1)

Suppose we have a particle in the $|j\,0\rangle$ J^2, J_z eigenstate which decays into two particles in $|j\,m_1; 1\,m_2\rangle$. We can show that $m_1 \neq 0$ and $m_2 \neq 0$ by showing that

$$\langle j, 0; 1, 0 | j\,0\rangle = 0$$

By equation 7.127, we have

$$\langle j_1, m_1; j_2\, m_2 | j_3\, m_3\rangle = (-1)^{j_1+j_2-j_3} \langle j_1 - m_1; j_2 - m_2 | j_3 - m_3\rangle$$

So

$$\langle j\,0; 1\,0 | j\,0\rangle = -\langle j\,0; 1\,0 | j\,0\rangle$$

Therefore,

$$\langle j\,0; 1\,0 | j\,0\rangle = 0$$

Clearly this depended on the fact that $m_3 = -m_3$, $m_1 = -m_1$, $m_2 = -m_2$.

2)

a)

$$R(\hat{y}\tfrac{\pi}{2})|1\,0\rangle = \sum_m |1\,m\rangle\langle 1\,m|R(\hat{y}\tfrac{\pi}{2})|1\,0\rangle$$

$$= \sum_m |1\,m\rangle\, d^{(1)}_{m0}(\hat{y}\tfrac{\pi}{2})$$

$$= d^{(1)}_{-10}(\tfrac{\pi}{2}\hat{y})|1\,-1\rangle$$

$$+\ d^{(1)}_{00}(\tfrac{\pi}{2}\hat{y})|1\,0\rangle$$

$$+\ d^{(1)}_{10}(\tfrac{\pi}{2}\hat{y})|1\,1\rangle$$

$$= \frac{\sin(\frac{\pi}{2})}{\sqrt{2}}|1\,-1\rangle + \cos(\tfrac{\pi}{2})|1\,0\rangle$$

$$- \frac{\sin(\frac{\pi}{2})}{\sqrt{2}}|1\,1\rangle$$

$$\boxed{R(\hat{y}\tfrac{\pi}{2})|1\,0\rangle = \frac{1}{\sqrt{2}}|1\,-1\rangle - \frac{1}{\sqrt{2}}|1\,1\rangle}$$

$$\boxed{R(\hat{y}\tfrac{\pi}{2}) = \begin{bmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix}}$$

b)

To turn the z axis into the y axis we make a rotation by $\alpha = \frac{\pi}{2}$ about $\hat{z}$ and $\beta = \frac{\pi}{2}$ about $\hat{y}'$.

So

$$R(-\tfrac{\pi}{2}\hat{x})|1\,0\rangle = \sum_m |1\,m\rangle\langle 1\,m|R(-\tfrac{\pi}{2}x)|1\,0\rangle$$

$$= \sum_m |1\,m\rangle\, d^{(1)}_{m0}(\tfrac{\pi}{2}\hat{y})\, e^{-im\frac{\pi}{2}}$$

$$= -\frac{e^{-i\frac{\pi}{2}}\sin(\frac{\pi}{2})}{\sqrt{2}}|1\,1\rangle + \cos(\tfrac{\pi}{2})|1\,0\rangle + \frac{e^{i\frac{\pi}{2}}\sin(\frac{\pi}{2})}{\sqrt{2}}|1\,-1\rangle$$

$$\boxed{= \frac{i}{\sqrt{2}}|1\,1\rangle + \frac{i}{\sqrt{2}}|1\,-1\rangle}$$

$$\boxed{R(-\tfrac{\pi}{2}\hat{x}) = \begin{bmatrix} -\frac{i}{2} & \frac{i}{\sqrt{2}} & -\frac{i}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{i}{2} & \frac{i}{\sqrt{2}} & \frac{i}{2} \end{bmatrix}}$$

