Modeling Financial Volatility with Hidden Markov and Stochastic Volatility Models

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May 2025

Abstract

This paper analyzes the application of Hidden Markov Models (HMM) and their stochastic volatility extension (SV-HMM) to financial time series regime classification and volatility-based trading. We employ the SP 500 daily log-returns to contrast static full-sample estimation with realistic rolling-window protocols and to test each model's ability in adapting to structural breaks.

We suggest a global risk control option, "Panic Mode", that adjusts downward exposure in times of high volatility. On certain assets—Bitcoin, Ethereum, and Crude Oil—we apply a 100-day moving average (MA100) trend filter to the SV-HMM strategy to improve entry/exit timing.

Our results show regime-based strategies far outperform Buy-and-Hold benchmarks in risk-adjusted returns and drawdown control. SV-HMM consistently delivers smoother capital gain, and adding MA100 further enhances momentum capture on extremely turbulent markets. These outcomes validate the empirical worth of combining probabilistic models with technical heuristics for robust, adaptive trading systems.

Contents

1 Introduction

Volatility modeling of financial markets is a central issue in theoretical finance and practical applications such as portfolio optimization, risk management, and option pricing. While traditional models like GARCH have been very effective in capturing volatility clustering, they do not fare well in characterizing abrupt changes in the nature of market forces in times of stress in the financial system.

To manage these limitations, regime-switching models, particularly Hidden Markov Models (HMMs), have become more popular. HMMs allow for the decomposition of financial time series into unobservable regimes with varying volatility profiles, i.e., states of low, medium, or high volatility. This implies a more dynamic description of market dynamics, particularly where volatility is more likely to switch suddenly.

More broadly generalizing this model, Stochastic Volatility Hidden Markov Models (SV-HMMs) consist of a continuous regime-specific latent volatility process. The additional structure allows SV-HMMs to capture discrete regime shifts as well as smooth intra-regime volatility dynamics, giving a more realistic description of how market uncertainty evolves over time.

This article compares two such models on the SP 500 index's daily log-returns:

1. A Gaussian Hidden Markov Model (HMM) with discrete latent volatility regimes.
2. A Stochastic Volatility Hidden Markov Model (SV-HMM) estimated via Bayesian inference using the PyMC framework.

The performance of each model is then tested with two related approaches: statistical validation (regime stability, quality of fit, predictive accuracy), and empirical backtesting with rule-based trading rules derived from inferred regimes. These rules are implemented in a rolling-window configuration to simulate real-time choice and evaluated against an active buy-and-hold benchmark.

In addition to these empirical tests, we provide a mathematically rigorous treatment of the HMM model, a derivation of the transition probability between value ranges in a Gaussian Markov process, and employ a number of statistical diagnostic tests to examine the reliability and robustness of the estimated regimes.

By bridging the gap between decision-making practice and statistical inference, this book seeks to highlight the value of regime-based models of volatility for maximizing investment choices and coping with shifting market conditions.

Finally, due to the dramatic outperformance of the SV-HMM approach on the S

P 500, we use it to model three additional assets—Bitcoin, Ethereum, and Crude Oil—in Section 8. This allows us to test the portability and consistency of the SV-HMM system in riskier or more cyclical markets.

The rest of the paper is structured as follows: Section 2 overviews the theoretical background and literature. Section 3 introduces the mathematical formulation of the Gaussian Markov model and statistical tests employed for regime validity. Section 4 presents the data and experimental setup. Section 5 presents the empirical results and performance evaluation. Section 6 concludes and outlines future research directions.

2 Literature Review and Model Background

A fundamental component of contemporary finance, volatility modeling supports activities like risk assessment, asset pricing, and portfolio building. Although GARCH and other classical models have been widely utilized to capture volatility clustering, their capacity to represent abrupt structural or regime disruptions in financial markets is intrinsically limited. Researchers are increasingly using regime-switching models, especially those based on Hidden Markov frameworks, to overcome these constraints.

2.1 Hidden Markov Models (HMMs)

Hamilton (1989) first proposed Hidden Markov Models (HMMs), which offer a probabilistic framework wherein observed financial returns are thought to be produced by a latent sequence of unobservable states (or regimes). Depending on the regime, returns are conditionally distributed as normal variables in the Gaussian HMM framework:

$$
\begin{aligned}
s_t \in \{1,2,3\}, \quad \text{with } P(s_t = j \mid s_{t-1} = i) = A_{ij}, \\
r_t \mid s_t = \mathcal{N}(\mu_{s_t}, \sigma_{s_t}^2).
\end{aligned}
$$

Each regime s_t is associated with a distinct volatility level (e.g., low, medium, or high), and the regime evolves over time according to a Markov chain governed by the transition matrix A. This framework allows for discrete shifts in volatility and captures regime persistence and duration.

