Addition/Subtraction Principle

Addition Principle and Disjoint Unions

Definition: Two sets are said to be *disjoint* if $X \cap Y = \emptyset$. Similarly, a collection $X_1, \ldots, X_n$ of sets are said to be disjoint if any pair of them is disjoint.

Note that one can have 3 sets A, B, and C with $A \cap B \cap C = \emptyset$ but A, B, and C are not disjoint.

Proposition: Let $X_1, X_2, \ldots, X_n$ be a disjoint collection of finite sets. Then

$$|X_1 \cup X_2 \cup \cdots \cup X_n| = \sum_{i=1}^{n} |X_i|.$$

Proposition: Suppose that $U \subset X$. Then $|X - U| = |X| - |U|$

Examples

(Example 3.6) How many even 5 digit numbers are there for which:

- no digit is zero
- the digit 6 appears exactly once.

Problem 3

(Problem 3 from section 3.3) Five cards are dealt from a 52-card deck and lined up in a row. How many such lineups are there in which all five cards are the same color (i.e. black or red)?

Problem 9

(Problem 9 from Section 3.3) Consider "words" of length 6 made from the letters A, B, C, D, E, F, G, H. How many such words are possible if each letter can occur at most one time, and the word must contain two consecutive vowels?