minimum for $E = \|Y - XM\|^2$
occurred when

$$\nabla E = 0$$

$$X^T Y - X^T X M = 0$$

$$X^T(Y - XM) = 0$$

X, Y constant matrices
M variables

$$M = \begin{bmatrix} m_1 \\ \vdots \\ m_{K+1} \end{bmatrix}$$

$X : N \times (K+1)$ data
Y target

X given matrix $N \times (K+1)$

column space of X is the subspace of $\mathbb{R}^N$ spanned by the columns of X.

colspace spanned by $K+1$ vectors

$$M = \begin{pmatrix} m_1 \\ \vdots \\ m_{K+1} \end{pmatrix}$$

XM is an $N \times 1$ vector

as M varies, XM gives elements of $\mathbb{R}^N$.

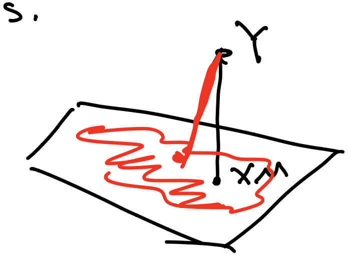
$$\begin{pmatrix} x_{11} & x_{12} & \cdots \\ x_{21} & & \\ \vdots & & \end{pmatrix} \begin{pmatrix} m_1 \\ \vdots \\ m_{K+1} \end{pmatrix} = \begin{pmatrix} m_1 x_{11} + m_2 x_{12} + \cdots + m_{K+1} x_{1,K+1} \\ \vdots \\ x_{N1} m_1 + \cdots + x_{N,K+1} m_{K+1} \end{pmatrix}$$

$$= m_1 X[:,1] + m_2 X[:,2] + \cdots + m_{K+1} X[:,K+1]$$

XM varies over colspace(X) as M varies.

$\cancel{\|Y-XM\|}$ $Y - XM$

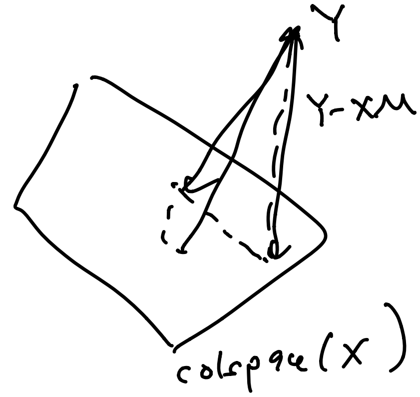
minimize $\|Y - XM\|^2$



colspace (X)

What is the point in colspace(X) closest to Y?

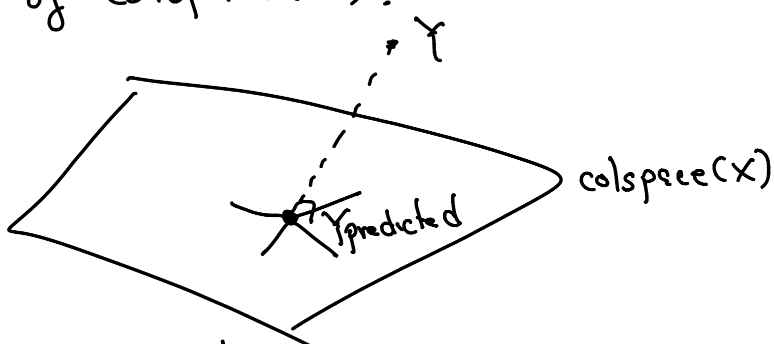
This happens for XM when $\underset{k+1 \times n}{X^T}(Y - XM) = 0$

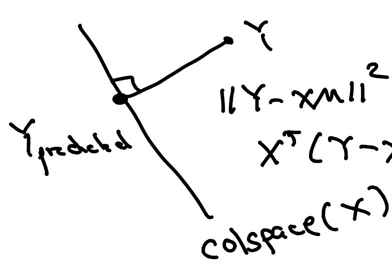


$$\left(\text{col of any } X \text{ matrix} \right) \cdot \begin{pmatrix} Y - XM \\ \\ \\ \end{pmatrix}$$

entries of $X^T(Y - XM)$ are the dot products of the colums of X with $Y - XM$ as M varies.

If all dot products are zero $\Leftrightarrow$ $Y - XM$ is perpendicular to a spanning set for colspace(X) $\Rightarrow$ perpendicular to any element of colspace(X).





$$\left.\begin{array}{l} \|Y - XM\|^2 \text{ minimized} \\ X^T(Y - XM) = 0 \end{array}\right\} \text{mean.}$$