1.8-1.9 Matrices and Linear Transformations

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Linear Transformations and Matrices

If A is an $n \times m$ matrix, and x is any vector in $\mathbf{R}^m$, then Ax is a vector in $\mathbf{R}^n$.

So we can define a function $T : \mathbf{R}^m \to \mathbf{R}^n$ by

$$T(x) = Ax.$$

For example if

$$A = \begin{bmatrix} 0 & -4 \\ 4 & 1 \\ 3 & 3 \end{bmatrix} \text{ and } v = \begin{bmatrix} x \\ y \end{bmatrix}$$

then

$$T(v) = Av = \begin{bmatrix} -4y \\ 4x + y \\ 3x + 3y \end{bmatrix}$$

Function terminology

In general if $f : X \to Y$ is a function then f is a "rule" that associates exactly one element $y \in Y$ to each element $x \in X$. The y corresponding to x is called $f(x)$. Furthermore:

- X is called the domain of f
- Y is called the codomain of f
- the set of $y \in Y$ so that there is an $x \in X$ with $f(x) = y$ is called the *range* of f.
- if $f(x) = y$, then y is called the *image* of x under f.

If A is an $n \times m$ matrix, then the domain of $f(x) = Ax$ is $\mathbf{R}^m$ and the codomain is $\mathbf{R}^n$.