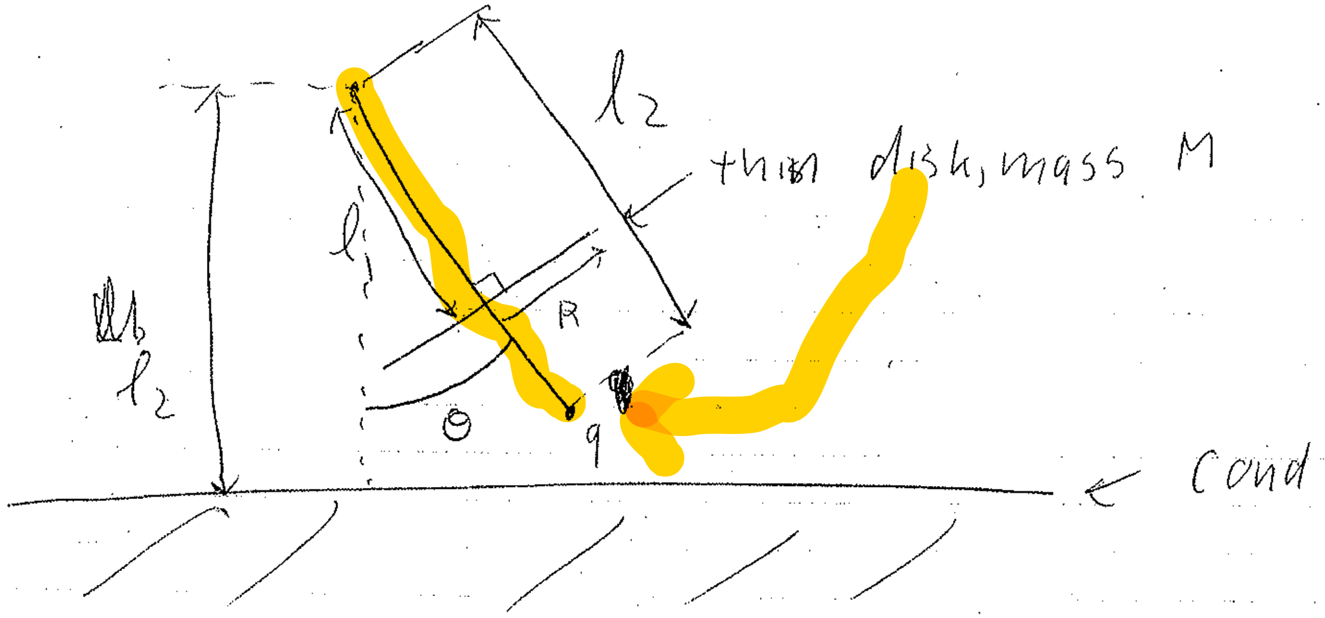
Physics 612/613 midterm

1)



A ~~pendulum~~ rigid top is made of a massless rod of length l_2 on which a very thin disk of radius R and mass M is mounted, a distance $l_1 < l_2 - R$ from one end. The rod is attached to a pivot

1) cont) fixed a distance exactly l_2 above a horizontal conducting plane, as shown in the figure.

On the end of the rod opposite the pivot is attached a charge q.

~~Ihini~~

Initially, the object is spinning about ~~its axis at~~ the rod at an anglular velocity ω_0. The rod initially makes some angle ~~θ_a~~ ~~with~~ $\theta_0 \neq 0$ with the vertical; this ~~rate~~ angle is initially changing

M3 ML

1) (cont) at some rate $\dot{\theta}_0$. The initial motion of the rod is in the plane of the figure.

Show that if ω_0 exceeds some critical value ω_c, the rod will never be pulled vertical (i.e., θ will never $= 0$), and find that critical value in terms of M, R_1, ℓ_1, ℓ_2, and q.

Hint: The ~~odd its~~ charge q is attracted towards the conducting surface by its image charge. You can use anything you know from E+M about image charges without rederiving it.

Neglect gravity.

Intentionally left blank (just to mess with your heads!)

