Relations

Relations

Examples of Relations

 \triangleright =, <, >, \leq , \geq , \neq , etc. are relations between numbers.

▶ ⊂ is a relation between sets

"is the parent of" or "is a child of" or "is a spouse of" are relations between people.

"comes earlier in the dictionary" is a relation between words.

Abstract Relations

Suppose we consider the relation < on \mathbb{N} . We can "abstract" this relation by considering all pairs $(x,y) \in \mathbb{N} \times \mathbb{N}$ where x < y. Let R be the set of such pairs.

So $(1,2) \in R$, but $(5,4) \notin R$.

Once we have the set R, we know everything about <. Namely

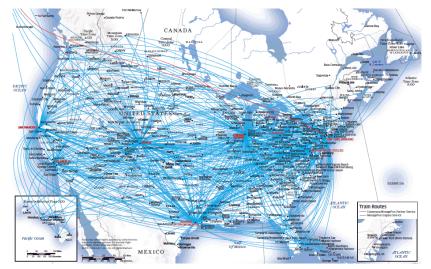
$$x < y \Leftrightarrow (x, y) \in R$$
.

Now we *identify* the relation < with this set R and we can study relations using set theory.

Pictures of relations

A big picture

Here the underlying set is "North American Cities" and the relation is $(x, y) \in R$ if there was a United flight joining the two cities in 2019.



Abstract Relations: formal definition

Definition: Let A be a set. A *relation* on A is a subset R of the Cartesian product $A \times A$. We abbreviate the statement $(x, y) \in R$ as xRy, and $(x, y) \notin R$ as xRy.

Abstract relations: A few examples

- (Example 11.1) $A = \{1, 2, 3, 4\}$ and R consists of
 - $\{(1,1),(2,1),(2,2),(3,3),(3,2),(3,1),(4,4),(4,3),(4,2),(4,1)\}\subseteq A\times A$
 - ► (Example 11.2) $A = \{1, 2, 3, 4\}$ and S consists of $\{(1, 1), (1, 3), (3, 1), (3, 3), (2, 2), (2, 4), (4, 2), (4, 4)\} \subseteq A \times A$

Abstract Relations

► (Example 11.3) The intersection of the two relations from the previous examples is a relation

$$\{(1,1),(2,2),(3,3),(3,1),(4,4),(4,2)\}$$

One more example

• (Example 11.4) $B = \{0, 1, 2, 3, 4, 5\}$ and

$$U = \{(1,3), (3,3), (5,2), (2,5), (4,2)\} \subseteq B \times B.$$

Problem 3, page 204.

Let $A = \{0, 1, 2, 3, 4, 5\}$. Write out the relation R that expresses \geq on A and illustrate it with a diagram.

Problem 5, page 204.

Write out the sets A and $R \subseteq A \times A$ described by this diagram.

