Equivalence Relations and partitions

Partitions

Definition: A partition of a set A is a set of non-empty subsets of A such that the union of all of the subsets is A and the intersection of any two of the subsets is the empty set.

Intuitively: a partition is a division of A into disjoint subsets.

Partitions (Examples)

- Integers divided into even and odd integers.
- Integers divided into congruence classes modulo 3.
- Books with one author divided up into classes by author.
- People grouped by their county of residence.

Partitions and Equivalence Relations

Theorem (11.2): Let R be an equivalence relation on a set A. Then the equivalence classes $\{[a] : a \in A\}$ form a partition of A.

Converse to Theorem 11.2

Proposition: Suppose P is a partition of a set A. Define a relation R on A by setting aRb if and only if a and b belong to the same element of the partition. Then R is an equivalence relation.

As a result, partitions of a set are "the same" as equivalence relations on a set.