Problem 1 (35 pts). A software company has n requests for developing software systems $\{R_1, R_2, \ldots, R_n\}$. For each system R_i, the company will get $\$p_i$ and will need m_i months for the development of R_i. On all projects, the company can work not more than m months and it is necessary to finish one system before starting another one. However, if the company will hire 10 more programmers, the time for each project will be half of the initial time but the company will have to pay each programmer $\$a$ per month. Note that the company either hire all additional programmers for all m months or does not hire them at all. Design an efficient algorithm allowing the company to get the largest profit developing these systems and estimate the complexity of this algorithm.

Problem 2 (35 pts). A computing facility has to perform n processing jobs $\{J_1, J_2, \ldots, J_n\}$. Each J_i demands three types of computers: at first, a supercomputer for the time r_i, then a regular desktop for the time q_i and after this, a specialized computer for the time t_i. The facility has only one supercomputer that can perform only one job after another but it has more than n desktops and more than n specialized computers. Design an efficient algorithm allowing finding the schedule to do all jobs with the minimal time and estimate the complexity of this algorithm.

Problem 3 (30 pts). Let G be an arbitrary set of n companies and their brand values are also given. Some of these companies compete with one another. A set A of companies from G is perfect if no two of them compete with one another. The problem MPP of finding a perfect set of companies with the maximal total brand value (that is, the sum of their brand values) is considered intractable although solvable in the general case. That is why a special case of G is considered.

Assume that the structure of G is cyclic (that is, each company competes exactly with two another companies) and find an efficient algorithm for solving MPP estimating the complexity of this algorithm. An example of a cyclic structure with brand values as weights is given in Figure 1.

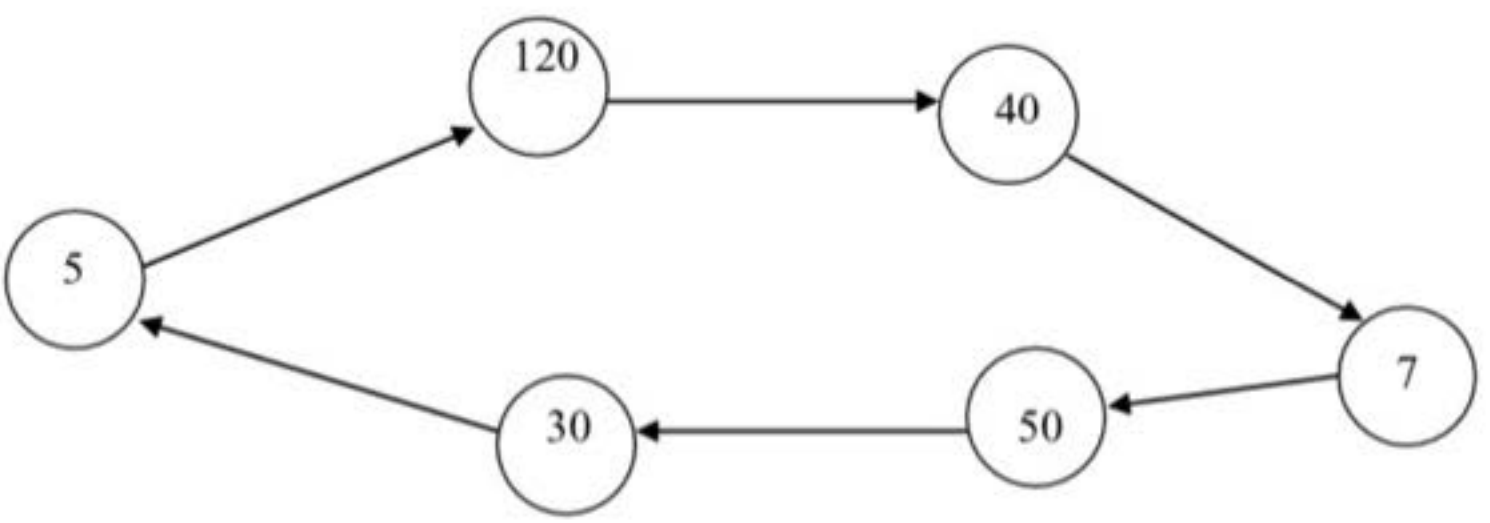

Figure 1. Brand values are given in millions.