Problem 1. There is a multiverse W of disconnected universes U_i (i = 1, 2, … , *n*) and it was found between which of these universes, it is possible to establish connections. Creation of each connection demands a lot of resources. Physicists and engineers constructed a function that estimates the price of each possible connection.

(50 pts). Build an algorithm that allows connecting the largest number of the universes for the minimal price. Prove that it is correct and estimate its complexity.

(50 pts). If it is impossible to do this, prove it.

(10 pts). Is it always possible to connect all universes in W? Explain your answer.

Problem 2 (15 pts). On the planet Alphaomega, there are *n* spaceships and *n* persons having the rank of a spaceship captain. Each captain has the preference list of spaceships and the crew of each spaceship has the preference list of captains. The goal is to find a Stable Spaceship Matching of pairs (*c, s*).

Decide whether the following statement is true or false.

In every instance of the Stable Spaceship Matching, there is a stable matching containing a pair (*c, s*) *such that, at least, one of them is ranked third on the preference list of the other.*

If it is true, give a short explanation and design an algorithm.

If it is false, give a counterexample and explain that it is correct.

Problem 3 (25 pts). Take the following functions and arrange them in descending order of growth rate indicating when functions have the same order of growth rate.

$\sqrt[3]{n}$

$\log_{10} 2^n$

$\sqrt[4]{n^3}$

$\sqrt{n^n}$

$\log_{10}{}^2 n + 5n$

$5n + 5^n$

$n^{7/2}$

$\log_{10}(n^5 + 5n)$