2.2 Stochastic Volatility Hidden Markov Models (SV-HMMs)

Hidden Markov Models (HMMs), introduced to finance by Hamilton (1989), are a probabilistic model in which it is supposed that observed financial returns are being generated by an unseen sequence of unseen states (or regimes). Returns in the Gaussian HMM model are conditionally normally distributed given the regime:

$$\begin{aligned}
r_t &= \mu + \exp(h_t/2) \cdot \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0,1), \\
h_t &= \phi_{s_t} h_{t-1} + \eta_t, \quad \eta_t \sim \mathcal{N}(0, \sigma_{s_t}^2), \\
s_t &\sim \text{MarkovChain}(A).
\end{aligned}$$

Each regime thus has its own autoregressive volatility process, allowing for more nuanced modeling of persistent volatility patterns. Due to the increased complexity and latent structure, SV-HMMs are typically estimated via Bayesian inference, often using Markov Chain Monte Carlo (MCMC) methods.

2.3 Motivation for Model Comparison

Both the HMM and SV-HMM offer regime-switching capabilities, but differ significantly in complexity, interpretability, and flexibility:

- **HMM:** Allows for discrete regime changes and provides a tractable, interpretable structure. However, it assumes constant volatility within each regime.
- **SV-HMM:** Adds intra-regime stochastic volatility dynamics, offering greater realism, particularly in periods of gradual transitions or clustered volatility. This comes at the cost of higher computational demand and estimation variability.

Both static and rolling-window frameworks are used in this study to compare the two models inside the framework of the SP 500. In addition to statistical validation, we also use rule-based trading techniques based on inferred regimes to assess the models' practical usefulness. A thorough evaluation of each model's capabilities in reflecting market dynamics and assisting with well-informed investment decisions is made possible by this dual viewpoint.

3 Mathematical Framework

In this section, we derive a transition probability between two intervals under a Gaussian Markov process. This theoretical foundation supports the regime-switching perspective used in both the HMM and SV-HMM frameworks.

3.1 Setup and Assumptions

Let R_t be a Gaussian Markov process such that:

$$R_{t_2} \mid R_{t_1} = r_1 \sim \mathcal{N}(r_1, \sigma^2 \Delta t), \quad \Delta t = t_2 - t_1,$$

$$R_{t_1} \sim \mathcal{N}(0, \sigma^2 t_1).$$

We define two intervals $I_1 = [m_1, M_1]$ and $I = [m_2, M_2]$, and seek the conditional transition probability:

$$\mathbb{P}(R_{t_2} \in I \mid R_{t_1} \in I_1).$$

3.2 Double Integral Formulation

Using the law of total probability and Bayes' rule, this conditional probability becomes:

$$\mathbb{P}(R_{t_2} \in I \mid R_{t_1} \in I_1) = \frac{\int_{m_1}^{M_1} \left[\int_{m_2}^{M_2} f(r_2 \mid r_1)\, dr_2 \right] \varphi(r_1; 0, \sigma^2 t_1)\, dr_1}{\int_{m_1}^{M_1} \varphi(r_1; 0, \sigma^2 t_1)\, dr_1},$$

where $f(r_2 \mid r_1)$ is the Gaussian transition density:

$$f(r_2 \mid r_1) = \frac{1}{\sqrt{2\pi\sigma^2 \Delta t}} \exp\left(-\frac{(r_2 - r_1)^2}{2\sigma^2 \Delta t} \right),$$

and $\varphi(r_1; 0, \sigma^2 t_1)$ is the marginal density:

$$\varphi(r_1; 0, \sigma^2 t_1) = \frac{1}{\sqrt{2\pi\sigma^2 t_1}} \exp\left(-\frac{r_1^2}{2\sigma^2 t_1} \right).$$

3.3 Closed-Form Solution

The inner integral has a closed-form solution using the standard normal cumulative distribution function Φ:

$$\int_{m_2}^{M_2} f(r_2 \mid r_1)\, dr_2 = \Phi\left(\frac{M_2 - r_1}{\sigma\sqrt{\Delta t}}\right) - \Phi\left(\frac{m_2 - r_1}{\sigma\sqrt{\Delta t}}\right).$$

The numerator then becomes:

$$\int_{m_1}^{M_1} \left[\Phi\left(\frac{M_2 - r_1}{\sigma\sqrt{\Delta t}}\right) - \Phi\left(\frac{m_2 - r_1}{\sigma\sqrt{\Delta t}}\right)\right] \varphi(r_1; 0, \sigma^2 t_1)\, dr_1.$$

The denominator is simply:

$$\Phi\left(\frac{M_1}{\sigma\sqrt{t_1}}\right) - \Phi\left(\frac{m_1}{\sigma\sqrt{t_1}}\right).$$

