Independence

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Definition: Two events A and B are independent if $P(A \cap B) = P(A)P(B)$. Alternatively, they are independent if $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

Informally, two events are independent if they don't influence each other; knowing that A happened doesn't give you any additional information about B.

Independence Example - Discrete

- Suppose that our sample space consists of N flips of a coin that has probability p of giving heads.
- The events corresponding to a H in position i and in position j are independent.
- The chance of getting k heads in N flips is

$$P(k, N) = \binom{N}{k} p^k (1-p)^{N-k}$$

The probability distribution on the set $0, \ldots, N$ given by this formula is called the *binomial distribution* for parameters p and N.

Independence Example - Continuous

Suppose we have a thermometer that measures the temperature t within an error $x = t - t_0$ from the true temperature, where x is normally distributed with standard deviation σ.

Suppose we make N independent measurements of the temperature. How are the errors distributed?

$$P(|x_1| < \delta, \ldots, |x_N| < \delta) =$$
$$\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^N \int_{x_1=-\delta}^{\delta} \cdots \int_{x_N=-\delta}^{\delta} e^{-(\sum_{i=1}^N x_i^2)/(2\sigma^2)} dx_1 \ldots dx_N$$

This is the *multivariate gaussian* distribution.

Non-independent events

Suppose we draw a pair of real numbers (x, y) from the plane R^2 controlled by the distribution

$$P((x,y) \in U) = A \int_U e^{(-x^2-xy-y^2)/(2\sigma^2)} dxdy$$

This density function has a bump at the origin and its level curves are ellipses.

The two coordinates are not independent of each other.

Non-independent events

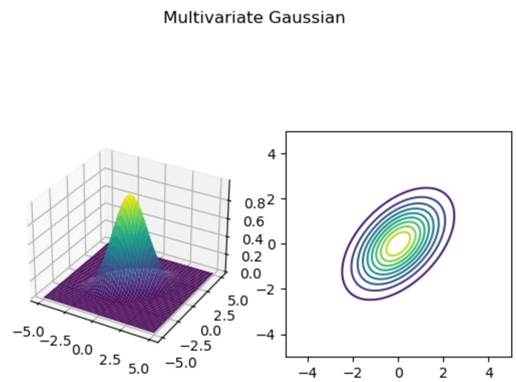

Figure 1: Multivariate Gaussian