Exercise 1:

$$G(z) = \frac{1}{z - H}, \quad G_0(z) = \frac{1}{z - H_0}$$

let $A$, $B$ operators

$$B = A + (B - A)$$

$$\rightarrow \quad B^{-1} B A^{-1} = B^{-1} A A^{-1} + B^{-1}(B - A) A^{-1}$$

$$\rightarrow \quad A^{-1} = B^{-1} + B^{-1}(B - A) A^{-1}$$

letting $A = z - H_0 - \lambda V$, $B = z - H_0$

$$\rightarrow$$

$$\frac{1}{z - H_0 - \lambda V} = \frac{1}{z - H_0} + \frac{1}{z - H_0} (\lambda V) \frac{1}{z - H_0 - \lambda V}$$

$$\boxed{G(z) = G_0(z) + G_0(z) \lambda V G(z)}$$

Exercise 2:

$$|\psi\rangle = \frac{P|\psi_0\rangle}{\sqrt{\langle\psi_0|P|\psi_0\rangle}}$$

$$P \approx P_0 + \lambda(P_0 V G_Q + G_Q V P_0)$$

$$|\psi\rangle \approx (P_0 + \lambda(P_0 V G_Q + G_Q V P_0))|\psi_0\rangle$$

since $Q_0|\psi_0\rangle = 0$, $G_Q|\psi_0\rangle = 0$

$$|\psi\rangle \approx |\psi_0\rangle + \lambda G_Q V P_0|\psi_0\rangle$$

$$|\psi\rangle \approx |\psi_0\rangle + \lambda \sum_{\alpha\neq 0} \frac{|\alpha\rangle\langle\alpha| V|\psi_0\rangle}{E_0 - E_\alpha}$$

$$|\psi\rangle \approx |\psi_0\rangle + \lambda \sum_{\alpha\neq 0} \frac{\langle\alpha|V|\psi_0\rangle}{E_0 - E_\alpha}|\alpha\rangle$$

$$\langle\psi_0|P|\psi\rangle = 1 + \lambda^2 \sum_{\alpha\neq 0} \frac{|\langle\alpha|V|\psi_0\rangle|^2}{E_0 - E_\alpha}$$

Exercise 3:

$$\Delta_n = \sum_{\substack{k_1,\dots,k_{n+1} \\ (k_1+\dots+k_{n+1}=n-1)}} S_{k_1} V S_{k_2} V \dots V S_{k_{n+1}}$$

$$E = \mathrm{Tr}[HP] = E_0 + \sum_{n=1}^{\infty} \lambda^n \mathrm{Tr}\,\Delta_n$$

$$= E_0 + \sum_{n=1}^{\infty} \delta E_n$$

$$\Delta_1 = \sum_{\substack{i,j \\ (i+j=0)}} S_i V S_j$$

$$= P_0 V P_0$$

$$\delta E_1 = \lambda \,\mathrm{Tr}\,\Delta_1$$

$$= \lambda \sum_{\alpha} \langle \alpha | P_0 V P_0 | \alpha \rangle$$

$$= \lambda \langle \Psi_0 | V | \Psi_0 \rangle$$

Exercise 4:

$$\Delta_2 = S_0 V S_0 V S_1 + S_0 V S_1 V S_0 + S_1 V S_0 V S_1$$

$$\begin{aligned}
\mathrm{Tr}\,\Delta_2 &= \mathrm{Tr}(S_0 V S_0 V S_1) + \mathrm{Tr}(S_0 V S_1 V S_0) + \mathrm{Tr}(S_1 V S_0 V S_1) \\
&= \mathrm{Tr}(S_0 V S_1 V S_0) \\
&= \sum_\alpha \langle \alpha | S_0 V S_1 V S_0 | \alpha \rangle \\
&= \langle \psi_0 | V S_1 V | \psi_0 \rangle \\
&= \langle \psi_0 | V \sum_{\alpha \neq 0} \frac{|\alpha\rangle\langle\alpha|}{E_0 - E_\alpha} V | \psi_0 \rangle \\
&= \sum_{\alpha \neq 0} \frac{|\langle \psi_0 | V | \alpha \rangle|^2}{E_0 - E_\alpha}
\end{aligned}$$

$$\begin{aligned}
\delta E_2 &= \lambda^2 \mathrm{Tr}\,\Delta_2 \\
&= \lambda^2 \sum_{\alpha \neq 0} \frac{|V_{0\alpha}|^2}{E_{0\alpha}}
\end{aligned}$$

