Image and preimage

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Key defintions

Definition: Let $f : A \to B$ be a function.

1. If $X \subseteq A$, then the **image** of X is the set $f(X) = \{f(x) : x \in X\} \subseteq B$.
2. If $Y \subseteq B$, then the **preimage** of Y is the set $f^{-1}(Y) = \{x \in A : f(x) \in Y\} \subseteq A$.

Note: $f^{-1}(Y)$ is defined *even when* f^{-1} is not a function, i.e. even when f is not bijective.

Example 12.13

Example 12.13 Let $f : \{s,t,u,v,w,x,y,z\} \to \{0,1,2,3,4,5,6,7,8,9\}$ be

$$f = \{(s,4),(t,8),(u,8),(v,1),(w,2),(x,4),(y,6),(z,4)\}.$$

This f is neither injective nor surjective, so it certainly is not invertible. Be sure you understand the following statements.

1. $f(\{s,t,u,z\}) = \{8,4\}$
2. $f(\{s,x,z\}) = \{4\}$
3. $f(\{s,v,w,y\}) = \{1,2,4,6\}$
4. $f(\emptyset) = \emptyset$
5. $f^{-1}(\{4\}) = \{s,x,z\}$
6. $f^{-1}(\{4,9\}) = \{s,x,z\}$
7. $f^{-1}(\{9\}) = \emptyset$
8. $f^{-1}(\{1,4,8\}) = \{s,t,u,v,x,z\}$

Problem 12.6.7

Problem: Prove that, if $f : A \to B$ is a function, and W and X are subsets of A, then

$$f(W \cap X) \subseteq f(W) \cap f(X)$$

Problem 12.6.9

Problem: Prove that, if $f : A \to B$ is a function, and W and X are subsets of A then

$$f(W \cup X) = f(W) \cup f(X)$$