PS3.1

Physics 611, Problem Set #3

Due: Thursday, ~~Oct. 22nd~~ Nov 16

1) Suppose the gravitational potential between the sun and the planets was a "Yukawa potential"

$$\boxed{(1.1)\quad U(r) = -m\mu\frac{e^{-kr}}{r}}$$

where μ and k are constants, and m is the mass of the planet in question.

a) Write down the effective potential for radial motion of the planet.

b) Plot this potential for a body in a circular orbit at radius r_c, when

1) $kr_c \ll 1$

2) $kr_c \gg 1$.

c) Determine the largest value r_L that r_c can take on for the circular orbit to remain stable.

1d) In a quantum field theory of gravity, the gravitational force is ~~mediated by~~ ~~the~~ carried by "gravitons": particles whose mass m_g is related to the constant k in the Yukawa potential by

$$\boxed{(1.2) \quad m_g = \frac{\hbar k}{c}}$$

where $\hbar = \frac{h}{2\pi}$, $h =$ Planck's constant, and c is the speed of light.

Given the observation that the essentially circular orbit of the planet pluto is stable (Pluto's still there, after all), find an ~~lower~~ upper bound ~~on~~ on the mass m_g of the graviton. Express your answer in kilograms. How does this compare with the mass of the electron?

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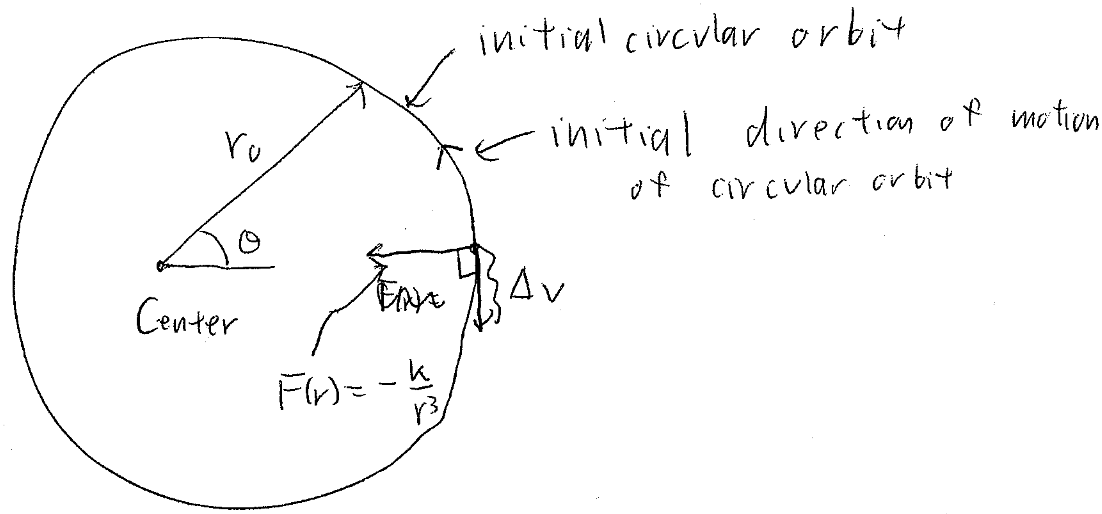
2) Consider a particle initially moving in a circular orbit of radius r_0 in an attractive ~~force~~ central force with $|F(r)| = \frac{k}{r^3}$. At time $t = 0$, the particle is given an impulse opposite its direction of motion that lowers its ~~velocity~~ speed by Δv (see figure:)



a) Calculate the ~~position of the~~ distance of the particle from the center as a function of time. Is there any time beyond which your expression breaks down? Why?

b) Calculate the trajectory $r(\theta)$ in polar co-ordinates.

2c) Repeat parts (a) and (b) for the case in which the initial ~~nudge Δv is~~ impulse Δv is in the opposite direction (i.e., <u>increases</u> the particle's velocity).

3) a) Repeat ~~all~~ parts (a) and (b) of problem (2) for the case of Δv directed radially inward.

b) The same, for Δv radially outward.

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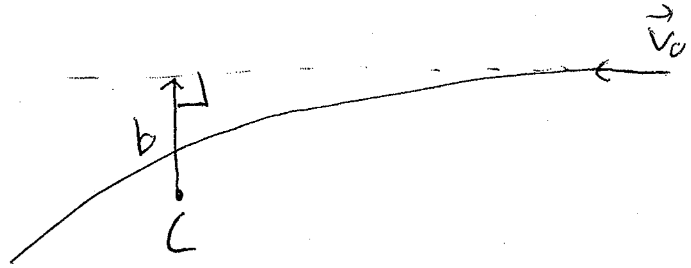
4) A particle of mass m moving in the same $\frac{1}{r^3}$ force as in the preceding two problems approaches the center from infinity at a speed v_0 with an "impact parameter" ~~(i.e., distan~~ b, as illustrated:



b ~~is~~ ~~[illegible]~~ is, by definition, the distance of closest

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4) (cont) approach of the extrapolated initial straight line trajectory of the particle to C.

a) Calculate the distance of closest approach r_{min} of the particle to C.

b) Show that there is some minimum impact parameter b_c such that, if $b < b_c$, the particle falls into the center, while for $b > b_c$, the particle ultimately escapes; and find an expression for b_c in terms of m, k, and v_0.

c) For $b > b_c$, find the asymptotic direction in which the particle ultimately (i.e., as $t \to \infty$) ~~[illegible]~~ moves off away from C