Induction, continued

Induction

In Section 10.1, the book proves the following propositions by applying the axiom of induction.

1. If $n \in \mathbb{N}$, then $1 + 3 + 5 + \cdots + (2n - 1) = n^2$
2. If n is a non-negative integer, then $5|(n^5 - n)$.
3. If $n \in \mathbb{Z}$, and $n \geq 0$, then $\sum_{i=0}^{n} i \cdot i! = (n + 1)! - 1$.
4. If $n \in \mathbb{N}$, then $2^n \leq 2^{n+1} - 2^{n-1} - 1$.
5. If $n \in \mathbb{N}$, then $(1 + x)^n \geq 1 + nx$ for all $x \in \mathbb{R}$ with $x > -1$.

YOU SHOULD CAREFULLY STUDY ALL OF THESE PROOFS

Two notes: Problem 3 has $n \geq 0$ and Problem 5 has an additional variable.

Triangular numbers (Exercise 1)

Proposition: Prove that $1 + 2 + 3 + \cdots + n = \frac{n^2+n}{2}$.

Geometric series

Proposition: For any $n \geq 0$,

$$1 + x + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1}$$

A result on sets (Problem 17)

Proposition: Suppose that $A_1, A_2, \cdots, A_n$ are sets contained in a universal set U and that $n \geq 2$. Then

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$$