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### 1. Patterns in yield over time

## a. Plot of yield vs. time by grain

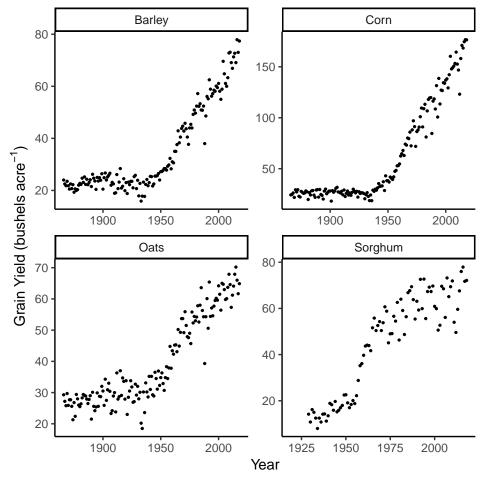


Figure 1 | Four plots showing grain yeild over time by crop Grain yeild is measured in bushels per acre. Barley, Corn and Oats look to have a linear low plateau for yield over time with positive slope starting in the 1940s. Sorghum looks to have a linear piecewise relationship for yiled over time with an increasing rate starting in the 1950s but then decreasing rate starting in the 1970s.

#### b. Statistical models to decribe Figure 1

Barley, Corn and Oats yield over time, seen in Figure 1, could be described by a linear low plateau model. For example, let x = year and  $f(x) = \text{Grain Yield (bushels acre}^{-1})$ . Then for Barley we have:

$$f(x) \approx \begin{cases} 25 & x \le 1950 \\ 25 + \frac{4}{5}x & x > 1950 \end{cases}$$

For Corn yield, the slope would be steeper when x > 1950 at approximately 2x. The slope for Oats yield when x > 1950 looks to be approximately  $\frac{3}{5}x$ . Sorghum shows a different trend that follows a linear piecewise relationship described by the following:

$$f(x) \approx \begin{cases} 15 + \frac{1}{5}x & 1925 \le x \le 1950\\ 20 + \frac{6}{5}x & 1950 < x \le 1975\\ 50 + \frac{3}{5}x & 1975 < x \end{cases}$$

#### 2. Fertilizer use

# a. Plot of fetilizer use over time by crop

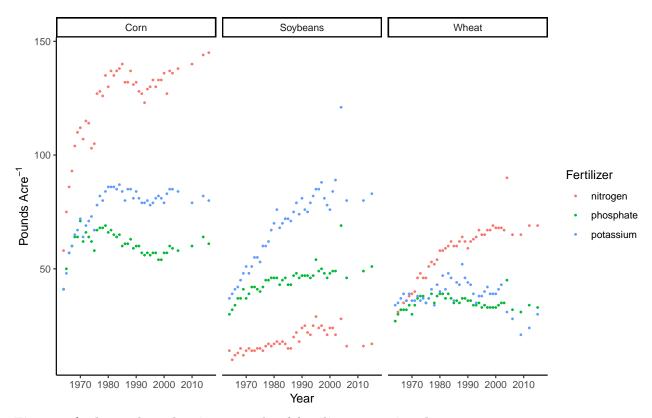


Figure 2 | Three plots showing pounds of fertilizer over time by crop.