Equivalence relations (finishing up)

1. Let $A = \{1, 2, 3\}$. List all possible equivalence relations on A.

Relations between sets/functions

Basic definitions

- A subset $R \subset A \times B$ defines a relation between sets A and B.
- A function F is a special kind of relation between two sets $A \times B$. When we write $F : A \to B$ we mean that
  - F is a relation on $A \times B$.
  - F has the special property that, for all $a \in A$, there is a unique $b \in B$ such that $(a, b) \in F$.
  - The sets A and B are called the domain and the codomain of F.
  - The range of F is the set $\{b \in B : \exists a \in A, (a, b) \in F\}$ Two functions are equal if they are equal as sets.

Basic problems

1. Suppose that $F = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x + 3y = 4\}$. Is F a function from $\mathbb{Z}$ to $\mathbb{Z}$?
2. Is the set $F = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x + 3y = 4\}$ a function from $\mathbb{R} \to \mathbb{R}$?
3. Is the set $F = \{(x, y) \in \mathbb{Z} \times \mathbb{R} : x + 3y = 4\}$ a function from $\mathbb{Z} \to \mathbb{R}$?
4. Is the set $F = \{(x, y) \in \mathbb{R} \times \mathbb{Z} : x + 3y = 4\}$ a function from $\mathbb{R} \to \mathbb{Z}$?

What might a graphical depiction of the problems above look like?

A few more

1. Let $A = \{1, 2, 3\}$ and $B = \{0, 1\}$. How many relations are there on $A \times B$? How many of them are functions?

2. More generally, let A be a finite set and let B be the set $\{0, 1\}$. How many functions are there from A to B? What is the relation between this and the power set of A?