Permutations

Factorials

Definition 1: For $n \in \mathbb{Z}$, $n \geq 0$, define $0! = 1$ and $n! = (1)(2)\cdots(n-1)(n)$. Alternatively, define $n!$ for non-negative integers n by setting $0! = 1$ and $n! = n(n-1)!$.

Proposition: The number of different lists of length n made up of elements from the set $\{1, 2, \ldots, n\}$, *without repetitions*, is $n!$.

Permutations

Definition: Let X be a set. A permutation of X is a list of length $|X|$ of the elements of X, without repetition. (**Note:** There are other definitions of permutations in other contexts, all related to this one).

Examples:

Definition: Let X be a set. A k-permutation of X is a list of k elements of X without repetition. $P(n,k)$ is the number of k permutations of a set with n elements.

Proposition: The number $P(n,k)$ of k-permutations of a set with n elements is $n(n-1)\cdots(n-k+1)$ or, equivalently

$$P(n,k) = \frac{n!}{(n-k)!}$$

Proof:

Examples of k-permutations

(Problem 7, page 84) How many 9-digit numbers can be made from the digits $1, 2, 3, 4, 5, 6, 7, 8, 9$ if repetition is not allowed and all the odd digits occur first, followed by all the even digits.

(Problem 15, page 84) In a club of 15 people, there is a president, vice-president, secretary, and treasurer. In how many different ways can this be done?