PS1.1

Physics 611, Problem Set #1

Due: In class, ~~Thursday 10/18~~ Tuesday, 10/14, 2 PM Sharp!

No exceptions!

1) Find, using calculus of variations, the shape of a "geodesic"; that is, the path of shortest length between two points on the surface of a sphere, as follows:

a) Write the arc length S as

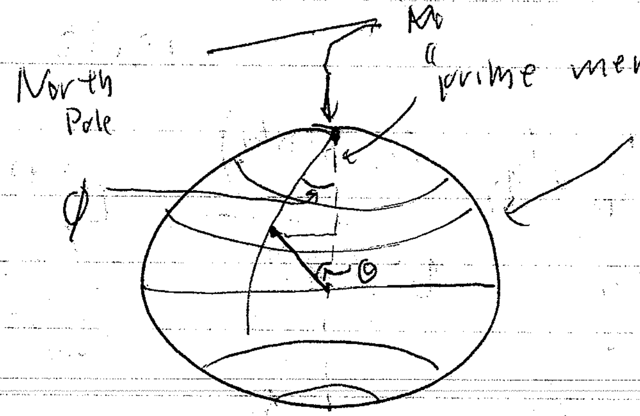

Fig. 1

an integral over ϕ of some function $\mathcal{L}(\theta(\phi), \dot{\theta}(\phi))$; i.e.,

$$\boxed{(1.1)\ S = \int_{\phi_0}^{\phi_f} \mathcal{L}(\theta(\phi), \dot{\theta}(\phi))\, d\phi}$$

where θ and ϕ are the ~~spherical co-ordin~~

1a) (cont) polar and azimuthal angles, ~~reske~~ on the surface of the sphere, respectively, as illustrated ~~in~~ rather ineptly in Figure 1 ~~at~~ on the preceding page, and $\dot{\theta}(\phi) \equiv \frac{d\theta}{d\phi}$.

You will have to figure out what $\mathcal{L}(\theta, \dot{\theta})$ is, explicitly, using geometry.

b) Write down the Euler-Lagrange equation for $\theta(\phi)$ that follows from minimizing S.

c) Show that the (somewhat nasty) Euler-Lagrange equation found in (b) is solved by a "Great circle route": that is, the path formed by the intersection of the sphere with a plane passing through its center.

1c)(cont) (Again, you'll have to use analytic geometry to find the $\theta(\phi)$ this construction implies), and then explicitly verify ~~that it so~~ by substitution into the Euler-Lagrange equation of part c that this $\theta(\phi)$ satisfies that equation.

1d) Compute the ~~geodesic tha~~ great circle 2 route from Eugene, Oregon to your home town. In particular, find the Northernmost point on this route.

(The following isn't part of the assignment, but rather a fun suggestion: by tracing this route out on a map, you can actually keep track, by comparing the view out the window with your map, of what you're flying over the next time you fly home - which

1d) cont) is much more interesting than watching the lousy Bruce Willis movie the airline is showing.

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2)

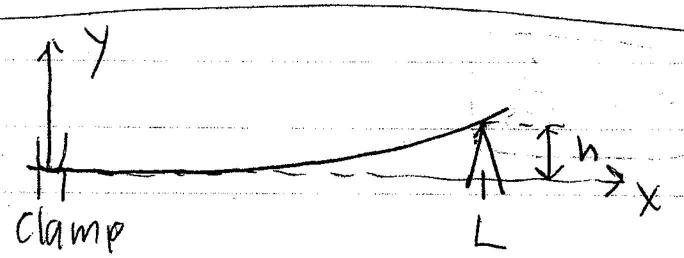


Fig 2

The figure above illustrates the configuration of a thin, flexible rod.

The energy needed to bend the rod is

$$(2.1)\quad E = \frac{1}{2} K \int_0^L \left(\frac{d^2 y}{dx^2}\right)^2 dx$$

where the constant K is a measure of the "stiffness" of the rod.