M4

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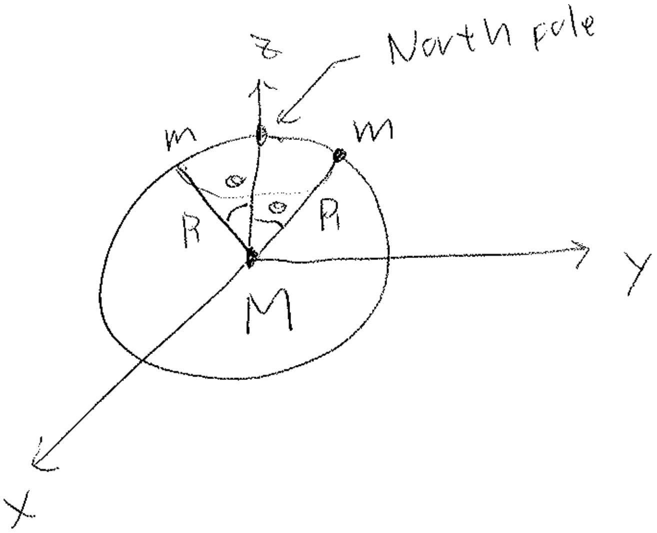
2) Model the Earth with its ~~wandering~~ ~~w~~ drifting continental plates as a sphere of mass M with two point masses $m \ll M$ (the "continents") on its surface. In a suitable cartesian co-ordinate system centered at the earth's center and ~~system~~ attached to the rotating earth, ~~the~~ one ~~them~~ point mass~~es~~ lies ~~in the~~ ~~at $x = 0$ plane, with~~ lines from the center

8) (cont) at $(x, y, z) = (0, \sin\theta, \cos\theta)$, while the other lies at the mirror image point on the other side of the x-z plane, $(x, y, z) = (0, -\sin\theta, \cos\theta)$ (see figure):



The radius of the ~~an~~ Earth is R.

a) Calculate the moment of inertia tensor of the Earth about the center of the sphere.

b) Calculate the moment of inertia tensor about the Earth's center of mass, and show that, for $m \ll M$, it differs

~~3~~ 2 (cont) M6 8

~~3~~ 2b) negligibly from that about the center of the sphere. Specifically, ~~tll~~ show that

$$(\cancel{3}2.1)\quad \boxed{\underset{\sim}{I}_{cm} - \underset{\sim}{I}_{\text{center of sphere}} \ll \underset{\sim}{I}_{\text{center of sphere}} - \underset{\sim}{I}_{\text{sphere}},}$$

where by $\underset{\sim}{I}_{\text{sphere}}$ I mean the moment of inertia tensor ~~about~~ of the sphere of mass M <u>without</u> the point masses m about <u>its</u> center of mass.

c) Neglecting the terms you showed to be negligible in part b, show that, as the continents drift south ~~northward~~ (i.e., as Θ increases ~~goes decreases~~), at some critical value of Θ, rotation about the z-axis becomes unstable. Find that critical value of Θ.

2d) Assume the continents are drifting ~~northward~~ south according to

$$\boxed{(2.2)\quad \theta(t) = \omega t}$$

, with $\omega \ll \Omega$, the rotation rate of the earth $\left(= \frac{2\pi}{1\,\text{day}}\right)$.

Find the linearized equations of motion for the angular momentum vector $\vec{L}$ in the Earth's co-rotating frame for rotation nearly (but not exactly) around the z axis), and estimate how long (it takes) after the instability sets in for $\vec{L}$ to wander an appreciable angle (say, 45°) over the earth's surface. Assume the initial angle between $\vec{L}$ and $\hat{z}$ is around 10^{-5} radians, and take $\frac{m}{M} \simeq 10^{-3}$, and $\omega = (10^{8}\text{ years})^{-1}$

M8

Hint: should you encounter an equation of the form

~~$\ddot{y} = f(t)\, y$,~~

~~a sol~~

(1) $\boxed{\ddot{y} = c\, t\, y}$ (c = constant)

try a solution of the form

(2) $\boxed{y = A e^{S(t)}}$

You'll find

(3) $\boxed{\ddot{S} + \dot{S}^2 = c\, t}$

<u>Assume</u> (and verify a posteriori) that one of the two terms on the left hand side of (3) dominates at "large" t (Here, "large" means large enough that $S \gg 1$, but may still be small compared to $\frac{1}{\omega}$).

~~then you should be~~