

$$\langle \sigma_x \rangle = c_+^* c_- + c_+ c_-^*$$

$$\langle \sigma_y \rangle = -i c_+^* c_- + i c_+ c_-^*$$

$$\langle \sigma_z \rangle = |c_+|^2 - |c_-|^2$$

$$\langle \vec{\sigma} \rangle = \langle \sigma_x \rangle \hat{x} + \langle \sigma_y \rangle \hat{y} + \langle \sigma_z \rangle \hat{z}$$

$$|\langle \vec{\sigma} \rangle|^2 = \left(|c_+^* c_- + c_+ c_-^*|^2 + |-i c_+^* c_- + i c_+ c_-^*|^2 + (|c_+|^2 - |c_-|^2)^2 \right)^{\frac{1}{2}}$$

$$|\langle \vec{\sigma} \rangle|^2 = \left(2|c_+|^2 |c_-|^2 + \cancel{c_+^2 c_-^{*2}} + \cancel{c_+^{*2} c_-^2} + 2\cancel{|c_+|^2 |c_-|^2} - \cancel{c_+^{*2} c_-^2} - \cancel{c_+^2 c_-^{*2}} + |c_+|^4 - 2\cancel{|c_+|^2 |c_-|^2} + |c_-|^4 \right)^{\frac{1}{2}}$$

$$|\langle \vec{\sigma} \rangle|^2 = \left(|c_+|^4 + 2|c_+|^2 |c_-|^2 + |c_-|^4 \right)^{\frac{1}{2}}$$

$$|\langle \vec{\sigma} \rangle|^2 = |c_+|^2 + |c_-|^2 = 1$$