PHYS 632: Quantum Mechanics II (Winter 2021)
Exercises 15 February 2021 (Monday, Week 7)
Due Monday, 22 February 2021

Exercise 1. Go through the Ramsey interferometry method (the basic method used in atomic clocks) mathematically for a spin-1/2 system, to show that the output state reflects the internal evolving phase of the atom, using the following steps:

1. Start in the $|+\rangle$ state.
2. Apply a $\pi/2$-pulse (i.e., Hadamard gate, like the beam splitter from lecture), which puts the spin in a superpositions state (which you should calculate).
3. Allow free evolution for time T, assuming the energy difference between $|\pm\rangle$ is $\hbar\omega$. To be definite, assume $E_+ = \hbar\omega$ and $E_- = 0$.
4. Apply another $\pi/2$-pulse.
5. In the final state, show that the probability of measuring the spin to be in, say, the $|+\rangle$ state is an oscillating function of ωT ("interference fringes" or "Ramsey fringes").

1. $|+\rangle$

2. $U_{BS} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$|+\rangle \rightarrow \frac{1}{\sqrt{2}} (|+\rangle - |-\rangle)$

3. $\frac{1}{\sqrt{2}} \left(e^{-i\omega T} |+\rangle - |-\rangle \right)$

4. $\frac{1}{2} \left((e^{-i\omega T} - 1)|+\rangle - (e^{-i\omega T} + 1)|-\rangle \right)$

5. $P_+ = \frac{1}{4} (e^{-i\omega T} - 1)(e^{i\omega T} - 1)$

$P_+ = \frac{1}{4} (2 - (e^{i\omega T} + e^{-i\omega T}))$

$P_+ = \frac{1}{2} (1 - \cos(\omega T))$

$P_+ = \sin^2\left(\frac{\omega}{2} T\right)$