Direct Proofs

If P, then Q

Proposition: If P is true, then Q is true.

We are asserting the truth of the implication $P \implies Q$. If P is false, then the implication is automatically true. So from a practical point of view, we must show that whenver P is true, Q is also true.

Proof: Suppose P.

(bunch of stuff)

Therefore Q.

Simple example

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The definition of even integer says that an integer x is even if there is an integer y so that $x = 2y$. We found that $a + b = 2(k + u + 1)$ where $k + u + 1$ is an integer.

Therefore $a + b$ is even.