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Fighting for light

Finding a Stable Tree Height Distribution

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″Life finds a way″

- *Ian Malcolm*

Abstract

In this project, we will outline the main characteristics that shape the behaviour of a forest, considering the context of trees fighting for the access to light. In an effort to focus on the struggle to be taller, to achieve more light than the others, we will balance this advantage with a great dependence on high quality soil and its nutrients coupled with a penalising factor for trees that are too large and too dislocated. We will show how this balance can be explained with mathematical equations implemented into computational simulations.

With these tools, we will try to identify some stable height distribution of the trees that lets future generations of trees that thrive in the same way, reproduce homogeneously and ideally, resembles any known distribution of trees of a real forest.

Despite the simulation´s focus on performing the usage of a single tree species, the construction of the model will be oriented. It will also try to guarantee the generalisability of its usage and so the conclusions, leading to the possibility of further research in the topic using different species, environment or scenarios.

Contents

1 Introduction

In our society and worldwide, many studies and research have marveled at the complexity of a multifarious ecosystem called *The Forest*. This complex ecosystem consists of mainly trees that safeguard the earth and support all forms of life. Plants and Trees, in this case, help to prived a unique environment that affects all kinds of life in the forest. They deliver unpolluted air, reduce the temperature on sizzling days, accumulate heat during the night and act as an excellent sound absorption from all forms of noise. In the forest, there is another unique, powerful and silent event happening among all plants and trees, that shapes the equilibrium among them, called *The War for Sunlight*.

Sunlight, to a plant or tree, is the primary source of energy that it needs to survive. Sunlight is used to manufacture foods from Carbon Dioxide and Water in a process called *Photosynthesis*. Photosynthesis is a process that occurs within the green pigment of chlorophyll of all plants, producing oxygen as a byproduct and converts sunlight into chemical energy, using it a source of energy for their survival.

In *The War for Sunlight*, there are several limiting factors that affect plants and trees within forest; as they develop their strategy in the competition for light. Any change to one limiting factor may change the rate of photosynthesis and thus the fate of that plant or tree. When *Fighting for Light*, the level of light is not necessarily the only limiting factor that determines the fate of a plant or tree.

In this project, we will study how *The War for Sunlight* shapes the relationship between the individuals within a forest, their environment and how the struggle for survival determines the equilibrium of the whole ecosystem.

1.1 Objective

Throughout this project, we aim to gain a better general understanding of the dynamics between trees in the struggle for sunlight and how they interact with their environment. This initial setting will be used to find a stable height distribution of trees that can be related to actual forests or to well-defined known distributions. In order to do so, we will perform a computational simulation of a forest, following the behaviour shaped by the rules that we will develop in the following sections.

This process will be organised as following:

- Firstly, we will outline the main characteristics that explains the general behaviour of a tree in a forest, for that we will sketch some general rules of the behaviour of a tree within a forest and its interaction with the environment.
- Nextly, this scheme of rules will be translated in a set of theoretical formulas that will aim to explain the same behaviour in a more concise - but mathematical - way. This step is crucial, for it may be used for some theoretical prediction whilst setting the conditions for any numerical simulation to follow.
- Finally, once we have a clear mathematical scheme, we will construct a simulated forest that will follow the rules, as defined in the formulas settled before.

Above all, the results of the simulation will be used to find a height distribution for a given species of trees that guarantees the equilibrium of the forest.

Hopefully, the results of this project may lead to a better understanding of how growth and death dynamics of a forest can be explained by the battle for light that is present in every plant-based ecosystem.

2 Growth and Death Dynamics of Trees within a Forest

When studying the nature and behaviour of a whole forest, the first and most simple approach is to understand the individual behaviour of every tree.

The first and most simple approach in order to understand the pattern of growth and death of a tree in a forest is the amount of sunlight it can get. One of the most critical consequences of coexisting with other plants in a forest is that some of them may impede access to light to other plants if they are taller. This way, only the taller ones get the necessary amount of light to keep growing, exacerbating this situation. The logic outcome of this approach is that a forest would tend to be extremely heterogeneous, with some trees growing higher and higher and plenty of others just shrinking to death. We know [1] that this extreme situation is not plausible for the sustainability of any forest and does not explain the usual height distribution of forests; a good representative example of such distribution can be seen in figure 1.

In order to better understand the dynamics of forests, we need to consider some other forces of nature, in addition to the access of sunlight. In the following chapter, we will outline the most relevant constraints and interdependences that shape the growth and death of trees in a forest.

The next step of the project is going to be the construction of a mathematical model, which should be cultivated by the insight extracted from this section. In order to ease this transition from qualitative understanding to mathematical specificity, we will summarise all dependencies in two sets of *rules*. So, these rules aim to set the bases for a proper mathematical model and to smooth out the transition between mathematical modelling and computing simulations.

In order to set this section as general as possible, the assumptions conclude from these rules will be settled with no regard of the tree species. This can limit the specificity of the rules and cap its accuracy, but it will ease the replication and tweaking of this model, and so its usage for different trees, environments and circumstances.

2.1 Growth Rules

The first set of rules are those related to *Growth*. These rules outline the growth pattern of trees and its dependence with the environment.

2.1.1 The quality of the soil affects the tree growth rate g.

Since the soil is a storage place for minerals, nitrogen and - most importantly - water, we have a high dependency and a very close relationship between tree size and the soil; in which they grow. The tree roots also serve to fix a tree on a place, just like an anchor [2].

What forms soil *quality*?

- **Climate** in that particular region. Temperature, humidity, wind, hours of sunlight. All these variables can determine the quality of soil.
- **Humidity**. Frequent rain in a region and the ability to retain water in the soil will determine the humidity of the soil.
- **Nutrients**. Formations of many different chemical compounds and the nutrients of soil can be altered by the influence of **animals** and *trees*. Also, more importantly, the presence of enough and valuable nutrients is a crucial variable for the quality of the soil.

