Exercise 1. If $[A, B] = 0$, show that

$$e^{A+B} = e^A e^B. \tag{1}$$

There is a brute-force way to do this by working with the product of two series, but I suggest instead approaching this proof as follows. Consider instead the series expansion of $e^{A+\lambda B}$, and compute the derivative with respect to λ, being careful to note where you use $[A, B] = 0$. It should then be obvious how to show that

$$\partial_\lambda^m e^{A+\lambda B} = \sum_{n=m}^{\infty} \frac{B^m (A + \lambda B)^{n-m}}{(n-m)!}. \tag{2}$$

Setting $\lambda = 0$ gives you an expression that lets you show the Taylor expansions of both sides of Eq. (1) are equivalent.