

X a sample space P probability

Def: A random variable on X is a $f: X \rightarrow \mathbb{R}$.

For example.

$$X = \{H, T\}$$

$$P(H) = p, \quad 0 \leq p \leq 1.$$

$$f: X \rightarrow \mathbb{R}$$

$$f(H) = 1 \quad f(T) = 0$$

This is a random variable

Bernoulli random variable with parameter p .

If $f: X \rightarrow \mathbb{R}$ is a R.V. and $U \subseteq \mathbb{R}$.

What is $P(f \in U)$?

$$\text{What is } P(\underbrace{\{x \mid f(x) \in U\}}_{f^{-1}(U)}) = P(f \in U).$$

f is Bernoulli with parameter p

$$P(f \in U) = P(\{x \in X \mid f(x) \in U\})$$

$$\begin{aligned} U &= \{0\} \\ \{x \in X \mid f(x) \in U\} &= \{x \mid f(x) = 0\} \\ &= \{T\} \end{aligned}$$

$$P(\{T\}) = 1 - p.$$

$$P(f \in \{0\}) = 1 - p$$

$$P(f \in [-\frac{1}{2}, \frac{1}{2}]) = 1 - p.$$

$$\begin{aligned} f(H) &\notin U \\ f(T) &\in U \end{aligned} \quad f^{-1}(U) = \{T\}$$

$$X = \{T, H\}^N = \{\text{sequences of heads and tails of length } N\}$$

$$P(x_1, \dots, x_N) = p^{\# \text{heads}} (1-p)^{\# \text{tails}}$$

$$f_i(x_1, \dots, x_N) = f(x_i) \quad f \text{ Bernoulli}$$

$$f_i(x_1, \dots, x_N) = \begin{cases} 1 & \text{if } x_i = H \\ 0 & \text{if } x_i = T. \end{cases}$$

$$f = f_1 + f_2 + \dots + f_N$$

$$f(x_1, \dots, x_N) = f_1(x_1) + f_2(x_2) + \dots + f_N(x_N) \\ = \# \text{ of Heads.}$$

Values of f are $0, 1, \dots, N$.

$$P(f=k) = P(f \in \{k\} \subseteq \mathbb{R})$$

$$= P(\{\text{sequences} \mid f(\text{seq}) = k\})$$

$$= P(\{\text{sequences} \mid k \text{ heads}\})$$

$$P(f=k) = \binom{N}{k} p^k (1-p)^{N-k}$$

f Binomial random variable with parameters N, p .
values $0, 1, \dots, N$