More Counting Problems

Counting

- How many eight digit decimal numbers have a one in either their second or fourth digit?
- (Problem 10, Section 3.3) Let $X = \{P, R, O, F, S\}$.
  - How many lists of length six can be made from elements of X (allowing repetition)
  - How many lists of length six can be made that end in S and contain more than one O?
- (10, Section 3.4) How may permutations of the digits 0 through 9 are there in which the digits alternate even and odd?
- (16, Section 3.5) How many 10 digit binary strings are there that do not have exactly 4 ones?

Pascal's Triangle/Binomial coefficients

The binomial coefficients $\binom{N}{k}$ have three defintions (at least).

1. a recursive definition where:

- for all $N \geq 1$, $\binom{N}{0} = \binom{N}{N} = 1$
- for all $0 < k < N$,

$$\binom{N}{k} = \binom{N-1}{k-1} + \binom{N-1}{k}$$

2. They also have a definition that comes from the binomial theorem; that is, you can define $\binom{N}{k}$ to be the coefficient of x^k in the expansion of $(x+y)^N$.

3. They also are given by a formula:

$$\binom{N}{k} = \frac{N!}{k!(N-k)!}$$

How do you show that all three definitions are the same?