Analysis of various Ankle Taping Methods

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Abstract

Inversion ankle sprains are a common type of injury for athletes. The BYU Exercise Science department conducted an experiment of 16 subjects measuring ankle inversion for four different treatments. The goals of the experiment are to find which treatment allows for the least inversion and how well the treatments hold up after exercise. We propose a random coefficients (RC) model with the treatment and exercise as fixed effects. The subjects were treated as random effects to capture the dependence within subjects. We will then use tests of significance to evaluate which of the four treatments is best and how they hold up after an hour of exercise. Based on the results of the analysis, we conclude that the Tape and Brace method is optimal to reduce inversions and that it holds up the best after exercise.

1 Introduction

Sprained ankles are a common problem among athletes. Inversion ankle sprains are the most common and occur when the ankle turns too far inward. The BYU Exercise Science department has a machine that can cause unanticipated ankle inversions without creating ankle injuries. This way, the degree of ankle inversion can be measured and compared. Exercise scientists are interested in what kind of ankle taping methods affect the severity of inversion. They are also interested in what kind of effect exercise has on inversion severity. An experiment was performed on sixteen subjects with four different ankle treatments (tape-cast, air-cast, tape and brace, and a control). Ankle inversion was measured before and after exercise.

Researchers desire to know what ankle treatment (or taping method) is best. In this analysis, we will assess (1) what treatment allows for the least inversion and (2) how well the taping methods hold up during exercise. These will be addressed by the use of a Random Coefficients (RC) model on the experimental results. Random Coefficient models are mixed models that allow the use of both fixed and random effects. By treating the subjects as random effects, we will be able to control for each subject and make inferences on the effects of taping technique and exercise.

2 Data

Before exercising, sixteen subjects were given each of the four different ankle treatments. They were all hooked up to the machine and five different measurements of ankle inversion were taken for each treatment. All of the subjects then exercised for an hour. After the exercise, ankle inversion was again measured five times per treatment for all subjects. Thus, there were a total of 40 measurements per subject, with a total of 640 measures in the experiment. The response variable is maximum ankle inversion. These are negative, with values away from zero being worse than values closer to zero. The time period was recorded as a 0 (before exercise) or 1 (after 1hr exercise).

A visual of the data is shown in Figure 1. A slight downward trend can be seen after exercise for each treatment, although some subjects appear to go against that trend. The subjects start in several different places for the four treatments, and for a single subject, the effect of exercise on max inversion is not consistent across treatments. A random coefficients model will allow us to capture these sources of variability. Using this model, we can think of the effect between time periods as a slope, the starting inversion value of each treatment before exercise as an intercept, and the subjects as being random effects.

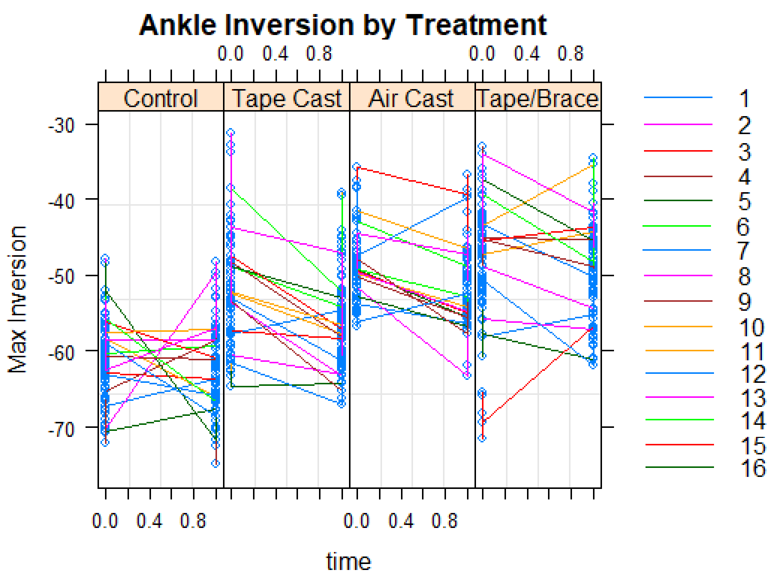

Figure 1: This plot shows each of the sixteen subjects across all four treatments. The line indicates the average effect before (time 0) and after (time 1) exercise for the subjects.

3 Methods

We will proceed with the Random Coefficients model mentioned earlier. We will include a treatment term and treatment:time interaction as fixed effects for later inference questions. The response variable was ankle inversion. In Figure 1, each subject's ankle inversions can behave differently after exercise across treatments. This can be seen from the different slopes/intercepts for any single subject. It is for this reason we will include a random slope and intercept for each treatment:subject interaction. There could also be an inherent subject effect that carries across the treatments. We have included a random intercept for each subject to capture this effect. We do not believe there is independence between treatments for a single subject, which would have happened if we had only included the treatment:subject interaction as a random effect.

