Chapter 1 Section 1

Sets

- A **Set** is collection of things, called the “elements” of the set.
- Two sets are the same means they have exactly the same elements. Knowing the elements means knowing the set.

Describing sets by listing elements

A set can be described by listing its elements using curly braces.

$$A = \{1, 2, 3\}$$

means A is the set whose elements are 1, 2, and 3. The symbols $\{$ and $\}$ are special and are used to describe sets.

Note: The sets $A = \{1, 2, 3\}$ and $B = \{3, 1, 2\}$ are the same because they have the same elements. So we write $A = B$.

The $\in$ symbol

The symbol $\in$, which looks a little like a backwards 3 and a little like a greek ϵ, means "is an element of."

- $1 \in A$ means 1 is an element of the set A.

The symbol $\notin$ means "is not an element of."

- $5 \notin A$ means that 5 is not an element of A.

Basic examples

- The natural numbers $\mathbb{N}$ is the set of counting numbers $1, 2, 3, \ldots$
$$\mathbb{N} = \{1, 2, 3, \ldots\}$$
- The integers are the are the positive and negative whole numbers, and zero:
$$\mathbb{Z} = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$
- The rational numbers $\mathbb{Q}$ are the positive and negative fractions and zero.

We take for granted addition, multiplication, commutative, associative, laws etc.

Other sets

- the alphabet
- the set of words in English
- the set of people now living
- the set of chairs in my house (what's a chair. . . .)

The empty set

There is exactly one set which nas no elements, called the *empty set.*
The empty set can be written $\emptyset$ or $\{\}$.

The cardinality of a set

- If A is a set, we write $|A|$ for the *number of elements* in the set if that number is finite.
- If $A = \{1, 2, 3\}$ then $|A| = 3$
- We will study cardinality in more detail at the end of the class; for now, we will take this idea for granted. We also take for granted that a set like $\mathbb{Z}$ has infinitely many elements.

The real numbers

- The real numbers is the set of all numbers with possibly infinite decimal expansions (positive or negative). A proper definition is hard to give and is usually done in analysis. We will work with the real numbers informally as we did in Calculus.