Appendix

```
## show code version

library(mvtnorm)
library(Matrix)

gen.XY=function(n,desigman,cor,rho){
  beta=c(0,0.5)
  sigmama=1

  if(desigman==1){
    # desigman matrix
    X=matrix(rep(c(8,10,12,14),each=n),nrow=n)
    if(cor=="exch"){
      R=matrix(rho,nrow=4,ncol=4)
      diag(R)=1
    }
    # check if it is pos definite.
    # solve(R)

    if(cor=="expo"){
      R=matrix(nrow=4,ncol=4)
      for(i in 1:4){
        for(j in 1:4){
          R[i,j]=rho^abs(X[1,i]- X[1,j])
        }
      }
    }
    # check if it is pos definite.1
    # solve(R)

    err=rmvnorm(n,sigmama=sigmama^2 * R)
    Y=X %*% beta+err
  }

  if(desigman==2){
    mm=list()
    cl=c(8,10,12,14)
    X=c()
    for (k in 1:4){
      mm[[k]]=matrix(rep(cl[-k],each=n/ 4),nrow=n/ 4)
      X=rbind(X,mm[[k]])
    }

    if(cor=="exch"){
      R=matrix(rho,nrow=3,ncol=3)
      diag(R)=1
      err=rmvnorm(n,sigmama=R)
    }
    if(cor=="expo"){

      mme=list()
      mmR=list()
```

```r
      err=c()
      for (k in 1:4){
        mmR[[k]]=matrix(nrow=3,ncol=3)
        for(i in 1:3){
          for(j in 1:3){
            mmR[[k]][i,j] <- rho^abs(mm[[k]][1,i]- mm[[k]][1,j])
          }
        }
        mme[[k]]= rmvnorm(n/ 4,sigmama=sigmama^2 * mmR[[k]])
        err=rbind(err,mme[[k]])

      }
    }
    Y=X %*% beta+err
  }
  return(list(X,Y))
}

var.est=function(residual,cor){
  n=nrow(residual)
  m=ncol(residual)

  if(cor=="exch"){
    alpha1=sum(residual^2)/m/n
    alpha2=c()
    for (mm in 1:m){
      alpha2=paste(alpha2,"+residual[,",mm,"] * residual[,",-mm,"]",sep="")
    }
    alpha2=sum(eval(parse(text=alpha2)))/n/m/(m-1)
  }

  if(cor=="expo"){
    sigmama2=sum(residual^2)/m/n
    d.lst=c(2,4,6)
    for (i in 1:3){
      assigman(paste("ct",d.lst[i],sep=""),0)
      assigman(paste("rho",d.lst[i],sep=""),0)
    }
    for(i in 1:n){
      for(j in 1:m){
        for(k in 1:m){
          if(j !=k){
             diff=abs(X[i,j] -X[i,k]) ## global var
             tmr=eval(parse(text=paste("ct",diff,sep="")))+1
             assigman(paste("ct",diff,sep=""),tmr)
#              tmr2= residual[i,j] * residual[i,k]
             tmr2=max(0,eval(parse(text=paste("rho",diff,sep="")))+residual[i,j] * residual[i,k])
            }

          }
        }
      }
    }
    alpha2=0
    for (i in 1:3){
      tmr=paste("(rho",d.lst[i],"/ ct",d.lst[i],"/ sigmama2)^{1/",d.lst,"} * ct",d.lst[i],sep="")
      alpha2=alpha2+eval(parse(text=tmr))/n/m/(m-1)
    }
    alpha1=sum(residual^2)/m/n
  }

  if(cor=="uncorrelated"){
    alpha1=sum(residual^2)/m/n
    alpha2=0
  }
  return(list(alpha1,alpha2))
}

gls=function(X,Y,cor){
  n=nrow(X)
  m=ncol(X)

  beta.OLS=coef(lm(c(Y) ~ c(X)))
  res=Y - X %*% beta.OLS
  alpha.hat=var.est(res,cor)
  alpha1=alpha.hat[[1]]
  alpha2=alpha.hat[[2]]

  Aleft=matrix(0,nrow=2,ncol=2)
  XWY=c(0,0)
  for(i in 1:n){
    if(cor=="exch"){
      R=matrix(alpha2,nrow=m,ncol=m)
      diag(R)=alpha1
    }

    if(cor=="expo"){
      R=matrix(nrow=m,ncol=m)
      for(j in 1:m){
        for(k in 1:m){
          R[j,k]=alpha1 * alpha2^abs(X[i,j]- X[i,k])
        }
      }
    }

    if(cor=="uncorrelated"){
      R=diag(alpha1,nrow=m,ncol=m)
    }

    Aleft=Aleft+t(cbind(rep(1,m),X[i,])) %*% solve(R) %*% cbind(rep(1,m),X[i,])
    XWY=XWY+t(cbind(rep(1,m),X[i,])) %*% solve(R) %*% Y[i,]
  }

  betahat=solve(Aleft) %*% XWY
  res=Y - X %*% betahat
  alpha.hat=var.est(res,cor)
  alpha1.2=alpha.hat[[1]]
  alpha2.2=alpha.hat[[2]]

  Bmid=matrix(0,nrow=2,ncol=2)
  var.quasi=solve(Aleft)
  for(i in 1:n){
    if(cor=="exch"){
      R=matrix(alpha2,nrow=m,ncol=m)
      diag(R)=alpha1
      sigma=matrix(alpha2.2,nrow=m,ncol=m)
      diag(sigma)=alpha1.2
    }

    if(cor=="expo"){
      R=matrix(nrow=m,ncol=m)
      for(j in 1:m){
        for(k in 1:m){
          R[j,k]=alpha1 * alpha2^abs(X[i,j]- X[i,k])
        }
      }
      sigma=matrix(nrow=m,ncol=m)
      for(j in 1:m){
