Subsets

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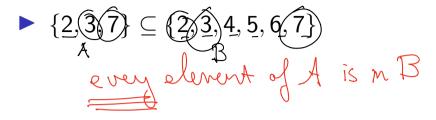
Definition

Suppose that A and B are sets.

- If every element of A is also an element of B, then we say that A is a **subset** of B. This can be written using the subset symbol $A \subseteq B$. $A \subseteq B$
- ▶ If at least one element of A is not an element of B, then A is not a subset of B. This can be written $A \nsubseteq B$.

If
$$A = \{-3, 15, \text{ purple}\}$$
 $B = \{-3, 15, \text{ purple}, \text{ elephant}\}$

then $A \subseteq B$
 $C = \{-3, 14, \text{ purple}\}$
 $C \notin B$ because $14 \in C$ but $14 \notin B$



►
$$\{2,3,11\} \not\subseteq \{2,3,4,5,6,7\}$$

11 ∈ A but 11 & B

one element of A is Not in B

► $\mathbb{N} \subseteq \mathbb{Z}$ $\mathbb{N} = \{1,2,3,...\} \subseteq \mathbb{Z} = \{2,...,-3,-2,-1,0,1,...\}$

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 $ightharpoonup \mathbb{R} imes \mathbb{N} \subset \mathbb{R} imes \mathbb{R}$ $R \times N = \{ (r, n) : r \in \mathbb{R}, n \in \mathbb{N} \}$ typical element might be $(\pi, 4) \in \mathbb{R} \times \mathbb{N}$ $\mathbb{R} \times \mathbb{R} = \begin{cases} (r_1, r_2) : r_1 \in \mathbb{R} \text{ and } r_2 \in \mathbb{R} \end{cases}$ sine IN ER every natural number. (r,n) e RXIN is also in CARXIR. SO RXN S RXR

ightharpoonup $\mathbb{N} \times \mathbb{R} \not\subseteq \mathbb{R} \times \mathbb{N}$ NxR & RXN $N \times IR = \{(N,r) : N \in \mathbb{N}, r \in \mathbb{R}\}$ RXN= { (r,n): reR, neNJ. TO see that INXIR &IRXIN We need to find an element of MXR which is NOT IIN IRXIN. (3,TT) ENXIR BY (3,77) ERXN althorh 3 CIR, TEN. Since there is an element of INTIR that is NOT in IRXIN, we know that NXR&RXX

▶ For any set A, $A \subseteq A$.

For any A is a subset of itself
set A)
Because every element of A
is an element of A.

The Empty Set

▶ The empty set is a subset of every set.

is Every element of ϕ is an element of A?

There is no lement of ϕ that isn't in A.

(because the empty A has no elements).