Problem Set 2

Instructions: Write up your solutions using LaTeX and submit them on HuskyCT by September 25, 2022.

Problem 1: Let $n \geq 2$. The group S_n is defined via its "natural" action on the set $[n] = \{1, 2, \ldots, n\}$. From this action, and given $1 \leq k \leq n$, we can deduce an action of S_n on the set $P_n(k)$ of k-element subsets of $[n]$ by defining $\sigma(\{a_1, \ldots, a_k\}) = \{\sigma(a_1), \ldots, \sigma(a_k)\}$. Since there are $\binom{n}{k}$ elements of $P(k)$ this action yields a homomorphism $\phi_{n,k} : S_n \to S_{\binom{n}{k}}$.

1. Let $n = 4$. Describe the maps $\phi_{4,1}$, $\phi_{4,2}$, and $\phi_{4,3}$.
2. When is $\phi_{n,k}$ faithful? In other words, when is $\phi_{n,k}$ injective?
3. Notice that $\binom{n}{n-1} = n$ so $\phi_{n,n-1}$ is a homomorphism from S_n to itself. Describe this map. Is it always an isomorphism?