

Constructive vs non-constructive proofs

Constructive Proofs

A constructive proof of an existence claim gives an example of an object with the desired properties.

For example, the $A/(B + C) + B/(A + C) + C/(A + B) = 4$ result had a constructive proof because I presented explicit values of A , B , and C .

Euclid's algorithm is constructive because it explains how to find x and y so that $\gcd(a, b) = ax + by$.

Non-constructive proofs

A non-constructive proof shows that something exists by “ruling out its non-existence” without necessarily explaining how to find the example.

See the proposition on page 154 for an example.

Here is an example of a theorem (the Intermediate Value Theorem) whose proof is not constructive.

Theorem: Let $f : [a, b] \rightarrow \mathbb{R}$ be a continuous function where $f(a) < 0$ and $f(b) > 0$. Then there exists a $c \in (a, b)$ such that $f(c) = 0$.

The proof (which you will learn when you take Analysis) shows that c exists without giving an algorithm for finding it.