# Checking cases

# Cases

**Proposition:** If $n$ is a natural number, then $1 + (-1)^n(2n - 1)$ is a multiple of 4.

# Cases

**Proof:** Let $K(n) = 1 + (-1)^n(2n - 1)$. We consider the cases where $n$ is odd and even separately. When $n$ is even, $K(n) = 1 + (2n - 1) = 2n$. Since $n$ is even, $n = 2m$ for some $m$, and therefore $K(n) = 2(2m) = 4m$. Therefore $K(n)$ is a multiple of 4.

When $n$ is odd, $K(n) = 1 - (2n - 1) = 2 - 2n$. Since $n$ is odd, $n = 2m + 1$ for some $m$, and therefore

$$K(n) = 2 - 2(2m + 1) = 2 - 4m - 2 = 4m$$

and again $K(n)$ is a multiple of 4.

# Cases

**Proposition:** (The Triangle Inequality) For any real numbers $x$ and $y$, we have

$$|x + y| \leq |x| + |y|$$

Note: $|x| = x$ if $x \geq 0$, otherwise $|x| = -x$.

# Cases

**Proof:** There are four cases to consider, depending on the signs of $x$ and $y$, and we take them in turn.

1. $x \geq 0$ and $y \geq 0$. Then $x + y \geq 0$. Therefore, in this case, $|x| = x$, $|y| = y$, and $|x + y| = x + y$ and so $|x + y| = |x| + |y|$.
2. If $x < 0$ and $y < 0$ then $|x + y| = -x - y = |x| + |y|$.

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Then $y < x + y < x$. If $x + y \geq 0$, then $|x + y| = x + y = |x| - |y| \leq |x| + |y|$. If $x + y < 0$, then $|x + y| = -x - y = -|x| + |y| \leq |x| + |y|$.

The 4th case, $x < 0$ and $y \geq 0$, follows by the same argument.