PHYS 631: Quantum Mechanics I (Fall 2020)
Exercises 16 November 2020 (Monday, Week 8)
Due Monday, 23 November 2020

Exercise 1. A qubit is a two-state quantum system, with states $|0\rangle$ and $|1\rangle$. Suppose the qubit's eigenstates under some Hamiltonian H are $\sqrt{2}\,|\pm\rangle = |0\rangle \pm |1\rangle$, and that the eigenenergies satisfy $|E_+ - E_-| = \hbar\omega$.

Reproduce the argument that if $|\psi(t=0)\rangle = |0\rangle$, then the state will oscillate to $|1\rangle$ and back to $|0\rangle$ as time progresses. What is the oscillation frequency? (That is, what is the frequency at which the probability associated with $|0\rangle$ oscillates?)

Note that this is a universal phenomenon: You just need a pair of split levels of opposite parity, and *voilà*, Rabi oscillations. This is something very useful to people in quantum information, quantum optics, laser physics, neutrino physics, etc. The argument here is one of the simplest you can make in physics, yet one of the most powerful.

Exercise 2. NH_3 has such a split pair of opposite-parity levels, associated with even and odd wave functions for the nitrogen atom. Recall that this is the famous 24.0-GHz microwave transition in the ammonia maser, which was historically an important frequency standard (now pretty much superseded by the hydrogen maser).

Suppose we denote the energy levels in the NH_3 maser transition by $|\pm\rangle$, and let M denote an operator that represents the interaction between NH_3 and maser radiation. It then follows that $\langle +|M|-\rangle \neq 0$, otherwise masers wouldn't exist (i.e., it must be possible to make a transition between the two states via maser radiation, otherwise masers wouldn't work).

Argue based on the above observation that M could anticommute with the parity operator Π (i.e., $[M,\Pi]_+ = 0$), but it's not possible for $[M,\Pi] = 0$.