

Disproof of existence

The proof of Fermat's Last Theorem is a disproof of existence; it shows that there are NO solutions to the Fermat equation.

The disproof of a statement

"There exists $x \in S$ such that $P(x)$ "

requires proving a universal statement:

"For all $x \in S$, not $P(x)$."

Disproof of existence

Claim: There exists a pythagorean triple (a, b, c) such that all of a , b , and c are odd.

The negation of this claim is

“For all pythagorean triples (a, b, c) , at least one of a , b , or c is even.”

Disproof of existence by contradiction

Proof by contradiction is often useful to prove “nonexistence” of something.

Claim: There is a real number x such that $x \in (x^4, x^2)$. (See Example 9.5).