

**Proof by contradiction**

1. Prove that  $\sqrt{3}$  is irrational.
2. If  $a$  and  $b$  are positive real numbers, then  $a + b \geq 2\sqrt{ab}$ .

**Other types of proofs**

1. Prove that  $a \equiv b \pmod{10}$  if and only if  $a \equiv b \pmod{2}$  and  $a \equiv b \pmod{5}$ .
2. Prove that there exists an integer  $n \in \mathbb{N}$  for which  $11 \mid (2^n - 1)$ .

**Problems to try**

1. Prove that the product of any five consecutive integers is divisible by 120.
2. Prove that the product of any  $n$  consecutive integers is divisible by  $n!$ .
3. Prove that, for all integers  $n \in \mathbb{Z}$ , the greatest common divisor of  $n$  and  $n + 1$  is 1.