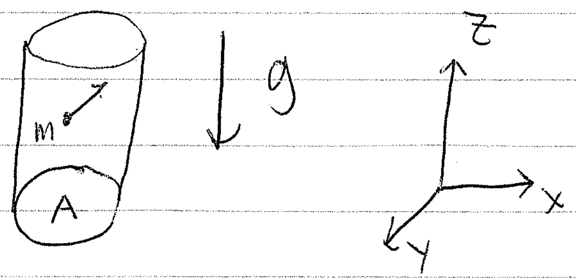
Physics 613 Problem Set #1, Due 2/18/09, in class 2PM sharp

1) Consider a single particle contained in a vertical cylinder of cross sectional area A in a gravitational field:



Study the statistical mechanics of this problem in the microcanonical ensemble. Specifically, calculate for fixed particle energy E the phase space volume $\Omega(E)$, the temperature $T(E)$, the energy as a function of temperature $E(T)$, and the specific heat $C(T)$; in terms of the particle mass m and the acceleration due to gravity g. Take the gravitational potential energy to be zero at the bottom of the

1) cont) cylinder. Also calculate the probability ~~P(z)~~ density $P(z)$, defined such that $P(z)\,dz$ is the probability that the particle lies between z and $z+dz$ ($z=0$ being the bottom of the cylinder). Does $P(z)$ vanish or diverge for any value of z? If so, why? (20 pts)

2) Calculate $\Omega(E)$, $T(E)$, $E(T)$, and $C(T)$ for ~~a single mass on a spring movi~~ simple one dimensional harmonic oscillator: a mass m ~~on a~~ free to move in one direction only while connected to a spring of spring constant k. Also calculate $P(x)$, the probability density defined so that $P(x)\,dx$ is the probability that the mass is

2) (cont) between x and $x+dx$. ~~Does P(x) diverge~~

(Define $x=0$ as the equilibrium position of the spring - the point at which it exerts no force). Does $P(x)$ diverge for any x? If so, why? Where is $P(x)$ a minimum? Again, why? (20 pts)

3) The same calculation for a point particle of mass m moving in the gravitation field of a much larger point mass M. Calculate, for 3 dimensional motion, the phase space volume $\Omega(E)$, ~~E(T) ble~~ $T(E)$, $E(T)$, and $C(T)$. Also calculate ~~the probability density $P(r)$ that the particle lies between r a dr anywhe~~

3) (cont) $P(r)$, defined ~~as the pro~~

so that $P(r)dr$ is the probability

that the particle of mass m lies

anywhere in the spherical shell between

r and $r+dr$ centered on the big mass M.

Does $\Omega(E)$ diverge at any E?

If so, why?

Does $P(r)$ diverge or vanish at any

r? Again, why?

Define the gravitational potential energy

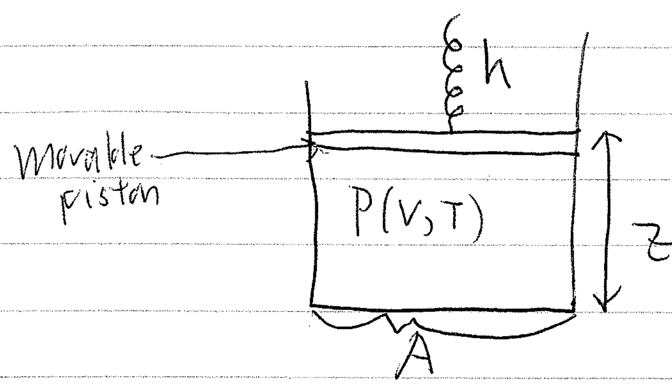
to be zero when the two masses are

infinitely far apart. Note that the

total energy may be negative in this case.

(20 pts)

4) Consider a gas with known equation of state $P(V,T)$ confined to a cell of area A by a movable ~~spring held~~ piston held is place by a spring of spring constant h:



Assume the spring is tensionless when $z=0$ (i.e., when the piston sits at the bottom of the cell). Calculate the specific heat ~~of the~~ C_h in this situation (the "specific heat at constant spring constant", to coin an ungainly moniker) in terms of the heat capacity at constant volume C_V and thermodynamic derivatives

4) (cont) that can be obtained from the equation of state.

(Hint: Use geometry and Hook's law to relate pressure and volume ~~area~~ to z; then eliminate z to obtain a relation between P, V, T, and the spring constant k.) (20 pts)

5) Prove, without using Jacobians, that

$$C_p = C_V - \frac{T\left(\frac{\partial V}{\partial T}\right)_P^2}{\left(\frac{\partial V}{\partial P}\right)_T}.$$

Use the fact that P always <u>decreases</u> upon isothermal expansion to show that $C_p > C_V$, always. (10 pts)

6) Consider a system obeying the ideal gas equation of state with ~~heat~~

6) (cont) constant heat capacity at constant volume $C_V = \gamma N k_B$, where N is the number of particles and γ a dimensionless constant. Derive the relation between temperature and volume during an adiabatic process in terms of γ. Verify that your result reduces to that derived in class for a monoatomic ~~[illegible]~~ ideal gas (Hint: what is γ for such a gas?) 20 pts