Inverse Relations and Inverse Functions

The identity function

Definition: Let A be a set. The identity function $i_A : A \to A$ is the function defined by $i_A(x) = x$ for all $x \in A$.

As a set of ordered pairs, $i_A \subset A \times A$ consists of all pairs (a, a) for $a \in A$.

The identity function

Proposition: The identity function is bijective.

The inverse of a relation

Definition: Let A and B be sets and let R be a relation on $R \subset A \times B$. The inverse relation R^{-1} to R is the relation on $B \times A$ defined by

$$R^{-1} = \{(b, a) \in B \times A : (a, b) \in R\}.$$

Examples of inverse relations

Example: Let $A = \mathbb{R}$ and let R be the relation $<$. Then R consists of all pairs $(a, b) \in \mathbb{R} \times \mathbb{R}$ with $a < b$. The inverse relation R^{-1} consists of all pairs $(b, a) \in \mathbb{R} \times \mathbb{R}$ with $(a, b) \in R$. Thus R^{-1} is the relation $>$.

Another example

Example: Let $A = \mathbb{Z}$ and let R be the relation "divides", so that R consists of pairs $(a, b) \in \mathbb{Z} \times \mathbb{Z}$ where $a|b$. The inverse relation R^{-1} consists of pairs (a, b) where $b|a$, or, in other words, where a is a multiple of b.

So the inverse relation to $a|b$, meaning a is a divisor of b, is the relation aRb when a is a multiple of b.