EXAMINERS

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Centre Number

3 2 5 4 4 1 8 5

Student Number

CSSA

CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW

Garlay

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2020
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

73

Mathematics Advanced

Morning Session
Thursday, 20 August 2020

General Instructions

- Reading time – 10 mins
- Working time – 3 hours
- Write using black pen
- Use Multiple Choice Answer Sheet provided
- NESA-approved calculators may be used
- A reference sheet is provided on a SEPARATE sheet
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks - 100

Section I Pages 2-6

10 marks 7
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II Pages 7-26

90 marks
- Attempt Questions 11-37
- Allow about 2 hours and 45 minutes for this section

Q	1	1
4		1
14		3
17		2
18ab		2
22		5

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6100-1

5 What is the value of $(\log a)^2 \div \log(a^2)$?

(A) 1

(B) $\dfrac{\log a}{2}$

(C) $\log a$

(D) $2\log a$

$\dfrac{2\log a}{2\log a} = 1$

6 Which of the following is equal to $\dfrac{4^n - 1}{2^n - 1}$?

(A) $2^n + 1$

(B) $2^n - 1$

(C) 2^2

(D) 2

$\dfrac{4^n - 1}{2^n - 1}$

$\cancel{2\times(2^n - 1)}$

$\dfrac{2\times 2^n - 1}{2^n - 1}$

$=$

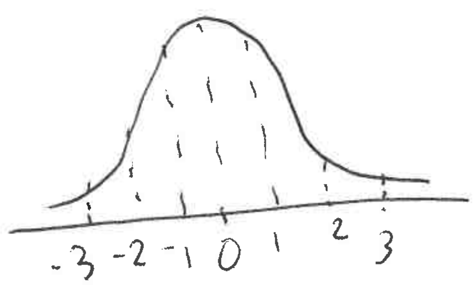
7 For a normally distributed set of scores, which of the following statements is FALSE?

(A) A z-score more than 2 has a higher probability than a z-score more than 1.

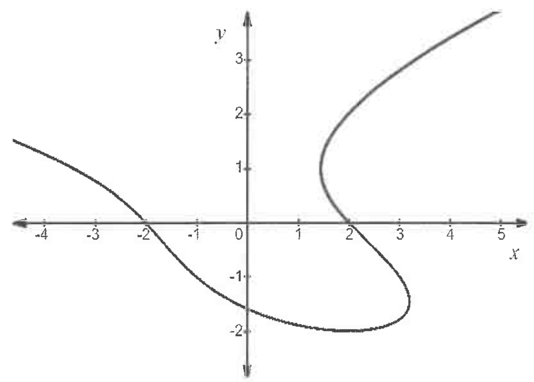
(B) The frequency distribution is symmetrical about the mean.

(C) The z-score of a score describes how far that score is from the mean.

(D) The mean and the median are equal.



8 Below is a graph of $x^2 + 2xy - y^3 = 4$.



What type of relation is this?

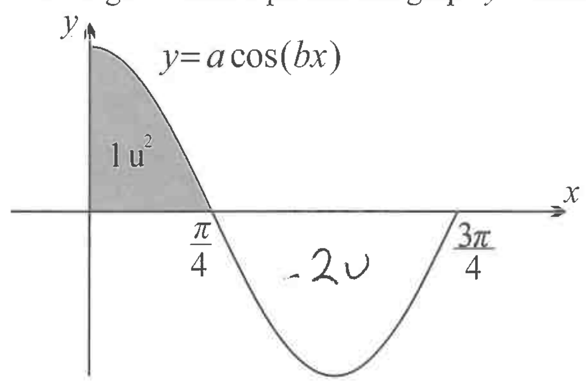
(A) one-to-one

(B) one-to-many

(C) many-to-one

(D) many-to-many

9 The diagram shows part of the graph $y = a\cos bx$.



$1 + 2 = 3$

The area of the shaded region is equal to 1 unit^2. What is the value of $\int_0^{\frac{3\pi}{4}} f(x)\,dx$?

(A) -2

(B) -1

(C) 1

(D) 3

10 The graph of $y = \frac{3}{x+1}$ is translated 4 units right and dilated vertically by a factor of $\frac{1}{2}$.
Which of the following gives the equation of the new function?

(A) $\frac{y}{2} = \frac{3}{x-3}$

(B) $2y = \frac{3}{x-4}$

(C) $2y = \frac{3}{x-3}$

(D) $\frac{y}{2} = \frac{3}{x-4}$

End of Section I

The paper continues over the page

Section II

90 marks
Attempt Questions 11-37
Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of your response.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Solve $|2x-3| \leq 1$. 2

Question 12 (2 marks)

Differentiate $\frac{e^x}{x^2+1}$. 2

u
v

$y' = \frac{u'v - uv'}{v^2}$

$y' = \frac{e^x(x^2+1) - 2xe^x}{(x^2+1)^2}$

2

2

Question 13 (2 marks) 2

Evaluate $\int_{-1}^{0} x(3x-4)\,dx$.

