# Random Variables - continuous case

Random Variable: Continuous Case

- We make two independent measurements of temperature t, where the true temperature is t_0 and the errors $x = t - t_0$ are independent normal random variables with variance 1.

- The sample space $X = \mathbf{R}^2$ and the probability density is the multivariate gaussian

$$p(x) = \frac{1}{2\pi} e^{(-x_1^2 - x_2^2)/2} = \frac{1}{2\pi} e^{-\|x\|^2/2}$$

Independent normally distributed errors
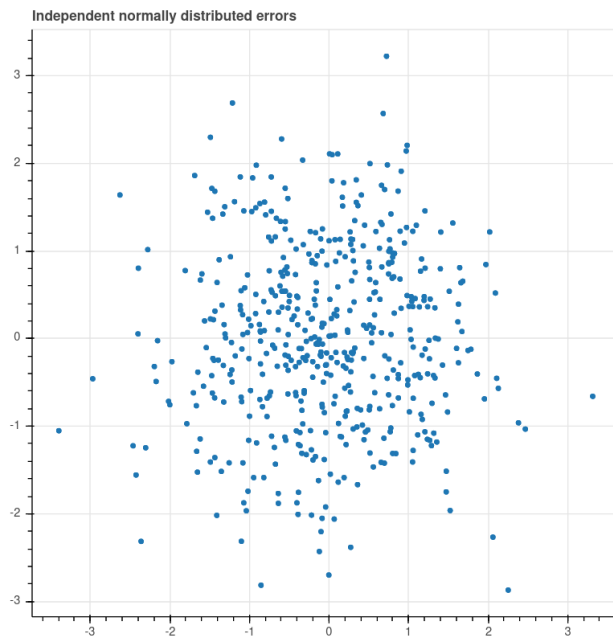
Distribution of norms

- How is $\|x\| = \sqrt{x_1^2 + x_2^2}$ distributed? $\|x\|$ is a random variable on X.

- What is the probability $P(\|x\| < r)$?

- Here is a histogram using the sample data above showing the distribution of the distances. Notice that as r increases, more and more of the points lie within distance r of the origin.

Cumulative fraction of total points within radius r
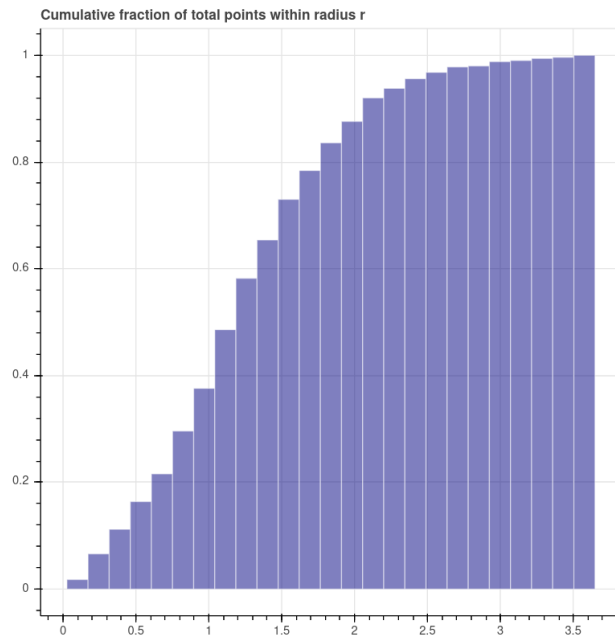
Distribution of norms (continued)

- By definition,

$$P(\|x\| < r) = P(\{(x_1, x_2) : x_1^2 + x_2^2 < r^2\}) = \frac{1}{2\pi} \int_{\|x\| < r} e^{-\|x\|^2/2}$$

- This is a doable integral using polar coordinates.

$$\frac{1}{2\pi} \int_{\|x\| < r} e^{-\|x\|^2/2} = \frac{1}{2\pi} \int_{\theta=0}^{2\pi} \int_{\rho=0}^{r} e^{-\rho^2/2} \rho \, d\rho \, d\theta$$
$$= \int_{\rho=0}^{r} \rho e^{-\rho^2/2} d\rho$$
$$= 1 - e^{-r^2/2}$$

Distribution of norms continued

- Here we superimpose our calculated cumulative density with the experimental data to see that they match.

Cumulative fraction of total points within radius r