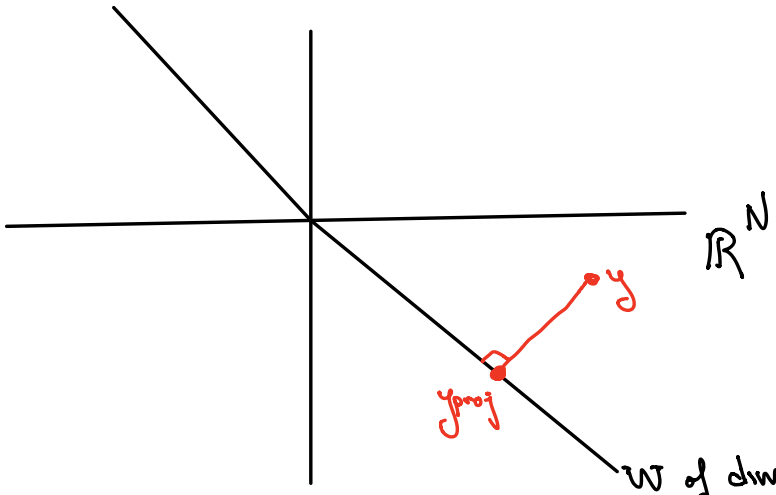

$(y - y_{proj}) \perp W$

$X^T(Y - XM) = 0$

as M varies, XM varies of W

$X^T(Y - XM) = 0$ means $Y - XM$ is $\perp$ all columns of X therefore to W.

$X^TY = \underbrace{X^TX}_{D} M$ $\qquad M = D^{-1}X^TY$

$y_{proj} = XM$

$= \underline{XD^{-1}X^T}Y$

Let $P_W = XD^{-1}X^T$ this is an $N \times N$ matrix.

① If $y \in \mathbb{R}^N$, then $P_W y \in W$

$P_W y = X(D^{-1}X^T y) \in W$

② $(y - P_W y) \perp W$.

$X^T(y - P_W y) = 0$

③ $P^2 = P$.

$$X D^{-1} \underbrace{X^T X}_{D} D^{-1} X^T = X D^{-1} D D^{-1} X^T = X D^{-1} X^T = P.$$

④ $y \in W$ $\quad y = Xm$ for some m.

$$P_y = X D^{-1} X^T X m = X D^{-1} D m$$
$$= Xm = y.$$