3)

if $\begin{pmatrix} a_1 \\ a_0 \\ a_{-1} \end{pmatrix} = U \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$

and $a_1 = -\frac{1}{\sqrt{2}}(a_x + i a_y)$

$a_{-1} = \frac{1}{\sqrt{2}}(a_x - i a_y)$

$a_0 = a_z$

then

$$\boxed{U = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \end{bmatrix}}$$

b)

$$\rho_s = U \rho U^\dagger$$

$$\rho_s = U\, R_z(-\gamma)\, R_y(-\beta)\, R_z(-\alpha)\, U^\dagger$$

$$\rho_s = \begin{bmatrix} \cos^2\left(\frac{\beta}{2}\right) e^{-i(\alpha+\gamma)} & \frac{1}{\sqrt{2}} e^{-i\gamma} \sin(\beta) & e^{i\alpha} e^{-i\gamma} \sin^2\left(\frac{\beta}{2}\right) \\ -\frac{e^{-i\alpha}}{\sqrt{2}} \sin(\beta) & \cos(\beta) & e^{i\alpha} \frac{\sin(\beta)}{\sqrt{2}} \\ e^{-i(\alpha-\gamma)} \sin^2\left(\frac{\beta}{2}\right) & -\frac{e^{i\gamma} \sin(\beta)}{\sqrt{2}} & \cos^2\left(\frac{\beta}{2}\right) e^{i\alpha + i\gamma} \end{bmatrix}$$

$$\boxed{\rho_s = \begin{bmatrix} e^{-i\alpha} \frac{1+\cos(\beta)}{2} e^{-i\gamma} & \frac{e^{i\gamma}}{\sqrt{2}} \sin(\beta) & e^{i\alpha} \frac{1-\cos(\beta)}{2} e^{-i\gamma} \\ -\frac{e^{-i\alpha}}{\sqrt{2}} \sin(\beta) & \cos(\beta) & \frac{e^{i\alpha}}{\sqrt{2}} \sin(\beta) \\ e^{-i\alpha} \frac{1-\cos(\beta)}{2} e^{i\gamma} & -\frac{e^{i\gamma}}{\sqrt{2}} \sin(\beta) & e^{i\alpha} \frac{1+\cos\beta}{2} e^{i\gamma} \end{bmatrix}}$$

$$\begin{bmatrix} e^{-i\alpha} \frac{1+\cos(\beta)}{2} e^{-i\gamma} & \frac{e^{i\gamma}}{\sqrt{2}} \sin(\beta) & e^{i\alpha} \frac{1-\cos(\beta)}{2} e^{-i\gamma} \\ -\frac{e^{-i\alpha}}{\sqrt{2}} \sin(\beta) & \cos(\beta) & \frac{e^{i\alpha}}{\sqrt{2}} \sin(\beta) \\ e^{-i\alpha} \frac{1-\cos(\beta)}{2} e^{i\gamma} & -\frac{e^{i\gamma}}{\sqrt{2}} \sin(\beta) & e^{i\alpha} \frac{1+\cos\beta}{2} e^{i\gamma} \end{bmatrix}$$

c)

$$P_s^T = \begin{bmatrix} e^{-i\alpha}\frac{1+\cos(\beta)}{2}e^{-i\gamma} & -\frac{e^{-i\alpha}}{\sqrt{2}}\sin(\beta) & e^{-i\alpha}\frac{1-\cos(\beta)}{2}e^{i\gamma} \\ \frac{e^{i\gamma}}{\sqrt{2}}\sin(\beta) & \cos(\beta) & -\frac{e^{i\gamma}}{\sqrt{2}}\sin(\beta) \\ e^{i\alpha}\frac{1-\cos(\beta)}{2}e^{-i\gamma} & \frac{e^{i\alpha}}{\sqrt{2}}\sin(\beta) & e^{i\alpha}\frac{1+\cos\beta}{2}e^{i\gamma} \end{bmatrix}$$

$$= \left[D^{(1)}_{m'm}(\alpha, \beta, \gamma)\right]$$