Direct Proof

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Basic terminology

- Definitions, Theorems, Proofs
- Axioms
- Propositions, Lemmas, Corollaries
- Conjectures

Theorems

- Theorem. A theorem is a mathematical statement that is proven to be true.

Theorem: The sum of the squares of the lengths of the sides of a right triangle is equal to the square of the length of the hypotenuse.

Theorem: The sum of two even integer is an even integer.

Definitions

- Definition. A definition is "an exact unambiguous explanation of the meaning of a mathematical word or phrase."

Definition: A "right triangle" is a triangle one of whose interior angles is a right angle.

Definition: A positive integer n is prime if it is greater than one and its only divisors are 1 and n.

Definition: A function $f : \mathbb{R} \to \mathbb{R}$ is continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$.

- Definitions are sometimes written as "If, Then" but they are really "If and only if" statements.

Proofs

A proof is a logical argument that establishes the truth of a theorem.

A true proof of a mathematical statement is almost never given because of length. In practice a proof describes the key steps that are needed to construct a formal proof. There is a social element in what constitutes a proof which depends on the audience.

Recently some mathematicians have been advocating for computer verified proofs because mistakes do occur in published results.

Lemma, Proposition, Corollary

- Lemmas and Propositions are words for "less important" theorems. "Lemma" usually refers to a small theorem that is needed to prove a bigger one. "Proposition" is bigger then "lemma" but smaller than "Theorem."
- Corollary is a word for a theorem that is an immediate consequence of a Theorem.

Lemmas and Propositions preceed theorems; corollaries follow them.

Theorem: Any polynomial function is continuous.

Corollary: Any quadratic function is continuous.