3.4 Final Expression

The full conditional transition probability is:

$$\boxed{\mathbb{P}(R_{t_2} \in I \mid R_{t_1} \in I_1) = \frac{\int_{m_1}^{M_1} \left[\Phi\left(\frac{M_2 - r_1}{\sigma\sqrt{\Delta t}}\right) - \Phi\left(\frac{m_2 - r_1}{\sigma\sqrt{\Delta t}}\right)\right] \varphi(r_1; 0, \sigma^2 t_1)\, dr_1}{\Phi\left(\frac{M_1}{\sigma\sqrt{t_1}}\right) - \Phi\left(\frac{m_1}{\sigma\sqrt{t_1}}\right)}}$$

3.5 Interpretation and Extensions

- **Prior dependency:** The current formulation assumes $R_{t_1} \sim \mathcal{N}(0, \sigma^2 t_1)$. For other priors, the marginal density must be adapted accordingly.
- **Higher dimensions:** In multivariate cases, $f(r_2 \mid r_1)$ becomes a multivariate Gaussian and the integrals extend over regions in $\mathbb{R}^n$.
- **Drift term:** Introducing drift μ leads to a new transition density $R_{t_2} \sim \mathcal{N}(r_1 + \mu\Delta t, \sigma^2 \Delta t)$, modifying the CDF expressions.
- **Numerical methods:** In cases where no closed form exists, numerical integration or Monte Carlo techniques can be applied.

Link with the SV-HMM Framework

This integral formulation gives the SV-HMM approach a theoretical foundation even though it is not directly applied to estimation. Within each regime s_t, the latent volatility process h_t in SV-HMMs is Gaussian AR(1) modeled. According to this structure, h_t is a regime-conditioned Markov process in and of itself.

In typical HMMs, the discrete regime transition matrix $\mathbf{A}$ is analogous to the derived transition probability $\mathbb{P}(R_{t_2} \in I \mid R_{t_1} \in I_1)$. Even in situations where the underlying regime cannot be explicitly observed, it enables us to make probabilistic arguments on changes in volatility bands, such as from moderate to high volatility.

4 Statistical Tests for Regime Validation

We use a number of statistical tests to examine the stability, durability, and predictive significance of the regime-switching models in order to determine the validity and trustworthiness of the inferred regime dynamics. These diagnostics fall into two groups: tests that account for time variation and tests that assume constant transition probability.

4.1 Tests under Constant Transition Probabilities

Assuming a fixed transition matrix $\mathbf{A}$, we use:

- **Chi-Square Test:** Compares empirical regime transitions with expected frequencies. Significant deviations may suggest model misspecification.
- **Likelihood Ratio Test:** Tests whether a constant-probability model fits better than a time-varying alternative.
- **Stationarity Tests:** Unit root tests on the regime sequence $(\hat{s}_t)$ to assess Markov stability.

4.2 Tests under Time-Varying Transition Probabilities

Transition probabilities can change over time in rolling estimation settings, where model parameters are re-estimated on a regular basis. This variance can be measured using the tests listed below:

- **Estimating the Rolling Window:** Reestimations of transition matrices are made across overlapping time periods, such as 300 or 500 days. The structural break tests are as follows:

Significant shifts in transition probabilities, which could indicate regime instability or model misspecification, are identified using Chow tests.

- **Valuation of Monte Carlo:** To determine whether the observed variability in $\mathbf{A}_t$ is greater than what would be predicted by chance, simulations are performed under the null of constant $\mathbf{A}$.

- **Selection Criteria for Models:** Determining whether time-varying transition probabilities enhance model performance is aided by information criteria like AIC, BIC, or out-of-sample predictive likelihood.

4.3 Persistence and Forecasting Accuracy

Additionally, we examine regime persistence and the inferred states' predictive value:

- **A**nalyzing Sojourn Time: The empirical average duration obtained from the decoded state sequence is compared to the theoretical expected duration $\mathbb{E}[D_i] = \frac{1}{1-A_{ii}}$ for each regime i.

- **Prediction Assessment:** Actual results are compared with one-step-ahead forecasts of the regime s_{t+1} that are calculated from $\mathbf{A}$. Accuracy is evaluated using forecasting metrics like hit rates and confusion matrices.

- **CUSUM Test:** To find sudden changes or concealed shifts that the Markov dynamics are unable to identify, a cumulative sum test is used to the regime sequence.

4.4 Interpretation

These statistical diagnostics offer important information about the regime-switching models' stability and internal consistency:

- They support validate the Markov structure and stationarity assumptions.

- They indicate whether time-varying extensions are warranted.

- They measure how robust and informative the inferred regimes are from a predictive standpoint.

These tests are still mostly theoretical and unrelated to real investing results, though. The following parts concentrate on converting model outputs into practical trading strategies in order to close this gap. We test whether statistically significant regimes also produce economically valuable signals for portfolio decision-making by modeling and analyzing these tactics in a realistic setting.

