Pascal's Triangle and the binomial theorem

Pascal's Triangle

We proved that $\binom{n}{k}$ counts the number of subsets with k elements that can be found in a set with n elements.

We know that

$$\binom{n}{k} = \frac{n!}{(n-k)!}k!$$

We set $\binom{n}{k} = 0$ if $k > n$ (there are no subsets with k elements in a set with n elements if $k > n$.)

In the inductive proof that $\binom{n}{k}$ counts subsets, we proved that

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

This relation defines "Pascal's Triangle".

Binomial Theorem

Theorem:

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

Proof: By induction.