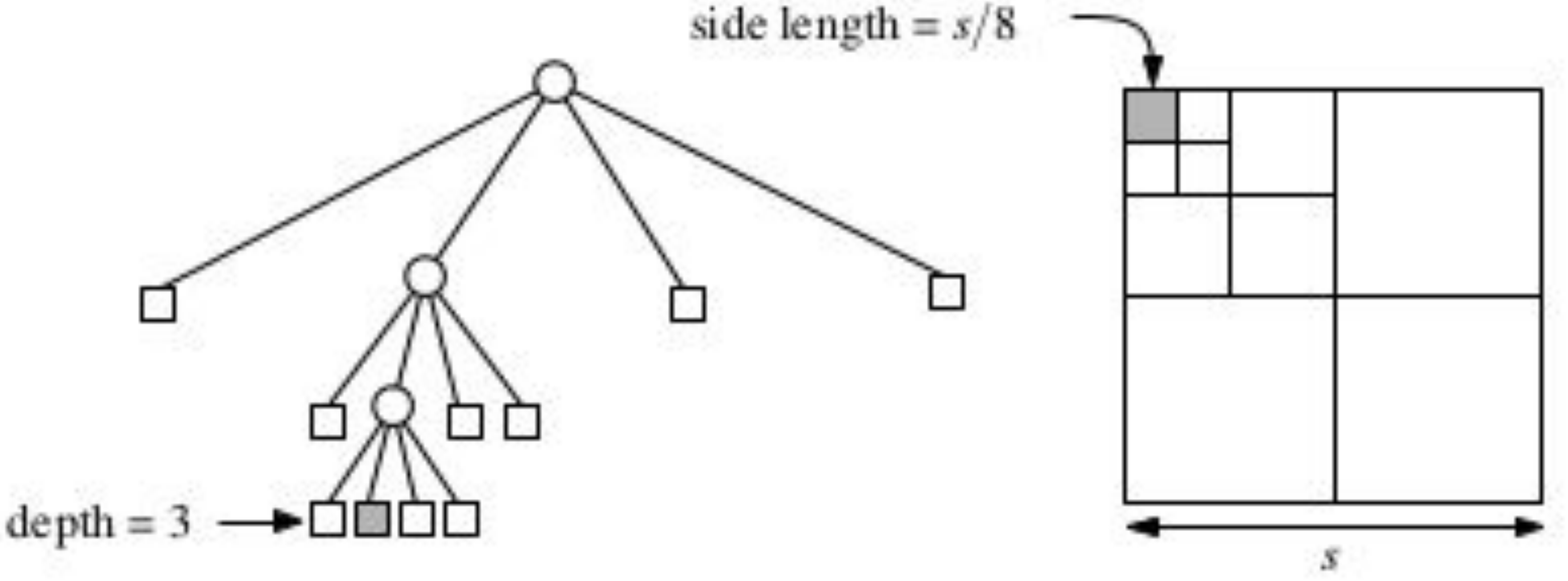
