Set Proofs

Elements of sets

Many types of theorems can be expressed as questions about the relationship between sets. Sometimes it's a question of membership.

Theorem: For any natural numbers a and b there exist integers k and l such that

$$\gcd(a, b) = ak + bl.$$

Theorem: Let a and b be natural numbers, and let $A = \{ax + by : x, y \in \mathbb{Z}\}$. Then $\gcd(a, b) \in A$.

More examples

General situation: $A = \{x \in S : P(x) \text{ is true}\}$. Then

$$x \in A \Leftrightarrow (x \in S) \wedge P(x).$$

- Let $A = \{3x + 2 : x \in \mathbb{Z}\}$. Then $14 \in A$.

- Let $A = \{3x + 2 : x \in \mathbb{Z}\}$. If $x \equiv 2 \pmod 3$, then $x \in A$.

More examples

- Let B be the set of $X \in \mathcal{P}(\mathbb{N})$ such that, for all $x \in X$ and $y \in X$, $|x - y| < 2$.

Is $\{-1, 2\} \in B$? Is $\{0, 1\}$?