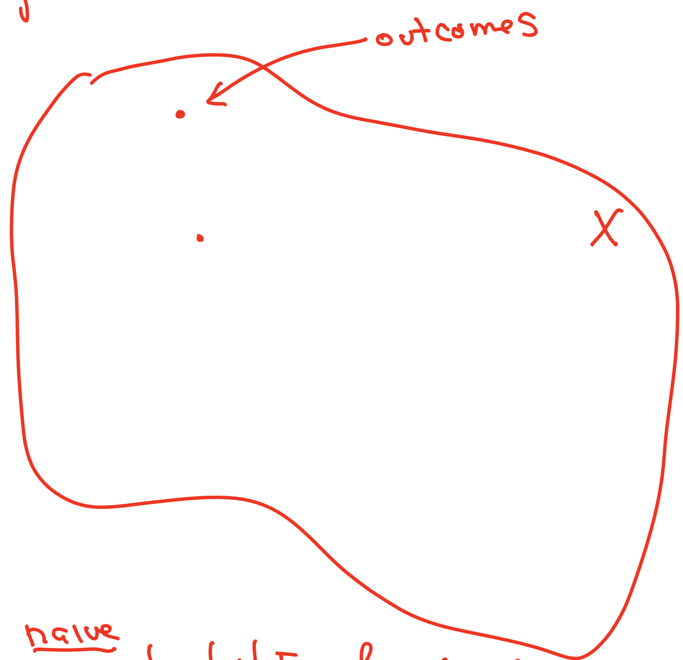
Probability starts with a set



sample space

An Event is a subset

$U \subseteq X$

We have a naive probability function*

$$P : \mathcal{P}(X) \longrightarrow \cancel{\mathbb{R}}\ [0,1]$$

$P(U) =$ "how likely is U"?

$P=0 \iff$ doesn't happen

$P=1 \iff$ always happens

$p \in (0,1)$ measures likelihood.

If $U_1, \dots, U_n$ are disjoint events

$$P(U_1 \cup \cdots \cup U_n) = \sum_{i=1}^{N} P(U_i)$$

$P(X) = 1$

$P(\emptyset) = 0$

★ Coin Flipping

$X = \{H, T\}$

U	$P(U)$
$\emptyset$	0
$\{H\}$	p
$\{T\}$	$1-p$
$\{H,T\}$	1

★ X = rolls of 2 6-sided dice

$X = \{(⚀,⚁), (⚁,⚁), \dots\}$ 36 total elements.

– $P(\{(x,y)\}) = 1/36$

Probability that sum of the dice is 5?

$E = \{(1,4), (2,3), (3,2), (4,1)\}$

$P(E) = 4/36 = 1/9$.

★ $X = \mathbb{R}$

we measure the temperature with an error-prone thermometer.
"True temperature" is t_0
Measurements are $t_0 + x$. $x \in \mathbb{R}$

Probability density $p(x) : \mathbb{R} \to \mathbb{R}$.

$U \subseteq \mathbb{R}$ $P(U) = \int_U p(x)\,dx$ = "chance that our measurement lands in U".

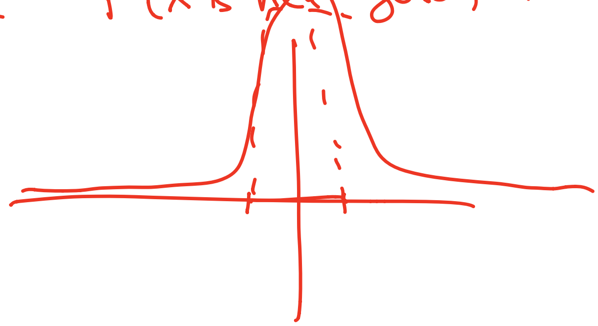
$U = (-\delta, \delta)$ for $\delta \neq 0$

$P(U) = \int_{-\delta}^{\delta} p(x)\,dx$

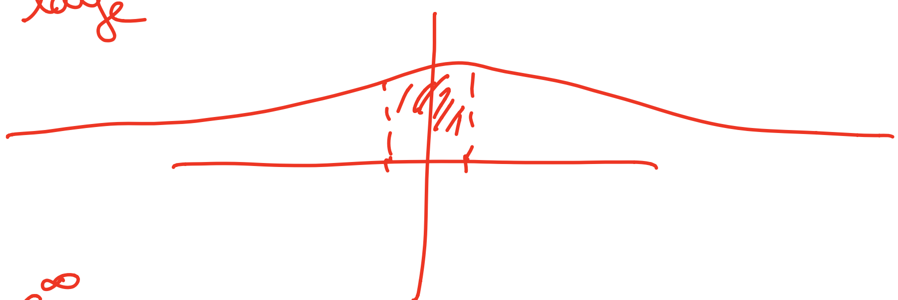
σ small

$p_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/(2\sigma^2)}$ $\sigma \in \mathbb{R}$

σ small P(x is near zero) is large



σ large



$$P(\overset{\mathbb{R}}{\overset{\|}{X}}) = \int_{-\infty}^{\infty} p(x)\,\partial x = 1.$$