In order to justify the random effects chosen earlier, we considered models with several different combinations of random effects, and found the model with lowest AIC/BIC. The results are shown in Table 1 with the lowest AIC/BIC values in bold. The notation (1 | tmt:subj) indicates fitting a random intercept for each treatment subject interaction and (time | subject) indicates fitting a random slope and intercept for each subject. Since the random effects chosen earlier had the lowest AIC/BIC, we included those effects in the final model.

Table 1: Random Effect Justification

	(time | subj)	(1 | tmt:subj)	(time | subj:tmt)	(1 | subject) + (time | subj:tmt)	(time | subject) + (time | subj:tmt)
AIC	3915.662	3858.861	3786.154	**3773.339**	3774.887
BIC	3969.2	3903.476	3839.691	**3831.338**	3841.809

Thus, the mixed model we used to fit the data is as follows:

$$\begin{aligned} Y =& \boldsymbol{X\beta} + \boldsymbol{Zu} + \boldsymbol{\epsilon} \quad \text{where} \\ \boldsymbol{u} \sim& N(0, \boldsymbol{G}), \quad \epsilon \sim N(0, \boldsymbol{R}) \end{aligned} \tag{1}$$

Where **X** is a 640x8 design matrix with the treatment and treatment:time interaction effects on the columns, and β is a 8x1 vector of the fixed effects. **Y** is a 640x1 vector of the Ankle Inversions, or the response variable. **Z** has columns indicating random effects for an overall subject effect, an intercept for each subject/treatment combination, and a slope for each subject/treatment combination. There are 64 combinations and 16 subjects, so Z ends up being a 640x144 matrix. The vector **u** represents the 144 random effects and **G** captures the correlation structure behind them. This can be created using the variance estimates of the RC model structure. **R**$= \sigma^2 I$ represents the residual error for all measurements.

In order to use the RC model, assumptions of linearity, independence, normality, and equal variance need to be met. Since all of the explanatory variables are categorical, we will assume linearity is satisfied. Since we have introduced random effects into the model, each of the 640 measurements will not be independent. Thus, we will decorrelate the residuals by left multiplying by the inverse of the lower Cholesky decomposition of the covariance matrix of $\mathbf{Y}$. The covariance matrix of Y was calculated by $\mathbf{ZGZ}' + \mathbf{R}$.

Since there is no obvious pattern between the Cholesky residuals and fitted values on the plot in Figure 2, we will assume that after taking out the correlation structure of the data, the residuals are independent. The normal QQ plot shows that the tails may be a little lighter/heavier than usual, but overall it's not unreasonable to assume normality. Lastly, we must show equal variance from the decorrelated residuals. Although it looks like the variance might be funneling out in the far right plot, the data are sparse for lower fitted values, so we do not have as much information on the variability of lower fitted values. However, most of the points are contained within 2 standardized residuals, with outliers occurring for various predicted ankle inversions. We will assume equal variance and thus conclude the RC model fits the assumptions.

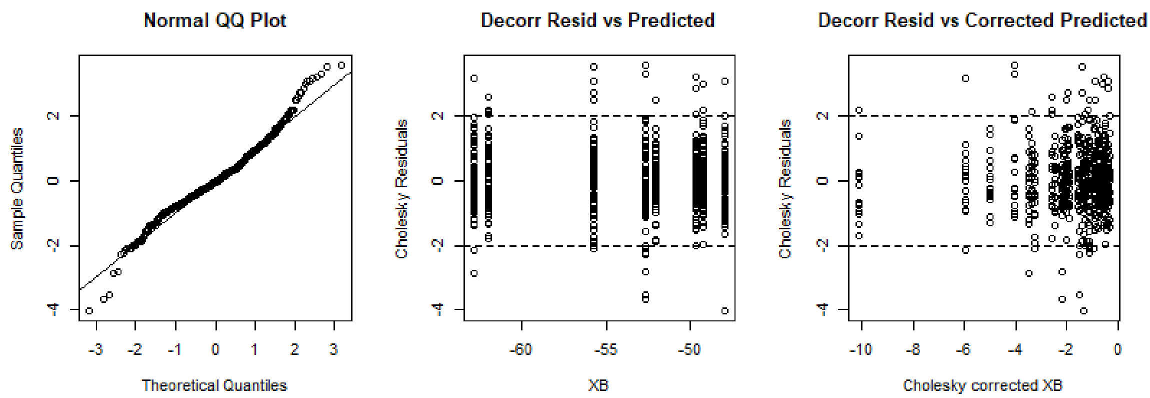

Figure 2: Assumption checking. These plots are for the decorrelated residuals. The rightmost plot contains the corrected residuals

4 Results

The estimated fixed effects for the model are shown in Table 2. The first four effects in the table are the ankle inversion estimates of each treatment before exercise. It appears that the control group had the worst inversion. The last four terms correspond to the slopes or effect of a specific treatment on ankle inversion after exercise. For example, we expect an change of -0.873 in ankle inversion after exercises for the control group, while being 95% confident that the true post-exercise effect is between [-3.288, 1.480]. The standard deviations and confidence intervals of the random effects are shown in Table 4. These explain the different sources of variability in the experiment.

