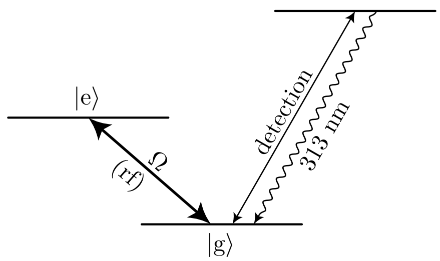
PHYS 631: Quantum Mechanics I (Fall 2020)
Exercises 16 November 2020 (Tuesday, Week 8)
Due Monday, 23 November 2020

Exercise 3. In the trapped-ion quantum Zeno experiment below,



a measurement confirming that the particle is in the ground state $|g\rangle$ inhibited Rabi oscillations to the excited state $|e\rangle$, because the detection of scattered photons gave the information required for the Zeno effect to take hold. However, it is worth reiterating that this *also* works when the atom is in $|e\rangle$—the atom *doesn't* scatter any photons, and the *lack* of photon detections over some time *still* provides the information needed for the Zeno effect to happen (so the atom is inhibited from making a transition to $|g\rangle$). This **null measurement result**, where the laser field makes a measurement, seemingly without even interacting with the atom, is a simple example of a class of measurements called **interaction-free measurements**.

Interaction-free measurements tend to be closely related to the Zeno effect. As one example, suppose that aliens visited Earth, and in a (square) potential well that they left behind, they may or may not have left behind a bomb. The bomb is of the highly contrived type, which is only detectable by a particular quantum particle, the **schmelectron** (symbol s^-). The problem is that, if a schmelectron interacts with the bomb, the bomb will absorb the schmelectron and detonate, which would be bad news for Earth. (That is, the bomb acts as a perfect absorber for schmelectrons.) So what to do?

The solution follows in the paragraph below. The exercise here is to understand the solution, and then to answer the following question. It should seem strange that the s^-/bomb interaction can be effectively made arbitrarily small, yet the measurement still works great; what is becoming arbitrarily large to compensate for the smallness of the interaction? (You may need to review how the transmission resonances work out in the square well.)

The idea is that the aliens (unwittingly) gave you an advantage by leaving the bomb in a potential well. You can take advantage of the transmission resonances for a quantum particle scattering from the potential well. The idea is to send in a delocalized schmelectron at some mean incident momentum (the momentum should be fairly well-defined to mimic the effect of the incident eigenstates that we analyzed), tuned to a transmission resonance. In the case where there is no bomb, the s^- should transmit with unit probability, so the lack of a backscattered s^- will be the indication that a bomb is not present. Since the bomb again acts as an absorber, it masks the effect of the second edge of the potential well. Recall that the transmission resonances of the well depend on a resonant wave circulating and building up inside the potential well. The bomb prevents this from happening, and so the s^- just reflects from the first edge of the well, with probability $R_1 = |r_1|^2$, where r_1 is the reflection amplitude from a single potential step. In this case, a reflected particle is an indication that there is a bomb in the well. But the problem is that, with probability $1 - R$, the s^- will still enter the well and detonate the bomb. Fortunately, in the limit of a deep well, R becomes arbitrarily close to 1, so the probability of setting off the bomb can be made arbitrarily small. But note that this probability can never be made strictly *zero*, because this would turn off the interaction completely, so we would never obtain *any* information about the bomb. Correspondingly, the s^- particle could never transmit through the well. In any case, the effect here comes from the (null) measurement that the bomb makes on the schmelectron, which is a spatial version of the Zeno effect—the measurement prevents the schmelectron from entering the potential well.