Properties of Relations

Reflexive Relations

Definition: A relation R is **reflexive** if, for all $x \in A$, $(x, x) \in R$. In other words, xRx for all $x \in A$.

- The '$=$' relation is reflexive, as is the $\leq$ relation.
- The $<$ relation is not reflexive.
- The "is an ancestor of" relation is not reflexive.
- The $\neq$ relation is not reflexive.

Symmetric Relations

Definition: A relation R is **symmetric** if, for all $x, y \in A$, $xRy \implies yRx$. In other words, if $(x, y) \in R$ then $(y, x) \in R$.

- The '$=$' relation is symmetric
- The $\leq$ relation is not symmetric
- The "is an ancestor of" relation is not symmetric.
- The $\neq$ relation is symmetric.

Transitive relations

Definition: A relation R is **transitive** if, for all $x, y, z \in A$, if xRy and yRz then xRz. In other words, if $(x, y) \in R$ and (y, z) in R then $(x, z) \in R$.

- The '$=$' relation is transitive
- The $\leq$ relation is transitive.
- The "is an ancestor of" relation is transitive.
- The $\neq$ relation is not transitive.

Example 11.7

Examine the properties reflexivity, symmetry, and transitivity when $A = \{b, c, d, e\}$ and

$$R = \{(b, b), (b, c), (c, b), (c, c), (d, d), (d, b), (b, d), (c, d), (d, c)\}$$

Example 11.7 continued

A picture

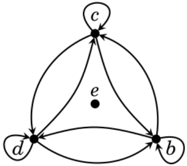

Figure 11.1. The relation R from Example 11.7

Congruence is reflexive, symmetric, and transitive.

Proposition: Let $n \in \mathbb{N}$. The relation R on $\mathbb{Z}$ defined by aRb if and only if $a \equiv b \pmod{n}$ is reflexive, symmetric, and transitive.