

PHYS 631: Quantum Mechanics I (Fall 2020)
Exercises 6 October 2020 (Tuesday, Week 2)
Due Monday, 12 October 2020

Exercise 3. Prove the uncertainty principle for the pair of observables P and Q , using the following outline.

(a) Start with the Cauchy–Schwarz inequality in the form

$$\langle \alpha | \alpha \rangle \langle \beta | \beta \rangle \geq |\langle \alpha | \beta \rangle|^2, \quad (1)$$

and use it to show

$$\langle \psi | P^2 | \psi \rangle \langle \psi | Q^2 | \psi \rangle \geq |\langle \psi | PQ | \psi \rangle|^2. \quad (2)$$

(b) For a complex number z , show that

$$|z| \geq \frac{1}{2} |z - z^*|. \quad (3)$$

(Do this by splitting z into its real and imaginary parts, and notice that the right-hand side above is magnitude of z 's imaginary part.)

(c) Combine your results from (a) and (b) to show that

$$\langle P^2 \rangle \langle Q^2 \rangle \geq \frac{1}{4} |\langle [P, Q] \rangle|^2. \quad (4)$$

(d) Finally, make the replacements $P \rightarrow P - \langle P \rangle$ and $Q \rightarrow Q - \langle Q \rangle$ to derive the uncertainty relation

$$V_P V_Q \geq \frac{1}{4} |\langle [P, Q] \rangle|^2. \quad (5)$$

Note that you are mainly showing that the commutator is invariant under this replacement.