$\int_{-1}^{0} 3x^2 - 4x \, dx$

$= \left[\frac{3x^3}{3} - \frac{4x^2}{2}\right]_{-1}^{0}$

$= \left[x^3 - 2x^2\right]_{-1}^{0}$

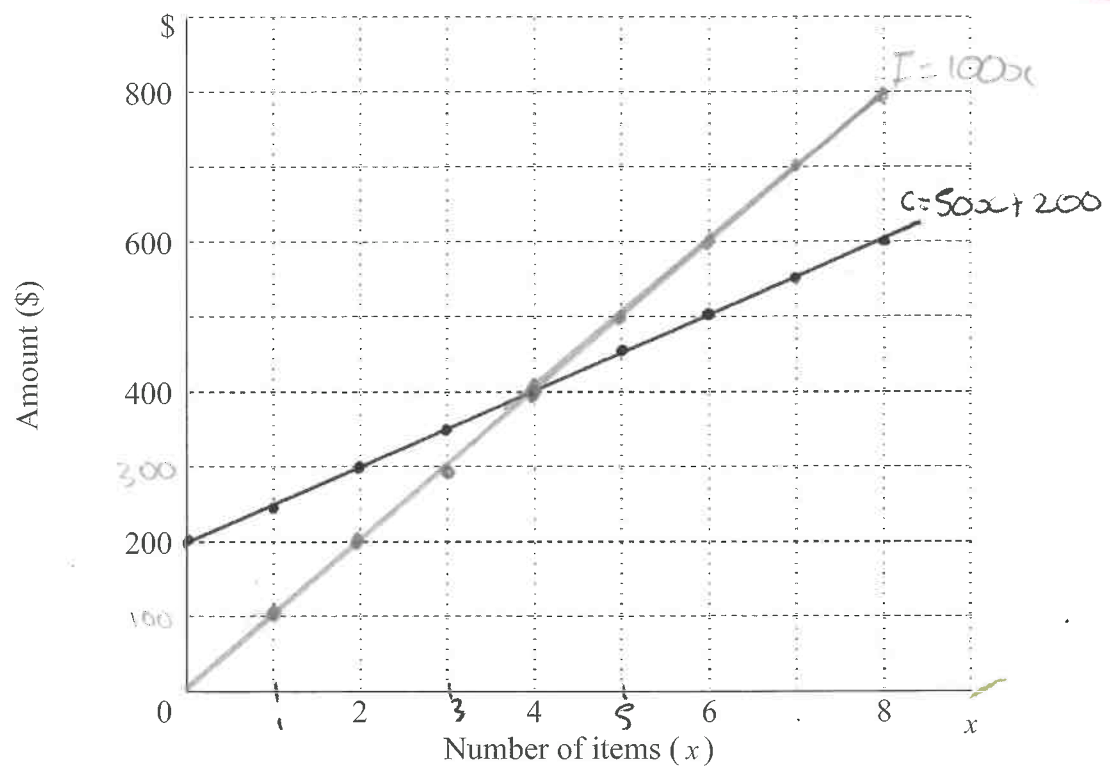
$= 0 - -3 \quad = 3$

Question 14 (3 marks)

Michael has a small manufacturing business.

The cost of manufacturing is given by the equation $C = 50x + 200$ and the income earned is given by the equation $I = 100x$, where x is the number of items that the business has manufactured.

(a) Graph each of the two equations on the grid below. 2



(b) How many items need to be manufactured for the business to break even? 1

4 items

5

$e^x \sin x + e^x \cos x$

Question 15 (2 marks) 2

Find the gradient of the tangent to the curve $y = e^x \sin x$ as it passes through the origin. $(0,0)$

$y' = u'v + uv'$ $\qquad x = 0$

$y' = e^x \sin x + e^x \cos x$ $\qquad m = e^0(\sin(0) + \cos(0))$

$y' = e^x(\sin x + \cos x) = m$ $\qquad m = 1$

m (tangent) $= 1$ at $x = 0$

Question 16 (2 marks) 2 0

Two ordinary 6-sided dice, one red and one blue, are rolled and the numbers on the uppermost face are recorded. The blue die shows a number less than five. What is the probability that the sum of the two numbers is greater than seven?

$p(<5) = \frac{4}{6}$ blue $\qquad p(>2)$ red $= \frac{4}{6}$

$\therefore p(>7 \text{ in total}) = \frac{4}{6} \times \frac{4}{6} = \frac{4}{9}$

5 3 4

Question 17 (2 marks) 2

A set of scores is normally distributed. A score of 12 has z-score of -2 while a score of 21 has a z-score of $+1$.

Calculate the mean of the scores.

$z = \frac{x - \mu}{\sigma}$ $\qquad 1 = \frac{21 - \mu}{\sigma}$

$-2 = \frac{12 - \mu}{\sigma}$

$-2\sigma = 12 - \mu$ $\qquad \sigma = 21 - \mu$ $\qquad \therefore \mu = 18$

$-2\sigma + \mu = 12$ (1) $\qquad \sigma + \mu = 21$ (2)

(1) − (2) $\qquad$ sub into (1)

$= -3\sigma = -9$ $\qquad -6 + \mu = 12$

$\sigma = 3$ $\qquad \mu = 18$

4

Question 18 (4 marks)

A cup of coffee is cooling according to the following exponential formula $C = 21 + (74 \times 3^{-0.2t})$, where C is the temperature in degrees Celsius and t is the time in minutes since the coffee was poured.

(a) Calculate the initial temperature of the coffee. 1

$t = 0 \quad C = 95°C$

(b) Calculate the temperature of the coffee after 10 minutes, correct to the nearest degree. 1

$t = 10$

$C = 21 + (74 \times 3^{-0.2(10)})$

$C = 29°C$

(c) After how many minutes, to the nearest minute, will the coffee first reach 50°C? 2

$C = 50$ solve for t

$50 = 21 + (74 \times 3^{-0.2t})$

$74 \times 3^{0.2t} = 29$

$3^{-0.2t} = \frac{29}{74}$

ln both sides

$-0.2t \ln 3 = \ln \frac{29}{74}$

$-0.2t = -0.85268$

$t = 4.26$

$\therefore t = 4$ minutes

$\theta + \sin^2\theta = 1$

$\theta = 1 - \sin^2\theta$

Question 19 (2 marks)

Show that $\dfrac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta} = \sin\theta\tan\theta$. 2

$\dfrac{\sec\theta(1-\cos^4\theta)}{2-\sin^2\theta}$

$\dfrac{\sec\theta(1-\cos^4\theta)}{1+\cos^2\theta} \rightarrow 1 + (1-\sin^2\theta) = 2 - \sin^2\theta$