Exercise 5:

$$\Delta_3 = S_0 V S_0 V S_0 V S_2 + S_0 V S_0 V S_1 V S_0 + S_0 V S_2 V S_0 V S_0 + S_2 V S_0 V S_0 V S_0 + S_1 V S_1 V S_0 V S_0 + S_1 V S_0 V S_1 V S_0 + S_1 V S_0 V S_0 V S_1 + S_0 V S_1 V S_1 V S_0 + S_0 V S_0 V S_1 V S_1 + S_0 V S_1 V S_0 V S_1$$

$$\begin{aligned}
\mathrm{Tr}\,\Delta_3 &= \mathrm{Tr}(S_0 V S_0 V S_2 V S_0) \\
&\quad + \mathrm{Tr}(S_0 V S_1 V S_0 V S_0) \\
&\quad + \mathrm{Tr}(S_1 V S_0 V S_0 V S_1) \\
&\quad + \mathrm{Tr}(S_0 V S_1 V S_1 V S_0) \\
&= -\mathrm{Tr}(S_0 V S_0 V S_2 V) \\
&\quad + \mathrm{Tr}(S_2 V S_0 V S_0 V) \\
&\quad + \mathrm{Tr}(S_0 V S_2 V S_0 V S_0) \\
&\quad + \mathrm{Tr}(S_0 V S_1 V S_1 V S_0)
\end{aligned}$$

$$\begin{aligned}
&= \mathrm{Tr}(S_0 V S_2 V S_0 V S_0) \\
&\quad + \mathrm{Tr}(S_0 V S_1 V S_1 V S_0)
\end{aligned}$$

$$= \langle \psi_0 | V S_1 V S_1 V | \psi_0 \rangle + \langle \psi_0 | V S_2 V S_0 V | \psi_0 \rangle$$

$$\begin{aligned}
&= \langle \psi_0 | V S_1 V S_1 V | \psi_0 \rangle \\
&\quad - \langle \psi_0 | V | \psi_0 \rangle \langle \psi_0 | V S_2 V | \psi_0 \rangle
\end{aligned}$$

$$\delta E_3 = \lambda^3 \, \mathrm{Tr}\, \Delta_3$$

$$= \lambda^3 \sum_{\alpha \neq 0} \sum_{\beta \neq 0} \frac{\langle 4_c | V | \alpha \rangle \langle \alpha | V | \beta \rangle \langle \beta | V | 4_c \rangle}{E_{0\alpha} E_{0\beta}}$$

$$- \lambda^3 V_{00} \sum_{\alpha \neq 0} \frac{|V_{0\alpha}|^2}{(E_{0\alpha})^2}$$

$$= \lambda^3 \sum_{\alpha \neq 0} \sum_{\beta \neq 0} \frac{V_{0\alpha} V_{\alpha\beta} V_{\beta 0}}{E_{0\alpha} E_{0\beta}} - \lambda^3 V_{00} \sum_{\alpha \neq 0} \frac{|V_{0\alpha}|^2}{(E_{0\alpha})^2}$$

Exercise 6:

If $\sum_{\alpha \neq 0} \frac{|V_{0\alpha}|^2}{E_{0\alpha}} = 0$ and $V_{00} = 0$

Then

$$\delta E_3 = \lambda^3 \sum_{\alpha \neq 0} \sum_{\beta \neq 0} \frac{V_{0\alpha} V_{\alpha\beta} V_{\beta 0}}{E_{0\alpha} E_{0\beta}}$$