More Mathematical Induction

Please hand this in sheet in via HuskyCT after class.

1. Prove that $3|(n^3 + 5n + 6)$ for every integer $n \geq 0$.
2. Prove that the number of n-digit binary numbers with no consecutive 1's is the Fibonacci number F_{n+2}. So for example, if $n = 3$, there are 8 different 3 digit binary numbers. Of these, 110, 011, and 111 have two consecutive 1's, so the remaining 5 don't. And, indeed, $F_5 = 5$.
3. Suppose you have equal numbers N of red and blue points in the plane, and no two of those points lie on a line. Then you can draw N non-intersecting line segments, each joining a red point to a blue point.
4. Let p be a prime number. Then there do not exist integers a and b such that $a^2 = pb^2$.

Proof: Suppose there were such a and b.

- There is a pair a and b such that both are positive and b is as small as possible.
- Since a^2 is divisible by p, we know that a is divisible by p.
- Let $a = pa_1$. Then $p^2a_1^2 = pb^2$ so $b^2 = pa_1^2$.
- Applying the same argument to b, we find $a_1^2 = pb_1^2$.
- However, $b_1 < b$, which contradicts b being the smallest possible choice.