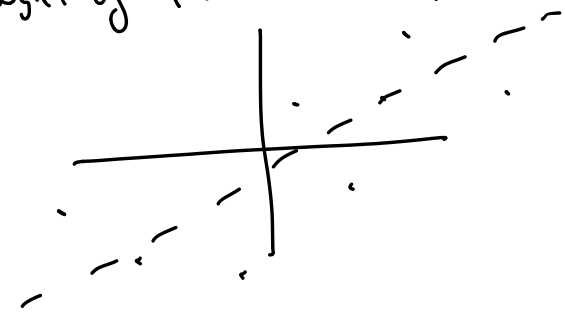
Given a set $\{(x_i, y_i)\}_{i=1}^{N}$ of points.

<u>Before</u>: thought of this as N pts in $\mathbb{R}^2$



<u>NOW</u>: $\vec{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$ $\quad \vec{Y} = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}$ $\quad \vec{E} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$

$\vec{X}, \vec{Y}, \vec{E} \in \mathbb{R}^N$

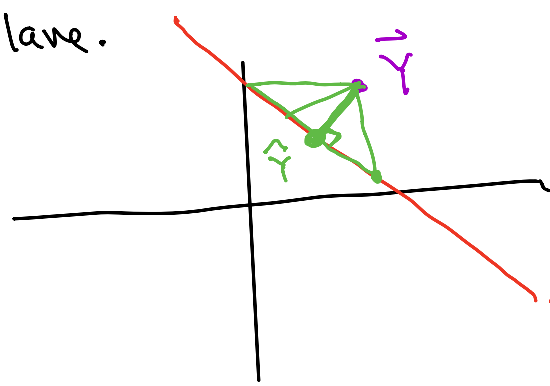
<u>NOTICE</u>: if $y_i = m x_i + b$ for all i then

$$\vec{Y} = m\vec{X} + b\vec{E}.$$

In other words the 3 vectors are linearly dependent.

But that's <u>NOT</u> <u>TRUE</u>.

$\vec{X}, \vec{E}$ span a plane (a 2-dim'l space) in $\mathbb{R}^N$. and $\vec{Y}$ doesn't belong to that plane.



$\hat{Y} = m\vec{X} + b\vec{E}$ for some m, b

$\hat{Y} = \begin{pmatrix} \hat{y}_1 \\ \vdots \\ \hat{y}_N \end{pmatrix}$ are predicted Y-values

plane spanned by $\vec{X}, \vec{E} = \{m\vec{X} + b\vec{E}\}$

$$\hat{Y}(m,b) = m\vec{X} + b\vec{E}$$

$$\|\hat{Y} - \vec{Y}\|^2 = \|m\vec{X} + b\vec{E} - \vec{Y}\|^2 \swarrow$$ minimize this

$$\|A\|^2 = A \cdot A$$

$$D = \|\hat{Y} - \vec{Y}\|^2 = (m\vec{X} + b\vec{E} - \vec{Y}) \cdot (m\vec{X} + b\vec{E} - \vec{Y})$$

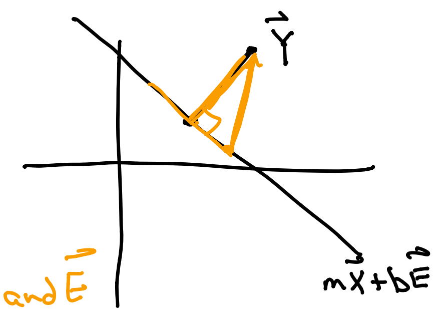
$$\frac{\partial D}{\partial m} = 2\vec{X} \cdot (m\vec{X} + b\vec{E} - \vec{Y})$$

$$\frac{\partial D}{\partial b} = 2\vec{E} \cdot (m\vec{X} + b\vec{E} - \vec{Y})$$

Set $\frac{\partial D}{\partial m} = 0$ and $\frac{\partial D}{\partial b} = 0$

$$\vec{X} \cdot \overbrace{(m\vec{X} + b\vec{E} - \vec{Y})}^{A(m,b)} = 0$$

$$\vec{E} \cdot \underbrace{(m\vec{X} + b\vec{E} - \vec{Y})} = 0$$



$A(m,b)$ is $\perp$ to both $\vec{X}$ and $\vec{E}$ and so is $\perp$ to the plane they span.

$$m(\vec{X} \cdot \vec{X}) + b(\vec{X} \cdot \vec{E}) = \vec{X} \cdot \vec{Y}$$

$$m(\vec{E} \cdot \vec{X}) + b(\vec{E} \cdot \vec{E}) = \vec{E} \cdot \vec{Y}$$

$\vec{X} \cdot \vec{X} = \sum x_i^2$

$\vec{X} \cdot \vec{E} = \sum x_i$

$\vec{X} \cdot \vec{Y} = \sum x_i y_i$

$\vec{Y} \cdot \vec{E} = \sum y_i$

$\vec{E} \cdot \vec{E} = N$

SAME equations we obtained by minimizing the MSE.