Exercise 2:

$\vec{L}$ does not need a symmetrized form because the commutator of each product in $\vec{r} \times \vec{p}$ is zero.

$$\boxed{\begin{aligned} &\text{i.e.} \quad y\partial_z = \partial_z y \quad \text{so} \\ &y\partial_z - z\partial_y = \partial_z y - \partial_y z \end{aligned}}$$

Exercise 3:

$$
\begin{aligned}
[L_x, p_y] &= [y p_z - z p_y, p_y] \\
&= [y p_z, p_y] - [z p_y, p_y] \\
&= [y p_z, p_y] \\
&= y[p_z, p_y] + [y, p_y] p_z \\
&= i\hbar p_z
\end{aligned}
$$

$$
\begin{aligned}
[L_y, p_x] &= [z p_x - x p_z, p_x] \\
&= [z p_x, p_x] - [x p_z, p_x] \\
&= -i\hbar p_z
\end{aligned}
$$

So, by (6), (7), and (8),

$$\boxed{[L_\alpha, p_\beta] = i\hbar\,\epsilon_{\alpha\beta\gamma}\,p_\gamma}$$

Exercise 4:

using $\alpha\beta\gamma$ as even and $\alpha\gamma\beta$ as odd,

$$[L_\alpha, L_\beta] = [L_\alpha, r_\gamma p_\alpha - r_\alpha p_\gamma]$$

$$= [L_\alpha, r_\gamma p_\alpha] - [L_\alpha, r_\alpha p_\gamma]$$

$$= [L_\alpha, r_\gamma] p_\alpha + r_\gamma [L_\alpha, p_\alpha]$$

$$- [L_\alpha, r_\alpha] p_\gamma - r_\alpha [L_\alpha, p_\gamma]$$

$$= [L_\alpha, r_\gamma] p_\alpha - r_\alpha [L_\alpha, p_\gamma]$$

$$= i\hbar(-r_\beta p_\alpha + r_\alpha p_\beta)$$

$$= i\hbar(r_\alpha p_\beta - r_\beta p_\alpha)$$

$$= i\hbar L_\gamma$$

So

$$\boxed{[L_\alpha, L_\beta] = i\hbar\,\epsilon_{\alpha\beta\gamma}\,L_\gamma}$$

Exercise 5:

Starting with

$$\vec{L} = i\hbar\left(\frac{1}{\sin\theta}\hat{\theta}\partial_\phi - \hat{\phi}\partial_\theta\right)$$

we can rewrite $\hat{\theta}$ and $\hat{\phi}$ in terms of $\hat{x}, \hat{y}, \hat{z}$ as

$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

$$\hat{\theta} = \cos\phi\cos\theta\,\hat{x} + \sin\phi\cos\theta\,\hat{y} - \sin\theta\,\hat{z}$$

So

$$L_x = \vec{L}\cdot\hat{x} = i\hbar\left(\cos\phi\frac{\cos\theta}{\sin\theta}\partial_\phi + \sin\phi\,\partial_\theta\right)$$

$$= i\hbar\left(\cos\phi\cot\theta\,\partial_\phi + \sin\phi\,\partial_\theta\right)$$

$$L_y = \vec{L}\cdot\hat{y} = i\hbar\left(\sin\phi\cot\theta\,\partial_\phi - \cos\phi\,\partial_\theta\right)$$

$$L_z = \vec{L}\cdot\hat{z} = i\hbar\left(-\frac{\sin\theta}{\sin\theta}\partial_\phi\right) = -i\hbar\,\partial_\phi$$

Exercise 6:

$$\Theta_{\ell}^{-\ell}(\theta) = \eta\,(\sin\theta)^{\ell}$$

$$(\ell\cot\theta - \partial_\theta)(\eta\,(\sin\theta)^{\ell})$$

$$= \eta\,(\,\ell\cot\theta\,\sin\theta^{\ell} - \ell\sin\theta^{\ell-1}\cos\theta)$$

$$= \eta\,\ell\,(\,\cos\theta\,\sin\theta^{\ell-1} - \sin\theta^{\ell-1}\cos\theta)$$

$$= 0 \qquad \checkmark$$

Exercise 7:

$$(L_+)^{\ell+m} |\ell\ (-\ell)\rangle$$

$$= (-1)^{\ell+m} \hbar^{\ell+m} e^{i(\ell+m)\phi} (1-\mu^2)^{m/2} \partial_\mu^{\ell+m} (1-\mu^2)^{\ell/2} |\ell\ (-\ell)\rangle$$

$$= (-1)^{\ell+m} \hbar^{\ell+m} e^{i(\ell+m)\phi} (1-\mu^2)^{\frac{m}{2}}$$

I am convinced of the result