Physics 611 ~~[illegible]~~ Exam

Due: ~~[illegible]~~
~~at 12 noon Thursday, Dec 10~~
In class, 2PM Sharp, Thursday, 12/4

1) In general relativity, a body moving around a black hole moves so as to minimize its "proper time" τ (i.e., the time measured by a clock moving with the body), which is given by

$$\boxed{(1.1)\quad \tau = \frac{1}{c}\int_{t_0}^{t_f} \sqrt{c^2\gamma(r) - \frac{1}{\gamma(r)}\left(\frac{dr}{dt}\right)^2 - r^2\left(\frac{d\phi}{dt}\right)^2}\, dt}$$

where

$$\boxed{(2)\quad \gamma(r) \equiv 1 - \frac{2GM}{rc^2}}$$

Here M is the mass of the black hole, c is the speed of light, G the gravitational

1) cont) θ constant, and (r, θ, ϕ) are the usual spherical co-ordinates. In (1.1), and throughout this problem, we'll ~~consider only~~ chose our $\theta = 0$ axis such that the body's motion lies in the "equitorial" (i.e., $\theta = \frac{\pi}{2}$) plane.

a) Derive the equations of motion that follow from minimizing τ.

b) Show that for $r \gg r_s \equiv \frac{2GM}{c^2}$, these equations reduce to those of the classical Kepler problem (i.e., standard Newtonian gravity). What is the physical significance of r_s?

c) For the full equations derived in (a), find two conserved quantities

1) cont) d) Using these two conserved quantities, find the effective potential for radial motion, and plot it. [If there are qualitatively different shapes of this potential for different values of the conserved quantities, plot all qualitatively different cases, and give the values of the conserved quantities that seperate different cases.

e) Describe, qualitatively, the different types of motion possible (By way of illustration, for the Kepler problem, the answer to this question would be: "closed orbits or escape orbits, <u>except</u> in the very special case angular momentum $L=0$, in which case some orbits plunge into the center. When $L \neq 0$, no orbits ever reach the center ($r=0$).

quantities are stable circular orbits possible.

1g) Calculate the frequency of small radial oscillations about a nearly circular orbit as a function of the radius ~~of the~~ r of that orbit.

1h) Find the angle $\Delta\theta$ between succesive ~~perihelia~~ perihelions (or peri-black-hole's), and calculate the precession rate $\frac{\Delta\theta}{T}$, where T is the time between succesive perihelions.

Compare your answer quantitatively with the observed ~~43 arc sec~~

$\frac{\Delta\theta}{T} = 43$ arc-seconds/century for mercury.

1i)

Black hole

A projectile is shot towards the black hole ~~a~~ from a great distance away at

1i) (cont) an initial velocity v_0. Show that if its "impact parameter" b (see figure) is $<$ some critical value, the projectile falls in to the black hole (i.e., all the way to $r = r_s$). Find b_c in terms of v_0 and r_s for $v_0 \ll c$.

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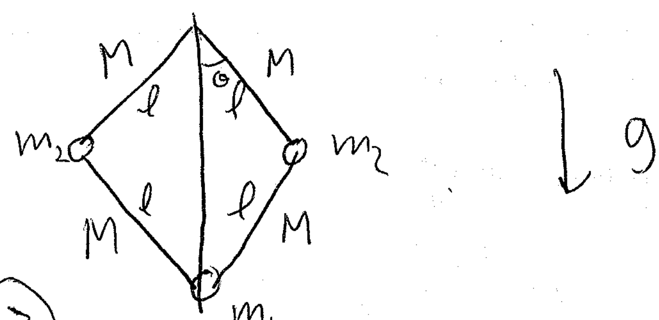
2) Consider the apparatus (sic) of problem 1 of problem set #2, but now with the rigid rods having identical masses M.



a) Repeat the (3) calculations requested in problem 1 of problem set #2 for this case. ~~The, derive the Lagrangian~~