Math 2710

Oct 21-25

Sequences

Definition: An (infinite) sequence with rational coefficients is a function $a : \mathbb{P} \to \mathbb{Q}$. Normally we view it as the sequence $a(1), a(2), \ldots$.

Some examples:

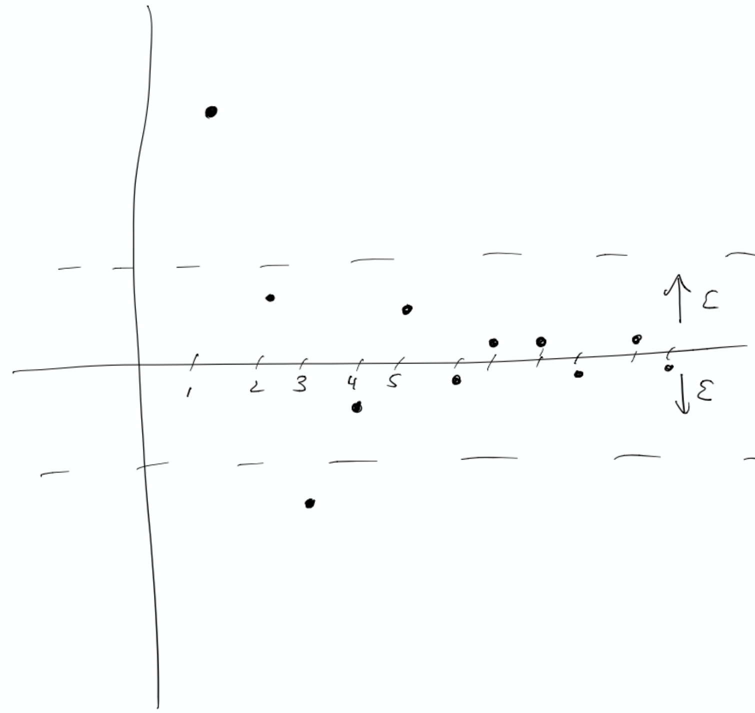
- $a(n) = 0$ for all n (the zero sequence).
- $a(n) = 1/n$ for $n = 1, 2, \ldots$
- $a(n) = n$ for $n = 1, 2, \ldots$
- $a(n) = (-1)^n$ for $n = 1, 2, \ldots$

We would like to have a way to speak about what happens to sequences as n gets larger and larger (so for example the sequence grows, it bounces around, or it approaches a particular number.)

Limit of a sequence

Definition: Let $a(n)$ be a sequence. Then we say that the limit of $a(n)$ is L if, for every $\epsilon > 0$, there is an integer N, so that $|a(n) - L| < \epsilon$ for all $n \geq N$. This is written:

$$\lim_{n \to \infty} a(n) = L.$$



and we say that the sequence *converges to L*.

Limits are about estimation

Examples

- Let $a(n)$ be the sequence defined by $a(1) = 1, a(2) = 1/2, and a(n) = 0$ for $n > 2$. Prove that the limit $\lim_{n\to\infty} a(n) = 0$.
- Let $a(n)$ be the sequence $a(n) = 1/n$. Prove that the limit of $a(n)$ as $n \to \infty$ is zero.
- Let $a(n) = (-1)^n$. Prove that the limit isn't 1. Then prove there is no limit.
- Let $a(n) = n$. Is there a limit?
- Let $a(n) = (n+1)/n$. Prove that the limit is 1.
- Let $a(n) = 4 + (-1/2)^n$.

Some other limit properties

Proposition: Let $a(n)$ be a sequence and let $b(n)$ be the sequence defined by $b(n) = a(n+5)$. (so $b(n)$ is the same as $a(n)$ but it "starts later.") Prove that

$a(n)$ converges if and only if $b(n)$ converges.