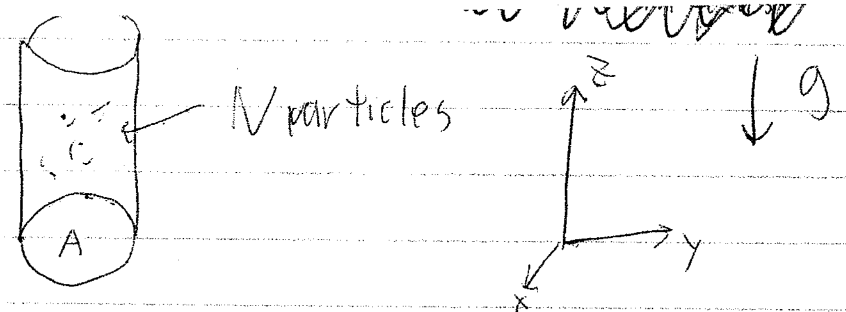
2)



Consider an infinitely tall cylinder ~~containi~~ of cross-sectional area A containing $N \gg 1$ identical particles ~~in a gravi~~ of mass m in a gravitational field g. In the microcanonical ensemble, ~~with~~ calculate ~~the temperature~~ ~~the entropy $S(E)$~~, the temperature $T(E)$, the specific heat $C(T)$, and the particle number density $\rho(z,T)$ for $z \ll N k_B T/mg$.

Hints: A) Rescale nasty multidimensional integrals to pull parameters out.

B) $(1-x)^y = e^{-xy}\left(1+O(x^2 y)\right)$

2) cont) Hints: c) ~~Drop all term~~ Calculate only to leading order in N.

---

3) Do the same problem in the canonical ensemble. Calculate $F(T)$, $E(T)$, $S(T)$, and $\rho(z,T)$. Do your results agree with (1)? Why, or why not? Do your results agree with the ~~ca~~ analogous 1 particle problem on problem set 1? Why or why not?

---

4) Consider a system of particles of mass m interacting gravitationally, in thermal equilibrium at temperature T, as a model for a "galactic ~~halo~~ dark matter halo". Solve this problem in "mean field theory" as follows:

4) cont) A) Pretend that each particle moves in a "mean field" $\phi(\vec{r})$ which satisfies Laplace's equation for gravity:

$$\boxed{(4.1) \qquad \nabla^2 \phi(\vec{r}) = 4\pi G m^2 \rho(\vec{r})}$$

where $\rho(\vec{r})$ is the mean number density of particles at the point $\vec{r}$. ~~determined~~

B) Determine $\rho(\vec{r})$ in terms of $\phi(\vec{r})$ from the canonical ensemble, treating the particles as non-interacting but moving in an external potential given by $\phi(\vec{r})$.

This leads to a 2nd equation relating ~~(c) Using this result, rewrite (3.1) as a~~ $\rho(\vec{r})$ and $\phi(\vec{r})$.

~~closed equation~~

C) Assume a spherically symmetric solution to the above two coupled equations of the form (next page→)

4) c) cont) $\boxed{(4.2)\ \rho(\vec{r}) = \rho_0 \left(\frac{r_0}{r}\right)^{\alpha}}$

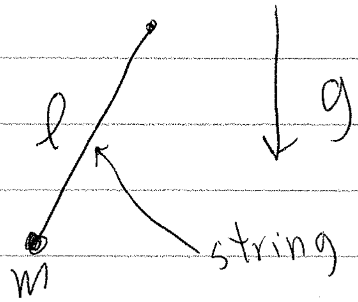
where ρ_0, r_0, and α are constants.

Determine α from the coupled equations.

~~Show that~~ show that the gravitational force experienced by a particle a distance r away from the center of the halo $(r=0)$ is $\propto r^{B}$, and find B.

Some astronomers have argued, based on rotation curve data for galaxies, that the gravitational force at long distances falls off as $\frac{1}{r}$, rather than $\frac{1}{r^2}$. ~~How would~~ What do you think of this idea, in light of the results of this problem?

#5) Consider a pendulum of mass m and length ℓ in a gravitational field g:



Treating the system classically, and assuming the pendulum is in thermal equilibrium with a heat bath at temperature T, calculate the partition function, free energy, entropy, and specific heat at constant length ℓ (C_ℓ) of the pendulum. Using the fact that tension τ and length ℓ are related to the free energy F in the same way as the pressure P and volume V are for an ideal gas,

5) Calculate the tension $\tau(\ell, T)$ in the string. What is the physical origin of this tension? Calculate the specific

5) ~~Do the same p~~

heat at constant <u>tension</u> τ ~~for this~~ (C_τ) for this system.

6) Repeat all of the above calculations quantum mechanically. Assume in <u>both</u> problems (4) and (5) that the thermal (and, in (5), quantum) fluctuations in the position of the pendulum are small compared to ℓ. For what ranges of m, g, ~~ω~~ and T will this be a good approximation? Do you recover the classical limit from the quantum problem at sufficiently high T? If so,

7b) For the classical pendulum above, calculate the temperature $T(\ell)$ if the pendulum starts at a temperature T_0 and length ℓ_0 and is then shortened adiabatically.