Prove: $4 \nmid (n^2-3)$.

FALSE PROOF.

Suppose $4 \mid (n^2-3)$.

$n^2-3 = 4k$ for some integer k

$n^2 = 4k+3$

$n = \sqrt{4k+3}$

But that can't be an integer!

So that's a contradiction

$4 \nmid (n^2-3)$

$7 \nmid (n^2-2)$ for all n.

Suppose $7 \mid n^2-2$

$n^2-2 = 7k$

$n^2 = 2+7k$

$n = \sqrt{7k+2}$

So that's a contradiction.

what if $k=1$?

$7k+2 = 9$

$n = 3$

$7 \mid 3^2-2 = 7$

GOOD PROOF.

$4 \mid n^2-3$ means $n^2 \equiv 3 \bmod 4$

$n^2 = 0, 1, 2^2 = 4 \equiv 0 \bmod 4,\ 3^2 \equiv 9 \equiv 1 \bmod 4$

$n^2 \equiv 0$, or $n^2 \equiv 1 \bmod 4$

therefore n^2 is never congruent to 3 and so $4 \nmid (n^2-3)$.

there are only 4 possibilities $n \equiv 0, 1, 2, 3$.

$$n = \begin{cases} 4k \checkmark \\ 4k+1 \checkmark \\ 4k+2 \checkmark \\ 4k+3 \checkmark \end{cases}$$

Case 1 n is even.

$n = 2k$

$n = 2k+1$

$4 \mid n^2$ n^2-3 can't be a multiple of 4

$n^2 = 4k^2$ $4k^2-3 = \underline{4(k^2-1)+1}$

$n^2 = 4k^2+4k+1$

$n^2-3 = 4k^2+4k+1-3$

$= \underline{4(k^2+4k-1)+2}$