Recommended Problems

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Problem 1 (DF, problem 6, p. 375)

Let R be an integral domain with field of fractions Q. Prove that $(Q/R) \otimes_R (Q/R) = 0$. In particular $\mathbb{Q}/\mathbb{Z} \otimes \mathbb{Q}/\mathbb{Z} = 0$.

Problem 2 (DF, problem 5, p. 375)

Let G be a finite abelian group and let p^k be the highest power of a prime p dividing $|G|$. Prove that $Z/p^k\mathbb{Z} \otimes G$ is the Sylow p-subgroup of G.

Ranks

Prove that, if M is a module over an integral domain R, and Q is the quotient field of R, then the rank of M is the dimension of the Q-vector space $Q \otimes M$.