PHYS 631: Quantum Mechanics I (Fall 2020)
Final Exam, 7–11 December 2020

Rules for this exam: You may take up to 12 hours after you download the exam to work on it, then you should upload your answers.

You may consult any inanimate object while taking the exam (including computers, *Mathematica*, and information posted on the internet). However, you may **not** discuss any part of the exam with **any human by any means** (except the instructor) until after the last due date.

Magic lifeline rule is still in effect for this exam (2 lifeline questions).

New rule: one solution per problem *please*. If you try multiple approaches to solve a problem, then I respect and applaud your tenacity. However **pick only one** to submit with your official solutions.

I have read and understand the rules.

____ $\longleftarrow$ **Sign your name here.**

Problem 1. (25 points total)
(a) (5 points) For a projection operator P, show that $e^P = 1 + (e-1)P$.

(b) (10 points) Suppose that $P_1, \ldots P_n$ are (nonidentical) projectors. Give a necessary and sufficient condition for $P_1 + \cdots + P_n$ to be a projection operator (and prove necessity and sufficiency).

(c) (10 points) An **elementary projector** is one that can be written $P = |a\rangle\langle a|$ in terms of some state vector $|a\rangle$. Suppose that $P_1, \ldots P_n$ are (nonidentical, elementary) projectors, and let Q be a linear combination of these. Give a sufficient condition for Q to be an elementary projector (and prove sufficiency).

Problem 2. (25 points total) Take the limits:

(a) (12 points)

$$L(x) = \lim_{a\to 0^+} \frac{1}{a} H_2(x/a)\, e^{-x^2/2a^2}. \tag{1}$$

(b) (13 points)

$$M(x) = \lim_{a\to 0^+} \left[\frac{1}{a^3} H_2(x/a)\, e^{-x^2/2a^2} - \frac{2}{a^3}\, e^{-x^2/2a^2}\right]. \tag{2}$$

Problem 3. (25 points) Quantum schmunneling in the schmymmetric schmouble-shmelta-schmell schmotential is just like quantum tunneling in the symmetric double-delta-well potential, except that the Hamiltonian in the representation of $|\mathrm{L}\rangle$ and $|\mathrm{R}\rangle$ states reads

$$H = \begin{bmatrix} E_0 & \hbar\Omega/2 \\ \hbar\Omega^*/2 & E_0 \end{bmatrix}, \tag{3}$$

where Ω is a complex number.

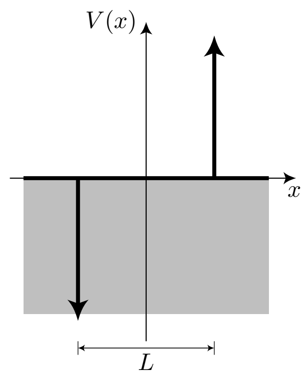
Show that Rabi (Schmabi) oscillations still occur, even when Ω is not a real number. Give an example of an initial state that gives rise to oscillations, and state explicitly the oscillation frequency.

(Don't forget the other side!)

Problem 4. (25 points total) Consider the delta well/barrier potential

$$V(x) = -\beta\,\delta(x + L/2) + \beta\,\delta(x - L/2), \qquad (\beta > 0), \tag{4}$$

as shown schematically below.



(a) (15 points) Derive an equation whose solution determines the energies of bound states of this potential (i.e., an equation for the wave number k).

(b) (10 points) Given some $\beta > 0$, how many bound states are there?