

Math 2710

Sep 2-6

1.4 Variable statements and quantifiers

First examples

Compare the following three statements

- ▶ *Helen* is a UConn student who has watched every minute of Game of Thrones.
- ▶ *There is a UConn student* who has watched every minute of Game of Thrones.
- ▶ *Every UConn student* has watched every minute of Game of Thrones.

All make assertions about the set U of UConn students

- ▶ The first asserts that a *particular* named element of U has a certain property (... has watched every minute of GoT)
- ▶ The second asserts that *There exists* an element of U with that property.
- ▶ The third asserts that *Every* element of U has that property.

Universal quantifier (For all, for every, for each)

A statement that includes a universal quantifier makes a claim about ALL objects of a particular type.

- ▶ For all x in the real numbers, $(x^2 - 1) = (x + 1)(x - 1)$.
- ▶ Every declared democratic presidential candidate will appear in the next official television debate.
- ▶ Each midterm exam in this course counts as 25% of your final grade.

Symbolic Form

- ▶ For all X , $P(X)$
- ▶ $\forall x, P(X)$.

Existential quantifier (There is, there exists, for some)

- ▶ There is a real number y so that $y^2 = 11$.
- ▶ There exists a car for sale in the United States that gets 50 mpg.
- ▶ There are some dogs that you should be afraid of.

Symbolic Form

- ▶ There exists X such that $P(X)$
- ▶ $\exists x$ such that $P(x)$.

Relation between universal and existential quantifiers

To show that the statement *Every UConn student has watched every minute of Game of Thrones* is FALSE, you must produce an example of a UConn student who has NOT watched every minute. So the negation of this claim is:

Some UConn student has not watched every minute of Game of Thrones or There is a UConn student who has not watched every minute of Game of Thrones

To show that the statement *There is a UConn student who has watched every minute of Game of Thrones* is FALSE, you must show that: **No student has watched every minute of Game of Thrones or All students at UConn have NOT watched every minute of Game of Thrones.**

Symbolic Form (page 11 of the text)

- ▶ $\text{NOT}(\forall x, P(x)) \leftrightarrow \exists x, \text{NOT } P(x)$
- ▶ $\text{NOT}(\exists x, P(x)) \leftrightarrow \forall x, \text{NOT } P(x)$

Second order statements

Second order statements have two quantifiers.

- ▶ For all x , there exists y , so that. . .
- ▶ There exists x , so that for all y , . . .

For all x , there exists y .

- ▶ For every even integer x , there exists an integer y so that $x = 2y$.
- ▶ For every real positive number x , there exists a real number y so that $x = y^2$.
- ▶ For every real $\epsilon > 0$, there exists a real $\delta > 0$ so that if $|x| < \delta$ then $x^2 < \epsilon$.

There exists y , so that for all x

- ▶ There exists an integer x so that, for all integers y , $xy = 0$.

An example

Definition: Given two integers n and d , we say that

- n is divisible by d

or

- n is a multiple of d

or

- d divides n

if there exists an integer m so that $n = dm$.