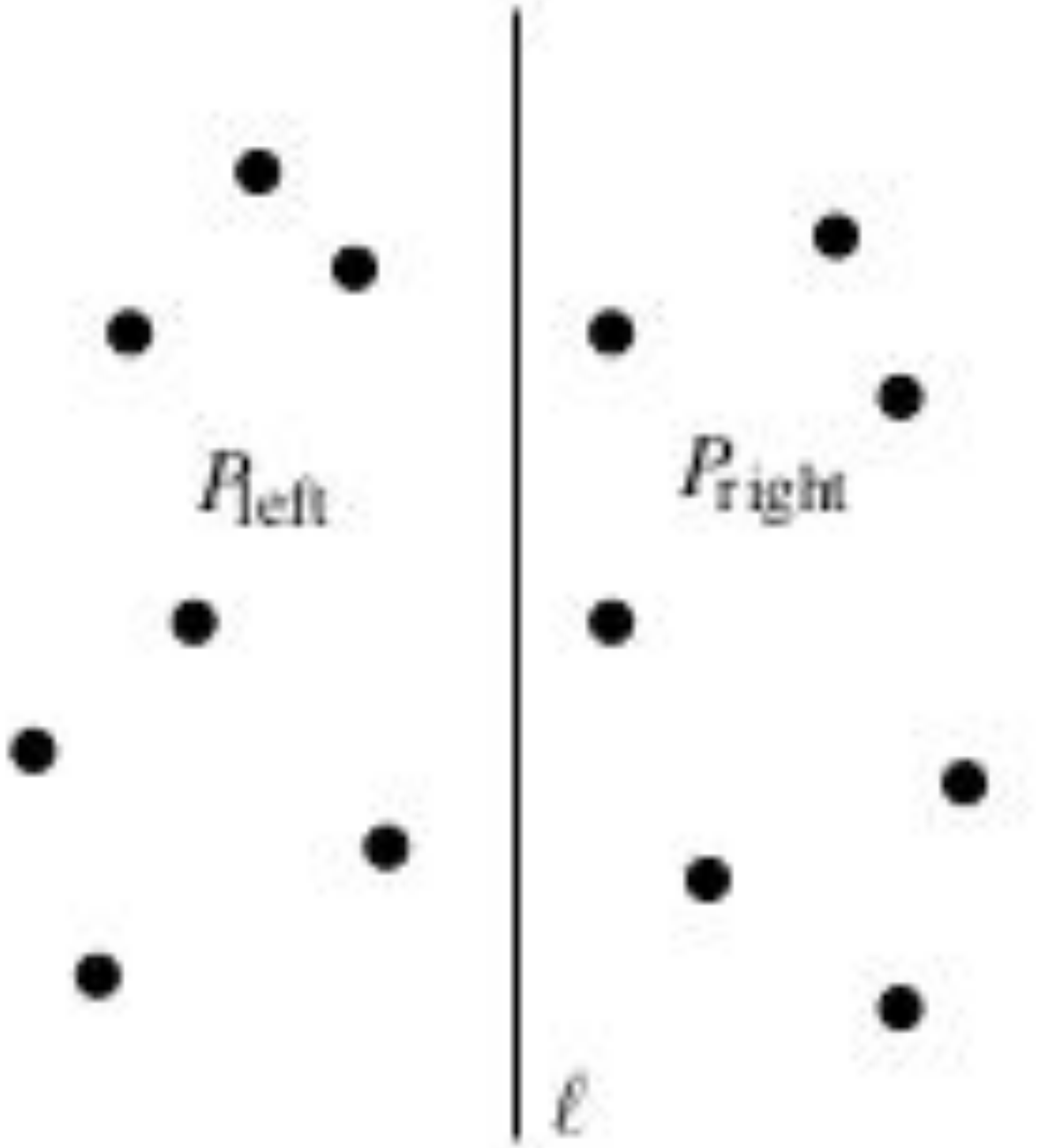



