Conditional Probability and Bayes Theorem

Conditional Probability

- Draw a card from a deck. $P(\text{king}) = 4/52 = 1/13$.
- Now suppose you *know* the card is a face card. Given that information, the probability of drawing a king is $4/12 = 1/3$, This is an example of *conditional probability.* $P(A|B)$.
- More generally

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Meaning the chance of getting A among conditions where B is known to hold.

Bayes Theorem

Theorem: Given events A and B in a sample space X, we have

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Proof: Substitute $P(B \cap A)/P(A)$ for $P(B|A)$.

An example: COVID testing

- A person can be *sick* (S) or *well* (W).
- Their covid test can be *positive* (+) or *negative* (-).

There are four possibilities:

- S+ – this is a true positive
- S- – this is a false negative
- W+ – this is a false positive
- W- – this is a true negative

An early CDC report estimated that $P(+|W) = 1/200$ and $P(-|S) = 1/4$.

COVID testing continued

- Suppose I get a covid test and it's positive. How likely am I to have the disease? In other words, what is $P(S|+)$?

The answer depends on the prevalence $p = P(S)$, the chance that I have COVID in the first place.

COVID testing continued

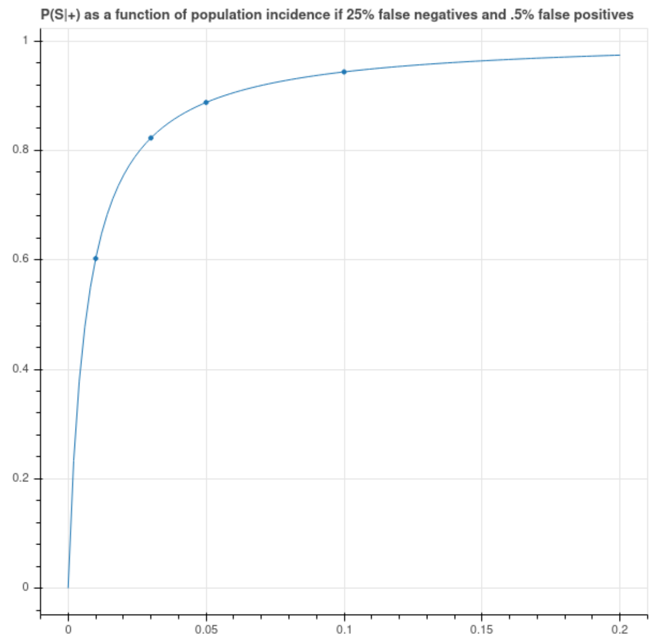

Figure 1: Chance I have COVID if I get a positive test vs prevalence