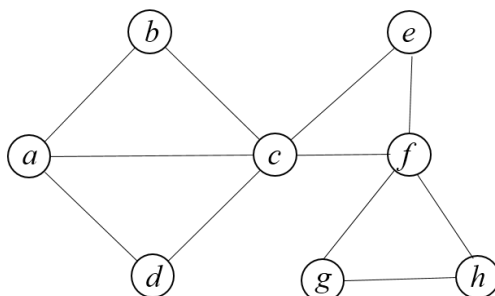


1. **Treewidth.** Give a minimum-width tree decomposition for the following graph.



2. **Closure.** Determine whether the following graph families are closed under the minor order for any fixed k . If the family is closed, provide justification. If the family is not closed, give a counterexample.
- Graphs that contain a path of length k .
 - Graphs that do not contain a path of length k .
 - Graphs that contain a dominating set of size k .
 - Graphs that do not contain a dominating set of size k .
 - Graphs that contain a k -vertex cycle
 - Graphs that do not contain a k -vertex cycle.
3. **Obstructions.** Graphs with pathwidth k turn out to be the same as those with gate matrix layout cost $k + 1$.
- What are the minor-minimal obstructions to pathwidth 1?
 - How many obstructions are there to pathwidth 2?
 - Is it feasible to isolate the obstruction set for pathwidth 3? Explain.
4. **Testing.** Testing for an immersed K_2 is easy. Just check whether a graph has an edge.
- How would you test for an immersed K_3 ?
 - How would you test for an immersed K_4 ?

Bring printed or handwritten answers to class on March 22. If you have any questions, please do not hesitate to email me or drop by during office hours.

¹ A pair of supplemental papers is attached. You may find them helpful.