Counting

Lists

Definition: a (finite) list is an element of the Cartesian product of sets $X = X_1 \times \cdots X_n$. A common counting problem is to determine the number of lists with certain properties whose entries are drawn from a Cartesian product like X.

Multiplication Principle

Fact 3.1 (Multiplication Principle) Suppose in making a list of length n there are a_1 possible choices for the first entry, a_2 possible choices for the second entry, a_3 possible choices for the third entry, and so on. Then the total number of different lists that can be made this way is the product $a_1 \cdot a_2 \cdot a_3 \cdots a_n$.

Figure 1: Multipication Principle (p. 69)

This informal principle can be applied in many settings, although in most cases there is a *hidden* proof by induction.

Example

Proposition: Suppose that $X_1, \ldots, X_n$ are finite sets. Then

$$|X_1 \times \cdots \times X_n| = |X_1||X_2| \cdots |X_n|.$$

Example

How many ways can you order a coffee if your choices are whole, skim, or soy milk; small, medium, or large size; and one or two shots of espresso?

Example

Consider lists of length 4 made with the symbols A, B, C, D, E, F, G.

Question: How many lists are there made up of these symbols (no special conditions)

Example continued

Question: How many lists are there if no letter is repeated?

Example continued

Question: How many lists are there if there are no repetitions, and at least one of the letters is an E?

Example continued

Question: How many lists are there if repetition *is* allowed, and the list contains at least one E?