

Relations

Relations

Examples of Relations

- ▶ $=, <, >, \leq, \geq, \neq$, *etc.* are relations between numbers.
- ▶ \subseteq is a relation between sets
- ▶ “is the parent of” or “is a child of” or “is a spouse of” are relations between people.
- ▶ “comes earlier in the dictionary” is a relation between words.

Abstract Relations

Suppose we consider the relation $<$ on \mathbb{N} . We can “abstract” this relation by considering all pairs $(x, y) \in \mathbb{N} \times \mathbb{N}$ where $x < y$. Let R be the set of such pairs.

So $(1, 2) \in R$, but $(5, 4) \notin R$.

Once we have the set R , we know everything about $<$. Namely

$$x < y \Leftrightarrow (x, y) \in R.$$

Now we *identify* the relation $<$ with this set R and we can study relations using set theory.

Pictures of relations

A big picture

Here the underlying set is “North American Cities” and the relation is $(x, y) \in R$ if there was a United flight joining the two cities in 2019.



Abstract Relations: formal definition

Definition: Let A be a set. A *relation* on A is a subset R of the Cartesian product $A \times A$. We abbreviate the statement $(x, y) \in R$ as xRy , and $(x, y) \notin R$ as $x \not R y$.

Abstract relations: A few examples

- ▶ (Example 11.1) $A = \{1, 2, 3, 4\}$ and R consists of

$$\{(1, 1), (2, 1), (2, 2), (3, 3), (3, 2), (3, 1), (4, 4), (4, 3), (4, 2), (4, 1)\} \subseteq A \times A$$

- ▶ (Example 11.2) $A = \{1, 2, 3, 4\}$ and S consists of

$$\{(1, 1), (1, 3), (3, 1), (3, 3), (2, 2), (2, 4), (4, 2), (4, 4)\} \subseteq A \times A$$

Abstract Relations

- ▶ (Example 11.3) The intersection of the two relations from the previous examples is a relation

$$\{(1, 1), (2, 2), (3, 3), (3, 1), (4, 4), (4, 2)\}$$

One more example

- ▶ (Example 11.4) $B = \{0, 1, 2, 3, 4, 5\}$ and

$$U = \{(1, 3), (3, 3), (5, 2), (2, 5), (4, 2)\} \subseteq B \times B.$$

Problem 3, page 204.

- ▶ Let $A = \{0, 1, 2, 3, 4, 5\}$. Write out the relation R that expresses \geq on A and illustrate it with a diagram.

Problem 5, page 204.

Write out the sets A and $R \subseteq A \times A$ described by this diagram.

