

Comments on homework problems

1. $\sqrt{6}$ is irrational.
2. Rational points on $x^2 + y^2 = 3$.
3. $a^2 + 4a + 5$ is odd if and only if a is even.
4. If $x, y \in \mathbb{R}$, then $x^3 + x^2y = y^2 + xy$ if and only if $y = x^2$ or $y = -x$.

Sets

1. Let m and n be integers. Prove that $\{x \in \mathbb{Z} : mn|x\} \subseteq \{x \in \mathbb{Z} : m|x\} \cap \{x \in \mathbb{Z} : n|x\}$. When does equality hold? Suppose A, B, C are sets. Show $A \cap (B \cap C) = (A \cap B) \cap (A \cap C)$.
2. Suppose $A = \emptyset$. Prove that $A \cap B = A \cap C$ if and only if $B = C$. What if A is empty?