Equivalence Relations

Key idea: An equivalence relation yields a partition of a set into disjoint subsets, and conversely.

1. Let f and g be polynomials with real coefficients. Say that fRg when they have the same derivative.

- Prove that this is an equivalence relation
- Describe the equivalence classes

2. Let R and S be equivalence relations on a set A. Prove that $R \cap S$ is also an equivalence relation. Suppose R is the relation "congruent modulo 3" and S is the relation "congruent modulo 5". What is the intersection of these two relations.

3. Prove or disprove: the union of two equivalence relations is an equivalence relation.

14. Suppose R is a reflexive and symmetric relation on a finite set A. Define a relation S on A by declaring xSy if and only if for some $n \in \mathbb{N}$ there are elements $x_1, x_2, \ldots, x_n \in A$ satisfying xRx_1, x_1Rx_2, x_2Rx_3, $x_3Rx_4, \ldots, x_{n-1}Rx_n$, and x_nRy. Show that S is an equivalence relation and $R \subseteq S$. Prove that S is the unique smallest equivalence relation on A containing R.

Figure 1: Problem 14