PHYS 631: Quantum Mechanics I (Fall 2020)
Exercises 20 October 2020 (Tuesday, Week 4)
Due Monday, 26 October 2020

Exercise 2. Recall that the position-representation propagator is just a representation of the unitary time-evolution operator:

$$K(x,t;x_0,t_0) := \langle x|U(t,t_0)|x_0\rangle. \tag{1}$$

Recall also that a common notation is

$$K(x,t;x_0,t_0) := \langle x,t|x_0,t_0\rangle. \tag{2}$$

(a) Show that this notation works out if

$$|x,t\rangle = e^{+iHt/\hbar}|x\rangle, \tag{3}$$

in the case of a time-independent Hamiltonian.

(b) Note that this is *backwards* from what you expect from ordinary time evolution, because forward time evolution has the form $U(t,0) = e^{-iHt/\hbar}$. So then what is $|x,t\rangle$? Show that it may be interpreted as the eigenstate of the *Heisenberg-picture* operator $x(t)$. What is the eigenvalue?

You'll want to transform to the Schrödinger picture here; if you don't remember how, try working it out rather than looking it up, remembering that $\langle\psi(t)|Q(0)|\psi(t)\rangle = \langle\psi(0)|Q(t)|\psi(0)\rangle$, and $|\psi(t)\rangle = U(t,0)|\psi(0)\rangle$. Here, $Q(t)$ and $|\psi(0)\rangle$ are in the Heisenberg picture, while $Q(0)$ and $|\psi(t)\rangle$ are in the Schrödinger picture.