Fighting for light

Finding a Stable Tree Height Distribution

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What does a tree need to survive?

- **Water** is a dynamic part of a tree's basic structure and is one of the key mechanisms of photosynthesis.

- **Soil** sustains and supports the tree. It holds water and contains essential nutrients the tree needs to grow.

- **Sunlight** gives energy to a tree by a process of photosynthesis. Each tree's leaves take in sunlight, air (CO_2) and mixing them with water and soil nutrients from the tree's roots.

- **Space** is essential for a tree to grow! Deprived of sufficient space, trees must compete with other plants for light, soil nutrients and water.

Our Objectives

- We will sketch some general rules of the behavior of a tree within a forest and its interaction with the environment.
- We will convert these rules into a set of theoretical mathematical formulas to help set the conditions for a forest simulation.
- We will construct a simulated forest that will follow the rules based on the formulas.
- We will analyze the results of the simulation used to find a height distribution for a given species of trees that guarantees the equilibrium of the forest.
- We will attempt to understanding of how growth and death dynamics of a forest can be explained by the battle for light that is present in every plant-based ecosystem.

Understanding nature

Real World

General rules of trees in nature

Experiment

Real tree size distribution

Growth **Death**

Abstraction

Validation

Mathematical model

Computation

Forecasting tree size distribution

Mathematics

Growth rules

The quality of the soil affects the tree growth rate

The tree modifies the quality of the soil around it

The amount of light modifies the tree growth rate

The age of the tree modifies its growth rate

Growth rules

The quality of the soil affects the tree growth rate

Quality
- Climate
- Humidity
- Nutrients

The higher is the quality of the soil, the stronger positive influence it has on the tree growth

Growth rules

The quality of the soil affects the tree growth rate

The tree modifies the quality of the soil around it

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Growth rules

The tree modifies the quality of the soil around it

Major soil modifications

- Structure
- Nutrients
- Erosion
- Toxicity

If soil nutrients are not counterbalanced within an environment, it can lead to forest destruction.

Growth rules

The quality of the soil affects the tree growth rate

The tree modifies the quality of the soil around it

The amount of light modifies the tree growth rate

The age of the tree modifies its growth rate

Growth rules

The amount of light modifies the tree growth rate

- Every species of tree is adapted to particular light conditions.

- Sudden changes will lead to a premature death.

Growth rules

The quality of the soil affects the tree growth rate

The tree modifies the quality of the soil around it

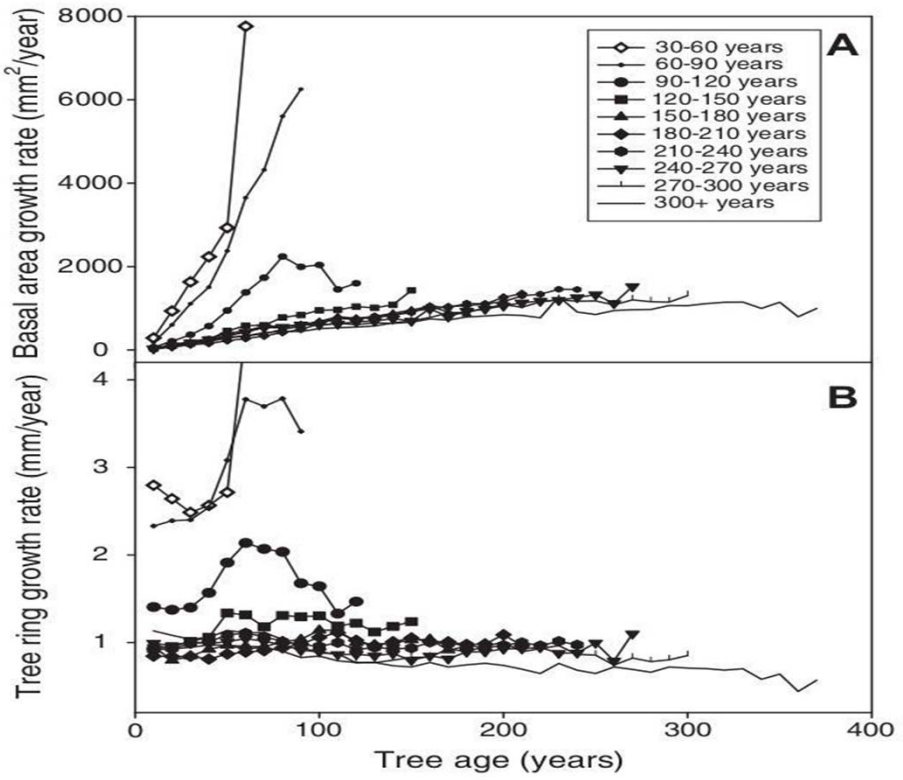
The amount of light modifies the tree growth rate

