

## Set Proofs Continued

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# Inclusion

Basic principle:  $A \subseteq B$  is equivalent to the statement  
 $x \in A \implies x \in B$ . One can prove this both directly and as  
 $x \notin B \implies x \notin A$ .

**Proposition:** Let  $A = \{4x + 2 : x \in \mathbb{Z}\}$ . Let  $B = \{2x : x \in \mathbb{Z}\}$ .  
Then  $A \subseteq B$ .

## More examples

**Proposition:** For all  $k \in \mathbb{Z}$ , let  $A = \{n \in \mathbb{Z} : n|k\}$  and let  $B = \{n \in \mathbb{Z} : n|k^2\}$ . Then  $A \subseteq B$ . (Note: this is problem 3 on page 171.)

## More examples

**Proposition:** Suppose  $A$ ,  $B$ , and  $C$  are sets. If  $B \subseteq C$ , then  $A \times B \subseteq A \times C$ . (This is problem 7 on page 171.)

## One more

**Proposition:** Let  $A$  and  $B$  be sets. Prove that  $A \subseteq B$  if and only if  $A - B = \emptyset$ . (This is problem 21 on page 171.)