The root tips of a tree push into the soil to give birth to many-branching root system and later the root hairs to absorb a big amount of water from the soil. The absorbed water already contains the minerals and nitrogen, which the tree needs every day in order to survive and keep growing [3].

All in all, we can say that the higher the quality of the soil, the stronger positive influence it has on the tree growth [4].

2.1.2 The tree modifies the quality of the soil surround it.

Trees improve our quality of life, provide beauty in landscape, gives us shade from the sunlight, gives us oxygen and bring animals to the surrounding area of the tree. They can produce fruit, flowers and other substances that are good for the environment. Its roots extend below the ground in the soil, which can modify the quality of the surrounding soil. Here we list some major negatively and positively modifications [5] [6].

- **Soil Structure:** Trees can improve soil structure by increasing fertility and prevent soil erosion. This leads to improving soil aeration and drainage.
- **Soil Nutrients:** In order to growth and build up all the structure of a tree, a huge amount of nutrients is needed. Trees' roots extract nutrients from the soil and transport it to the whole plant. When the tree dies, its corpse acts as substrate to more organisms to thrive and to provide nutrients to the soil.
- **Soil Erosion:** Trees with deep and strong roots help to prevent soil erosion from heavy rain and flooding systems. When a tree dies or is cut down, it roots still are in the soil preventing soil erosion.
- **Soil Toxicity:** Some trees like black walnut trees and butternut trees releases a substance called 'juglone' that makes the surrounding soil toxic for many other plants and trees. These trees provides a toxic ring around the tree and grow bigger as the tree get older.

We can say that a community of trees helps to the environment and provide the forest with security, humidity and many healthy circumstances that makes the tree and the whole ecosystem to thrive. Despite that, we must not forget that the growth of a tree needs a pump of nutrients out of the soil, and if this mechanism is not counterbalanced within a stabilised environment, it can lead to the destruction of the forest [7].

2.1.3 The amount of light modifies the growth rate g.

Different trees are able to live in a many light environments, and some species are even optimised to live in very unusual light conditions. Nonetheless, no species of plants is able to live in complete dark; they all need some amount of light in order to survive and thrive.

During the day, trees are receiving light from the sun, and at night they enjoy it from the moon and the stars, so basically tree is never completely disconnected from the light. Trees are evolving with a predictable transition from day to night. It is interesting, how the trees are able to measure light and shadow, which helps them to get information about the day length and distance to other trees. In order words, every species of tree in the forest adapts to its environment according to a particular light conditions [8].

With sudden change in the amount of light, trees can begin to either lose their leaves, grow strange branches or even slowly dying.

2.1.4 The age of the tree modifies the growth rate g.

As in most living individuals, the growth rate of a tree may vary through its lifetime. In particular, the growth of a tree is mainly calculated using *Tree Annual Radial Growth* (TARG), which is a method that measures trees' ring widths [9].

The measure of ring widths as an estimation of growth leads to a slightly simplistic, yet quite accurate approximation, which is the assumption of some equivalence between the diameter of an individual of a given species and its height. This relationship could be defined as

$$H = \alpha D \tag{1}$$

.

Where H is the height of a tree, D its diameter, and α is a parameter that changes from species to species.

As shown in figure 1, this metric (ring's width) has been used to outline the height distribution of a forest [1].

Using the very same metric, studies has been made where the yearly change in the ring widths has been measured, for different species of trees. And following analogous criteria, the total area covered (Basal Area) by that species is extrapolated [10].

In figure 2 we can see the relationship between this two metrics.

The point of all this measures, is that allows us, by measuring the development of a forest, to infer the growth rate of a tree species. If we understand the relationship between Ring Width and Height, and we comprehend how Ring Width is related to the Basal Area of that tree in a forest, then we are able to deduct the function that defines the time

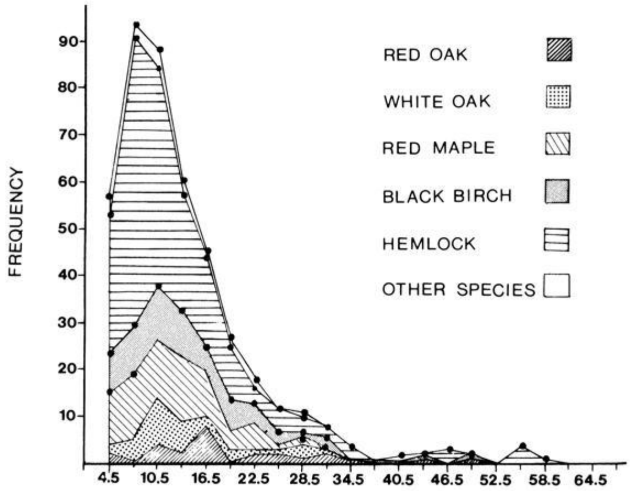

Figure 1: Distribution of all trees of a forest by diameter and species [1]

evolution of the growth rate of a tree, based on the observation of the Basal Area of that trees in a given forest. As shown in figure 3.

Nevertheless, this logic have some issues that need to be addressed. This logic and relationship are constructed under the assumption that the forest does not experiences major disturbances, and thus the growth of a forest is an extrapolation of the growth of its trees, where an ecological equilibrium is already established and undisrupted through the lifetime of its trees. An observation of the effect of this disturbances can be shown within [1], in its Figure 5.

Surprisingly enough, based on the observations of [10], shown in 2, we can conclude that the growth of a tree can be reasonably simplified as steady, without change through the lifetime of a tree. This leads us to dismiss the *Age* of a tree as a variable to explain its growth rate g, and consider it as a constant or fixed parameter instead.

2.2 Death Rules

The second set of rules are those related to *Death*. The life of a tree can be ended as a consequence of multiple reasons and situations, and so these following rules try to abridge the patterns that can lead to the death of a tree.