The age of the tree modifies its growth rate

Growth rules

The age of the tree modifies its growth rate

- Growth rate does not change with age



Death rules

The life expectancy of a tree

Soil quality affects the tree's life expectancy

Lack or excess of light modifies the tree's life expectancy

Death rules

The life expectancy of a tree

- A tree’s lifespan depends strongly on its species.
- However, there are external factors influencing:
    - Climate
    - Surrounding ecosystem
    - Size
    - Adaptation to the environment

Death rules

The life expectancy of a tree

Soil quality affects the tree’s life expectancy

Lack or excess of light modifies the tree’s life expectancy

Death rules

Soil quality affects the tree's life expectancy

- Soil dependence changes dramatically from one species to another.

- The growth and size of a given tree can be limited by a lack of quality of the soil.

Death rules

The life expectancy of a tree

Soil quality affects the tree’s life expectancy

Lack or excess of light modifies the tree’s life expectancy

Death rules

Lack or excess of light modifies the tree's life expectancy

Mathematical equations

Tree Growth Process

$$S_{t+\Delta t} = S_t + g$$

$$g = P\left(\frac{1 + R(Q) + R(L)}{3} + \frac{1}{2}\frac{h^2 - h^3}{h^2 + |h^3|}\right)$$

S_t – *size of the tree at time t*

g – *growth rate*

Q – *soil quality*

L – *Light available in the area*

h – *Difference in size with biggest neighbour*

P – *speed of growth (different for different species)*

Mathematical equations

Functions of Soil and Light

$$R(Q) = \alpha(Q - d)$$

$$R(L) = 1 - L^{\beta}$$

$$Q_{t+\Delta t} = (1 - \frac{S}{k_s} - \frac{g}{k_q})Q_t$$

$\alpha - sensitivity\ to\ soil$

$\beta - sensitivity\ to\ light$

$k_s - coefficient\ \ for\ size$

$k_g - coefficient\ \ for\ growth$

$d - soil\ agressivity$

Simulation

Forest matrix
- uniform distribution between 0-1

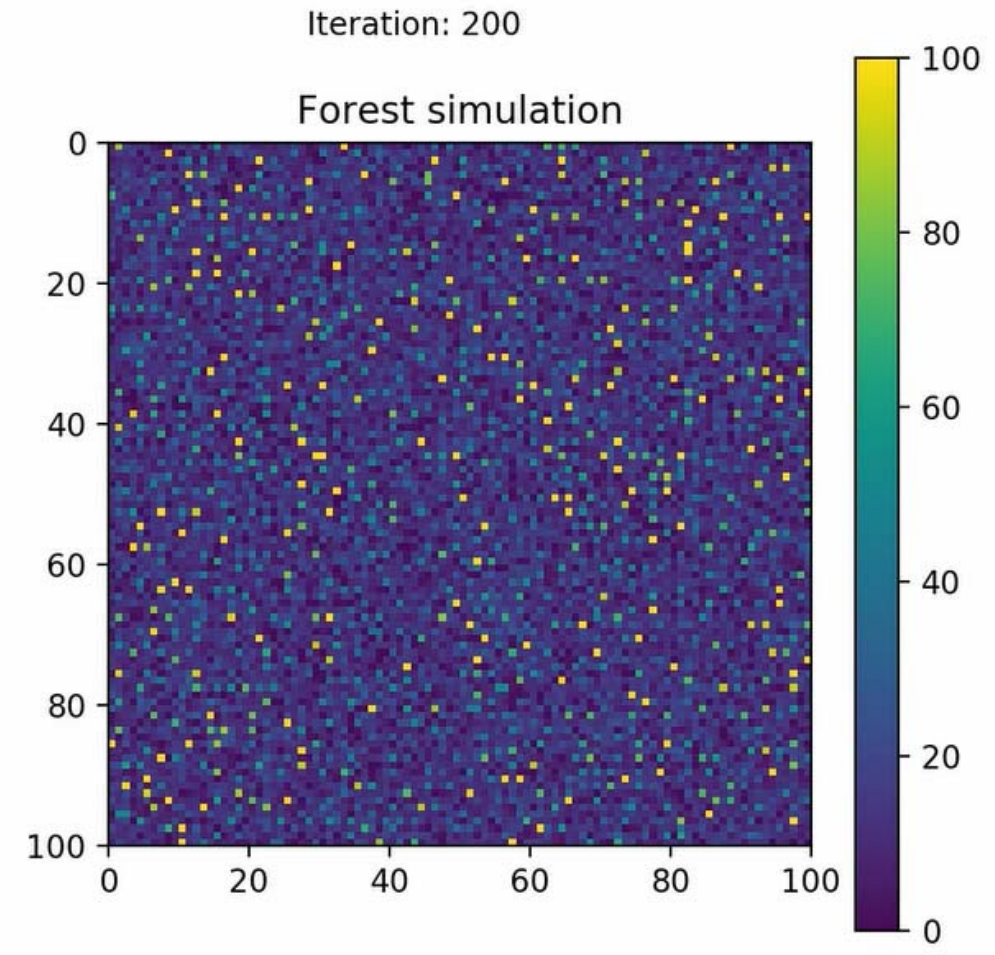
Soil matrix
- uniform distribution between 0-1

Parameters

- 1 iteration = 7 real years
- Max size = 100 meters
- Newborn probability = 0.4
- Lifetime = 57 iterations = 400 years

