Functions, Injectivity, Surjectivity

Key Definitions

1. A function $F : A \to B$ is *injective* if, whenever a and a' are two different elements of A, then $F(a)$ and $F(a')$ are two different elements of B. (Sometimes called "one-to-one").
2. A function $F : A \to B$ is *surjective* if, for all $b \in B$, there exists $a \in A$ with $F(a) = b$. Alternatively, F is surjective if its range coincides with its codomain. (also called "onto").
3. A function is *bijective* if it is both surjective and injective.

Problems

1. Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be the function defined by $f(n, m) = 3n - 4m$. If f injective? Is f surjective?

2. Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be defined by $f(m, n) = (m + n, 2m + n)$. Is f injective? Is f surjective?

3. Let X be a set with m elements, where $m \geq 2$, and let Y be a set with 2 elements. How many surjective functions are there from X to Y?

4. Let X and Y be sets and let $f : X \to Y$ be a surjective function. Define a relation R on X by xRy whenever $f(x) = f(y)$. Prove that R is an equivalence relation. Describe the equivalence classes in terms of Y.

Pigeonhole principle

Let A and B be finite sets and let $f : A \to B$ be a function. iIf $|A| > |B|$, then f is not injective. If $|A| < |B|$ then f is not surjective.

Problem: Given five points inside a square with side length one, at least two are within $\sqrt{2}/2$ of each other.