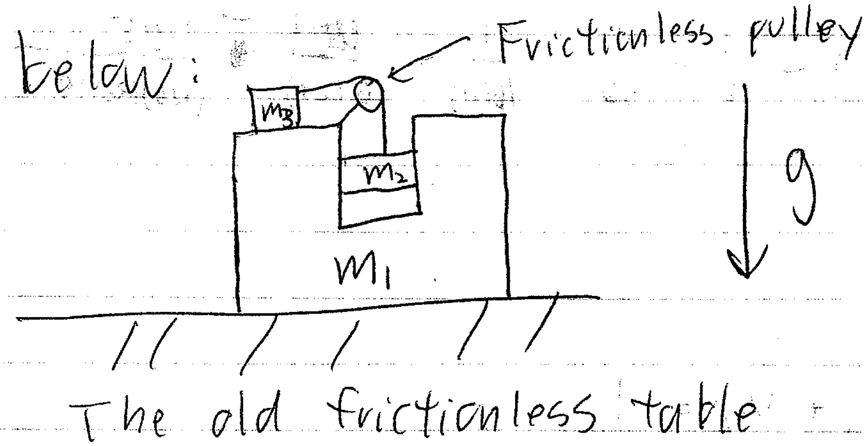
a) Derive the Euler-Lagrange equation that ~~minimizes this~~ determines the minimum energy configuration of the rod.

2a) (cont) (Note that the one we derived in class is insufficient, since the ~~Lagrang~~ quantity being minimized only involved a function and its first derivatives; here we have second derivatives. So you'll have to first extend our derivation of the E-LG equations to include this case.

2b) Solve the E-LG equation of part a for the boundary conditions illustrated

a clamp at $x=0$ holds the rod at $y=0$ and keeps it horizontal ($y'(x=0)=0$), while a wedge holds the far end of the rod up a distance h.

3) a) Write down the Lagrangian for the "pedagogical machine" illustrated below:



Gravity g acts vertically; the slot the mass m_2 slides in is frictionless, and you needn't worry about what happens when m_2 hits the bottom of this slot.

b) Find the acceleration (magnitude and direction) of the mass m_1.

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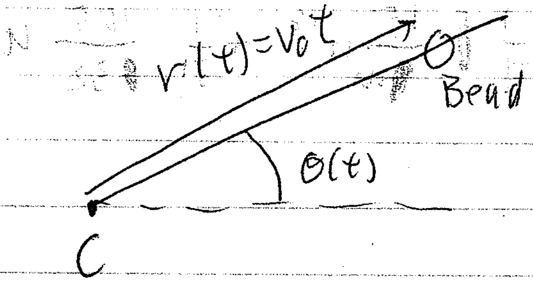
4) a) Write down the Lagrangian of a bead of mass m on a massless wire when the wire is completely free

4a) to rotate in a horizontal plane, and the bead is moved along the wire at a constant speed V_0 (see figure:) (No gravity)



(free pivot)

4b) Derive the equation of motion for $\theta(t)$ from this Lagrangian

4c) Solve this equation for $\theta(t)$ given the initial conditions $\dot{\theta}(t_0) = \omega_0$, $\theta(t_0) = 0$.

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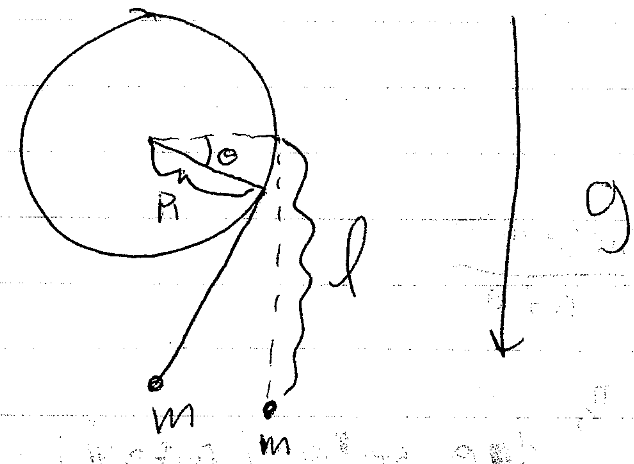
5) A mass ~~dangles~~ is suspended in a gravitational field by a string wrapped around a cylinder (of radius R), as shown: (next page:)

5) (cont)



a) Assuming the mass only moves in a vertical plane, and that the string always remains taut, write down the Lagrangian for this system taking the angle θ between the line from the center of the cylinder to the last point of string-cylinder contact and the horizontal as your independent dynamical variable. The length of string <u>not</u> in contact with the cylinder when $\theta = 0$ is l.

b) Derive the equation of motion for θ.

c) Simplify this equation in the limit $R \ll \ell$. Do you recognize the result?