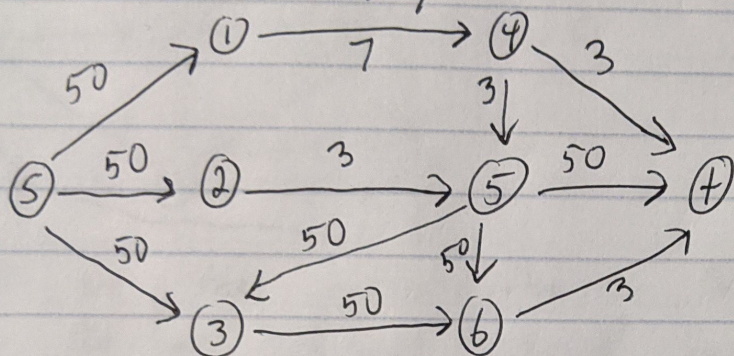


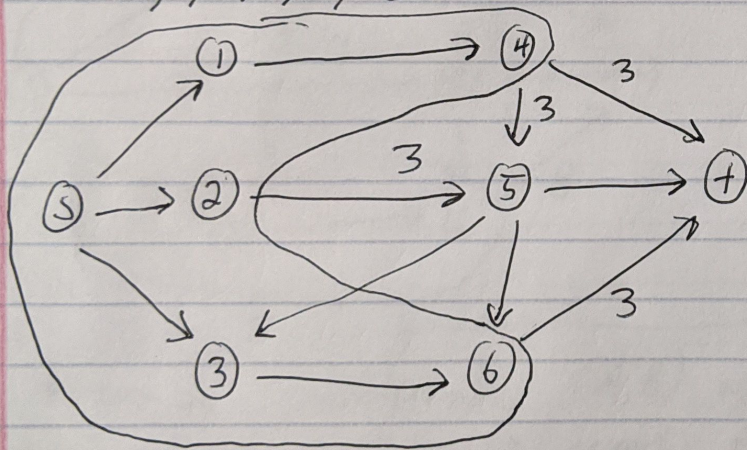
CS 180 Homework 5

Jeremy Cristobal 604982952 Discussion 1B

No, (A, B) will not necessarily be a minimum s - t cut after the innovation and new capacities. Let us use the following network to show why:

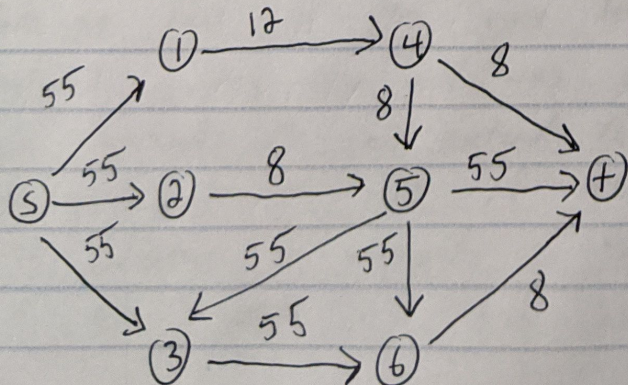


In this case, the minimum s - t cut is clearly $A = \{s, 1, 2, 3, 4, 6\}$:



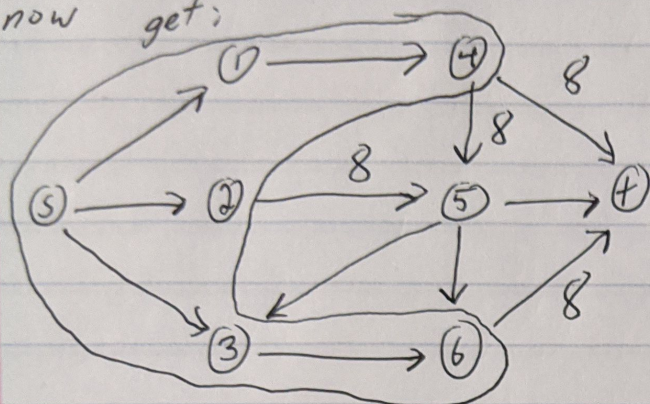
Capacity = 12

After the innovations and we add 5 to each edge, our new network now looks like:



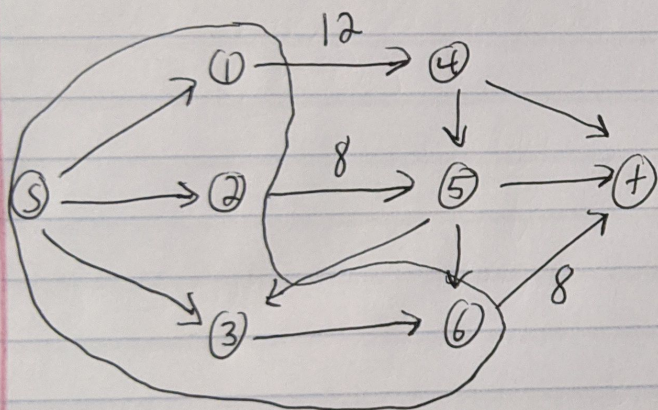
CS 180 Homework 5 Part II

If we use the s - t cut we found for the old network, we now get:



Capacity = 32

We see the capacity here is 32. However, the minimum s - t cut for the new network should actually be $A = \{s, 1, 2, 3, 6\}$:



Capacity = 28

Therefore, by counterexample, (A, B) will not necessarily be a minimum s - t cut with respect to the new capacities after the innovation.

We see this is the case because the new s - t cut has 1 fewer outward pointing edge and was within 5 units capacity-wise on the old network. We can extend this to say (A, B) will not be a minimum s - t cut if there exists another cut such that the difference in capacity in the old network is $5x - 1$ or less and it has x fewer outward pointing edges.