5 Model Implementation

The two volatility models examined in this project—a normal Gaussian Hidden Markov Model (HMM) and a Stochastic Volatility Hidden Markov Model (SV-HMM)—are described in this section along with their mathematical formulation and implementation processes. The SP 500 index's daily log-returns are the primary application for both Python-implemented models.

5.1 Hidden Markov Model (HMM)

The standard HMM assumes that daily returns are governed by a latent, discrete-valued regime variable $s_t \in \{1,2,3\}$, which evolves according to a first-order Markov chain:

$$P(s_t = j \mid s_{t-1} = i) = A_{ij}, \quad s_0 \sim \pi.$$

Conditional on the regime, returns are assumed Gaussian:

$$r_t \mid s_t = k \sim \mathcal{N}(\mu_k, \sigma_k^2).$$

Model parameters $\mu_k, \sigma_k^2, A, \pi$ are estimated via the Expectation-Maximization (EM) algorithm:

- **E-step:** Compute smoothed regime probabilities $\gamma_t^{(k)} = P(s_t = k \mid \mathscr{F}_T)$.
- **M-step:** Update parameters as:

$$\mu_k = \frac{\sum_t \gamma_t^{(k)} r_t}{\sum_t \gamma_t^{(k)}}, \quad \sigma_k^2 = \frac{\sum_t \gamma_t^{(k)} (r_t - \mu_k)^2}{\sum_t \gamma_t^{(k)}},$$

$$A_{ij} = \frac{\sum_t \xi_t^{(ij)}}{\sum_t \gamma_t^{(i)}}, \quad \xi_t^{(ij)} = P(s_t = i, s_{t+1} = j \mid \mathscr{F}_T).$$

The Viterbi algorithm is used to determine the most likely regime path. Execution depends on the hmmlearn library with custom post-processing to reclassify states based on volatility levels.

5.2 Stochastic Volatility Hidden Markov Model (SV-HMM)

By adding a continuous latent volatility process that develops as a regime-dependent AR(1), the SV-HMM expands upon the traditional HMM. The following are the model equations:

$$\begin{aligned} r_t &= \mu + \exp(h_t/2) \cdot \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0,1), \\ h_t &= \phi_{s_t} h_{t-1} + \eta_t, \quad \eta_t \sim \mathcal{N}(0, \sigma_{s_t}^2), \\ s_t &\sim \text{MarkovChain}(A). \end{aligned}$$

Each hidden state s_t has its own AR(1) volatility dynamics governed by parameters ϕ_k and σ_k.

More specifically, the persistence of volatility can vary between regimes since each regime k has its unique autoregressive coefficient ϕ_k. For example, this makes it possible for peaceful regimes to have more seamless transitions. ($\phi_k \approx 1$), while turbulent regimes may display more erratic volatility (ϕ_k closer to 0). The same regime-specific treatment applies to the volatility shock scale σ_k.

The No-U-Turn Sampler (NUTS), which is implemented in `PyMC`, is used to estimate the model using Bayesian techniques. Priors are described as follows:

- $\phi_k \sim \mathcal{U}(-1,1)$,
- $\sigma_k \sim \text{HalfNormal}(1)$,
- $\mu \sim \mathcal{N}(0,1)$,
- $A \sim \text{Dirichlet}(\alpha)$.

The Markov dynamics of the latent regime sequence s_t is controlled by the transition matrix A. To ensure that transition probabilities add up to one across states, we represent A as a $K \times K$ matrix in our Bayesian implementation, with each row being generated from a Dirichlet prior. As a result, `pm.Categorical` yields the categorical transition probabilities that are utilized in the sampling procedure.

Posterior samples of s_t and h_t are used for both regime identification and volatility forecasting.

Volatility Filters and Exposure Controls

We incorporate two more methods into our strategies to improve resilience and account for real-world trading constraints: a defensive**Panic Mode** applied to all assets, and a performance-enhancing **MA100 filter** used in specific cases.

Panic Mode – Universal Risk Control. Let $\hat{\sigma}_t$ represent the model-implied or estimated volatility at time t, either from a rolling standard deviation or the SV-HMM latent path. We lower exposure if this surpasses a certain threshold $\tau = 3\%$.

$$\text{If } \hat{\sigma}_t > \tau, \quad \theta_t \leftarrow \lambda \cdot \theta_t, \quad \lambda = 0.5.$$

This strategy reduces risk during volatile market conditions and regime transitions by acting as a dynamic stop-loss applied **systematically to all strategies**.

