## Direct Proofs Example

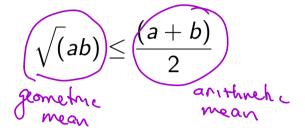
**Definition:** The arithmetic mean of two real numbers a and b is (a+b)/2.

**Definition:** The geometric mean of two positive real numbers a and b is  $\sqrt{(ab)}$ .

**Proposition:** If a and b are positive real numbers, then the geometric mean of a and b is less than or equal to their arithmetic mean.

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**Proposition:** If a and b are positive real numbers, then:



## Problem Solving Phase

Show:  $\sqrt{ab} \leq (a+b)$   $\sqrt{2}$  ?? Lema:  $a \geq b$  then  $a^2 > b^2$  ??  $ab \leq (a+b)^2$  Need to know:  $4ab \leq (a+b)^2 = a^2 + 2ab + b^2$   $5a \geq b^2$ 

## Isolating the needed lemma

**Lemma:** If a and b are positive, and  $a \le b$ , then  $\sqrt{a} \le \sqrt{b}$  where  $\sqrt{x}$  denotes the positive square root of x.

Given 
$$b-\alpha \ge 0$$
 Want  $\sqrt{b}-\sqrt{a} \ge 0$ 

KNOW  $\sqrt{a}+\sqrt{b} \ge 0$ 
 $(\sqrt{a}+\sqrt{b})(\sqrt{b}-\sqrt{a}) = b-a = (\sqrt{b})^2-(\sqrt{c})^2$ 
 $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = b-a$ 
 $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = b-a$ 
 $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = b-a$ 

Given and bya. Therefore b-a >0. Therefore

b-a = (15+12)(15-12)>0. Since 15+12>0

we can divid both sides of this acquily by 15+12

To Obtain 15-12>0 50 15>12.

## The proof

**Proposition:** If a and b are positive real numbers, then:

$$\sqrt(ab) \le \frac{(a+b)}{2}$$

Proof:

We know that  $(a-b)^2 \ge 0$ . Therefore  $a^2 + b^2 - 2ab \ge 0$  and so  $a^2 + b^2 \ge 2ab$ . Add 2ab to both sides to obtain  $a^2 + 2ab + b^2 \ge 4ab$  so  $(a+b)^2 \ge 4ab$ . Both sides of this inequality are positive, since the left side is a square the right side is a product of positive numbers. Now apply the lemma to take the square root of both sides to obtain

$$(a+b)\geq 2\sqrt{ab}.$$

Dividing both sides by 2 yields the desired result.