2.2.1 The life expectancy of the tree.

The first question to be addressed when studying the lifetime of a tree is the simplest one: How long does a tree live? Much like animals, the average age of trees depends strongly on its species. If a tree has enough water, food and sunlight throughout its life, then we can fairly assume that it will live until the end of its natural lifespan. That said, no amount of care can make an elm live as long as a sequoia. Some of the shorter-lived

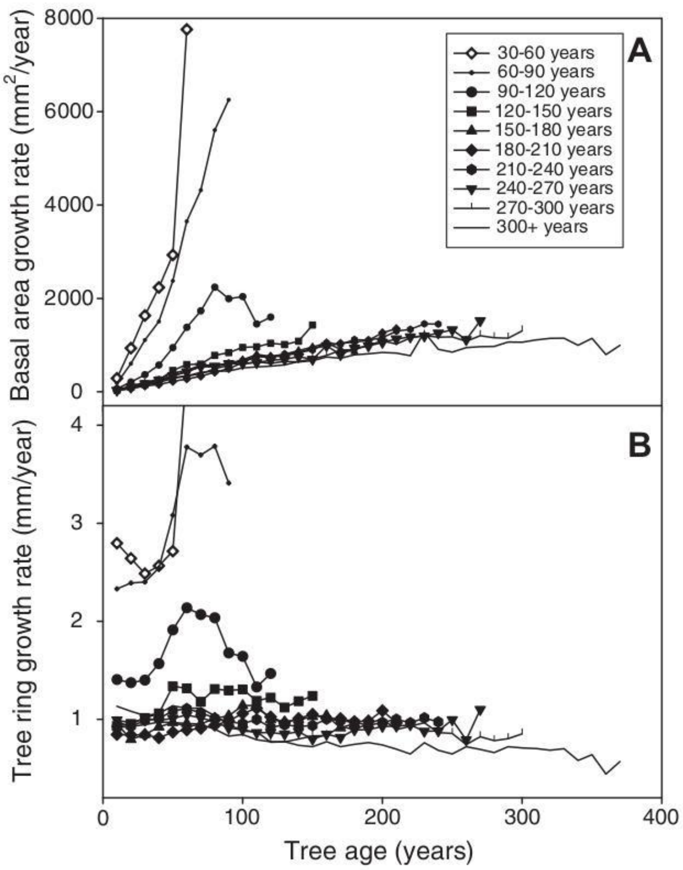

Figure 2: Decadal average growth rates in BAI (A), tree ring growth rate (B) [10].

trees include palms, which can live for around 50 years. The persimmon has an average lifespan of 60 years, and the black willow will probably survive for around 75 years. On the other hand, Alaska red cedar can live up to 3500 years; giant sequoias can last over 3000 years; and at least one Bristlecone pine is estimated to be almost 5000 years old [11] [12].

It is rare that a tree will get to live out its full, natural life span. Just like humans, a disease can kill a tree; the photosynthesis process can be impeded by an insect infestation or some icy storm; some fungal infection can rot away the wood; or the tree can be strucked down by lightening. Many trees are, of course, cut down by humans before they can live out their days.

In fact, there are many factors contributing to the length of the life of tree species:

- **Climate:** You can see that trees that have a reputation of becoming really old live in environments that have low moisture levels and much sunlight over the course of the year. For example, you can see that the most long-lived trees in America are located in California, where the temperatures are relatively high, while moisture levels are low. If you have one without the other trees don't tend to live that long.

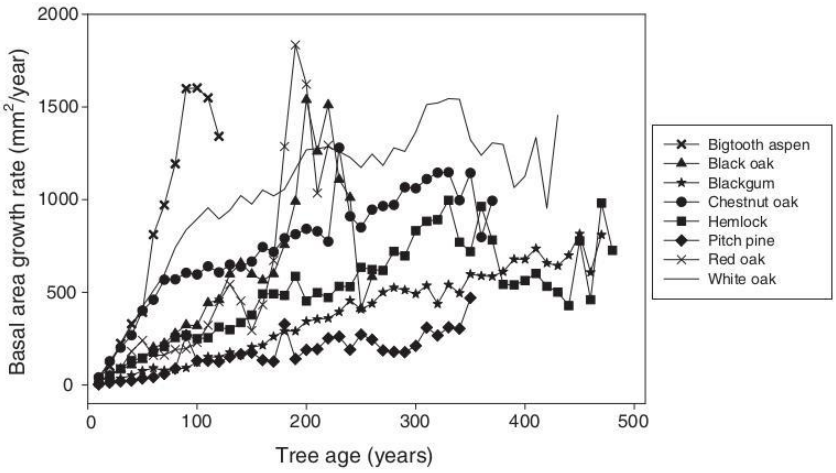

Figure 3: Average Basal Area trends of each studied species [10].

Cold climates make growth harder for each individual, yet hot and moist climates tend to help trees grow easier, but also die easier. Think of the amazon. It is one of the richest forests in the world, yet the growth of bacteria and high competition in the areas of the equator make life a lot harder for the longest living and slowest growing species. As it can be noticed, the vast majority of the longest-living trees are located in California, or other regions of the same latitude.

- **Other forms of life around its Ecosystem:** Individual trees tend to be less likely to be destroyed by a fire, as they are less likely to catch fire. At some instances, forest trees or dried-out grass can catch fire and pass it on to other individuals. Microbes and bugs found in some parts of the world are known to their abilities to destroy trees.

- **Size of the tree:** Eager trees are generally less likely to die of causes like drought or damage to their trunks, as they have greatly extensive nets of roots and thick trunks and thus are able to, even partly, recover these incidences. If a rockfall for example causes damage to one side of the tree, some of the brunches will probably die, but the rest of them, that have their own network to the roots, will probably survive.

  This means that luck is involved and if the tree is lucky enough to avoid dying for a certain amount of time, regular fluctuations in the environment do not kill it as they would kill a younger, less robust tree.

- **Distillation:** The way that the tree is adapted to its environment determines its longevity in it. Planting an olive tree in the amazon is a great way of making sure it will not survive. Some trees are better at surviving in a certain climate (even if it is at your own garden) simply because they have evolved to live there. That means that the trees probably don't have a defined lifetime in general, but the life expectancy is defined in each environment, depending on the circumstances.

