PHYS 631: Quantum Mechanics I (Fall 2020)
Exercises 27 October 2020 (Tuesday, Week 5)
Due Monday, 2 November 2020

Exercise 2. Suppose that a particle is in a bound state of a finite square-well potential. What if we perform a position measurement, and find the particle in the forbidden region? Clearly, a negative energy makes no sense, so it must be that $E > 0$ after the measurement, even though we started with $E < 0$. So what gives? Did we just violate energy conservation?

Show that the act of measurement must have transferred enough energy to the particle (i.e., via "quantum measurement back-action") to resolve any contradiction, using the following outline. First, recall that the wave function in the forbidden region has the form $\psi \sim e^{-k_{\text{I}}x}$, where

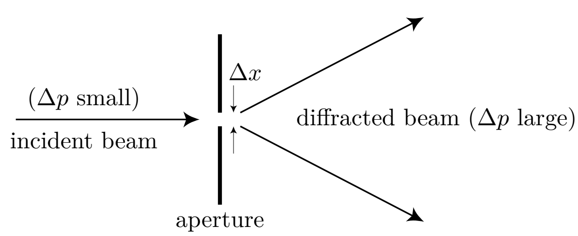
$$k_{\text{I}} = \sqrt{\frac{2m|E|}{\hbar^2}}, \tag{1}$$

and here $|E|$ is the energy "deficit" associated with finding the particle in the forbidden region. Thus, for this issue to be of any concern, the position measurement must localize the particle to a width of order

$$\Delta x \sim \frac{1}{k_{\text{I}}}. \tag{2}$$

Use the uncertainty principle to estimate the increase in the momentum width of the state, and then estimate the accompanying increase of the kinetic energy, showing that it is of order $|E|$.

Exercise 3. Another concrete (and classic) example of a position measurement is the single-slit diffraction problem.



Consider a beam of quantum particles (electrons, neutrons, photons, etc.) incident on an aperture, and we'll focus on the position transverse to the beam. The aperture acts as a position measurement, because any particles making it through the slit of width Δx have been effectively measured to be within this position interval (assume that the initial beam width is large compared to Δx, and the transverse momentum Δp is close to zero). This implies that the transverse momentum $\Delta p \sim \hbar/\Delta x$ is much larger than before the aperture, leading to diffraction. But this also implies a large kinetic energy associated with the transverse momentum, $(\Delta p)^2/2m$. Where did this energy come from? (Not the aperture, which we can assume has no effect other than to absorb particles that hit it.)