Chapter 1 Section 1

Section 1.1

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Sets

- A **Set** is collection of things, called the "elements" of the set.
- Two sets are the same means they have exactly the same elements. Knowing the elements means knowing the set.

Describing sets by listing elements

A set can be described by listing its elements using curly braces.

$$A = \{1, 2, 3\}$$

means A is the set whose elements are 1, 2, and 3. The symbols $\{$ and $\}$ are special and are used to describe sets.

Note: The sets $A = \{1, 2, 3\}$ and $B = \{3, 1, 2\}$ are the same because they have the same elements. So we write $A = B$.

The $\in$ symbol

The symbol $\in$, which looks a little like a backwards 3 and a little like a greek ϵ, means "is an element of."

- $1 \in A$ means 1 is an element of the set A.

The symbol $\notin$ means "is not an element of."

- $5 \notin A$ means that 5 is not an element of A.

Basic examples

- The natural numbers $\mathbb{N}$ is the set of counting numbers $1, 2, 3, \ldots$
$$\mathbb{N} = \{1, 2, 3, \ldots\}$$
- The integers are the are the positive and negative whole numbers, and zero:
$$\mathbb{Z} = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$
- The rational numbers $\mathbb{Q}$ are the positive and negative fractions and zero.

We take for granted addition, multiplication, commutative, associative, laws etc.

Set builder notation (predicates)

Set builder notation picks elements out of another set.

Example:

$$\{x \in \mathbb{Z} : x \geq 0\}$$

means "pick from the set $\mathbb{Z}$ those elements that satisfy the condition $x \geq 0$".

The condition is called a *predicate*.

Other sets

- the alphabet
- the set of words in English
- the set of people now living
- the set of chairs in my house (what's a chair. . . .)

The empty set

There is exactly one set which nas no elements, called the *empty set.*
The empty set can be written $\emptyset$ or $\{\}$.

The cardinality of a set

- If A is a set, we write $|A|$ for the *number of elements* in the set if that number is finite.
- If $A = \{1, 2, 3\}$ then $|A| = 3$
- We will study cardinality in more detail at the end of the class; for now, we will take this idea for granted. We also take for granted that a set like $\mathbb{Z}$ has infinitely many elements.

The real numbers

- The real numbers is the set of all numbers with possibly infinite decimal expansions (positive or negative). A proper definition is hard to give and is usually done in analysis. We will work with the real numbers informally as we did in Calculus.

Intervals $\mathbb{R}$

See page 7 of the text.

- $(a, b) = \{x \in \mathbb{R} : x > a \text{ and } x < b\}$ "open"
- $[a, b) = \{x \in \mathbb{R} : x \geq a \text{ and } x < b\}$ "half open"
- $(a, b] = \{x \in \mathbb{R} : x > a \text{ and } x \leq b\}$ "half open"
- $[a, b] = \{x \in \mathbb{R} : x \geq a \text{ and } x \leq b\}$ "closed"
- $[a, \infty) = \{x \in \mathbb{R} : x \geq a\}$ "infinite"
- $(a, \infty) = \{x \in \mathbb{R} : x > a\}$ "infinite"
- $(\infty, a) = \{x \in \mathbb{R} : x < a\}$ "infinite"
- $(\infty, a] = \{x \in \mathbb{R} : x \leq a\}$ "infinite"

Closer look: Example 1.1

Claim: $\{n^2 : n \in \mathbb{Z}\} = \{0, 1, 4, 9, 16, 25, \ldots\}$

- Say this in words
- What about $\{n^2 : n \in \mathbb{N}\}$?
- What about $\{n^2 : n \in \mathbb{Q}\}$?

Closer look: Example 1.2

Problem: Describe the set $A = \{7a + 3b : a, b \in \mathbb{Z}\}$.

Closer look: Problem 1.1.7

Problem: Describe the set $\{x \in \mathbb{R} : x^2 + 5x = -6\}$.

- The back of the book gives the answer $\{-2, -3\}$. Why is this the answer?
- What about if we replace $\mathbb{R}$ with $\mathbb{Q}$?
- What about if we replace $\mathbb{R}$ with $\mathbb{N}$?