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CS161 Discussion 1B

Homework 5

1. Key: s = Smoke, f = Fire, h = Heat

a) The sentence is neither valid nor unsatisfiable.

s	f	¬s	¬f	s ⇒ f	¬s ⇒ ¬f	(s ⇒ f) ⇒ (¬s ⇒ ¬f)
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

b) The sentence is neither valid nor unsatisfiable.

s	f	h	s ∨ h	s ⇒ f	(s ∨ h) ⇒ f	(s ⇒ f) ⇒ ((s ∨ h) ⇒ f)
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	F	F	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	F	F
F	F	F	F	T	T	T

c) The sentence is valid.

s	f	h	s ∧ h	s ⇒ f	h ⇒ f	(s∧h) ⇒f	(s⇒f)∨(h⇒ f)	((s∧h)⇒f)⇔((s⇒f)∨(h⇒ f))
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	F	F	T
T	F	F	F	F	T	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	F	T	T	T
F	F	F	F	T	T	T	T	T

2.

Key: y = unicorn is m**y**thical
i = unicorn is **i**mmortal
a = unicorn is a m**a**mmal
h = unicorn is **h**orned
g = unicorn is ma**g**ical

a) Knowledge Base:

1. $(y \Rightarrow i) \wedge (\neg y \Rightarrow (\neg i \wedge a))$
2. $(i \vee a) \Rightarrow h$
3. $h \Rightarrow g$

b) $(y \Rightarrow i) \equiv (\neg y \vee i)$
$(\neg y \Rightarrow (\neg i \wedge a)) \equiv (y \vee (\neg i \wedge a)) \equiv (y \vee \neg i) \wedge (y \vee a)$
$(i \vee a) \Rightarrow h \equiv \neg(i \vee a) \vee h \equiv (\neg i \wedge \neg a) \vee h \equiv (h \vee \neg i) \wedge (h \vee \neg a)$
$(h \Rightarrow g) \equiv (\neg h \vee g)$

CNF Knowledge Base:

1. $\neg y \vee i$
2. $y \vee \neg i$
3. $y \vee a$
4. $h \vee \neg i$
5. $h \vee \neg a$
6. $\neg h \vee g$

c) (1-6 is the knowledge base found in part b)

(i) Mythical: KB $\wedge \neg\alpha$ where $\alpha = y$

7. $\neg y$ $(\neg\alpha)$
8. a (resolution of 3 and 7)
9. h (resolution of 5 and 8)
10. g (resolution of 6 and 9)
11. $\neg i$ (resolution of 2 and 7)

We have reached the end of new, useful statements we can derive. Because we cannot show (KB $\wedge \neg\alpha$) $\models$ false, we **cannot** derive that the unicorn is mythical from the knowledge base.

(ii) Magical: KB $\wedge \neg\alpha$ where $\alpha = g$

7. $\neg g$ $(\neg\alpha)$
8. $\neg h$ (resolution of 6 and 7)
9. $\neg i$ (resolution of 4 and 8)
10. $\neg a$ (resolution of 5 and 8)
11. $\neg y$ (resolution of 1 and 9)
12. y (resolution of 3 and 10)
13. empty (resolution of 11 and 12)

Because we can show (KB $\wedge \neg\alpha$) $\models$ false, we know that (KB $\wedge \neg\alpha$) is UNSAT. Therefore, we **can** derive that the unicorn is magical from the knowledge base.

(iii) Horned: KB $\wedge \neg\alpha$ where $\alpha = h$

7. $\neg h$ $(\neg\alpha)$
8. $\neg i$ (resolution of 4 and 7)
9. $\neg a$ (resolution of 5 and 7)
10. $\neg y$ (resolution of 1 and 8)
11. y (resolution of 3 and 9)
12. empty (resolution of 10 and 11)

Because we can show (KB $\wedge \neg\alpha$) $\models$ false, we know that (KB $\wedge \neg\alpha$) is UNSAT. Therefore, we **can** derive that the unicorn is horned from the knowledge base.

3.

Probability of neither and positive: (.3) * (.1) = .03
Probability of only gas and positive: (.2) * (.3) = .06
Probability of only oil and positive: (.5) * (.9) = .45

So, the total probability of a positive test is .03 + .06 + .45 = .54

Therefore, the probability that oil is present when a test returns positive is:

.45 / .54 = **.8333**