Recommended Problems

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1. (DF, 14.2, problem 14) Show that $\mathbb{Q}(\sqrt{2+\sqrt{2}})$ is an extension of degree 4 with cyclic Galois group.

2. (DF, 14.2, Problem 16) Show that $x^4 - 2x^2 - 2$ is irreducible and that its roots are $\pm\sqrt{1 \pm \sqrt{3}}$. Let $\alpha_1 = \sqrt{1+\sqrt{3}}$ and $\alpha_2 = \sqrt{1-\sqrt{3}}$. Show that $K_1 = \mathbb{Q}(\alpha_1)$ and $K_2\mathbb{Q}(\alpha_2)$ are different, and that their intersection is the field $F = \mathbb{Q}(\sqrt{3})$. Then show that K_1K_2 has Galois group $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ over F. Finally show that $x^4 - 2x^2 - 2$ has galois group equal to the Dihedral group of the square.

3. (DF, 14.2, Problem 17-18) These problems derive some basic properties of the galois norm and trace for an algebraic element defined as:

- $\mathrm{Tr}(\alpha) = \sum_\sigma \sigma(\alpha)$ where the sum is over the set of Galois conjugates of α
- $\mathrm{N}(\alpha) = \prod_\sigma \sigma(\alpha)$ where the product is over the set of Galois conjugates of α