Fundamental Theorem of Arithmetic

First Step (Prop 10.1 pg 186)

Recall that, if a and b are natural numbers, there are integers k and l so that

$$\gcd(a, b) = ak + bl.$$

Proposition: Suppose that $n \geq 2$ and that $a_1, \ldots, a_n$ are n integers. Let p be a prime number. If $p|(a_1 \cdot a_2 \cdots a_n)$ then p divides at least one of the a_i.

Proof:

Second Step (Theorem 10.1, page 192)

Proposition: Any integer $n > 1$ has a unique prime factorization, meaning it can be written as a product of prime numbers, and any two such products differ only up to the order of the factors.

Step 1: Every integer has a prime factorization (strong induction).

Step 2: The prime factorization is unique (minimal counterexample).