        for(k in 1:m){
          sigma[j,k]=alpha1.2 * alpha2.2^abs(X[i,j]- X[i,k])
        }
      }
    }

    if(cor=="uncorrelated"){
      R=diag(alpha1,nrow=m,ncol=m)
      sigma=diag(alpha1.2,nrow=m,ncol=m)
    }
    Bmid=Bmid+t(cbind(rep(1,m),X[i,])) %*% solve(R) %*% sigma %*% solve(R) %*% cbind(rep(1,m),X[i,])
    var.semi=solve(Aleft) %*% Bmid %*% solve(Aleft)
  }
  return(list(betahat[2],var.quasi[2,2],var.semi[2,2]))
}

setwd("f:/")
n.lst=c(8,20,60)
desigman.lst=c(1,2) # 1:balanced completem,2: unbalanced
rho.lst=c(0.5,0.9)
cor.lst=c("exch","expo") #1: exchangeable 2:exponential

aa=expand.grid(rho.lst=rho.lst,cor=cor.lst,desigman=desigman.lst,n=n.lst)
nrows=nrow(aa)

df=data.frame(n=aa$n,
                  desigman=aa$desigman,
                  cor=aa$cor,
                  rho=aa$rho,
                  cov.uncor=rep(NA,nrows) ,
                  cov.exch=rep(NA,nrows)  ,
                  cov.expon=rep(NA,nrows) ,
                  var.uncor=rep(NA,nrows) ,
                  var.exch=rep(NA,nrows)  ,
                  var.expon=rep(NA,nrows) ,
                  re.uncor=rep(NA,nrows) ,
                  re.exch=rep(NA,nrows)  ,
                  re.expon=rep(NA,nrows) ,
                  sd.uncor=rep(NA,nrows) ,
                  sd.exch=rep(NA,nrows)  ,
                  sd.expon=rep(NA,nrows) ,
                  quasi.uncor=rep(NA,nrows) ,
                  quasi.exch=rep(NA,nrows)  ,
                  quasi.expon=rep(NA,nrows) ,
                  semi.uncor=rep(NA,nrows) ,
                  semi.exch=rep(NA,nrows)  ,
                  semi.expon=rep(NA,nrows)
)
n.sim=2000
ind=1
for ( n in n.lst){
  for (  desigman in  desigman.lst){
    for ( cor in cor.lst){
      for ( rho in rho.lst){
        betahat.uncor=c()
        se.semi.uncor=c()
        se.quasi.uncor=c()

        betahat.exch=c()
        se.semi.exch=c()
        se.quasi.exch=c()

        betahat.expon=c()
        se.semi.expon=c()
        se.quasi.expon=c()

        print(paste(ind,"/",nrows))
        for(i in 1:n.sim){
          data=gen.XY(n,desigman,cor,rho)
          X=data[[1]]
          Y=data[[2]]

          gls.result=gls(X,Y,"uncorrelated")
          betahat.uncor[i]=gls.result[[1]]
          se.semi.uncor[i]=sqrt(gls.result[[2]])
          se.quasi.uncor[i]=sqrt(gls.result[[3]])
          gls.result=gls(X,Y,"exch")
          betahat.exch[i]=gls.result[[1]]
          se.semi.exch[i]=sqrt(gls.result[[2]])
          se.quasi.exch[i]=sqrt(gls.result[[3]])
          gls.result=gls(X,Y,"expo")
          betahat.expon[i]=gls.result[[1]]
          se.semi.expon[i]=sqrt(gls.result[[2]])
          se.quasi.expon[i]=sqrt(gls.result[[3]])
          print(i)
        }

        var.comp=c(var(betahat.uncor,na.rm=TRUE),
                    var(betahat.exch,na.rm=TRUE),
                    var(betahat.expon,na.rm=TRUE))
        re=rep(0,3)
        re=var.comp/var.comp [ which(var.comp==min(var.comp)) ]

        ## table combined
        fill.in=5:length( df[ind,])
        df[ind,fill.in]=c(
          sum(betahat.uncor+1.96 * se.semi.uncor > 0.5 &betahat.uncor - 1.96 * se.semi.uncor < 0.5,na.rm
          sum(betahat.exch+1.96 * se.semi.exch > 0.5 &betahat.exch - 1.96 * se.semi.exch < 0.5,na.rm=TRU
          sum(betahat.expon+1.96 * se.semi.expon > 0.5 &betahat.expon - 1.96 * se.semi.expon < 0.5,na.rm

          var.comp,

          re,

          sd(betahat.uncor,na.rm=TRUE),
          sd(betahat.exch,na.rm=TRUE),
          sd(betahat.expon,na.rm=TRUE),
          mean(se.quasi.uncor,na.rm=TRUE),
          mean(se.quasi.exch,na.rm=TRUE),
          mean(se.quasi.expon,na.rm=TRUE),
          mean(se.semi.uncor,na.rm=TRUE),
          mean(se.semi.exch,na.rm=TRUE),
          mean(se.semi.expon,na.rm=TRUE)
        )

        ind=ind+1

      }
    }
  }
}
df
write.table( df,
             file="data2.csv",
             sep=",",
             row.names=T)


df= datahw5

plot(df)
```