

# Math 2710

Aug 26-28

## Course Info

# Key links

- ▶ Syllabus
- ▶ Tests
- ▶ Homework
- ▶ Piazza

# Grading

- ▶ Two midterms (25 points) tentatively Sep 30 and Nov 5.
  - ▶ Notify me by Sep 20 if you need an alternate date for the first exam because of Rosh Hashanah.
- ▶ Final Exam (40 points)
- ▶ Homework (8 points)
- ▶ Piazza participation (2 points)

# Homework

- ▶ daily assignments
- ▶ periodically collected and graded with short lead time
- ▶ assorted short quizzes or other assignments from time to time

## 1.1 What is this course about?

# Mathematics as a discipline

This course is about

- ▶ *how mathematics is done*
- ▶ *how mathematics is communicated.*

The actual mathematics we will learn in this course is less important than the approach

## A very simple example

**Assertion:** The sum of two even numbers is an even number.

Goal: find a mathematical proof of this fact.



# Mathematical Proof

A *mathematical proof* of this assertion is an argument that starts from known facts and definitions and establishes the the truth of the assertion using the tools of logic.

A proof in *formal logic* starts from explicit hypotheses or axioms and applies the rules of deductive logic to reach a conclusion. Proofs of even simple facts in formal logic are extremely long and mostly not readable by humans.

In principle, a mathematical proof contains enough information to produce a formal logical proof.

# Good Mathematical Proofs

A good mathematical proof is

- ▶ *rigorous*, meaning it gives a complete logical argument,
- ▶ *informative*, meaning that it provides enough information to explain why the assertion is true
- ▶ *efficient*, meaning that it is as short as possible while still being rigorous and informative.

## Example, continued

To construct a proof of this assertion, we need:

- ▶ to know exactly what the terms mean (what is an even integer?)
- ▶ to establish in our own minds that the assertion IS true, and figure out why
- ▶ communicate our understanding of why the assertion is true rigorously and efficiently.

# Discussion

- ▶ Define *even number*.
- ▶ Explain why the assertion about even numbers is true, as rigorously and efficiently as you can.

# Key Vocabulary

- ▶ theorem
- ▶ lemma
- ▶ proposition
- ▶ corollary
- ▶ example
- ▶ algorithm
- ▶ definition
- ▶ proof
- ▶ statement
- ▶ proposition
- ▶ converse
- ▶ contrapositive
- ▶ conditional statement

## 1.2 Logic

# Statements

A statement is a sentence that is either **True** or **False**

Compound statements are built up using logical operators **AND**, **OR**, **NOT**, and others.

The truth of a compound statement depends on the individual statements and the properties of the operators.

# AND, OR, NOT

P	Q	AND
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	OR
T	T	T
T	F	T
F	T	T
F	F	F

P	NOT
T	F
F	T



# Implications/Conditionals

P	Q	$\Rightarrow$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\Leftrightarrow$
T	T	T
T	F	F
F	T	F
F	F	T

## Discussion

- ▶  $P$  and  $Q$  are equivalent means  $P \iff Q$  or “ $P$  if and only if  $Q$ ”.
- ▶ Show:  $P \implies Q$  is equivalent to  $(Q \text{ OR NOT } P)$ .
- ▶ Exclusive OR is the operator that is TRUE when one of two statements is True, but not both. Express it in terms of AND, OR, and NOT.
- ▶ If my basement is wet, then it is either very rainy or a pipe has broken. Express this using the various operators and test its truth under different conditions.

::: notes

Section 1.3 starts here, Friday Aug. 30

## 1.3 Sets

# Sets

We rely on a “naive” notion of set, meaning a collection of objects.  
For example:

- ▶ the set of integers
- ▶ the set of words in the English language
- ▶ the set of people in the world
- ▶ the empty set

# Subsets

We can construct sets by selecting elements of another set, yielding a *subset* of the original set.

## Explicit specification

$A = \{1, 3, 5, 8, 9\}$ , a subset of the integers.

## Selection by a property

Suppose  $P$  is the set of people. Then

$$\{p \in P : p \text{ is a legal resident of Chicago}\}$$

is shorthand for the set consisting of people  $p$  for which the statement “ $p$  is a legal resident of Chicago” is True.

# Set operations

If  $A$  and  $B$  are both subsets of some huge (and usually unmentioned) set  $U$ , then:

- ▶  $A = B$  means that  $A$  and  $B$  have the same elements.
- ▶  $A \subset B$  means that every element of  $A$  is also an element of  $B$ .



## More set operations

- ▶  $A \cup B$ , the union of  $A$  and  $B$ , is  $\{x \in U : x \in A \text{ OR } x \in B\}$
- ▶  $A \cap B$ , the intersection of  $A$  and  $B$ , is  $\{x \in U : x \in A \text{ AND } x \in B\}$
- ▶  $A \times B$ , the product of  $A$  and  $B$ , is the set of all ordered pairs  $(x, y)$  where  $x \in A$  and  $y \in B$ .

## Discussion

Suppose  $A$  and  $B$  are two sets contained in some big set  $U$ . Prove the following; truth tables may be helpful.

- ▶  $A = B$  if and only if  $A \subset B$  and  $B \subset A$ .
- ▶  $((A \cap B) = A)$  implies  $A \subset B$ .