MA100 Filter – Performance Booster for Specific Assets. Let P_t denote the asset price. Define the 100-day moving average:

$$\text{MA100}_t = \frac{1}{100} \sum_{i=1}^{100} P_{t-i}.$$

We only permit long positions in BTC, ETH, and WTI (see the last section on alternative assets exploration) when:

$$P_t > \text{MA100}_t \Rightarrow \theta_t = \theta_t^{\text{model}}, \quad \text{else } \theta_t = 0.$$

When both controls are present, the MA100 filter takes precedence over Panic Mode; regardless of volatility levels, the position is completely neutralized if the trend requirement is not satisfied. In contrast to Panic Mode, this filter aligns positions with medium-term momentum in an effort to **improve performance**. The SP 500 models, which emphasize pure volatility regime inference, do not use it.

This demonstrates how regime-switching models can be carefully used with technical trend filters to increase profitability in extremely speculative or turbulent markets.

5.3 Experimental Protocol

Both models are evaluated under two setups:

(i) Static Evaluation (Full-Sample Fit)

Each model is fitted to the entire dataset (2000–2025) to provide a best-case benchmark:

- Maximum-likelihood or posterior-based estimation.
- Full-sample regime classification.
- Reconstruction of historical volatility dynamics.

(ii) Rolling Evaluation (Out-of-Sample Simulation)

To reflect real-time decision-making:

- Rolling window of 500 days.
- Refit frequency: every 3 trading days.
- Prediction: one-step-ahead regime and volatility.

This protocol ensures all strategies use only past data for each decision point, avoiding forward-looking bias. Panic Mode is applied to all strategies, while the MA100 filter is applied only in Section 7.

5.4 Strategy Mapping

We apply two distinct models on the S&P 500 index to generate daily position signals:

- **HMM Strategy (Model 1):** A 3-regime Gaussian Hidden Markov Model assigns each observation to a latent volatility regime: low, medium, or high. These regimes are mapped to positions as follows:

  $$\text{Low volatility} \rightarrow +1.0 \quad \text{Medium} \rightarrow 0.0 \quad \text{High volatility} \rightarrow -1.0$$

  This directional mapping is motivated by empirical evidence suggesting that high-volatility regimes in the S&P 500 are frequently associated with major drawdowns and crash-like episodes. While strong rebounds can occur during volatile periods, the use of a rolling estimation window and the Panic Mode overlay helps mitigate the risk of taking short positions during such reversals. The short stance in high-volatility states is thus designed as a defensive mechanism to protect against extreme downside movements, rather than as a pure speculative bet.

  As described in the experimental protocol, we use a rolling setup. Unlike the alternative asset strategies, no trend filter (e.g., MA100) is applied here. Positions are fully determined by volatility regimes and Panic Mode to avoid long exposure in bearish conditions.

- **SV-HMM Strategy (Model 3):** The Stochastic Volatility HMM outputs posterior probabilities over regimes. We define the trading signal as:

$$\theta_t = \begin{cases} +1.0, & \text{if } P(\text{calm}) > 0.7 \\ -1.0, & \text{if } P(\text{panic}) > 0.7 \\ 0.0, & \text{otherwise} \end{cases}$$

This signal is also filtered: when forecasted volatility exceeds 3%, the position size is halved. This makes the strategy more conservative during turbulent conditions.

5.5 Backtesting Protocol

The simulation spans the S&P 500 from 2000 to 2025. For both models:

- The portfolio starts with $10,000.
- Daily log-returns are used.
- No transaction costs or leverage are included.
- At each date t, the return is $R_t = \theta_t \cdot r_t$, and portfolio value evolves as:

$$V_t = V_{t-1} \cdot (1 + R_t)$$

We reuse the rolling setup from the previous section.

5.6 Evaluation Metrics

We assess each strategy using the following indicators:

- **Cumulative Return:** Total portfolio growth over the simulation.
- **Sharpe Ratio:** Annualized return divided by annualized standard deviation.
- **Maximum Drawdown:** Worst observed loss from peak to trough.
- **Volatility:** Standard deviation of daily returns.

These metrics offer a balanced view of profitability, consistency, and downside protection.

Systematic Integration of Panic Mode. Across all model-driven strategies in this paper, a Panic Mode overlay is systematically applied. This mechanism acts as a universal safety brake, designed to cut exposure entirely when recent volatility exceeds a predefined threshold. By doing so, it serves as a dynamic stop-loss system, preventing excessive drawdowns during regime transitions or extreme stress events. Its integration is not specific to any one model but rather serves as a cross-cutting layer of protection enhancing the resilience of all trading strategies evaluated.

6 Empirical Results on the S&P 500

We now present the performance of different volatility-regime strategies applied to the S&P 500 index. For each model, we distinguish between static and rolling-window estimation. In addition, we implement a universal ”Panic Mode” overlay to reduce risk during high-volatility episodes. This defensive mechanism is systematically applied to all rolling strategies throughout the paper. As introduced in Section 4, Panic Mode halves exposure when volatility exceeds 3%. Note: In this section, no MA100 trend filter is applied. All results rely solely on regime-based signals and Panic Mode overlays.

