Test1

Jeremiah Groom

2024-06-24

To find the mean and variance of proportions where the number of values for each proportion are not necessarily equal, you can follow these steps:

Mean of Proportions

1. **Collect Data**: Suppose you have multiple groups, each with a different number of observations. Let n_i be the number of observations in the i-th group, and let p_i be the proportion of successes in the i-th group.

2. **Total Observations and Successes**: Calculate the total number of observations N and the total number of successes S across all groups.

$$N = \sum_{i=1}^{k} n_i$$

$$S = \sum_{i=1}^{k} (p_i \times n_i)$$

3. **Overall Proportion (Mean of Proportions)**: The mean of the proportions is the overall proportion of successes across all groups.

$$\hat{p} = \frac{S}{N} = \frac{\sum_{i=1}^{k} (p_i \times n_i)}{\sum_{i=1}^{k} n_i}$$

Variance of Proportions

1. **Calculate Individual Variances**: For each group, the variance of the proportion p_i is:

$$\text{Var}(p_i) = \frac{p_i(1 - p_i)}{n_i}$$

2. **Weighted Variance**: Since the groups have different sizes, you need to weight the variances by the group sizes. The overall variance of the proportions can be calculated as follows:

$$\text{Var}(\hat{p}) = \frac{1}{N} \left(\sum_{i=1}^{k} n_i \times \text{Var}(p_i) \right)$$

where $\text{Var}(p_i) = \frac{p_i(1-p_i)}{n_i}$.

3. **Plugging in Individual Variances**: Substitute the individual variances into the formula.

$$\text{Var}(\hat{p}) = \frac{1}{N} \left(\sum_{i=1}^{k} n_i \times \frac{p_i(1-p_i)}{n_i} \right)$$

$$\text{Var}(\hat{p}) = \frac{1}{N} \left(\sum_{i=1}^{k} p_i(1-p_i) \right)$$

Summary

- **Mean of Proportions**:

$$\hat{p} = \frac{\sum_{i=1}^{k}(p_i \times n_i)}{\sum_{i=1}^{k} n_i}$$

- **Variance of Proportions**:

$$\text{Var}(\hat{p}) = \frac{1}{\sum_{i=1}^{k} n_i} \left(\sum_{i=1}^{k} p_i(1-p_i) \right)$$

By following these steps, you can accurately calculate the mean and variance of proportions, even when the number of values for each proportion are not necessarily equal.