Determinants

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The determinant as a linear function

Suppose that A is an $n \times n$ matrix whose i^{th} row is of the form $u + kv$ where u and v are vectors and k is a constant:

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ u + kv \\ \vdots \\ A_n \end{bmatrix}$$

Here $A_1, A_2, ...$ are the rows of A with $A_i = u + kv$.

Linearity of the determinant

Then

$$\det A = \det \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ u \\ \vdots \\ A_n \end{bmatrix} + k \det \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ v \\ \vdots \\ A_n \end{bmatrix}$$

This is sometimes expressed by saying that the determinant is a linear function of each row (holding the others fixed).