*Figure 14.4*
A node at depth $i$ corresponds to a square of side length $s/2^i$

# Quadtrees

**Theorem 5.5** *A kd-tree for a set $P$ of $n$ points in the plane uses $O(n)$ storage and can be built in $O(n \log n)$ time. A rectangular range query on the kd-tree takes $O(\sqrt{n}+k)$ time, where $k$ is the number of reported points.*

# kd-trees



# kd-trees

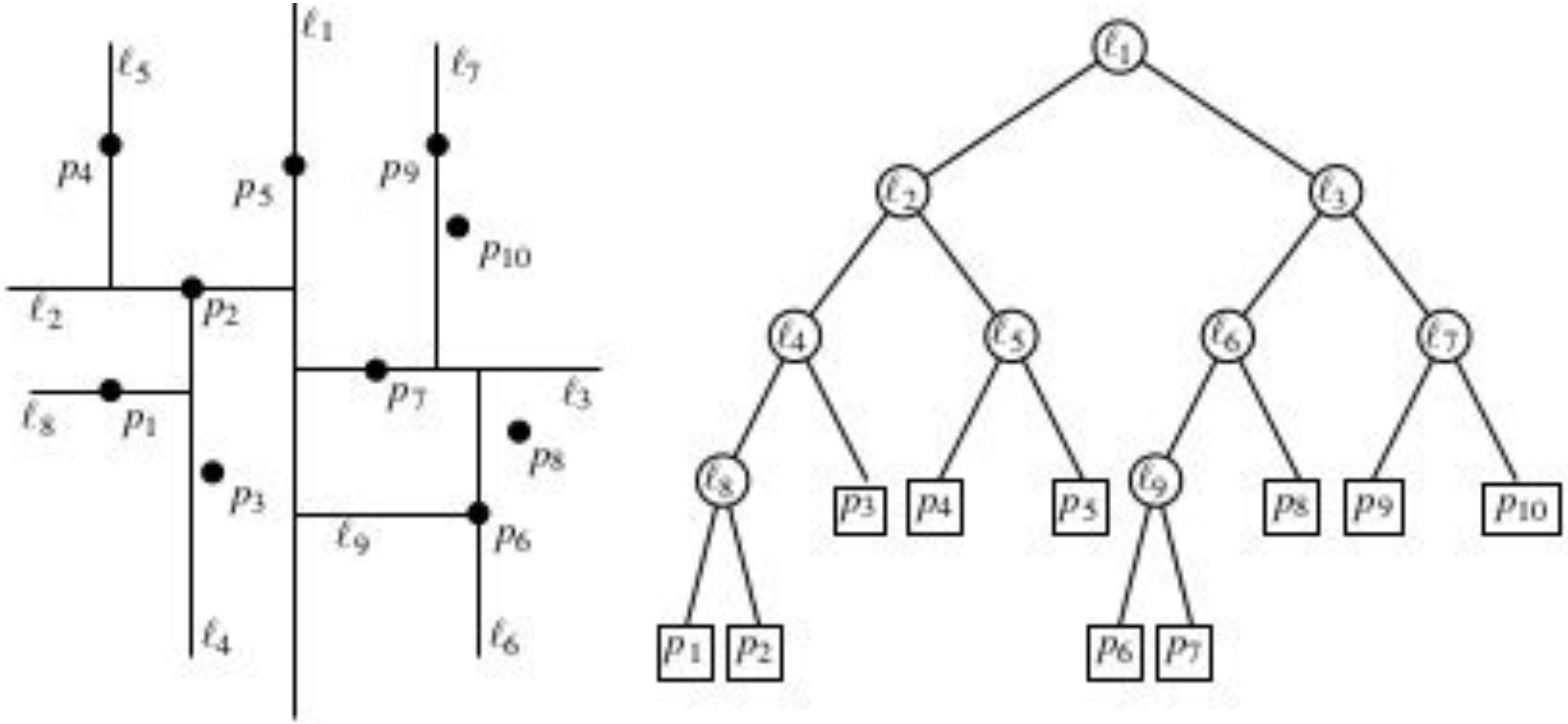



*Figure 5.3*
A kd-tree: on the left the way the plane is subdivided and on the right the corresponding binary tree

