

Sets

1. Let m and n be integers. Prove that

$$\{x \in \mathbb{Z} : mn|x\} \subseteq \{x \in \mathbb{Z} : m|x\} \cap \{x \in \mathbb{Z} : n|x\}.$$

When does equality hold?

2. Suppose A, B, C are sets. Show $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
3. Suppose $A \neq \emptyset$. Prove that $A \times B \subseteq A \times C$ if and only if $B \subseteq C$. What if A is empty?
4. Let $A = \{x : x \equiv 1 \pmod{4}\}$, $B = \{x : x \equiv 1 \pmod{3}\}$ and $C = \{x : x \equiv 1 \pmod{12}\}$. Prove that $A \cap B = C$.

Euclid's algorithm and congruence equations

1. Solve $7x \equiv 3 \pmod{41}$.
2. Solve $6x \equiv 2 \pmod{24}$.