Support Vector Machines in real life

Three ideas for support vector machines

1. Sets that aren't linearly separable.
2. Kernel functions and non-linear boundaries
3. Multiclass classification

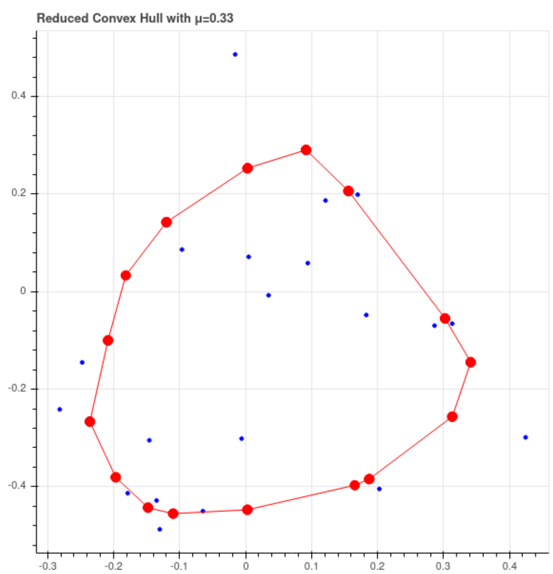
Reduced convex hulls and non-separable sets

In practice, we typically DO NOT have linearly separable sets.

In this case, we can look for a classifying hyperplane that puts **most** of the positive points on one side and **most** of the negative points on the other.

One approach is to do this by using the *reduced convex hulls*

$$C(A, r) = \{\sum_{i=1}^{n} \lambda_i x_i : 0 \leq \lambda_i \leq r, \sum_{i=1}^{n} \lambda_i = 1\}$$



RCH

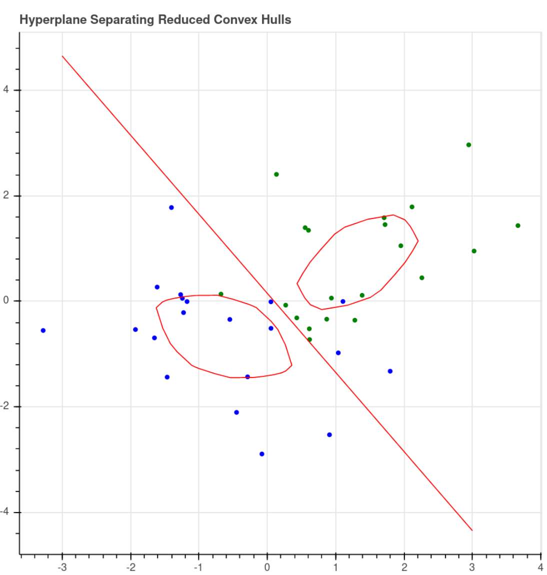
Reduced convex hulls and SVM

In practice, we choose a value r and find the closest points between the reduced convex hulls. This can be done by the SMO algorithm, the only change being the constraint on δ which becomes:

$$\delta \geq \max\{-\lambda_i^+, -\lambda_j^-\}$$

and

$$\delta \leq \min\{r - \lambda_i^+, r - \lambda_j^-\}.$$



RCH - SVM

Kernel functions

Recall that the function that we were trying to minimize depended only on the inner products $(x_i^{\pm}, x_j^{\pm})$. This allows for something called the *kernel trick.*

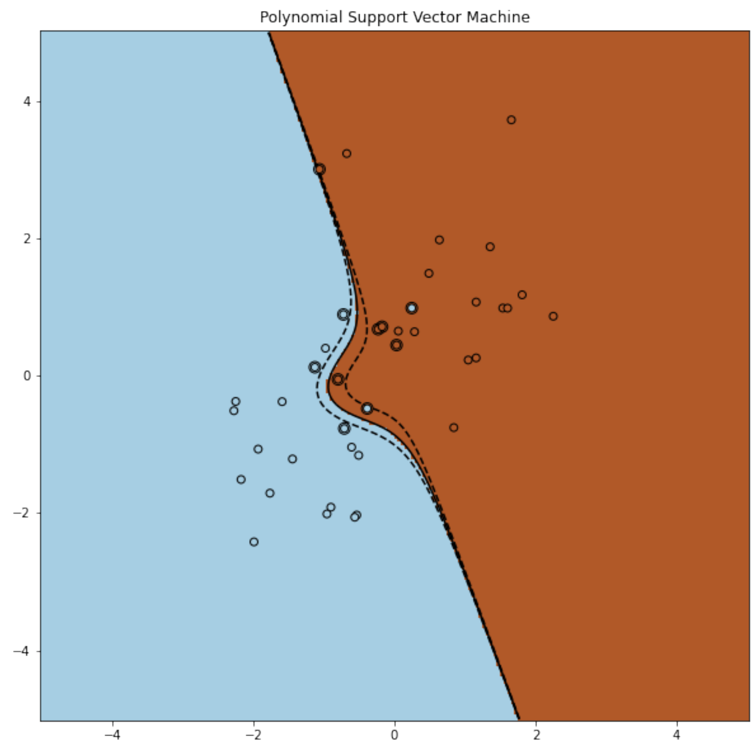
We can choose a **different** set of values for the dot products – basically, any collection of dot products so that the symmetric matrix $(x_i^{\pm}, x_j^{\pm})$ is positive (semi)-definite – any redo the analysis. This matrix is called a *kernel.*

This amounts to embedding the points in a high dimensional space, possibly by a non-linear map, and finding the classifying hyperplane there.

Common choices of kernels:

- polynomial: finds separating polynomial curves instead of lines.
- radial basis kernel (rbf): this sets the "distance" between x and y to be
  $$(x, y) = e^{-\|x-y\|^2/2\sigma^2}.$$
  This greatly rescales the distance between points.

Example



Polynomial Kernel

Multiclass classification

In general, the idea is to train classifiers that discriminate each class from all of the others.

This requires $\binom{n}{2}$ classifiers.