From The Expert: One-way Analysis of Variance

From the Expert: One-way Analysis of Variance

Experimental Design offers a way to examine the output variables with manipulated input variables. Experimental problems can broadly divided into these categories:

- **Treatment comparisons**. The goal is to compare several treatments and select the most promising one.
- **Variable screening**. A number of factors have influence on the output. We want to concentrate only on important factors, while removing others.
- **Response surface exploration**. The relationship between output and few important factors will be explored using response surface. Simple mathematic formulas that represent this relationship may be derived.
- **System optimization**. The ultimate goal of most statistical experiments is to find the best setting for input variables. To accomplish this goal, the output needs to be analyzed.
- **System robustness**. The most optimal way for setting the input variables should be weighted with system robustness because setting the specific input value can be very costly. A setting with the least deterioration (values of input deviated from the settings) is considered the most robust.

Analysis of variance (ANOVA) provides the framework for comparing *more than* two means of independent populations such as different treatments, methods, machines, etc. The name (ANOVA) came from the fact that the variances are partitioned according to the sources of variations (within groups or between groups). The fundamental ideas of ANOVA include estimating group means, partitioning the variation according to sources, assessing magnitude variation according to sources.

Common terminology in ANOVA or Experimental design

Factor – refers to independent variable
Level – different aspects (amounts) of the independent variable
Cell – a specific combination of different levels of the independent variable

There are different types of ANOVA

- One-way ANOVA

Concerns only one factor, for instance, blood pressure (high, low, normal)
The analysis goal is on group differences. This is the extension of the t-test to more than 2 samples. Note that a one-way ANOVA with two levels is similar to performing a t-test.

- Two-way ANOVA

Concerns on two factors such as gender and blood pressure

- Three-way ANOVA

Concerns on three factors such as gender, blood pressure, and sugar levels

In two-way ANOVA and higher, the analysis goal is on the interaction between factors. In other words, we want to explore what effect has on one factor when the level of another factor changes.

One-way Analysis of Variance or One-Way ANOVA

It is an analysis concerning one independent variable and one dependent variable. In addition, it can test the effect of different levels or subgroups of the independent at once. The term one-way refers to the fact that only one variable defines the groups.

1. Partitioning of variance

Total variance in data (SS total) = Between group variance (SS Between) + Within group Variance (SS within)

In shorthand:

`SST = SSB + SSW`

= +

`sum_1^n (()^2 = n_(group) sum_(group)`

Explanation:

SST is the sum of the squared deviations between each observation and the overall (grand) mean.

SSB is the sum of the squared deviations between each group mean and the overall (grand) mean, using sample size of each group as the weight.

SSW is the sum of the squared deviations between each observation and the mean of its own group.

2. ANOVA table

Source of variation	Sum of Squares	df	Mean Squares	F
Between groups	SSB	k-1 one less than number of groups	SSB/(k-1)= MSB	MSB/MSW
Within groups	SSW	N-k	SSW/(N-k)=MSW	
Total	SST=SSB+SSW	N-1 one less than number of observations		

3. Hypothesis

Analogous to linear model, the random error are independent, normal distribution with mean 0 and constant variance. The hypothesis tests the difference in the groups' means.

$H_0: \mu_1 = \mu_2 = \mu_3 = ... = \mu_k$

)

When μ_k is the mean of the *k*-th treatment.
These hypotheses are called omnibus hypothesis, and tests of these hypotheses are omnibus F-tests.

4. Test Statistic

When *F* is less than 1, variability between groups is small compared to variability within groups. This implies that groups do not explain much variation in the data

When *F* is large, variability between groups is big compared to variability within groups. This suggests that groups explain a lot of variation in the data.

- Reject H_0 if $F_{(obs)} >= F a$
- Fail to reject H_0 if

If p-value is provided, then reject H_0 when p-value < a.

When H_0 is rejected, it means that at least one group means is different. So, which groups are different? To answer this question, we need to do the post hoc tests or a test after a significant omnibus F-test is found. There are many alternatives for post hoc tests depending on the research fields.

Multiple comparisons tend to increase the probability of Type I error. The more comparisons, the higher the probability of type I error. For instance, if $\alpha = 0.05$ for each comparison. This means that there is a 5% chance it will be falsely significant for each comparison. For *n* comparisons, the probability of making a type I error becomes $1 - (1 - a)^n = 1 - 0.95^n$. For 4 comparisons, the Type I error is 0.19 not 0.05. If multiple comparisons are conducted, some adjustment is needed to make sure that type I error does not exceed α. Using *Bonferroni adjustment*, dividing the significance level by the number of comparisons. Thus, 4 comparisons at is 0.0125 (. Therefore, Reject H_0 if $p < 0.0125$.

One of the alternatives is the Scheffe Test. It is a multiple comparisons method in one step. With the reason that the Scheffe test conducts multiple tests on the same data, it controls experiment-wise error rate and does not increase the probability of Type I error. Another alternative is Tukey's honest significant test or Tukey's HSD. This method is pairwise comparisons (compare one mean with another mean) and keeps the maximum experiment-wise error rate to .

5. Assumptions

- Data in each group should be normally distributed. This can be detected by plotting histogram or using statistical tests such as Anderson-Darling test, Shapiro-Wilk test, Kolmogorov-Smirnov, etc.
- Variance of groups should be approximately equal or homogeneity of variance. This can be examined by Levene statistic test.
- Each outcome variable should be independent of each other value. This can be investigated by how the data is collected.

6. One-way ANOVA procedure

- Verify assumptions (homogeneity, normality, independence). Perform transformation if there is violation in homogeneity

- Verify assumptions (homogeneity, normality, independence). Perform transformation if there is violation in homogeneity.
- Do F-test (test all group means).
- If means differ (reject `H_0`), continue with post hoc test such as Tukey's HSD to examine which groups are different.

7. Useful residuals plots for checking the model assumptions

- Normality - Checking the normality assumption by normal probability plot or a quantile-normal plot of the residuals. The residuals should be normally distributed.
- Homogeneity of variance -This can be investigated using a scatterplot between residuals and predicted values. This variability of the residuals should not depend on the value of responses. The residuals should have constant variance.
- Plot of the residuals in time sequence

The purpose is to check whether the skill of the experimenter is changed over time (e.g. better skill)

Examples of one-way ANOVA using R can be found at:

- http://www.stat.columbia.edu/~martin/W2024/R3.pdf
- http://homepages.inf.ed.ac.uk/bwebb/statistics/ANOVA_in_R.pdf

References:

Eckel, S (2008) ANOVA. University of Southern California. Retrieved from: http://www-hsc.usc.edu/~eckel/biostat2/notes/notes7.pdf

Lindquist, M.A.(2011). Applied Linear Regression Analysis. Columbia University. Retrieved from: http://www.stat.columbia.edu/~martin/W2024/R3.pdf

Trinity (2002) Introduction to Statistical Modeling, University of Oxford. Retrieved from:http://www.stats.ox.ac.uk/pub/bdr/IAUL/ModellingLecture4.pdf

Webb, B. ANOVA in R. School of Informatics, University of Edinburgh. Retrieved from: http://homepages.inf.ed.ac.uk/bwebb/statistics/ANOVA_in_R.pdf

Activity Details

Task: View this topic