The Power Set of a Set

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Definition

Definition: If A is a set, the **power set** of A, written $\mathcal{P}(A)$, is the set whose elements are all subsets of A. In set builder notation,

$$\mathcal{P}(A) = \{X : X \subseteq A\}$$

Example

$$A = \{0, 1, 3\}$$

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$$\mathcal{P}(A) = \{\emptyset, \{0\}, \{1\}, \{3\}, \{0, 1\}, \{0, 3\}, \{1, 3\}, \{0, 1, 3\}\}$$

Example

$$\mathcal{P}(\emptyset) = \{\emptyset\}$$

Notice that $|\emptyset| = 0$ and $|\mathcal{P}(\emptyset)| = 2^0 = 1$.

Example

$$\mathcal{P}(\{a\}) = \{\emptyset, \{a\}\}$$

Example – some common mistakes

$\mathcal{P}(1)$ makes no sense because 1 is not a set.

Example – some common mistakes 2

$\mathcal{P}(\{1,\{1,2\}\} = \{\emptyset, 1, \{\{1,2\}\}, \{1,\{1,2\}\}\}$. Notice that $\{1,2\}$ is not an element of $\mathcal{P}(\{1,\{1,2\}\})$ but $\{\{1,2\}\}$ is.

Infinite case

THe power set $\mathcal{P}(\mathbb{N})$ is very large and can be identified with infinite sequences of I's and O's.

The set $\mathcal{P}(\mathbb{R}^2)$

$\mathcal{P}(\mathbb{R}^2)$ is huge and includes every graph of every function plus lots of other things, more than we can really comprehend.

Problem 1.4.15

What is $\mathcal{P}(A \times B)$ if $|A| = m$ and $|A| = n$?