Mathematical Induction

1. Write

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

   in summation notation, and then prove it. Emphasize the inductive hypothesis.

2. Prove that $3|(n^3 + 5n + 6)$ for every integer $n \geq 0$.

3. Prove that

$$\sum_{k=1}^{n} F_k^2 = F_n F_{n+1}$$

   where F_k is the k^{th} Fibonacci number. (Note that $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-2}$ for integers $n \geq 3$.)

4. Prove that the number of n-digit binary numbers with no consecutive 1's is the Fibonacci number F_{n+2}. So for example, if $n = 3$, there are 8 different 3 digit binary numbers. Of these, 110, 011, and 111 have two consecutive 1's, so the remaining 5 don't. And, indeed, $F_5 = 5$.