~~$3n+7 = 2a + (n-1)d$~~

~~$3n + 7 = 2a + dn - d$~~

Question 20 (3 marks)

The sum of the first n terms of an arithmetic series is given by

$$S_n = \frac{n(3n+7)}{2}.$$

$S_n = \frac{1}{2}n(2a + (n-1)d)$ $\quad -30$

$S_2 = 1(10 + 1d) = 13$

$10 + 1d = 13 \quad d = 3$

(a) Find the first three terms of the sequence. 2

$S_1 = 5$ $\quad$ ~~$S_3 = 24$~~ $\quad S_1 = 5 = \frac{n(3n+7)}{2}$

~~$S_2 = 13$~~ $S_2 = 13$ $\quad \frac{1}{2}(2a$ $\quad 10 = n(3n+7)$

$S_3 = 24$ $\quad \frac{1}{2}(2a) = 5$ $\quad 10 = 3n^2 + 7n$

$3n^2 + 7n - 10 = 0$

$2a = 10$

$a = 5$ $\quad (3n-3)(3n+10) = 0$

$\boxed{T_1 = 5 \\ T_2 = 8 \\ T_3 = 11}$ ← answer

$n = 1 \quad n = -\frac{10}{3}$ wrong

(b) Find an expression for the n-th term of the sequence. 1

$a = 5 \quad d = 3$ $\quad T_n = 5 + (n-1)3$

$T_n = a + (n-1)d$ $\quad T_n = 5 + (3n-3)$

$T_n = 5 + 3n - 3$

$T_n = 2 + 3n$

Question 21 (3 marks)

Solve $\sin\left(x + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ for $0 \le x \le 2\pi$. 3

$\rightarrow \frac{\pi}{6} \le x + \frac{\pi}{6} \le \frac{13\pi}{6}$

$\frac{\pi}{6} = 30°$ $\quad -0.866$

~~$x + \frac{\pi}{6} = \sin^{-1}($~~

$x + \frac{\pi}{6} = 240, 300$

$x = 210, 270$

$x = \frac{7\pi}{6}, \frac{3\pi}{2}$

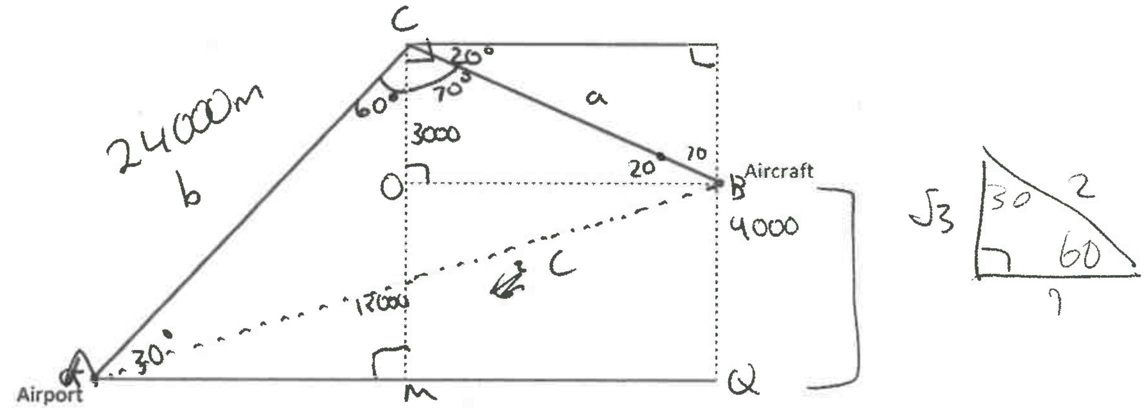
Question 22 (5 marks)

5

An aircraft took off from an airport on an angle of elevation of 30° until it reached a maximum height of 12 000 metres.

The aircraft then descended on an angle of depression of 20° to a new height of 9000 metres.

Find the straight-line distance of the aircraft to the airport when it first reaches the new height of 9000 metres, correct to the nearest metre.



In $\triangle AMC$; $\sin 30 = \frac{12000}{b}$

$b = \frac{12000}{\sin 30}$

$b = 24000\,m$

~~$\triangle COB$; $a =$~~

$\overrightarrow{MC} = 12000m$

$\overrightarrow{QB} = 9000m$

$\overrightarrow{OC} = 12000 - 9000 = 3000m$

$\triangle COB$; $\sin 20 = \frac{3000}{a}$

$a = 8771 \cdot 4132\,m$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (8771 \cdot 4132)^2 + (24000)^2 - 2(a \times b)\cos 130$

$c^2 = 923569164 \cdot 3$

$c = 30390 \cdot 28\,km$ ~~$= 30 \cdot 39\,km$~~

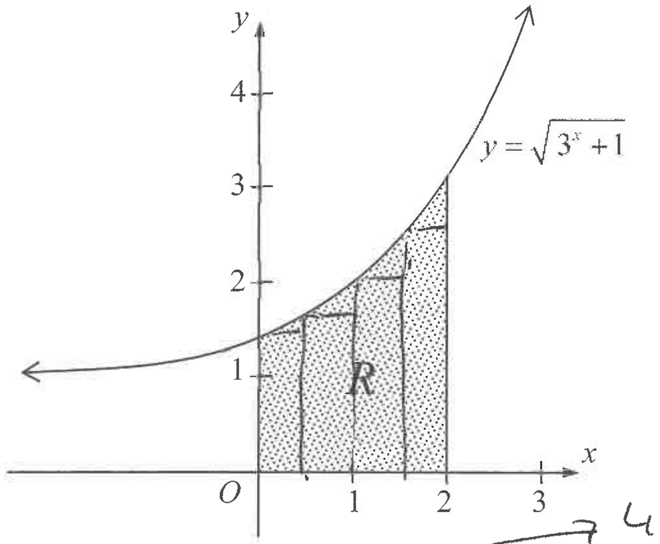
$c = 30390m$

$\therefore$ aircraft is now 30390m away from airport at the new height of

5

Question 23 (3 marks)

The diagram below shows the region R which is bounded by $y = \sqrt{3^x + 1}$, the x-axis and the lines $x = 0$ and $x = 2$.