2.2.2 Soil poor in nutrients affects the life expectancy of a tree.

The dependence of a tree on the quality of the soil follows a relationship that can change dramatically from one species to another. Despite that, there are some assumptions that can be considered as general rules for every species of tree. Many different factors determine how the characteristics of the soil influences the growth of a tree and these factors can vary greatly from one species to another. Despite that, is reasonable to assume that - for every kind of species - the more nutrients the better for the tree [5]. For the sake of simplicity, we can condense all kind of factors that determine the *quality* of the soil and construct a general variable of quality that contains the information of amount of different nutrients, humidity or other forms of life.

Henceforth, we can assume that the growth and size of a given tree can be limited by a lack of this *quality* of the soil, among other things.

2.2.3 The lack or excess of sunlight modifies the tree life expectancy.

Light availability is a key factor in forest dynamics as interspecific differences in shade tolerance underpin species coexistence and species succession in forests. Light is often the limiting factor in tree survival or growth in both northern hemisphere hardwood forests and tropical rainforests - tree individuals exhibit plasticity in terms of tree architecture to utilise light resources and allocate then into growth and survival.

Plasticity of canopy architecture for maximising light resources may also be seen as asymmetric competition for light and can explain the evolution of height. Apart from plasticity, competition for light affects tree demography in terms of both mortality and recruitment.

The role of light is further emphasised by findings reporting that under low light, small differences in tree growth at early life stages result in variation in rates of mortality of at least two levels of magnitude between species. The effect of light availability across tree life stages is not homogeneous; species with low mortality in the absence of light at early life stages may have slow growth rates at intermediate or late life stages [13].

3 Mathematical Modelling

Once we have settled the general rules that shape the behaviour of trees within a forest, we will translate them into mathematical formulas.

An assumption that can be reasonable extracted from the previous chapter is that the main behaviour of a forest can be explained by the *Size* of every tree and the *Soil Quality* surround them. Hence, the following section will be focused on the interdependence of these two variables.

3.1 Tree Size

In the fight for light, one characteristic that determines the success or failure of every tree is its size. Thus, the crucial variable that this project is concern of is the *size* of each tree. In order to properly simulate the evolution of this variable through the lifetime of a tree, we will assume that it changes from one step of time to another, following the formula below:

$$S_t = g + S_{t-1} \tag{2}$$

Where S_t is the size of a tree at a given time t and g is a *growth* parameter that will determine how much a tree grows in a step of time.

Once we have this equation, the main question that arises from it is regarding the definition of g. Following the logic and conclusions extracted from our previous section, where *Growth Rules* and *Death Rules* were established, this growth function is defined as following:

$$g = P\left(\frac{1 + R_Q + R_L}{3} + \frac{1}{2}\frac{(\Delta h)^2 - (\Delta h)^3}{(\Delta h)^2 + (\Delta h)^3}\right) \tag{3}$$

Where

- P is a constant parameter specific for every tree species.
- Q is a variable that defines the *quality* of the soil where that tree is placed. And R_Q is a function that explains how a given tree species grows in that type of soil.

$$R_Q = \alpha(Q - d) \tag{4}$$

  Where α is a parameter, specific for each species and d is a parameter that reflects the aggressiveness of a low quality soil. This way, the higher the quality of the soil, the greater the growth of the tree.

- L is a measure of the "amount" of light, and R_L is a function, defined between -1 and 1 that explains how a given tree species grows with that quantity of light.

$$R_L = 1 - L^{\beta} \tag{5}$$

  Where β is a parameter, specific for each species. This way, as stated within the previous chapter, every tree needs an specific amount of light and its growth is penalised if there is either too much light or not enough. The spectrum of acceptable of light is determined by every species' light sensitivity, expressed by the β.

- Δh is a measure of the height difference between the taller neighbour and the tree.

This way, the growth of a tree benefits from a nutrient rich soil and has a crucial dependence on the amount of light. Moreover, all of this can be affected by the height differences between that tree and its neighbours. The point of the cubic relationship between heights is that, as it is explained in previous sections, the growth of a tree can be benefited from little variations of height, since it can use help of bigger surrounding trees as subjection and support for its own growth, but too high neighbours can shade the tree and greatly threaten its access to light and maybe even other resources.

3.2 Soil Quality

Another essential variable that depicts the behaviour of our forest is the effect that the trees impose on the soil. It is known that the growth of plants has a remarkable effect in the components of the soil [6, 4], that monotonous plantations cause an excess of this effect and it is usually pervasive for plant's growth [14], and that this effect is more severe the larger the plants are and the faster they grow [4].

Thus, in order to properly simulate the evolution a variable that brings some general sense of the *Quality* of the soil, a formula that sets this relationship between size and growth of trees and soil quality is defined as following:

$$Q = \left(1 - \frac{S}{k_{size}} - \frac{g}{k_{growth}}\right) Q_{t-1} \tag{6}$$

Where k_{size} and k_{growth} are constants to control the magnitude of the effect of their variables.

4 Simulation

The aim of this project is to simulate "computationally" a forest, analyse its behaviour and studying its statistical properties. To obtain a simulation, first of all we have to determine what kind of mathematical object we want simulate and what property will evolve in 'time'. In our case, the forest will be a $N \times N$ matrix that will contain a single tree in each of its positions. So, our $N \times N$ matrix will contain at each position the height of the tree in such place.

In mathematical modelling is always recommended to keep things as simple as possible. So, we defined a few properties in order to describe each tree species, as life expectancy, maximum height, growth rate, light tolerance, etc. Each tree species has its own values for these properties.

Moreover, as we have stated in previous sections, not only the trees need to be simulated in order to properly simulate a forest, but its soil too. For the state of the soil is an important part in the ecosystem balance of a forest, and it greatly affects the success of each tree.

In the simulation code, these extra fundamental properties are encoded in constants which can be found in the functions that carry the evolution of our forest. This functions will simulate the behaviour of each tree and of the forest as a whole.

