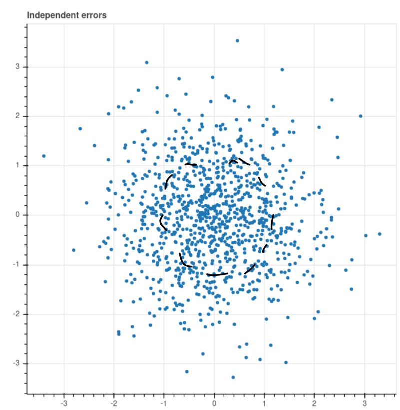
$X = \mathbb{R}^2$ measurements with randomly distributed errors normally with $\sigma^2 = 1$.

$(x_1, x_2) \in \mathbb{R}^2$

$$P(U) = \frac{1}{2\pi} \int_U e^{-\|x\|^2/2} \, dx_1 dx_2$$

$x = (x_1, x_2)$

levels courses are $\|x\|^2 = A$ Circles.



How does $\|x\| = \sqrt{x_1^2 + x_2^2}$ behave?

$f: X \longrightarrow \mathbb{R}$ $\quad f(x) = \sqrt{x_1^2 + x_2^2} = \|x\|^2$

What is the chance that $\|x\| < r$?

$$P(\|x\| \in [0, r)) = P(\{(x_1, x_2) \mid \|(x_1, x_2)\| < r\})$$

$$= \frac{1}{2\pi} \int_{\|(x_1, x_2)\| < r} e^{-\|x\|^2/2} \, dx_1 dx_2$$

$$= \frac{1}{2\pi} \int_{\theta=0}^{2\pi} \int_{\rho=0}^{r} e^{-\rho^2/2} \rho \, d\rho \, d\theta$$

$$= \int_0^r \rho e^{-\rho^2/2} \, d\rho \qquad u = -\rho^2/2$$

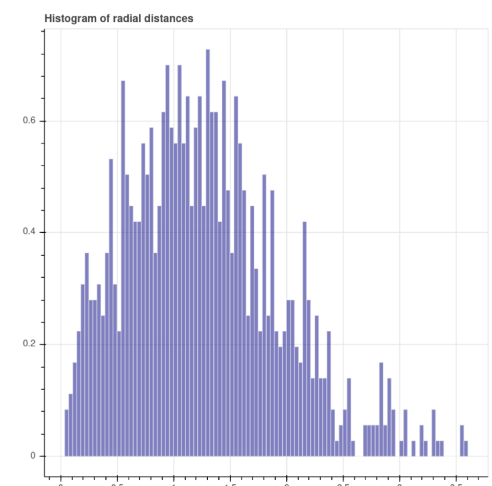
$$du = -\rho \, d\rho$$

$$= -\int_0^{\rho=r} e^u \, du = 1 - e^u \Big|_{\rho=0}^{\rho=r} = 1 - e^{-\rho^2/2} \Big|_{\rho=0}^{\rho=r}$$

$$\star \quad P(\|x\| < r) = 1 - e^{-r^2/2}$$

density f^{on} $= \underline{r e^{-r^2/2}}$

$$P(\|x\| < R) = \int_{r=0}^{R} r e^{-r^2/2} \, dr \quad \checkmark$$



χ- distribution