4 sub intervals

(a) Use the Trapezoidal Rule with five function values to find an approximation for the area R. Give your answer to two decimal places.

2

x	0	0·5	1	1·5	2
y	1·414	1·653	2	2·489	3·162

Trap rule $= \sum \frac{b-a}{2}(f(a) + f(b))$

$\therefore A = 4 \cdot 22\,u^2$

(b) With reference to the curve, give a reason whether your approximation is an over-estimation or underestimation.

1

The area is an ~~under~~estimation. Since graph is increasing, there are small areas missed using lower rectangles.

2

Question 24 (5 marks)

Seven students, who are still on their Learner's Permit, recorded the number of driving hours they have logged so far and the mark they scored in their Visual Art assessment task.

The results are recorded in the table below:

Name	Ally	Bree	Cain	Dan	Elle	Frank	Guy
Hours of driving (x)	63	68	72	76	82	84	91
Visual Art mark (y)	73	71	71	85	92	80	81

(a) Calculate Pearson's correlation coefficient for the data, correct to two decimal places. **1**

$r = 0.63$

(b) Calculate the value, to two decimal places, of the gradient and the y-intercept. Use these values to write the equation of the least-squares line of best fit. **2**

$y = a + bx$ $a = y\text{-intercept} = 40.16$ $y = 40.16 + 0.507x$

$y = 40.16 + 0.507x$ $b = \text{gradient} = 0.507$

(c) Another student, who has logged 55 hours of driving and who scored 94 in Visual Art, is added to the data. **1**

Describe the impact this addition to the data will have on Pearson's correlation coefficient.

the new value of r = 0.0091968.

This means that the effect ~~is that it~~ on the value ~~of~~ of pearson's coefficient value is much lower showing ~~tegs of a~~ a weaker correlation between data.

(d) A student made the statement, "Doing well in Visual Art depends on how much driving you have done." **1**

Based on the data, justify whether you agree or disagree with the statement.

I disagree. As the last student did the least hours of driving and got a 94.

there is no correlation between the two variables

$r = 0.009$

0

4

Question 25 (4 marks)

The displacement of a particle moving along the x-axis is given by $x = t + e^{-2t}$, where x is the displacement from the origin in metres and t is the time in seconds, for $t \geq 0$.

(a) Find an expression, in terms of t, for the velocity of the particle. **1**

$x = t + e^{-2t}$

$\dot{x} = v = 1 - 2e^{-2t}$

STOP HERE $1 - 2e^{-2t} = v$

$-2e^{-2t} = v - 1$

$2e^{-2t} = 1 - v$

~~$e^{-2t} = \frac{1-v}{2}$~~

$e^{-2t} = \frac{1}{2}(1-v)$ ln both sides

$-2t \ln e = \ln\left(\frac{1}{2}(1-v)\right)$

$-2t = \ln\left(\frac{1-v}{2}\right)$

$2t = -\ln\left(\frac{1-v}{2}\right)$

$t = \dfrac{-\ln\left(\frac{1-v}{2}\right)}{2}$

(b) Find the time when the particle is closest to the origin. **3**

~~Let $x = 0$~~ $t = 0$ must set $v = 0$

~~$t + e^{-2t} = 0$~~ $x = 1$ m away from origin.

when $x = 1$ $t(\ln 1 - 2) = 0$

$t + e^{-2t} = 1$ when $x = 0$ $t = 2$

$\ln t - 2t = 0$ ~~$v = 1 - 2e^{-2t}$~~

$2t = \ln t$ $x = -1$ ~~$t + e^{-2t} = -1$~~

$t = \frac{\ln t}{2}$ ~~$e^{-2t} = -1 - t$~~

~~$-2t = \ln(-1-t)$~~

~~$t = \frac{\ln(-1-t)}{2}$~~

1

Question 26 (2 marks)

Let $f(x) = 5x - 2$ and $g(x) = \frac{1}{\sqrt{x}}$. State the domain of $g(f(x))$. **2**

$g(f(x)) = \dfrac{1}{\sqrt{5x-2}}$ domain $= \left(\frac{2}{5}, \infty\right)$

$5x - 2 > 0$

$5x > 2$

$x > \frac{2}{5}$

4

Question 27 (7 marks)

$(x+1)(x+1) \cdot (x+5)$
$(x^2+2x+1)(x-5)$

Consider the curve $y=(x+1)^2(x-5)$.

(a) Find the stationary points and determine their nature. 3

$(x^2+2x+1)(x-5)$

$= x^3-5x^2+2x^2-10x+x-5$

$= x^3-3x^2-9x-5$

~~$y'=3x^2$~~ $y'=3x^2-6x-9$

Let $y'=0$ to find stationary points.

$3x^2-6x-9=0$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3, -1$

x	2	3	4
y'	−	0	+
y''	\	—	/

$(3,-32)$ is a minimum turning point

x	−2	−1	0
y'	+	0	−
y''	/	—	\

$(-1,0)$ is a maximum turning point.

(b) Given that the point $(1,-16)$ lies on the curve, show that it is a point of inflection. 2

$y'=3x^2-6x-9$

$y''=6x-6$

Let $y''=0$

$6x-6=0$

$6x=6$

$x=1 \rightarrow$ sub int y

$\therefore y=(1+1)^2(1-5)$

$y=-16$

$\therefore (1,-16)$ is a point of inflection

table of values?