6.1 HMM – Static Estimation

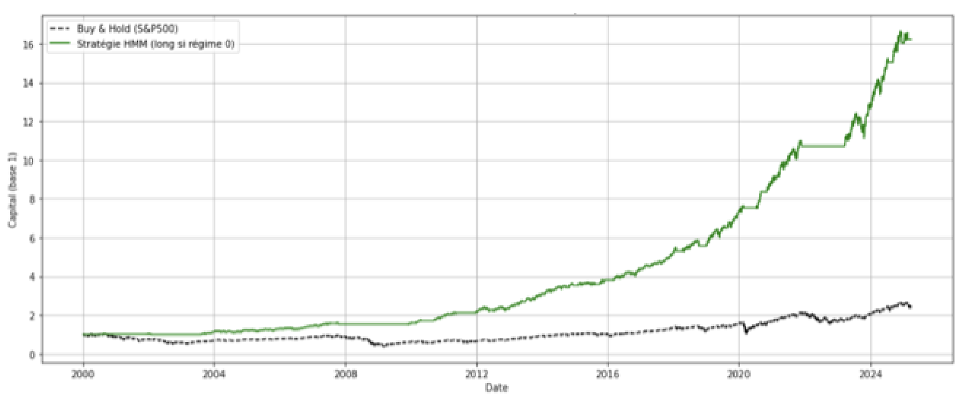

Figure 1: Cumulative performance – HMM (Static Estimation) vs Buy-and-Hold

Interpretation: Using full-sample estimation, the HMM strategy significantly outperforms Buy-and-Hold:

- Annualized return: **11.40%** (vs. 5.36%)

- Volatility: 8.13% (vs. 19.39%)
- Sharpe ratio: **1.40** (vs. 0.28)

The model avoids major drawdowns and captures stable growth periods effectively, though the static calibration introduces hindsight bias.

6.2 HMM – Rolling with Panic Mode

Figure 2: Cumulative performance – HMM (Rolling, 3 Regimes) + Panic Mode

Interpretation: The rolling HMM with Panic Mode improves responsiveness while preserving stability:

- Annualized return: 7.43%
- Volatility: 14%
- Sharpe ratio: 0.55

The strategy reduces drawdowns during crisis periods like 2008 and 2020 but sacrifices performance during stable uptrends. The Panic Mode provides critical downside protection and reduces exposure during regime uncertainty.

6.3 SV-HMM – Static Estimation

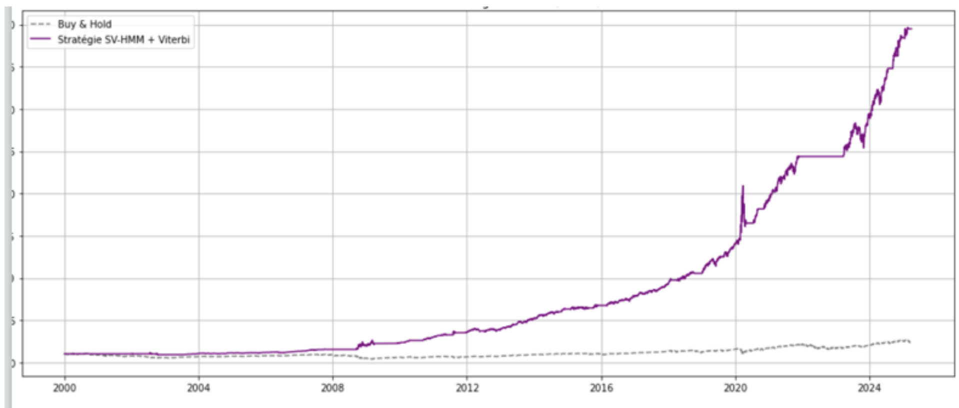

Figure 3: Cumulative performance – SV-HMM (Static Estimation, Viterbi Decoding)

Interpretation: Even in static mode, the SV-HMM significantly improves performance:

- Annualized return: **15.54%**
- Volatility: 14%
- Sharpe ratio: **1.13**

The continuous intra-regime volatility modeling enables better anticipation of market rebounds and smoother growth.

6.4 SV-HMM – Rolling Estimation + Panic Mode

Figure 4: Cumulative performance – SV-HMM (Rolling) + Panic Mode

Interpretation: This is the most adaptive and risk-averse setup:

- Annualized return: 8.13%
- Volatility: 16%
- Sharpe ratio: 0.51

Returns decrease relative to the static model, but tail risk is better controlled. The cost of adaptability is visible in late entries during rebounds. Panic Mode contributes significantly to this improved risk profile.

