Probability Basics

Outcomes and Sample Space

Probability begins with a set X of “outcomes”. This set may be continuous or discrete.

- $X = \{H, T\}$, the result of a single coin flip. (discrete)
- X is the possible results of throwing two six-sided dice – ordered pairs. (discrete)
- X is the set of real numbers, where a value x means measuring the temperature $t_0 + x$ where t_0 is the “true” temperature. (continous)

The set X of possible outcomes is called the *sample space.*

Event

An “event” is a subset of the sample space – a collection of outcomes.

The probability function P takes values between 0 and 1 and measures the “chance” that an event “occurs.”

If a sequence of events are disjoint, then the probability of them all happening is the sum of their probabilities.

$$P(U_1 \cup \cdots \cup U_n) = \sum_{i=1}^{n} P(U_i)$$

Events - discrete examples

- $P(\{H\}) = 1/2$
- $P({(⚀,⚄)}) = 1/36
- the probability of the event E consisting of throwing two dice that sum to 5:

  $$E = \{(\text{⚀}, \text{⚃}), (\text{⚁}, \text{⚂}), (\text{⚂}, \text{⚁}), (\text{⚃}, \text{⚀})\}$$

  is $(4)(1/36) = 1/9$

Events - continuous example

- $X = \mathbf{R}$.
- Probability arises from a density function $p(x)$
- $P(U) = \int_U p(x)dx$
- $\int_{-\infty}^{\infty} p(x)dx = 1$.

Normal distribution

- Measure temperature t using a thermometer.
- True temperature is t_0.
- Error $x = t - t_0$

  $$P(|t - t_0| < \delta) = \int_{x=-\delta}^{\delta} p_\sigma(x) dx$$

  where

  $$p_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/(2\sigma^2)}.$$

σ is called the "standard deviation".

Normal distribution cont'd

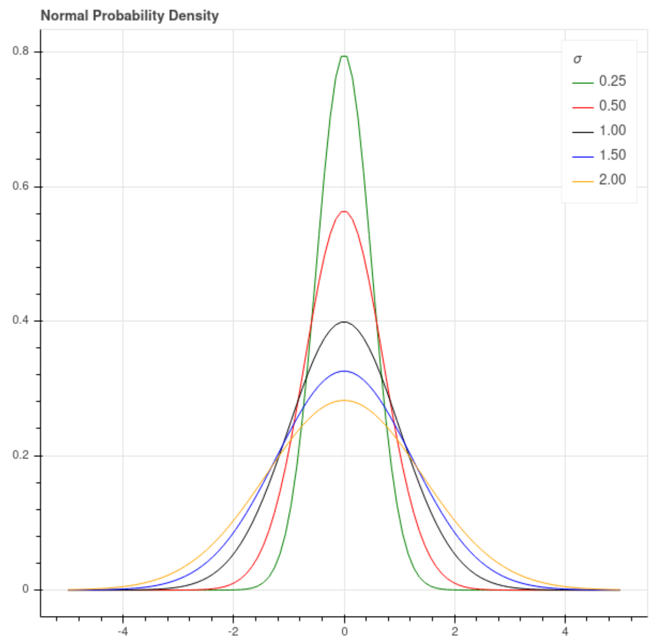

Figure 1: Normal Distributions