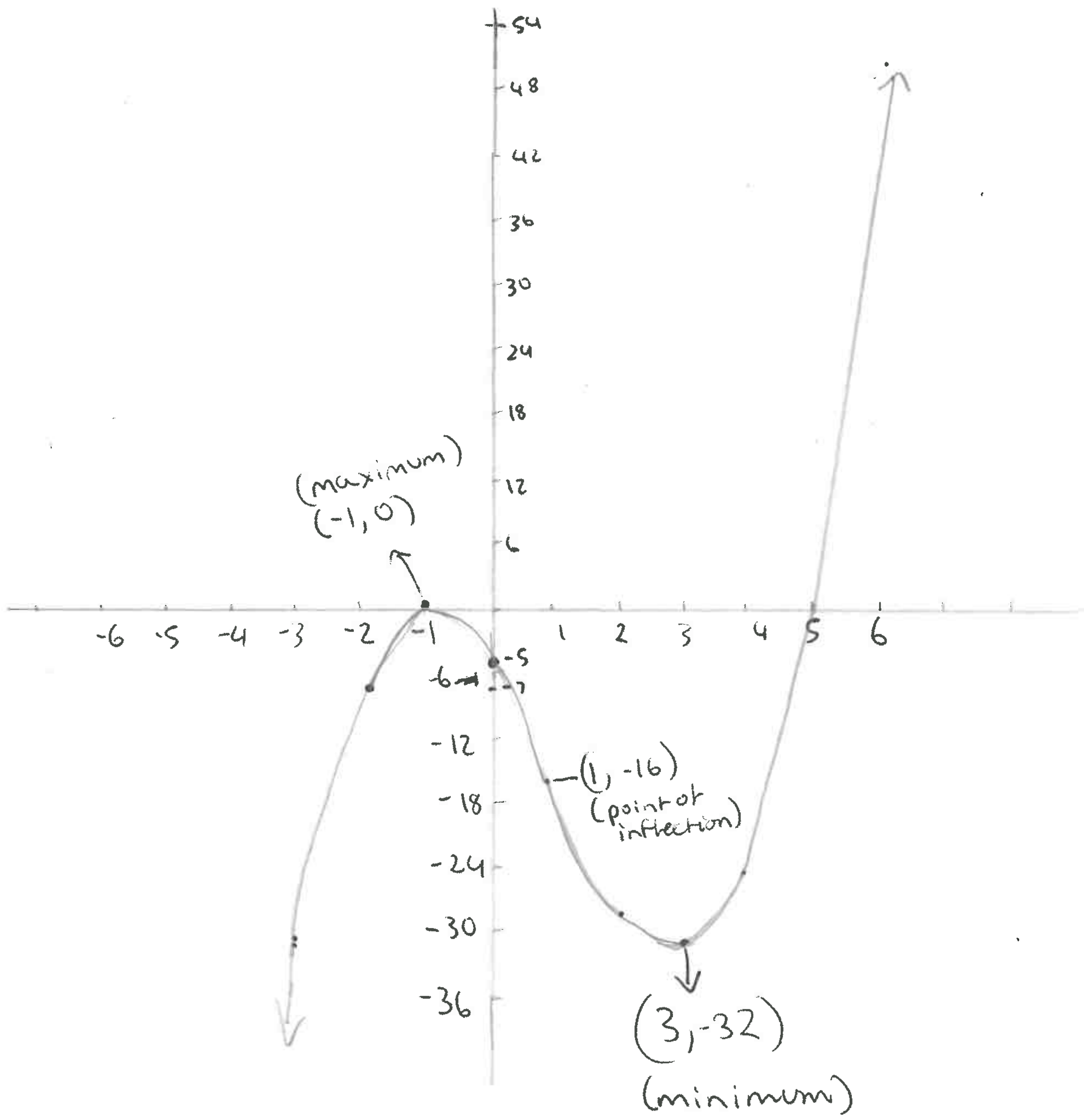
1

Question 27 continues over the page

4

Question 27 (continued)

(c) Sketch the curve $y=(x+1)^2(x-5)$, showing the stationary points and the point of inflection. 2



2

Question 28 (3 marks)

Consider the circle given by the equation $x^2+8x+y^2-4y-29=0$.

Give the equation of the circle in the form $(x-h)^2+(y-k)^2=r^2$, if it is translated up by three units and right by five units.

$x^2+8y+y^2-4y-29=0$

$(x^2+8y+16)+(y^2-4y+4)-16-4-29=0$

$(x+4)^2+(y-2)^2-49=0$

$(x+4)^2+(y-2)^2=49$ $r=7$

right by 5 : $(x-5)$ up by 3 = $y+3$

$(x-1)^2+(y+1)^2=49$

$(x-1)^2+(y+1)^2=7^2$

~~3~~ 2

2

Question 29 (6 marks)

The time t minutes that a customer spends waiting in a queue has a probability density function:

$$f(x)=\begin{cases}\dfrac{x(k-x)}{36} & \text{for } 0\le x\le k\\ 0 & \text{otherwise}\end{cases}$$

(a) Show that the value of k is 6. **2**

(b) Write down the value of $E(X)$. **1**

(c) Calculate $\mathrm{Var}(X)$. **3**

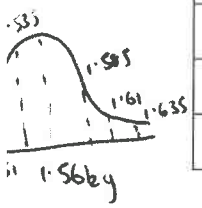
Question 30 (3 marks)

$<1.5 = \text{size } 14$

~~3~~ 2

Chickens are sized according to their weight. For example, a size 16 chicken weighs 1.6 kg. At a particular farm, the chickens they sell as size 15 have weights which are normally distributed with a mean of 1.56 kg and a standard deviation of 0.025 kg. Chickens that weigh less than 1.5 kg should be classified size 14 and chickens that weigh more than 1.6 kg should be classified size 16.