Despite the fact that we are calling them functions, they are not necessarily mathematical functions. Another words, they are used to describe the statements that shape the behaviour of the elements of the model called '*rules*'. Each rule can be the function of the size of a tree or the function of the soil quality, but can also be a condition that allows a tree to born or a probability of an individual to die for no apparent reason.

4.1 Parameters Definition

Since this project aimed to develop a mathematical model that could be run for any species or group of species, all formulae data and hypothesis were done without considering any species of tree. However, in order to run the model and obtain some results, we shall now choose a tree species and set all parameters according to its particular characteristics, as an example.

Therefore, the chosen tree to run the simulations is the Holm Oak (*Quercus ilex*). The election of this species is because it the most common specie in Spain, and it pertains to the *white oak* section of the genus. This should make our results quite significant, since this kind of oaks is one of the most common species of trees in North America [15].

Properties of the tree:

- Lifespan of 400 years
- Between 300 and 600 litres of water per year
- Can grow in most type of soils

- Has an average height of 20m, reaching a maximum of 40m.
- Slow growth but at a constant rate, reaching 10cm per year at optimal conditions.
- Acorns germinate very easily

After knowing which tree species will simulate and how it behaves, the actual code can be explained.

4.2 Code

In this section, a presentation of an explanation of the *python* code used to simulate the forest. The code is structured in different sections (called *blocks*) that will now be developed.

Block 1

Package declaration and initialisation, where the packages to be used are loaded to python interpreter.

```
import matplotlib.pyplot as plt
import matplotlib.figure as fig
from matplotlib.animation import FuncAnimation
import numpy as np
import random

random.seed(None)
```

`Matplotlib` is a library which provides tools to plot 2D static and animated figures. `Random` is a pseudorandom number generator with functions providing uniform distributions, random integers, etc. `Numpy` is the fundamental package for scientific computing with Python.

Block 2

Function declaration, where functions to be used are defined.

```
def dh1(x ,ii ,jj ):
    temp=x[ii][jj]
    i=ii
    j=jj
    sp=(ii+1)%ntrees
    sm=(ii-1)%ntrees
    tp=(jj+1)%ntrees
    tm=(jj-1)%ntrees
    if temp < x[sp][j]:
        temp=x[sp][j]
    if temp < x[sm][j]:
        temp=x[sm][j]
```

```
    if temp < x[i][tp]:
        temp=x[i][tp]
    if temp < x[i][tm]:
        temp=x[i][tm]
    if temp < x[sp][tp]:
        temp=x[sp][tp]
    if temp < x[sm][tm]:
        temp= x[sm][tm]
    if temp < x[sm][tp]:
        temp= x[sm][tp]
    if temp < x[sp][tm]:
        temp= x[sp][tm]
    return ((x[i][j]-temp)**2-(temp-x[i][j])**3) / ((x[i][j]-temp)**2+((temp-x[i][j]))**3+1)


def distribution (x,n_dif_sizes,ntrees,max_size):
    y=np.linspace(0.0,max_size,num=n_dif_sizes+1)
    z=np.zeros(n_dif_sizes+1)
    for i in range(ntrees):
        for j in range (ntrees):
            for s in range (n_dif_sizes):
                    if (x[i][j]>y[s])and(x[i][j]<y[s+1]):
                        z[s]=z[s]+1
            if x[i][j]>max_size:
                z[n_dif_sizes]=z[n_dif_sizes]+1
           # if x[i][j]==0:
               # z[0]=z[0]+1
    return z/(ntrees**2)

def lifetime (age):

     return 1/age


def newborn():

    return 0.4

Rl=lambda l,beta: 1;#1-pow(l,beta);

Rq=lambda q, alpha: (q-0.1)*alpha;

def g_ij (x,p,q,l,apha,beta,i,j):

    return p*((1+( Rl(l,beta)+Rq(q,alpha) ))/3+dh1(x,i,j)*1/2)
```

First of all, the location of the `dh1` function, which essentially computes and returns a function, depends on the size differences between a tree in a determined position (`x[i][j]`) and its highest neighbour (`temp`).
The `distribution` function is responsible of computing the distribution frequency function of the tree size and it works comparing the value of each position of the matrix

x, which will be the `forest` matrix presented in the next block, with a pair of two consecutive values of the vector `y` to determine the size interval in which it belongs. Why do someone need an interval to represent the size? Well! Each tree has a size and this is a float-type number, a decimal one, if the purpose is to be able to represent EACH position, then it will need a vector of size 10 to the power of the number of decimals plus the maximum integer value that the python interpreter is capable to store : a really big vector. It is a matter of precision, the maximum it can achieve is the proposed previously notation but, it is not useful at all, there is a need for more practical approach. For example, as in above case, it is noted a precision of 10 meters. In other words, trees with size in the range (0,10) will belong to the same size interval. `Max_size` and `n_dif_sizes` will determine how wide this interval will be: if the purpose is to achieve a very high precision, an increase the `n_dif_sizes` parameter is needed. Again, it is a matter of precision tolerance.

Block 3

Initialisation block, where the parameters of the rules and all the multidimensional arrays (or matrices) in which is codified our forest are declared and initialised.

```
#General tree parameters
max_age=10
max_size=100
min_size=1
max_warning=3
life_time=57

#Model parameters
l=0.5
alpha=3
beta=0.4
p=1
soil_coef=10
forest_coef=100

#Simulation parameters
ntrees=100
iterations=201 #lifetime of our forest
dif_sizes=10 #how many differents sizes we accept
dead_counter=np.zeros(iterations+2)
newborn_counter=np.zeros(iterations+2)

#arrays initiallization
warnings=np.zeros((ntrees,ntrees))

forest=np.random.rand(ntrees,ntrees)*min_size
nforest=forest
total_forest=np.zeros((iterations+1,ntrees,ntrees))

soil=np.random.rand(ntrees,ntrees)
```

```
nsoil=np.random.rand(ntrees,ntrees)
total_soil=np.zeros((iterations+1,ntrees,ntrees))

age=np.zeros((ntrees,ntrees))
total_dist=[]
```

In this section, an initialisation of all the parameters of the simulation and their initial value were established. As seen in the code, it is divided in three blocks, each one of them with parameters of the same kind and a fourth one with the array initialisation. `forest` and `nforest` store the tree size: `forest` before the iteration and `nforest` after it, but this trick will be explained later. `soil` and `nsoil` work as the matrix before but with the soil quality. `total_forest` and `total_soil` are vectors of matrices or 3D matrices and store `forest` and `soil` matrices respectively at each iteration. This is because due to the need to perform an animated figure, a static part could remove them. Also, `age` is simply a counter to each tree to know its age, in-which there are no interactions between ages; as an independent parameter. Finally, `total_dist` works as the `total_` vectors but for the distribution frequency function.

