Day 11

Ring example day

Fields and division rings

- $\mathbb{Q}$
- $\mathbb{R}$
- $\mathbb{C}$
- $\mathbb{Z}/p\mathbb{Z}$ (p prime)
- $\mathbb{H}$ (quaternions, with real coefficients)

Domains

- $\mathbb{Z}$
- $\mathbb{Z}[x]$
- $\mathbb{R}[x]$
- $\mathbb{H}(\mathbb{Z})$ – integer quaternions
- quadratic rings $\mathbb{Z}[\omega]$ where D is square free and

$$\omega = \begin{cases} \sqrt{D} & D \equiv 2, 3 \pmod 4 \\ \frac{1+\sqrt{D}}{2} & D \equiv 1 \pmod 4 \end{cases}$$

Proposition: Finite integral domains are fields.

(**Remark:** Finite division rings are fields, but this is a harder theorem).

Zero divisors and units

- $\mathbb{Z}$
- $\mathbb{Z}/n\mathbb{Z}$ with n composite - non-units are zero divisors.
- Continuous real valued functions on the interval - non-units aren't zero divisors.
- quadratic rings - finite and infinite unit groups.
- The "dual numbers" $F[\epsilon]$ where $\epsilon^2 = 0$.
- Units of $R[x]$ when R is an integral domain

Matrix rings

- $M_n(\mathbb{R})$
- $M_n(\mathbb{Z})$
- $M_n(R[x])$

Group rings

- Convolution