The following table gives the probability of scores with a *z*-score less than the given value.



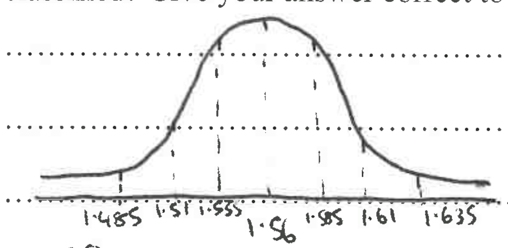
z	*first decimal place*									
	.0	**.1**	**.2**	**.3**	**.4**	**.5**	**.6**	**.7**	**.8**	**.9**
0.	0.500	0.540	0.579	0.618	0.655	0.692	0.726	0.758	0.788	0.816
1.	0.841	0.864	0.885	0.903	0.919	0.933	0.945	0.955	0.964	0.971
2.	0.977	0.982	0.986	0.989	0.992	0.994	0.995	0.997	0.997	0.998
3.	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

For example, the probability of a score with a *z*-score of less than 2.6 is 0.995, as highlighted above.

$16 = 1.6\text{kg}$ $\quad \mu = 1.56\text{kg} \quad \sigma = 0.025\text{kg}$

~~$>1.5 = 16$~~ $\quad <1.5 = 14 \quad >1.6 = 16$

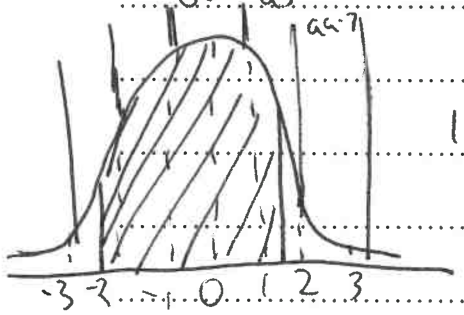
What is the probability that a randomly selected size 15 chicken has been incorrectly classified? Give your answer correct to the nearest 3 decimal places.



size 15 chicken $= 1.5 \leq x \leq 1.6$

$z = \dfrac{1.5 - 1.56}{0.025}$ $\quad z = \dfrac{1.6 - 1.56}{0.025}$

$z = -2.4$ $\quad z = 1.6$ ✓



$P(-2.4 \leq Z \leq 1.6)$ ✓

$1 - [P(Z > 1.6) + P(Z > 2.4)]$

$= 1 - (0.055 + 0.008)$

$= 0.937$ ✗

$0.055 + 0.008 = 0.063$

2

Question 31 (2 marks)

The sketch shows $y = f(x)$. Draw a sketch of $f(2-x)$.

odd

$\therefore f(-x) = -f(x)$

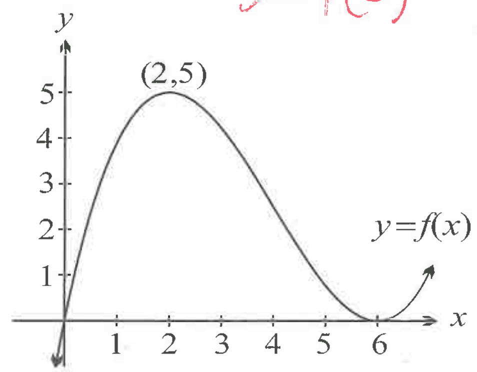
2

$5 = f(2)$

$f(-x+2)$

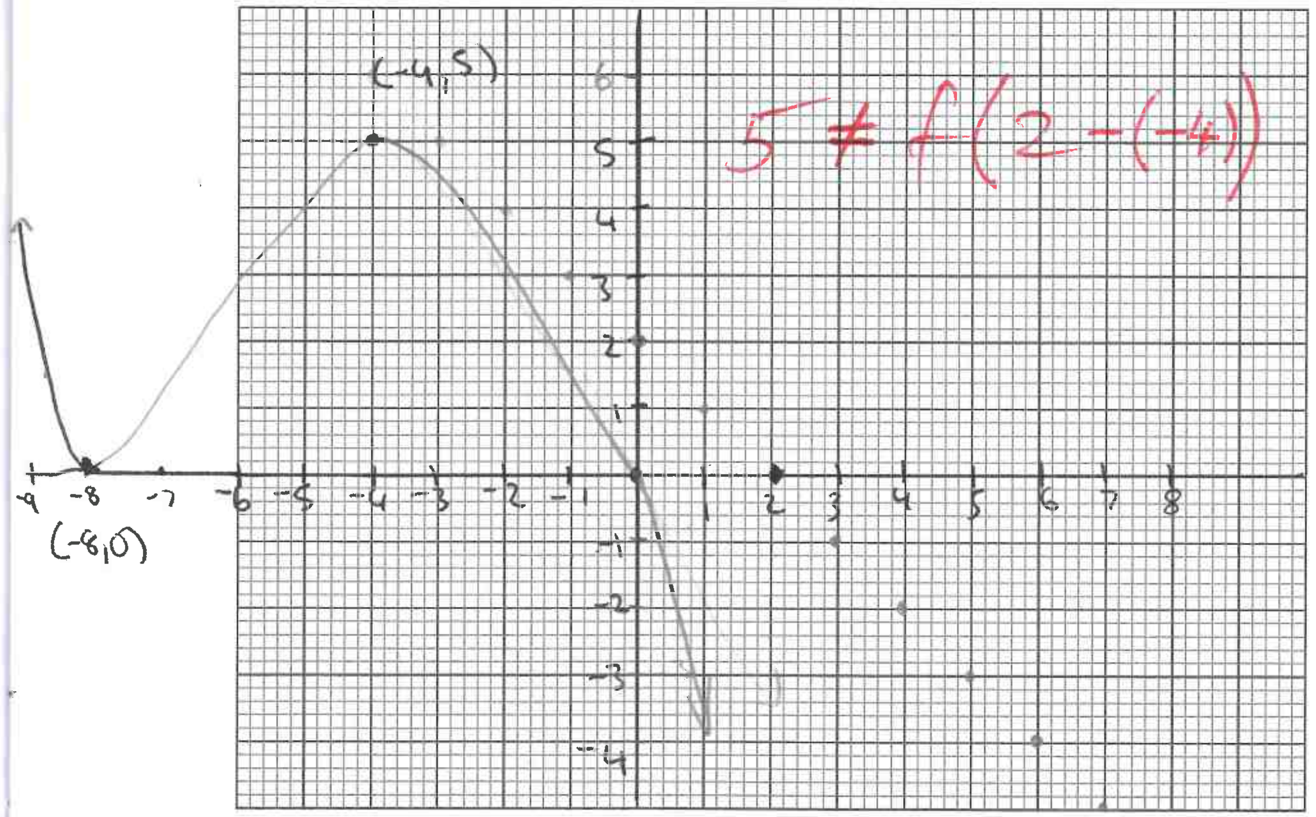
$\therefore f(2-x)$
$= f(-x+2)$
$= -f(x-2)$

