Chapter 4 section 1-2 cont'd

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Axioms

Our book does not mention axioms but it should. Axioms are statements that are asserted to be true for purposes of constructing a theory. For example:

Axiom: Given a line L, and a point P not on L, there is exactly one line through P parallel to L.

Axiom: An empty set exists.

Axioms in this course

- Existence of integers, natural numbers, rational numbers, and real numbers.
- Properties of addition, multiplication such as commutative and associative laws, including closure.
- Properties of $>$ and $<$

The Division Algorithm

The Division Algorithm: Given $a, b \in \mathbb{Z}$ with $b > 0$, there are unique integers q and r with $0 \leq r < b$ so that $a = bq + r$.

The Fundamental Theorem of Arithmetic

Theorem: Every natural number greater than one is a product of prime numbers, and this factorization into primes is unique up to rearranging the terms.

Some fundamental definitions: divisibility

Definition: Suppose a and b are integers. We say that a **divides** b, written $a|b$, if $b = ac$ for some $c \in \mathbb{Z}$. In this case we also say that a is a divisor of b and that b is a multiple of a.

GCD and LCM

Definition: The greatest common divisor of integers a and b, written $\gcd(a, b)$, is the largest integer that divides both a and b.

Definition: The least common multiple of integers a and b, written $\mathrm{lcm}(a, b)$, is the smallest integer that is a multiple of both a and b.