MAKE-UP EXAM
MATH 2710, FALL 2019

Instructions: For full credit you must do all of problems 1-4, and any three of the four problems 5-8. You will receive you score on 1-4 plus your best three scores from among 5-8.

1. Let X, Y, and Z be sets. Prove that $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$.

2. Let P be the proposition "If p is prime, then for all integers n we have p divides $n^p - n$."

1. Write the contrapositive of P.

2. Write the converse of P.

3. Write the number 102 in base 8.

4. Use the Euclidean algorithm to find the greatest common divisor of 88 and 12.

5. Let a, b and d be integers, with $d \neq 0$ and suppose (x_0, y_0) and (x_1, y_1) are solutions to the Diophantine equation

$$ax + by = d.$$

Prove that (b/d) divides $(x_0 - x_1)$.

6. Prove that there are infinitely many prime numbers.

7. Prove that if p is a prime number and p divides a or p divides b then $p | \text{lcm}(a, b)$.

8. Prove that if a, b, and c are nonzero integers and $a|b$ and $b|c$, then $a|c$.