# kd-trees

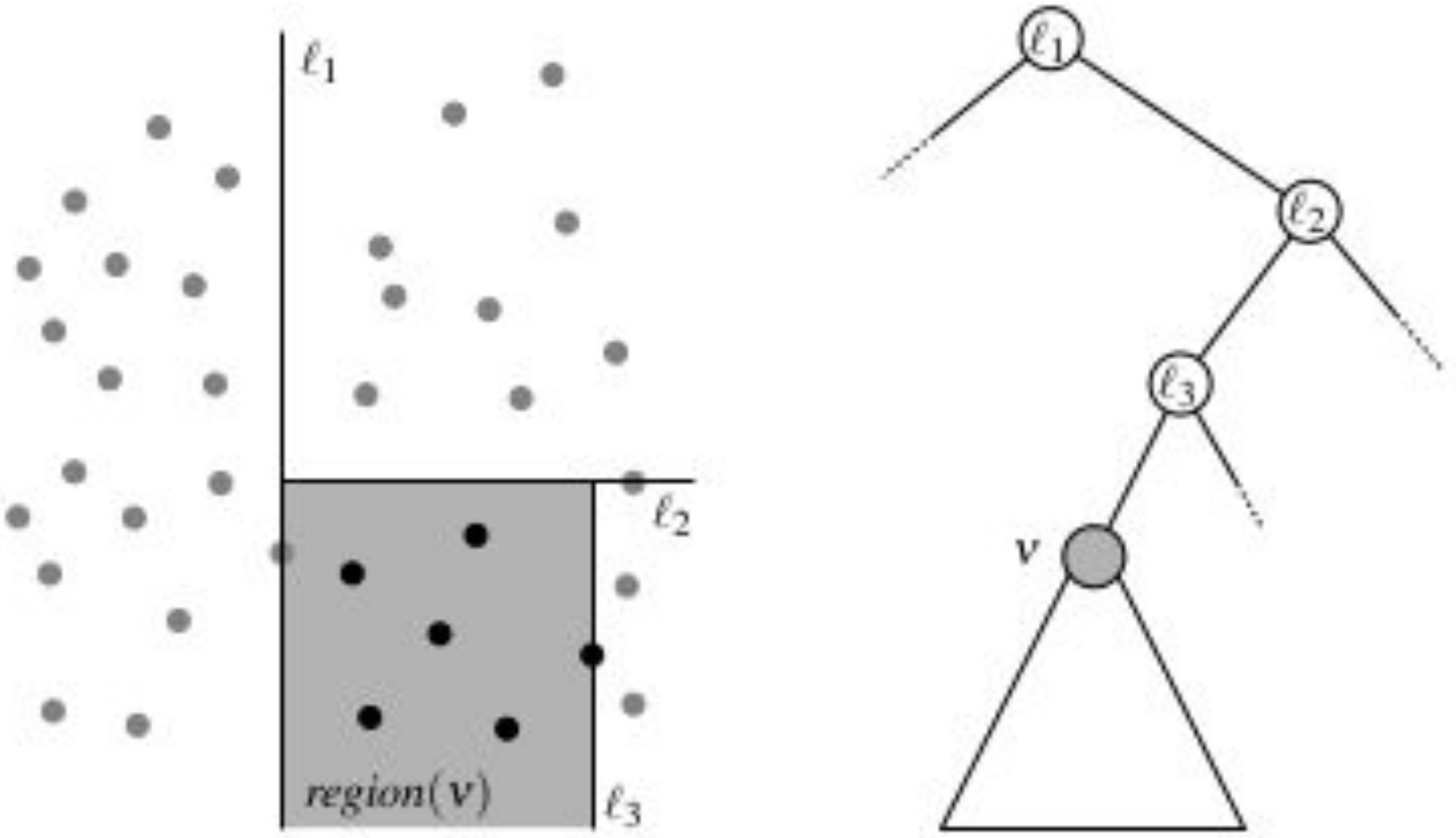
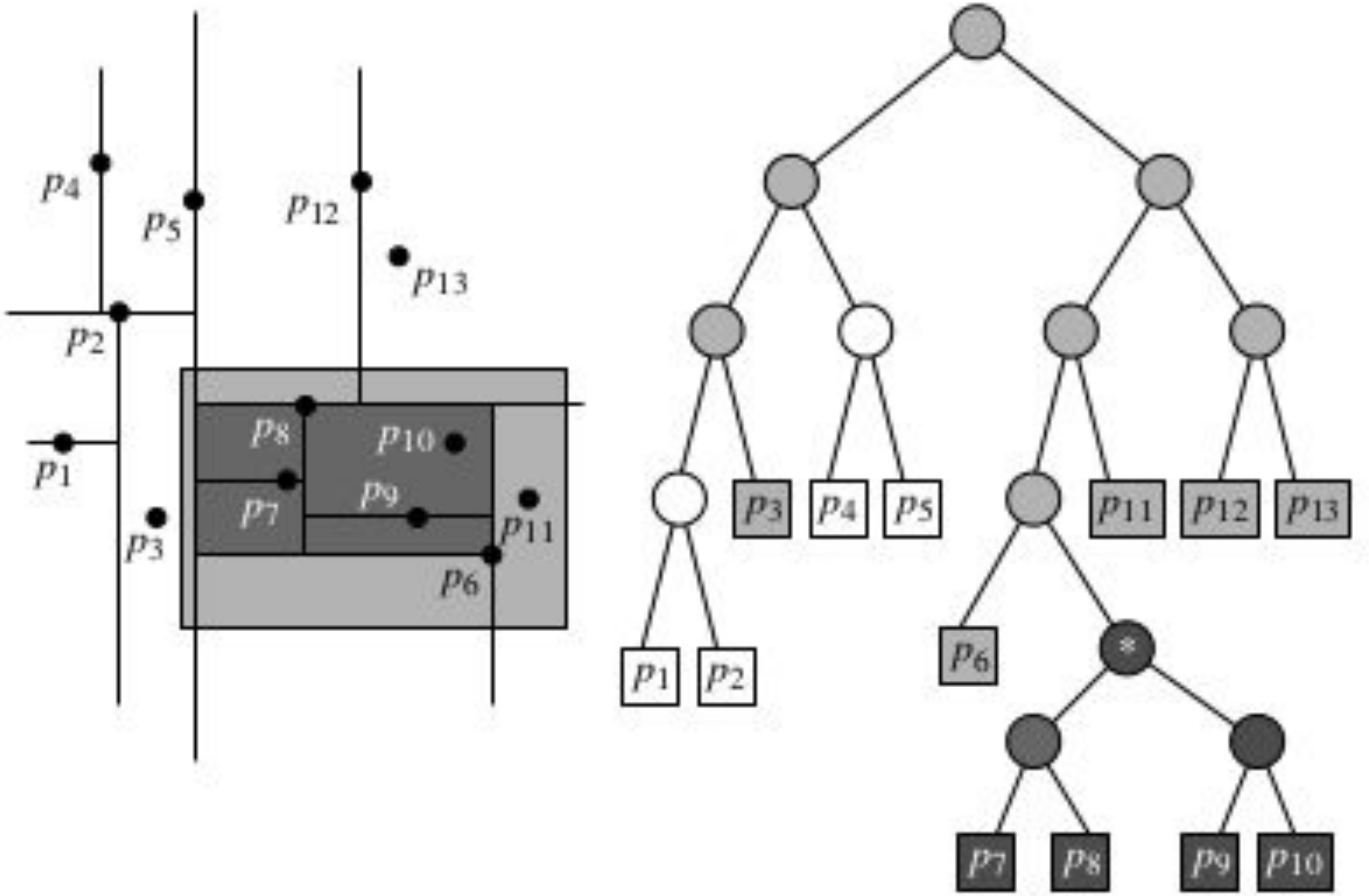



*Figure 5.4*
'orrespondence between nodes in a kd-tree and regions in the plane

# kd-trees




**Section 5.2**
KD-TREES

*Figure 5.5*
A query on a kd-tree

# kd-trees

**Theorem 5.5** *A kd-tree for a set $P$ of $n$ points in the plane uses $O(n)$ storage and can be built in $O(n \log n)$ time. A rectangular range query on the kd-tree takes $O(\sqrt{n}+k)$ time, where $k$ is the number of reported points.*

## Exercises (100 points is the perfect score. You can exceed the perfect score by doing the additional exercises)

- (50 points) Find the closest pair of points in a set of N points using quadtrees.
- (50 points) And using a two dimensional kd-tree.
- (100 points) How many pizzerias are in miraflores?