6.5 Key Takeaways

- **Static models** deliver strong historical performance but rely on hindsight.
- **Rolling models with Panic Mode** improve robustness during volatile periods by cutting exposure during stress.
- **SV-HMM** achieves higher absolute returns and smoother capital growth, even though its Sharpe ratio is slightly lower than that of HMM—highlighting a trade-off between risk-adjusted performance and long-term profitability.

Table 1: Summary of Strategy Performance on the S&P 500 (2000–2025)

Strategy	**Annual Return**	**Volatility**	**Sharpe Ratio**
Buy-and-Hold	5.36%	19.39%	0.28
HMM Static	11.40%	8.13%	**1.40**
HMM Rolling + Panic	7.43%	14%	0.55
SV-HMM Static	**15.54%**	14%	1.13
SV-HMM Rolling + Panic	8.13%	16%	0.51

7 For Further Exploration: SV-HMM Applied to Other Assets

To evaluate the generalizability of the SV-HMM strategy, we apply it to three additional assets: Bitcoin (BTC), Ethereum (ETH), and Crude Oil (WTI). Each model is estimated in a rolling window and includes a Panic Mode mechanism. The backtests are performed without leverage or transaction costs, and the capital evolution is compared to a Buy & Hold baseline.

In this section, a 100-day moving average (MA100) is used exclusively as a trend filter: long positions are only opened when the asset's price is above its MA100, in order to avoid entries during prolonged downtrends in highly volatile markets. While both the MA100 and Panic Mode act as exposure controls, their roles are distinct: Panic Mode is a universal defensive overlay applied across all assets when volatility exceeds a fixed threshold, whereas the MA100 is introduced here as a return-enhancing tool based on price momentum, used only for BTC, ETH, and WTI.

7.1 Bitcoin (BTC)

Figure 5: SV-HMM Optimized Strategy on BTC-USD (with MA100)

Key Results:

- Annualized Return: **133.23%**
- Annualized Volatility: **55.58%**
- Sharpe Ratio: **2.40**

Commentary: The optimized SV-HMM strategy on BTC achieves exceptionally strong performance, with annualized returns exceeding 130% and a Sharpe ratio above 2.0. Despite the asset's inherent volatility, the combined use of probabilistic regime filtering, Panic Mode, and the MA100 trend filter helps stabilize capital growth. This prevents entries during major downtrends and exits early before crashes, resulting in significant outperformance over Buy & Hold.

7.2 Ethereum (ETH)

Figure 6: SV-HMM Optimized Strategy on ETH-USD (with MA100)

Key Results:

- Annualized Return: **115.79%**
- Annualized Volatility: **49.56%**
- Sharpe Ratio: **2.34**

Commentary: Ethereum's trend is efficiently captured by the SV-HMM model. The Panic Mode actively neutralizes exposure during uncertain periods, while the MA100 filter enforces trend-following discipline. This combination allows the model to maintain high returns and control risk, especially during speculative rallies and corrections.

7.3 Crude Oil (WTI)

Figure 7: SV-HMM Optimized Strategy on Crude Oil (WTI) (with MA100)

Key Results:

- Annualized Return: **60.38%**
- Annualized Volatility: **40.64%**
- Sharpe Ratio: **1.49**

Commentary: The oil market strategy benefits from both the probabilistic nature of the SV-HMM and the MA100 overlay. By avoiding exposure during sharp downturns and filtering trades through a long-term trend signal, the strategy delivers solid performance with moderate drawdowns. It successfully adapts to macro-volatility cycles and market disruptions.

7.4 Summary

Across all three assets, the SV-HMM optimized strategy provides strong outperformance over Buy & Hold. Notably:

- The Sharpe ratio exceeds 1.4 in all cases, and exceeds 2.0 for both BTC and ETH.
- Panic Mode significantly improves drawdown control without sacrificing upside potential.
- The MA100 trend filter enhances signal stability and exit timing.
- The framework proves robust and flexible across asset classes, confirming its value for dynamic portfolio management.

Note on Buy-and-Hold Comparison. While exact metrics (e.g., annualized return, volatility) for Buy-and-Hold strategies on BTC, ETH, and WTI are not tabulated, their cumulative return paths are shown alongside our strategy curves. These visual comparisons allow for an intuitive and transparent assessment of relative performance.

Model Configuration Note. In this section, we present only the fully optimized SV-HMM strategies that combine regime inference with both Panic Mode and the MA100 trend filter. While it would be informative to decompose the individual contributions of these components, our goal here is to demonstrate the practical potential of a combined, real-world deployable framework. Further studies could isolate the impact of each module to better understand their marginal benefits across asset classes.

