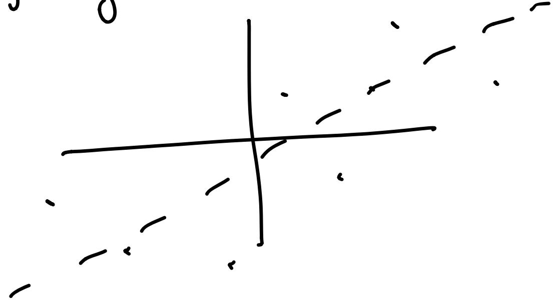
Given a set $\{(x_i, y_i)\}_{i=1}^{N}$ of points.

__Before__: thought of this as N pts in $\mathbb{R}^2$



__NOW__: $\vec{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$ $\quad \vec{Y} = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}$ $\quad \vec{E} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$

$\vec{X}, \vec{Y}, \vec{E} \in \mathbb{R}^N$

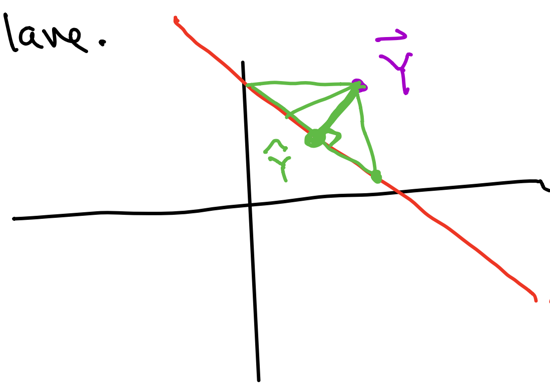
__NOTICE__: if $y_i = m x_i + b$ for all i then

$$\vec{Y} = m\vec{X} + b\vec{E}.$$

In other words the 3 vectors are linearly dependent.

But that's __NOT__ __TRUE__.

$\vec{X}, \vec{E}$ span a plane (a 2-dim'l space) in $\mathbb{R}^N$ and $\vec{Y}$ doesn't belong to that plane.



$\hat{Y} = m\vec{X} + b\vec{E}$ for some m, b

$\hat{Y} = \begin{pmatrix} \hat{y}_1 \\ \vdots \\ \hat{y}_N \end{pmatrix}$ are predicted Y-values

plane spanned by $\vec{X}, \vec{E} = \{m\vec{X} + b\vec{E}\}$

$$\hat{Y}(m,b) = m\vec{X} + b\vec{E}$$

$$\|\hat{Y} - \vec{Y}\|^2 = \|m\vec{X} + b\vec{E} - \vec{Y}\|^2$$ minimize this

$$\|A\|^2 = A \cdot A$$

$$D = \|\hat{Y} - \vec{Y}\|^2 = (m\vec{X} + b\vec{E} - \vec{Y}) \cdot (m\vec{X} + b\vec{E} - \vec{Y})$$

$$\frac{\partial D}{\partial m} = 2\vec{X} \cdot (m\vec{X} + b\vec{E} - \vec{Y})$$

$$\frac{\partial D}{\partial b} = 2\vec{E} \cdot (m\vec{X} + b\vec{E} - \vec{Y})$$

set $\frac{\partial D}{\partial m} = 0$ and $\frac{\partial D}{\partial b} = 0$

$$\left.\begin{aligned} m(\vec{X}\cdot\vec{X}) + b(\vec{X}\cdot\vec{E}) &= \vec{X}\cdot\vec{Y} \\ m(\vec{E}\cdot\vec{X}) + b(\vec{E}\cdot\vec{E}) &= \vec{E}\cdot\vec{Y} \end{aligned}\right]$$

same as Least squares equations derived earlier.

$$\vec{X}\cdot\vec{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} = \sum x_i^2 = N S_{xx}$$

$$\vec{E}\cdot\vec{E} = N$$

$$\vec{X}\cdot\vec{E} = \sum x_i = N\bar{x}$$

$$\vec{E}\cdot\vec{Y} = N\bar{y}$$

$$\vec{X}\cdot\vec{Y} = \sum x_i y_i = N S_{xy}$$