Subsets

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Definition

Suppose that A and B are sets.

- If every element of A is also an element of B, then we say that A is a **subset** of B. This can be written using the subset symbol $A \subseteq B$. $A \subseteq B$
- If at least one element of A is not an element of B, then A is not a subset of B. This can be written $A \not\subseteq B$.

if $A = \{-3, 15, \text{purple}\}$

$B = \{-3, 15, \text{purple}, \text{elephant}\}$

then $A \subseteq B$

$C = \{-3, 14, \text{purple}\}$

$C \not\subseteq B$ because $14 \in C$ but $14 \notin B$

Example

- $\{2, 3, 7\} \subseteq \{2, 3, 4, 5, 6, 7\}$
  A B
  every element of A is in B

  A B
- $\{2, 3, 11\} \not\subseteq \{2, 3, 4, 5, 6, 7\}$
  $11 \in A$ but $11 \notin B$
  one element of A is NOT in B

Example

- $\mathbb{N} \subseteq \mathbb{Z}$

$$\mathbb{N} = \{1, 2, 3, \ldots\} \subseteq \mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, \ldots\}$$

- $\mathbb{Z} \subseteq \mathbb{Q}$

$$\mathbb{Z} \subseteq \mathbb{Q}$$

$$\{\ldots -3, -2, -1, 0, 1, 2, \ldots\} \subseteq \mathbb{Q}$$

$$-3 = \frac{-3}{1} \in \mathbb{Q}$$

$$15 \in \mathbb{Z} \qquad 15/1 \in \mathbb{Q}$$

Example

- $\mathbb{R} \times \mathbb{N} \subseteq \mathbb{R} \times \mathbb{R}$

$\mathbb{R} \times \mathbb{N} = \{ (r, n) : r \in \mathbb{R}, n \in \mathbb{N} \}$

typical element might be $(\pi, 4) \in \mathbb{R} \times \mathbb{N}$

$\mathbb{R} \times \mathbb{R} = \{ (r_1, r_2) : r_1 \in \mathbb{R} \text{ and } r_2 \in \mathbb{R} \}$

sine $\mathbb{N} \subseteq \mathbb{R}$ every natural number is a real number.

$(r, n) \in \mathbb{R} \times \mathbb{N}$ is also in $\mathbb{R} \times \mathbb{R}$.

So $\mathbb{R} \times \mathbb{N} \subseteq \mathbb{R} \times \mathbb{R}$

Example

- $\mathbb{N} \times \mathbb{R} \not\subseteq \mathbb{R} \times \mathbb{N}$

$\mathbb{N} \times \mathbb{R} \not\subseteq \mathbb{R} \times \mathbb{N}$

$\mathbb{N} \times \mathbb{R} = \{ (n, r) : n \in \mathbb{N}, r \in \mathbb{R} \}$

$\mathbb{R} \times \mathbb{N} = \{ (r, n) : r \in \mathbb{R}, n \in \mathbb{N} \}$.

To see that $\mathbb{N} \times \mathbb{R} \not\subseteq \mathbb{R} \times \mathbb{N}$ we need to find an element of $\mathbb{N} \times \mathbb{R}$ which is NOT in $\mathbb{R} \times \mathbb{N}$.

$(3, \pi) \in \mathbb{N} \times \mathbb{R}$ but $(3, \pi) \notin \mathbb{R} \times \mathbb{N}$

although $3 \in \mathbb{R}$, $\pi \notin \mathbb{N}$.

Since there is an element of $\mathbb{N} \times \mathbb{R}$ that is NOT in $\mathbb{R} \times \mathbb{N}$, we know that

$\mathbb{N} \times \mathbb{R} \not\subseteq \mathbb{R} \times \mathbb{N}$

Example

- For any set A, $A \subseteq A$.

For any set A, A is a subset of itself

Because every element of A is an element of A.

The Empty Set

- The empty set is a subset of every set.

$\emptyset \subseteq A$ for any set A.

is Every element of $\emptyset$ is an element of A ?

There is no element of $\emptyset$ that isn't in A.

(because the empty set has no elements).