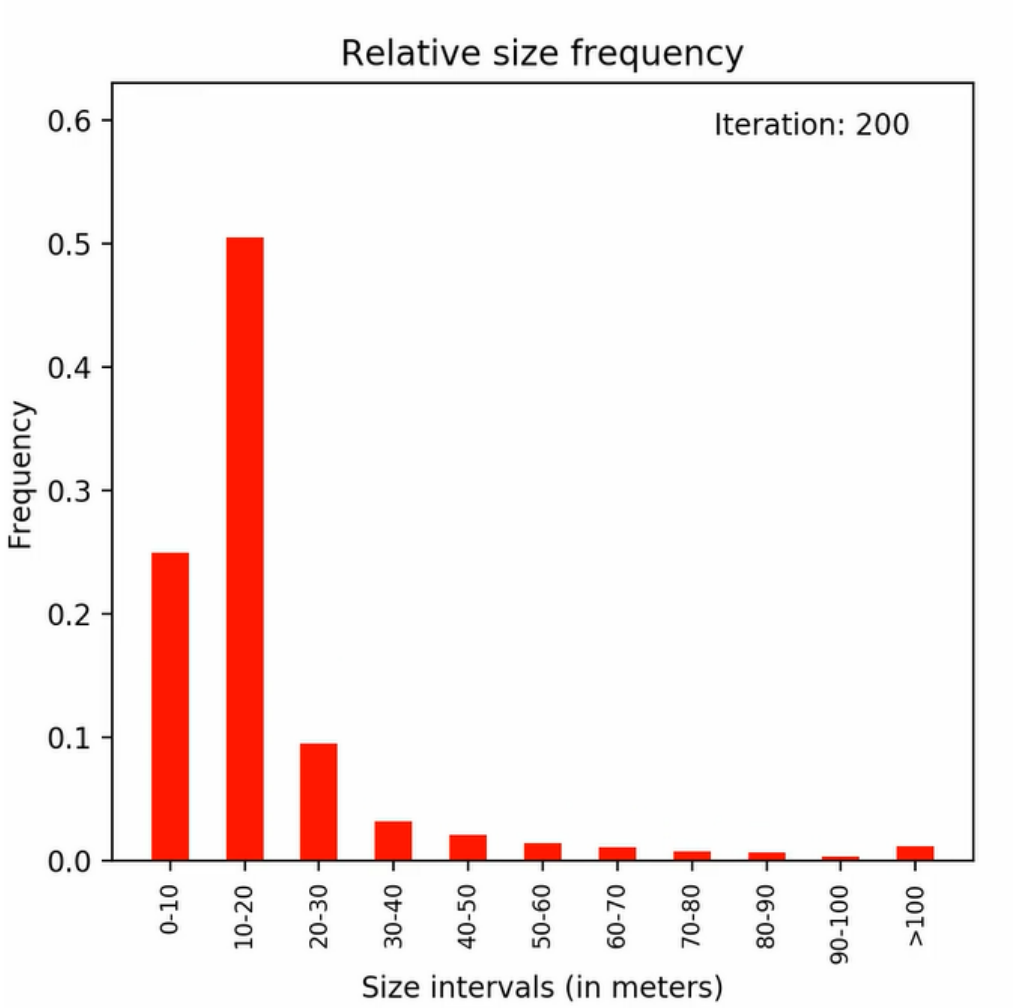
Results

Simulated forest



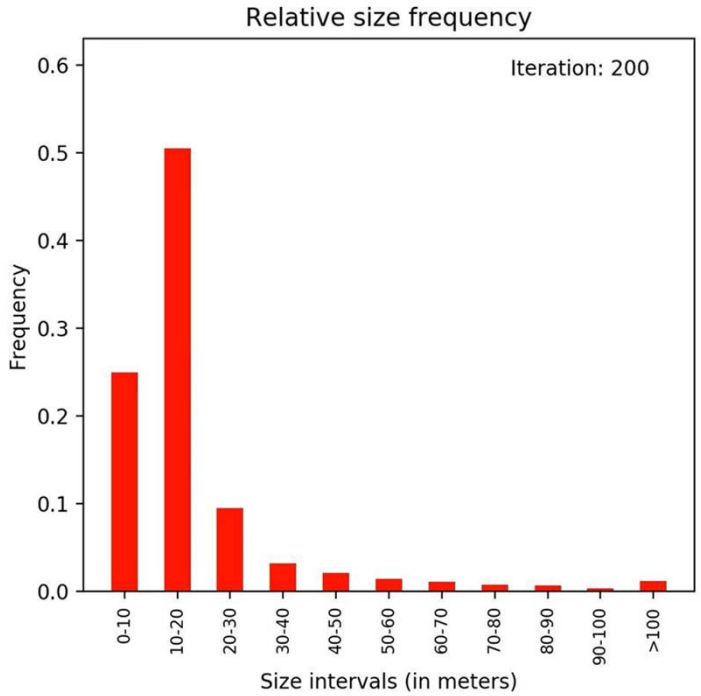
Results

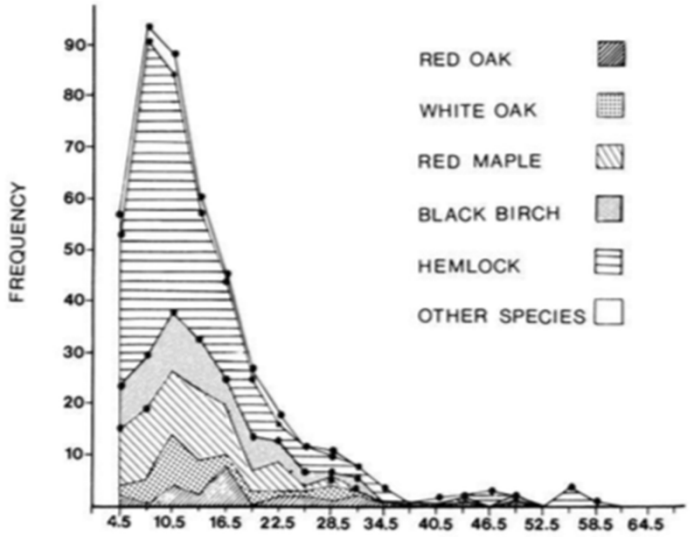
Simulated height distribution in our forest



Results

Simulated height distribution in our forest





Real diameter distribution in a real forest

Future research

- Simulation with more than one species
- Study deeply the relation between diameter and height
- More accurate model:
  - non-opaque leaves
  - variable shadows
  - variable sunlight over time
  - periods with no light: night
  - ...
- Size and diameter tree simulation