Indexed sets

Indexed sets

Summation notation

"Recall" that we can write a long sum of a bunch of numbers using summation notation.

$$a_1 + a_2 + \cdots + a_n = \sum_{i=1}^{n} a_i$$

We can even write infinite sums:

$$1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^i} + \cdots = \sum_{i=1}^{\infty} \frac{1}{2^i}$$

Indexed sets

Suppose we have a bunch of sets $A_1, A_2, \ldots, A_n$. Then we can write:

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

and

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$$

Indexed sets

If $A_1, A_2, \ldots, A_n$ are all sets, then

$$\bigcup_{i=1}^{n} A_i = \{x : x \text{ belongs to at least one set } A_i\}$$

- $A_1 = \{1, 4, 10, 12\}$
- $A_2 = \{5, 12, 15\}$
- $A_3 = \{1, 4, 15, 35\}$

What is $\bigcup_{i=1}^{3} A_i$?

Indexed sets

$$\bigcap_{i=1}^{n} A_i = \{x : x \text{ belongs to every set } A_i\}$$

- $A_1 = \{1, 4, 10, 12\}$
- $A_2 = \{5, 12, 15\}$
- $A_3 = \{1, 4, 15, 35\}$

What is $\bigcap_{i=1}^{3} A_i$?

Indexed sets

One can also take the union and intersection of infinitely many sets.

$\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

Example. For each $i \in \mathbb{N}$, let

$$A_i = \{-i, 0, i\}$$

What is $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.?

Index sets

Instead of numbering the sets, one can label them with elements of any set I called an index set.

$\bigcup_{i \in I} A_i$ is the set of elements that belong to *at least one* of the sets A_i.

$\bigcap_{i \in I} A_i$ is the set of elements that belong to *every one* of the sets A_i.

Index sets example

Let C be the set of Counties in the state of Connecticut (there are 8 of these). For each county $c \in C$, let $T(c)$ be the set of Towns in that County.

For example, if c is Tolland County, then the elements of $T(c)$ are Andover, Bolton, Columbia, Coventry, Ellington, Hebron, Mansfield, Somers, Stafford, Tolland, Union, Vernon, and Willington.

What is $\bigcup_{c \in C} T(c)$?

Index sets

Let

$$\mathbb{R}_+ = \{r : r \in \mathbb{R}, r > 0\}.$$

For every real number $r \in \mathbb{R}_+$, let

$$A_r = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < r^2\}.$$

Index sets

What is $\bigcap_{r \in \mathbb{R}_+} A_r$?

Index Sets

What is $\bigcup_{r\in\mathbb{R}_+} A_r$?

Example

What is $\bigcap_{i \in \mathbb{N}} [0, i+1]$?

Example

Suppose that I and J are sets, that $J \neq \emptyset$, and that $I \subseteq J$. Is

$$\bigcap_{a \in I} A_a \subseteq \bigcap_{a \in J} A_a?$$

Explain.