To address what treatment allows for the least inversion (or inversion value closest to zero), we will do F-tests on the coefficients of the fixed effects. Clearly the control group had the worst inversion since the upper confidence bound was not contained in any other treatment interval. When testing the hypothesis that all 3 pre-exercise non-control treatments are the same, a p-value of 0.0039 was obtained using 488 as the denominator degrees of freedom (this is the total size of the data, minus the number of fixed effects, minus the number of columns in $\mathbf{Z}$), suggesting that some non-control treatments have different effects. Testing the hypothesis that treatments 2 and 3 are the same yields a p-value of 0.018, but comparing 3 and 4 yields a p-value of 0.386. Thus we would conclude that all non-control treatments are better than the control and treatments 3 and 4 have the least inversion. While treatment 2 is better than the control, it's not the best treatment.

To determine which taping measures hold the best under exercise, we will examine the confidence intervals for the exercise effects. If the intervals are below zero, exercise has a significant worsening effect for that treatment group. If they contain zero, exercise does not have a significant impact on inversion and if they are above zero, exercise tends to improve inversion. The control group and 4th treatment's exercise effect confidence intervals contain zero, so we would conclude these hold up best under exercise. The 2nd and 3rd treatment tend to get worse after exercise. A visual of the fixed effects is shown in Figure 3.

Table 2: Fixed effects with 95% confidence intervals

(#) Treatment	2.5 %	Estimate	97.5 %
(1) Control	-64.321	-61.881	-59.441
(2) Tape-Cast	-55.119	-52.679	-50.239
(3) Air-cast	-51.683	-49.244	-46.804
(4) Tape & Brace	-50.425	-47.985	-45.545
(1) Control-Exercise	-3.227	-0.873	1.480
(2) Tape-Exercise	-5.401	-3.048	-0.694
(3) Air-Exercise	-5.224	-2.870	-0.517
(4) Tape/Brace-Exercise	-4.046	-1.692	0.661

Table 3: Random Effects

	2.5 %	Estimate	97.5 %
Tmt/Subj Intercept	2.783	3.723	4.613
Tmt/Subj Slope	3.195	4.243	5.146
Int/Slope Correlation	-0.796	-0.633	-0.374
Subject Intercept	1.768	2.955	4.559
Residual	3.608	3.832	4.079

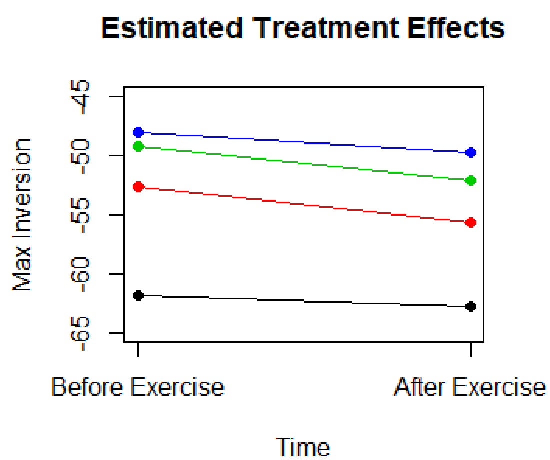

Figure 3: Estimated Inversions for each ankle treatment. Treatments 2, 3, and 4 (red, green, and blue respectively) are compared against the control group (black)

5 Discussion

Based on the results of the study, we would conclude that treatments 3 and 4 (Air-cast and tape & brace) allow for the least ankle inversion. Although not as good as the previously mentioned treatments, treatment 2 (Tape-Cast) still performed significantly better than the control group before exercise. Treatment 4 and the control held up well after exercise, while treatments 2 and 3 had slightly worse inversions after exercise. Treatment 4 not only allowed for the least inversion before exercise, but it also held up just as well as a control group after exercise. Thus, we would recommend the 4th treatment, Tape and Brace, to reduce ankle inversion.

One shortcoming of this experiment is that we only considered subjects after 1 hour of exercise. While extrapolation or interpolation could be used for other time periods, it is unclear if the ankle taping methods hold up in a linear fashion as time increases. It might be useful to experiment at different exercise time intervals so the results can be generalized for any amount of exercise. Although treatment 4 seems to be the best ankle taping method, this was only tested on 16 subjects and against 3 other treatments. Having more subjects or a greater variety of treatments will further our confidence that this treatment is best.