1



flip about y-axis

left by 2



1

Question 32 (6 marks)

Mimi borrows \$650 000 to buy a house at 6 % per annum with interest compounded monthly. The loan is to be repaid with equal monthly repayments of \$4200. Let A_n be the amount owing after n months.

(a) Show that $A_n = 840\,000 - 190\,000 \times 1.005^n$. 3

$A_1 = 650000(1.005)^1 - 4200$

$A_2 = [650000(1.005)^1 - 4200](1.005) - 4200$

$A_2 = 650000(1.005)^2 - 4200(1.005) - 4200$

$A_2 = 650000(1.005)^2 - 4200(1 + 1.005)$

$A_n = 650000(1.005)^n - 4200(1 + 1.005 + \dots 1.005^n)$

$A_n = 650000(1.005)^n - 4200\frac{(1.005^n - 1)}{0.005}$

$A_n = 650000(1.005)^n - 840000(1.005^n - 1)$

$A_n = 650000(1.005)^n - 840000(1.005)^n + 840000$

$A_n = 840000 - 190000(1.005)^n$

$\frac{1(1.005^n - 1)}{0.005}$

(b) After 15 years, the amount owing is \$373 722 to the nearest dollar. At this time Mimi borrows a further \$200 000 to build an extension. She adds this onto her previous mortgage. If the monthly repayments remain the same, what is the minimum remaining number of months it will take her to pay off the balance of the loan? 3

1

$n = 15$ $A = 373722$

$A = 573722$

$573722 = 840000 - 190000(1.005)^n$

$-190000(1.005)^n = -266278$

$1.005^n = 1.401463158$

$n \ln 1.005 = \ln(1.401463158)$

$n = 67.67$

$n = 68$ months

4

Question 33 (6 marks)

(a) Expand and simplify the expression $(x+4)(x-1)(x-3)$. 1

$x^2 - x + 4x - 4 = (x^2 + 3x - 4)(x - 3)$

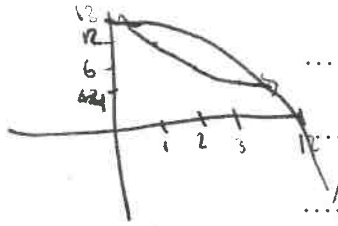
$= x^3 - \cancel{3x^2} + \cancel{3x^2} - 9x - 4x + 12$

$= x^3 - 13x + 12$

(b) Consider the curves defined by the equations $y = \frac{12}{x}$ and $y = 13 - x^2$. 5

Using part (a), or otherwise, find the points of intersection between these two curves and find the exact area enclosed between the two curves in the first quadrant.

4



$\frac{12}{x} = 13 - x^2$

$x^2 = 1$

$x = \pm 1$

$\int \frac{12}{x}$

$12 = 13x - x^3$

$= 12 \int \frac{1}{x}$

$x^3 - 13x + 12 = 0$

$x^3 - 13x = -12$

$= 12 \ln x + c$

$x(x^2 - 13) = -12$

$x = -12$ $x = \pm 1$

$x = 1$ $y = 12$

$\int_1^{12} 13 - x^2 - \frac{12}{x}$

$= \cancel{13x - \frac{x^3}{3} + 12}$

$= \left[13x - \frac{x^3}{3} - 12\ln x\right]_1^{12}$ 3

$= (-420 - 12\ln 12) - (\frac{38}{3} - 0)$

$= -\frac{1298}{3} - 12\ln 12 \ u^2$

5

Question 34 (2 marks)

Solve $\log_3 x + \log_9 x = 12$.

$\frac{\ln x}{\ln 3} + \frac{\ln x}{\ln 9} = 12$

$\frac{\ln x + \ln x}{\ln 3 \quad \ln 3 + \ln 3} = 12$

$\frac{\ln x}{2\times \ln 3} + \frac{\ln x}{2\ln 3} \quad \frac{2\ln x + \ln x}{2\ln 3} = \frac{3\ln x}{2\ln 3} = 12$

$3\ln x = 24\ln 3$

$\ln x = 8\ln 3$

~~$e^x = 8e^3 \times 3$~~

~~$x = 24$~~ $\quad x \approx 3e^8$

2

1

Question 35 (2 marks)

Sophie rolls two dice and observes that the sum of the uppermost numbers is six.
What is the probability that the number four has appeared at least once?