Block 4

Main block (or loop block), within which all the computations are made, that is the actual simulation of the forest, can be consider the 'core' of our code.

```
total_forest[0]=forest
total_soil[0]=soil
total_dist.append(distribution(forest,dif_sizes,ntrees,max_size))

for z in range (iterations):
    for i in range(ntrees):
        for j in range(ntrees):
            if forest[i][j]==0:
                if random.uniform(0.0,1.0)< newborn():
                    nforest[i][j]=min_size
                    newborn_counter[z]=newborn_counter[z]+1

            temp=g_ij(forest,p,soil[i][j],l,alpha,beta,i,j)
            if temp<0:
                warnings[i][j]=warnings[i][j]+1
                temp=0
                temp=temp+random.uniform(0.01,0.1)
            if forest[i][j]!=0:
                nforest[i][j]=forest[i][j]+temp*p
                nsoil[i][j]=(1-temp/soil_coef-forest[i][j]/forest_coef)*soil[i][j]

            if warnings[i][j]==max_warning:
                nforest[i][j]=0
                age[i][j]=0
                nsoil[i][j]=1
                dead_counter[z]=dead_counter[z]+1
```

```

            if nforest[i][j]>0:
                age[i][j]=age[i][j]+1

            if random.uniform(0.0,1.0)< lifetime(life_time):
                nforest[i][j]=0
                age[i][j]=0
                nsoil[i][j]=1 #maxima calidad
                if z!=0:
                    dead_counter[z]=dead_counter[z]+1
    dead_counter[z+1]=dead_counter[z]
    newborn_counter[z+1]=newborn_counter[z]
    forest=nforest
    soil=nsoil
    total_forest[z+1]=forest
    total_soil[z+1]=soil
    total_dist.append(distribution(forest,dif_sizes,ntrees,max_size))
```

The very first three lines copy the initial configuration of `forest`,`soil` and distribution frequency function to the its `total_` vectors respectively. They could be added to the iteration loop but that will complicate unnecessarily the code, so its kept without them. Then the iteration loop was found, which corresponds to the time flowing, in-which each iterations is equal to 7 real years. Next, there are `i` and `j` index which go over the `forest` and `soil` matrices to compute their new values.

Once the computation enters the `j` loop, it finds first the rule to the newborns. For each position with size equal 0 there is a probability of `newborn()`; that is to becomes a tree with size equal to `min_size`. In case of a newborn, a '1' was added to the `newborn_counter`. At the beginning, the function `newborn()` was intended to carry a computation considering amount of nearby trees and their distance, but in such small scenarios as carrying on, such computations would lead redundant results, making no difference in the final result whatsoever. Taking this into account, and the fact that the tree species that are considered has a great capability spreading seeds and that those seeds have a humongous success if they have the conditions, it is decided to simplify the code and the model and just set a constant probability of spore at every iteration of 0.4.

If one keep following the code, it will be confirmed that the `temp` function which is basically the $g_{i,j}$ function or rule that controls the growth of every `i,j` tree.

Four lines below there is the *warning* rule: This rule tries to implement some of the consequences of the *Death Rules*, in which a lack of necessary conditions could lead to an early death. Since the conditions that set whether a tree dies or survives are very similar to those who set how much it grows, this rule just checks if `temp` (g) is negative and if it is, adds a warning to this position and set `temp` to random value between 0.01 and 0.1. This functionality prevents the tree to stop growing nor left to die immediately after every slight misadventure. The point is that, after a fixed amount of uninterrupted warnings, the tree dies.

The next `if` statement updates the size of each tree and the soil quality with the temp function while the previous size is different from 0. Next tree size generation is store in a different array because if it isn't like this the present generation will be affected by the next generation and it would have no sense, it needs a completely new matrix which after the iteration will be the new `forest` matrix.

Again finding the *warning*, but in this case just to confirm, an establishment of the `max_warning` is reached, the tree dies and its size is set 0, as its age, and the quality of the soil becomes its maximum value, 1. After, an update to the `dead_counter` value.

Near to the final `j` loop, there is the tree age updating, only if the tree size is greater than one, or in other words, if there is a tree.

At the end of the loop, it is the lifetime checking. In fact, the lifetime is the mean value of the life time, so in our interpretation each tree has a probability of dying equal to the inverse of `lifetime` at every year. So, it is not an unattainable maximum, instead it is a probability, this way is possible to find trees some older than the `lifetime` value; as we should expect in reality.

Out of matrix loop, in the iteration one, it is found the part which stores the matrices in the `total_` vectors necessary to visualise the simulation.

