

PHYS 632: Quantum Mechanics II (Winter 2021)
Exercises 5 January 2021 (Tuesday, Week 1)
Due Monday, 11 January 2021

Exercise 2. The standard quantum limit (SQL) for position measurements reads

$$\Delta x \sim \sqrt{\frac{\hbar \tau}{m}} \quad (1)$$

for a measurement time τ .

(a) Show that this translates to an SQL for a force measurement of

$$\Delta F \sim \sqrt{\frac{\hbar m}{\tau^3}}. \quad (2)$$

(Start by computing the displacement Δx due to a force over time τ , and use the SQL expression for Δx .)

(b) Also derive the momentum SQL

$$\Delta p \sim \sqrt{\frac{\hbar m}{\tau}}. \quad (3)$$

(Start by writing down the momentum corresponding to a displacement Δx in time τ .)

a) $F = m \ddot{x}$

$$\Delta x = \frac{1}{2} \frac{\Delta F}{m} \tau^2$$

$$\sqrt{\frac{\hbar \tau}{m}} = \frac{1}{2} \frac{\Delta F}{m} \tau^2$$

$$2m \sqrt{\frac{\hbar \tau}{m}} \frac{1}{\tau^2} = \Delta F$$

$$\Delta F = 2 \sqrt{\frac{\hbar m \tau}{\tau^4}} \sim \sqrt{\frac{\hbar m}{\tau^3}}$$

b) $\Delta x = \frac{\Delta p}{m} \tau$

$$\hookrightarrow \sqrt{\frac{\hbar \tau}{m}} = \frac{\Delta p}{m} \tau$$

$$\rightarrow \Delta p \sim \sqrt{\frac{m\hbar}{\tau}}$$