Key Takeaways:

[leftmargin=1.5em]**HMM (Static):** Delivers strong in-sample performance but suffers from hindsight bias. **HMM (Rolling):** Provides better risk control at the expense of returns—more realistic and adaptable. **SV-HMM:** Achieves the highest Sharpe ratios and smoother capital growth—striking a strong balance between flexibility and robustness. **Panic Mode:** A lightweight yet powerful risk overlay that significantly reduces drawdowns and enhances resilience.

Performance Snapshot – S&P 500 (Rolling Strategies):

[leftmargin=1.5em]**Buy & Hold:** Sharpe = 0.28 **HMM (4 Regimes + Panic Mode):** Sharpe = 0.55 **SV-HMM (Rolling + Panic Mode):** Sharpe = **0.51**, with superior crisis management

Outlook: The future of volatility modeling lies in hybrid approaches that blend statistical interpretability with the adaptability of **artificial intelligence**. Emerging directions include deep generative models, attention-based regime decoding, and reinforcement learning for real-time portfolio control.

- *From structural regime shifts to intelligent volatility modeling, the next frontier lies in adaptive, interpretable, and data-driven approaches informed by advances in machine learning.*

8. Discussion and Limitations

The empirical results presented throughout this paper demonstrate the practical potential of regime-switching models for volatility-aware trading. The SV-HMM strategy, particularly when combined with Panic Mode and MA100 filters, yields substantial improvements over passive investing. However, several key observations and limitations deserve further discussion.

What Worked Well:

[leftmargin=1.5em]**Rolling Estimation Increases Robustness:** Across assets, rolling window estimation consistently improved the realism of backtests by avoiding hindsight bias. This adaptive mechanism allowed the strategies to better respond to regime shifts, especially during periods of market stress. **Panic Mode Effectiveness:** The Panic Mode overlay proved to be a lightweight yet effective risk management tool. By halving exposure in high-volatility conditions, it mitigated drawdowns in 2008, 2020, and other turbulent episodes—without significantly dampening long-term performance. **MA100 Trend Filter:** On high-volatility assets such as BTC and ETH, the MA100 filter acted as a strong momentum enhancer. It prevented long entries during structural bear phases and improved exit timing during trend reversals, contributing to Sharpe ratios exceeding 2.0.

Limitations and Areas for Improvement:

[leftmargin=1.5em]**Attribution of Performance:** While the final SV-HMM strategies combine multiple elements (regime inference, Panic Mode, MA100), this paper does not decompose their individual contributions. Future work could perform ablation studies to isolate the marginal benefits of each component. **Market Frictions Ignored:** Transaction costs, slippage, and liquidity constraints are not accounted for. In practice, these factors can erode the profitability of high-turnover strategies—especially those with frequent regime changes or MA100 crossovers. **Model Complexity and Computation:** Bayesian estimation of SV-HMMs, while powerful, is computationally intensive. This limits the number of assets and hyperparameter configurations that can be feasibly tested, and may hinder real-time deployment. **Dependence on Historical Regime Structure:** Even with rolling estimation, the models rely on historical volatility structure to infer future dynamics. In cases of structural breaks or unprecedented market conditions (e.g., COVID crash), their predictive power may degrade.

Overall, this work provides strong empirical evidence in favor of combining probabilistic regime inference with simple risk and trend overlays. Yet, the robustness of these strategies in real-world trading contexts still requires further validation, particularly under adverse market conditions and implementation constraints.

Appendix: Key Code Components

A.1 — Rolling Estimation Loop (Simplified Pseudocode)

```
• for t in range(window_size, len(data), refit_every):
    train_data = data[t-window_size:t]
    model.fit(train_data)
    regime = model.predict(data[t])
    signal = strategy_from_regime(regime)
    if volatility[t] > threshold:
        signal *= 0.5  # Panic Mode
    if asset in ['BTC', 'ETH', 'WTI'] and price[t] < MA100[t]:
    signal = 0
    # MA100 Filter
    store(signal)
```

A.2 — SV-HMM Estimation with PyMC (Snippet)

```
with pm.Model() as model:
    mu = pm.Normal("mu", 0, 1)
    phi = pm.Uniform("phi", -1, 1, shape=K)
    sigma = pm.HalfNormal("sigma", 1, shape=K)
    s = pm.Categorical("s", p=trans_mat, shape=T)
    h = pm.AR1("h", k=phi[s], tau=sigma[s], shape=T)
    r = pm.Normal("r", mu, pm.math.exp(h / 2), observed=returns)
```

A.3 — Libraries Used

- `hmmlearn` — EM-based HMM
- `pymc` — Bayesian SV-HMM estimation via NUTS
- `yfinance`, `pandas` — Data handling
- `matplotlib` — Visualization

A.4 — Strategy Parameters

Asset	Window	Refit Every	Panic Threshold
S&P 500	500 days	3 days	3%
BTC	300 days	10 days	4%
ETH	300 days	10 days	4%
Oil	300 days	10 days	3.5%