Block 5

And finally the visualisation block which takes care of the translation of our number-base data to a graphical one that is easier to analyse and interpret.

```
fig2 = plt.figure(figsize=(5,5), dpi=100 )


plt.title('Relative size frequency')
plt.xlabel('Size intervals (in meters)')
plt.ylabel('Frequency')
x=np.linspace(0,1,num=dif_sizes+1)
y=np.linspace(0,0.6,num=dif_sizes+1)

hist=plt.bar(x,y, color='red', align='center' , width=0.05, bottom=0)

xdiv=np.linspace(0.0,max_size, num=dif_sizes+1)
for i in range(dif_sizes+1):
    xdiv[i]=int(xdiv[i])
x2labels=[]
for i in range (dif_sizes):
    temp=str(int(xdiv[i]))+'-'+str(int(xdiv[i+1]))
    x2labels.append(temp)
x2labels.append('>100')
plt.xticks(x,x2labels, rotation='vertical',fontsize=8)

time_count=fig.text(0.7,0.85,'')
plt.subplots_adjust(top=0.90, bottom=0.15, left=0.12, right=0.95, hspace=0.25,
                    wspace=0.35)

```

```
def init ():
    for i,b in enumerate(hist):
        b.set_height([])
    time_count.set_text('')

def animation(s):
    time_count.set_text('Iteration: %d'%s)

    for i,b in enumerate (hist):
        b.set_height(total_dist[s][i])#s es la posicion en la lista, i es la posicion en el vector



anim = FuncAnimation(fig2,animation,init_func=init, interval=75, frames=iterations+1)

#anim.save('/Users/cristian/Desktop/WMM/project/forest_simulation/forest_evolution.mp4', fps=15, extra_args=['-vcodec', 'libx264'])
```

Finally, this block carries the visualisation part as an animated simulation. The simulation can be performed in real time: the loop is computed, the matrices will be updated and visualised through the `matplotlib.animate` package. An important drawback to be considered is that the simulation is extremely slow. In order to see a fluid simulation, it is better to compute all the iteration first and then perform its visualisation; and this way it is also easier to store it.

First the figure-type object must be defined, then set the title, x-axis label and y-axis label. Because of how `marplotlib` works it need a layer of the simulation for that it have to set how many x and y ticks will be and their separation. This ticks are created by the `np.linspace ([start, stop), step)` function, with `step` the number of steps between `start` and `stop` float-type (or integer if wished) numbers. Until now, the title, x-label, y-label and its axis ticks all are established with a need for the bar graphic itself. It is then provided by the `matplotlib` library as `bar(...)` function. Following the bar graphic initialization the label was found for each x-axis ticks, because they are not simply numbers but intervals.

Finally, the simulation: the function `FuncAnimation(...)` requires the initial layer, defined as `def init ()` function and the function that will depend on time, `def animation (s)`. The `s` variable grows in steps of 1 and uses it to go over the `total_func` vector which, remembering, stores successively the forest distribution frequency function; hence, each frame of all different positions were visualised from the `total_func`.

The `anim.save` function saved the simulation as a .mp4 file.

Code improvements

In pursuit of simplicity, our code only simulates one tree species at the same time. But it would not be too difficult to modify in order to simulate more than one species. The best way would be introducing a ‘tree class variable’ which contains characteristics as: high, diameter (in case one wants to reproduce faithfully a forest), age, and species; a ‘species class variable’ which will collect the specific properties of each species. This last class will set the species of each tree and can be created initially by the user.

5 Results & Discussion

The entire purpose of this project has been, within the context of fighting for light, to find a stable trees height distribution that leads all kind of trees to grow, develop, reproduce and form some kind of equilibrium that populates the entire forest and lets future generations to develop in the same way.

A problem that may arise in survival simulation is that the individuals do not scatter homogeneously, forming some kind of clusters of 'big guys' surrounded by a desert of smaller individuals doomed to death. This is not a stable geographical distribution, since it does not let future generations to thrive in the same way as their parents, and it discards some parts of fertile terrain without population. So, the foremost result to check is the geographical distribution of the individuals of the simulated forest.

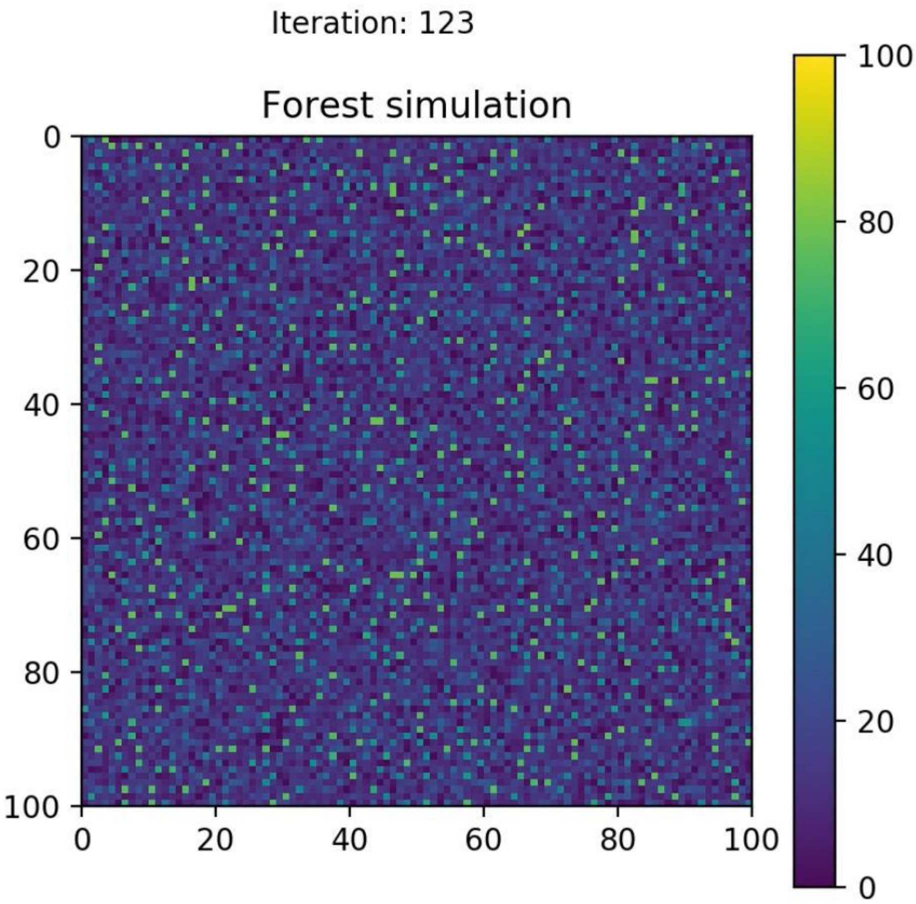

Figure 4: Screencapture of the video showing the time evolution of the simulated forest. The colour refers to the height of each tree.

