PHYS 631: Quantum Mechanics I (Fall 2020)
Exercises 5 October 2020 (Monday, Week 2)
Due Monday, 12 October 2020

Note: Remember that you'll submit the exercises *together for the entire week, as a single pdf file*, by next Monday.

Exercise 1. For a Hermitian operator Q with *distinct* eigenvalues q_1 and q_2, prove that the corresponding eigenvectors $|q_1\rangle$ and $|q_2\rangle$ are orthogonal.

Hint: compute $\langle q_2|Q|q_1\rangle$ two ways.

Exercise 2. For a Hermitian operator Q with eigenvalues q_n (assume distinct eigenvalues for simplicity), prove that the eigenvectors are complete: any vector can be written as a superposition of the form

$$|\psi\rangle = \sum_n c_n|q_n\rangle. \tag{1}$$

To to this, define

$$|\psi'\rangle := |\psi\rangle - \sum_n \langle q_n|\psi\rangle|q_n\rangle, \tag{2}$$

and compute $\langle\psi'|\psi'\rangle$. You should be able to show that $|\psi'\rangle$ is the null vector, which proves the result.