The sum of even numbers is even.

$2 + 2 = 4$ $6 + 8 = 14$

What is an even number?

    ends in $0, 2, 4, 6, 8$

Suppose x and y are even numbers.

$$x = d_1 d_2 \cdots d_n \qquad d_n = \text{one of } 0, 2, 4, 6, 8$$
$$y = s_1 s_2 \cdots s_k \qquad s_k = \text{one of } 0, 2, 4, 6, 8$$

$$x + y = u_1 \cdots \cdots u_n$$

$u_r = $ ones digit of $d_n + s_k$

is u_r one of $0, 2, 4, 6, 8$?

u_r is the ones digit of one of
the sums

$0 + 0$	$2 + 0$
$0 + 2$	$2 + 2$
$0 + 4$	$\vdots$
$\vdots$	

$\vdots$

Better definition: an integer
  x is even if $x = 2y$ for some integer y.

Suppose:
  x even
  z even

Then
  $x = 2y$ for some y y, k are integers
  $z = 2k$ for some k

$$x + z = 2y + 2k = 2(y+k)$$

$x+z$ is $2(y+k)$ so $x+z$ is even.