# Eigenvalues and Eigenvectors

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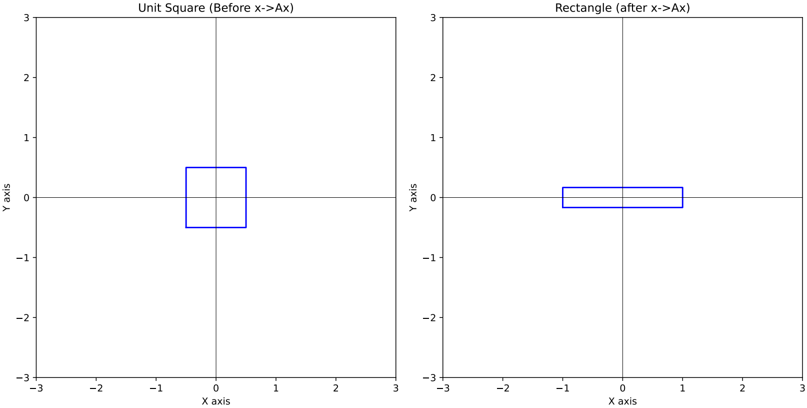
# Eigenvalues and Eigenvectors

If $A$ is a diagonal matrix:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1/3 \end{bmatrix}$$

then the linear transformation $x \mapsto Ax$ "stretches" along the $x$-axis and "shrinks" along the $y$-axis.

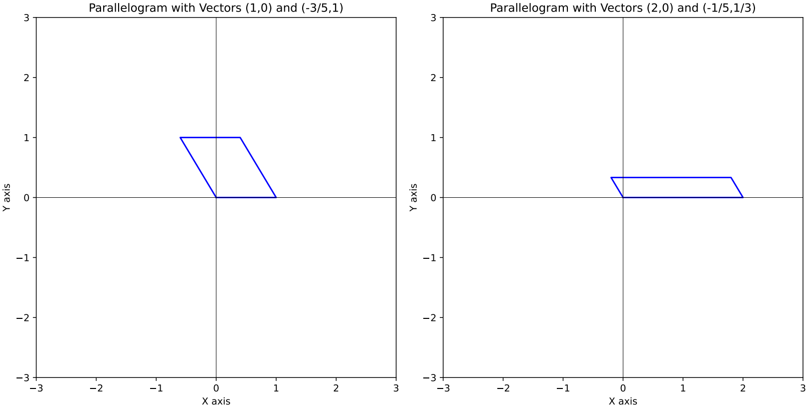


# Eigenvalues and Eigenvectors

If $A$ is upper triangular, say

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1/3 \end{bmatrix}$$

then $A$ stretches along the $x$-axis by $2$ as before. Less obviously, it shrinks along the direction given by the vector $(-3/5, 1)$.



# Eigenvalues and Eigenvectors

An **eigenvector** for a matrix $A$ is a vector $v$ which gets shrunk or lengthened by $A$ by some factor $\lambda$.

The factor $\lambda$ is called the **eigenvalue**.

More formally, a vector $v$ is called an eigenvector for $A$ (with eigenvalue $\lambda$) if $v$ is not zero and

$$Av = \lambda v.$$

# Eigenvalues and Eigenvectors

In the example above, the vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3/5 \\ 1 \end{bmatrix}$ are eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1/3 \end{bmatrix}$$

with eigenvalues $2$ and $1/3$ respectively.

$$\begin{bmatrix} 2 & 1 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} -3/5 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/5 \\ 1/3 \end{bmatrix} = (1/3) \begin{bmatrix} -3/5 \\ 1 \end{bmatrix}$$