In figure 4 it can be seen how evenly the distributed individuals of the forest are located. A result showing such homogenous distribution of the individuals (*trees*) is quite satisfactory, since it lets future generations to thrive and develop.

Once all checks have been made to the simulated conditions, it is easy to see that a great achievement has been create with the most interesting result for this project '*establishing the height distribution of the forest*'. As for now, it has been seen that the geographical distribution of the trees of difference heights, but the main question is about the frequency of those heights. Some extremely competitive environments can lead to great uneven populations; that would mean some enormous trees and plenty of very small ones.

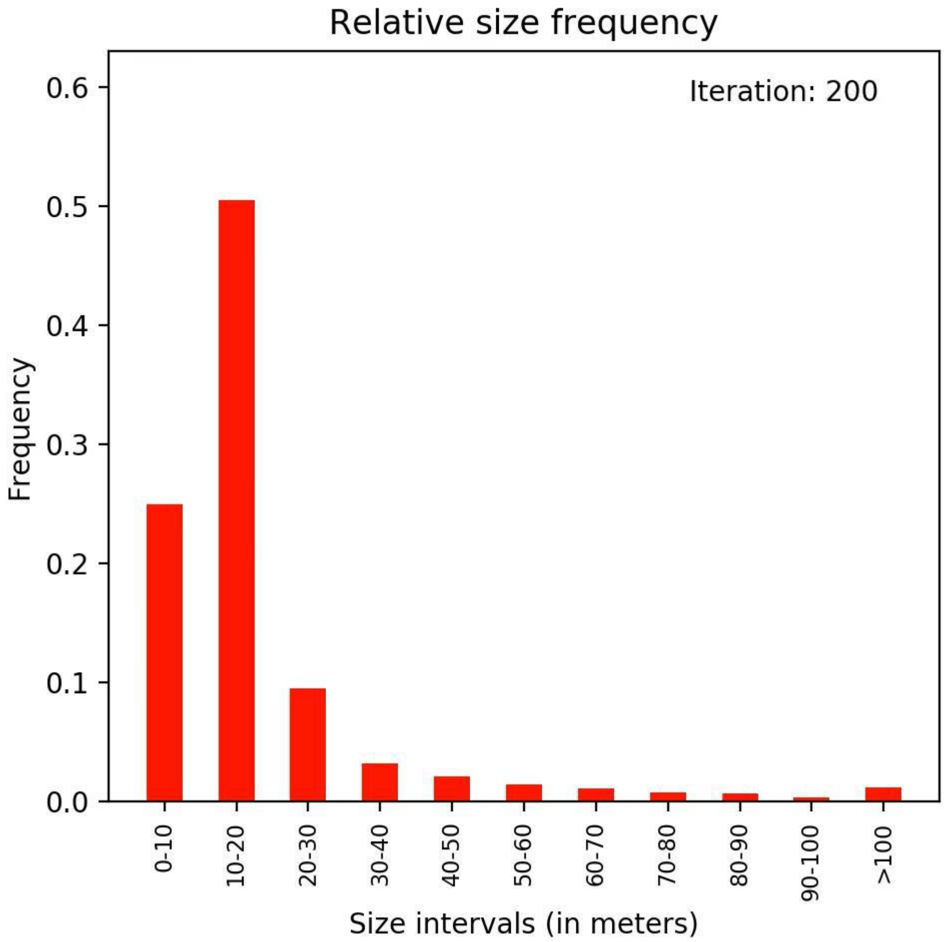

Figure 5: Frequency histogram of the trees height distribution of the simulated forest.

After some initial oscillations, the height distribution of our forest stabilises as shown in figure 5, and it stays this way from then on, even for future generations of trees.

Despite the elegance of such distribution, the most interesting point is its resemblance to the distribution of an actual forest, as measured in [1] and shown in figure 1.

The result shown in figure 5, when compared with theh actual measured forest in figure 1, confirms the value and accuracy of the methodology and assumptions employed throughout this project. The model developed, with very few assumptions regarding the interdependence between the *Size* of a tree and the *Soil Quality* surround it, in the context of Fighting for Light, has shown roughly the same height distribution of an actual Oak's forest of North America.

6 Conclusions

The development of this model intended to set a standard starting point for further study of the dynamics of a forest using survival simulation in the context of the fight for light. Considering the main characteristic of this model, the strong interdependence is between the *Size* and *Growth* of trees and the *Soil Quality* surround it. The balance between these two equations have revealed to be accurate representatives of the dynamics of a forest, yet simple; since we have shown a considerable resemblance between the predictions of the model and measure of an actual forest.

More research on this topic could establish its starting point in this main idea of balance. Further exploration is possible in establishing the height distributions that would lead to a stable forest and their resemblance with an actual forest.

Continued research could also focus more on the selection of the probability of death for unknown reasons (or random), perhaps with seasonal changes or extreme events (fire, weather disasters etc.) or how this balance can be disrupted by external forces i.e. human interface/actions.

Different species of trees could also be studied using the main scheme constructed on this model. This is possible by complicate a little the spawn probability of newborns, adding a dependence of near trees or taking into account the quality of that soil.

Additional research on this topic would be of great interest when studying new species introduced in an already stable environments and adding competitiveness between species of different reproduction patterns. Even though there is already a mathematical modelling-oriented research completed on this topic [16], it was mainly focused on species of animals in a far more complex environment, complex assumptions, low accuracy and predictability power.

In conclusion, when applying survival models to species of trees in a forest, simple approaches such as the balance between two interdependent factors, *Size/Growth* and *Soil Quality*, can lead to accurate results and be used as a basis to further a more complex research.

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