

## Conditional Probability

$X$  = deck of 52 cards  
draw a card at random.

$$P(\{\text{king}\}) = 4/52 = 1/13$$

You somehow learn that your card is a face card.

$$P(\{\text{king}\} \mid \{\text{face card}\}) = 4/12 = 1/3$$

12 face cards

Def:  $A, B$  be 2 events

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Theorem: (Bayes Theorem)

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Proof  $P(\underline{B} \mid \underline{A})P(\underline{A}) = P(\underline{A} \cap \underline{B})$

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

