X N rows
K+1 colums
Last column all 1's
col feature

row
sample

centered coordinates: change each feature so that average value of the modified features is zero.

[Assume that every one of the first k columns of X sums to zero.]

$$X^T(Y - XM) = 0$$

$$M = D^{-1} X^T Y$$

$$D = X^T X$$

$$\underset{K+1}{\underset{\sim}{}} \underset{K^T}{} \underset{N}{} \quad \underset{D}{}$$

$$\begin{pmatrix} x_{11} & x_{21} & x_{31} & \cdots \\ x_{12} & x_{22} & x_{32} & \cdots \\ & & & \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & x_{13} & & 1 \\ x_{21} & & & & 1 \\ \vdots & & & & \vdots \\ & & & & 1 \end{pmatrix} = \begin{pmatrix} & & & 0 \\ & D_0 & & 0 \\ & K \times K & & 0 \\ & & & 0 \\ 0 & 0 \cdots & 0 & N \end{pmatrix}$$

X_0 = N×K matrix of data without last column

$$D_0 = X_0^T X_0$$

$$D^{-1} = \begin{pmatrix} D_0^{-1} & \vdots & 0 \\ 0 \cdots & 0 & 1/N \end{pmatrix}$$

$$M = D^{-1} X^T Y \qquad \begin{pmatrix} D_0^{-1} & \vdots & 0 \\ 0 \cdots & 0 & N^{-1} \end{pmatrix} \underset{K \times N}{\begin{pmatrix} & X_0^T & \\ 1 & 1 & 1 \end{pmatrix}} \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}$$

$$= \begin{pmatrix} D_0^{-1} & & 0 \\ & \ddots & \\ 0 & & N^{-1} \end{pmatrix} \begin{pmatrix} \\ \\ \Sigma y_i \end{pmatrix}$$

$$M \quad \rightarrow \quad \begin{pmatrix} * \\ * \\ \vdots \\ * \\ \frac{\Sigma y_i}{N} \end{pmatrix} \qquad m_{K+1} = \bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

$$\begin{pmatrix} m_1 \\ \vdots \\ m_K \end{pmatrix} = D_0^{-1} X_0^T Y \qquad m_{K+1} = \bar{y} = \frac{1}{N} \sum y_{i\cdot}$$

$$D_0 = X_0^T X_0$$