

# Conditional Probability and Bayes Theorem

## Conditional Probability

- ▶ Draw a card from a deck.  $P(\text{king}) = 4/52 = 1/13$ .
- ▶ Now suppose you *know* the card is a face card. Given that information, the probability of drawing a king is  $4/12 = 1/3$ . This is an example of *conditional probability*.  $P(A|B)$ .
- ▶ More generally

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Meaning the chance of getting  $A$  among conditions where  $B$  is known to hold.

## Bayes Theorem

**Theorem:** Given events  $A$  and  $B$  in a sample space  $X$ , we have

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

**Proof:** Substitute  $P(B \cap A)/P(A)$  for  $P(B|A)$ .

## An example: COVID testing

- ▶ A person can be *sick* (S) or *well* (W).
- ▶ Their covid test can be *positive* (+) or *negative* (-).

There are four possibilities:

- ▶ S+ – this is a true positive
- ▶ S- – this is a false negative
- ▶ W+ – this is a false positive
- ▶ W- – this is a true negative

An early CDC report estimated that  $P(+|W) = 1/200$  and  $P(-|S) = 1/4$ .

## COVID testing continued

- Suppose I get a covid test and it's positive. How likely am I to have the disease? In other words, what is  $P(S|+)$ ?

The answer depends on the prevalence  $p = P(S)$ , the chance that I have COVID in the first place.

# COVID testing continued

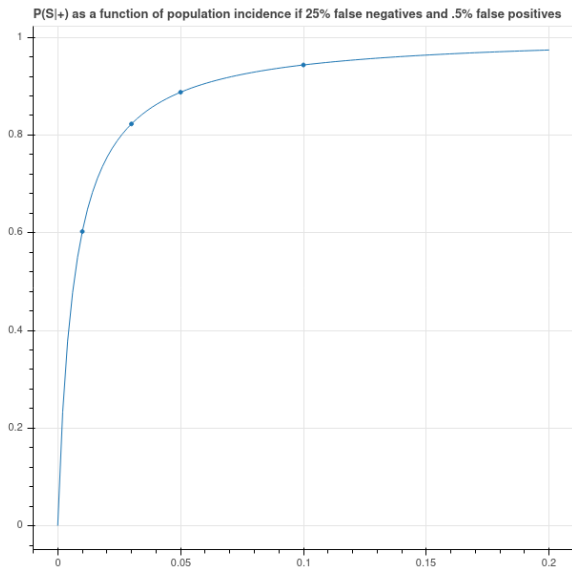


Figure 1: Chance I have COVID if I get a positive test vs prevalence