Combinations

Counting Subsets

Our next problem is counting subsets of a given size chosen from a set of a given size.

Question: How many different k element subsets does a set with n elements have?

Theorem on counting subsets

Proposition: The number of k element subsets of a set with n elements is called $\binom{n}{k}$. This number is read "n choose k" and it is called a "binomial coefficient". The formula for $\binom{n}{k}$ is:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

Proof: First we give the book's proof.

Proof: now we give a proof by strong induction.

Examples

Problem 2, page 89: If a set has 100 elements, how many subsets of A have 5 elements? How many have 10 elements? How many have 99 elements?

Problem 5, page 89: How many 16 digit binary strings contain exactly seven 1's?

Problem 11, page 89: How many positive 10 digit integers contain no zeros and exactly three 6's?

Problem 19, page 89: A 5-card poker hand is called a *flush* if all cards are the same suit. How many different flushes are there?