~~P(Goth)=~~

P(at Least 1, 4) = $\frac{11}{36}$

if 1 is 4 ∴ the other number is 2

$\frac{11}{36}$

$\frac{2}{36} = \frac{1}{18}$

2

1

1st → 6

2 ↓

	1	2	3	4	5	6
1	2	3	4	5	$\boxed{6}$	7
2	3	4	5	6	7	8
3	4	5	$\boxed{6}$	7	8	9
4	5	6	7	8	9	10
5	$\boxed{6}$	7	8	9	10	11
6	7	8	9	10	11	12

2

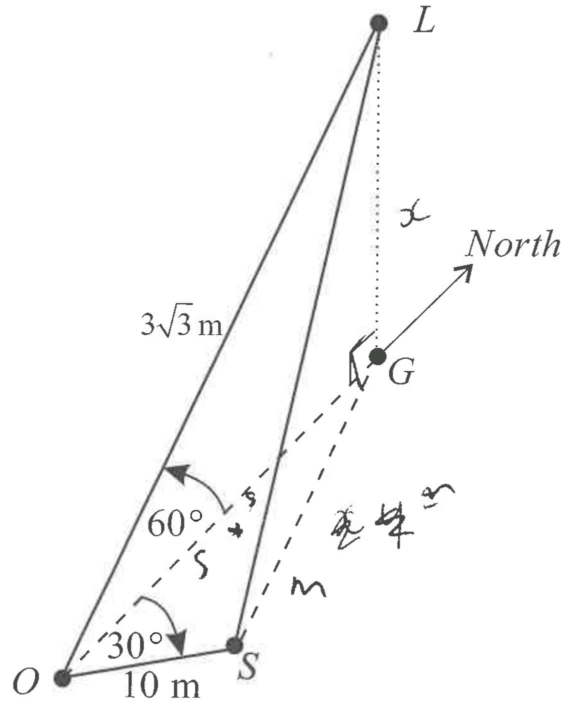
Question 36 (3 marks)

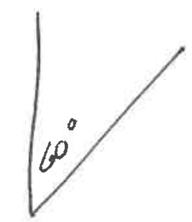
During the festival of lanterns Oscar releases a lantern from point O. It floats up into the sky due north in a straight line with an angle of elevation of $60°$. Sarina is located at S, 10 metres from O at a bearing of N$30°$E from O.

3

1

The lantern has travelled $3\sqrt{3}$ metres from O when Sarina first observes it at L. The point G is on the ground directly below L.





Let SL be y. Find the distance y. Give your answer correct to the nearest metre.

$\sin 60 = \frac{x}{3\sqrt{3}}$

~~$m: \tan 30 = \frac{m}{10} \quad m = 5.77m$~~

$x = 4.5m$

$m =$

$\angle LSG = \tan\alpha = \frac{4.5}{5.77}$

$\alpha = 37°57'$

$s = \cos 60 = \frac{s}{3\sqrt{3}}$

$s = 2.598m$

T

Question 37 (4 marks)

Two dangerous goods long-haul drivers are contracted to share the driving on one truckload of chemicals over a distance of 1200 km. They are each paid \$55 per hour for the time spent on the road. The truck consumes fuel at a rate of $10+\dfrac{v^2}{100}$ litres per hour where the average speed of the truck is v km per hour. Diesel fuel costs \$1.50 per litre. Show that to complete the journey the cost \$$C$ of fuel and wages will be given by $C=\dfrac{150\,000}{v}+18v$, and determine the minimum cost. Give your answer correct to the nearest dollar.

minimum cost =

$d = 1200\text{km}$ $\quad v^2$

$v/h = C = 10 + \frac{v^2}{100}$

$C = \frac{150\,000}{v} + 18v$

$C =$

0

Question 29 (6 marks)

Consider the random variable X whose probability density function is

$$f(x)=\begin{cases}\dfrac{x(k-x)}{36} & \text{for } 0\le x\le k\\ 0 & \text{elsewhere}\end{cases}$$

(a) Show that the value of k is 6. **2**

$\int_0^k \frac{x(k-x)}{36}\,dx = 1$

$\frac{1}{36}\int_0^k x(k-x)\,dx = 1$

$\frac{1}{36}\int_0^k kx - x^2$

$\frac{1}{36}\left[\frac{kx^2}{2} - \frac{x^3}{3}\right]_0^k = 1$

$\frac{1}{36}\left[\frac{k^3}{2} - \frac{k^3}{3}\right] = 1$

$\frac{k^3}{2} - \frac{k^3}{3} = 36$

$\frac{3k^3 - 2k^3}{6} = 36$

$k^3 = 216$

$k = 6$

2

(b) What is the mode and why? 1 **2**

3

(c) Find the cumulative distribution function of X. **2**

$\frac{1}{36}\int_0^x 6x - x^2$

$\frac{1}{36}\left[\frac{6x^2}{2} - \frac{x^3}{3}\right]_0^x$

$\frac{1}{36}\left[3x^2 - \frac{x^3}{3}\right]_0^x$

$= \frac{1}{36}\left(3x^2 - \frac{x^3}{3}\right)$

$= \frac{1}{12}x^2 - \frac{1}{108}x^3$

$= \frac{1}{108}(9x^2 - x^3)$

= CDF

0·07

5

Centre Number

3 2 5 4 4 1 8 5

Student Number

CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NEW SOUTH WALES
YEAR 12 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS ADVANCED – MULTIPLE CHOICE ANSWER SHEET

Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you have changed your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

A ⊗ B ⊗ *correct* C ○ D ○

ATTEMPT ALL QUESTIONS

Question	A	B	C	D
1	○	○	○	●
2	○	○	○	●
3	○	○	●	○
4	●	○	○	○
5	●	○	○	○
6	○	○	○	●
7	●	○	○	○
8	○	○	○	●
9	○	○	○	●
10	○	○	●	○

7

6100-4

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✓✓

Question 11 (2 marks)

Solve $|2x-3|=1$.

$2x-3=1$ $\qquad$ $2x-3=-1$

$2x=4$ $\qquad$ $2x=2$

$x=2$ $\qquad$ $x=1$

2