2. DTFT

$$H(e^{jw}) = \sum_{n=-\infty}^{n=\infty} 0.9^{|n|} e^{-jwn}$$

$$= \underbrace{\sum_{n=0}^{\infty} 0.9^{n} e^{-jwn}} + \sum_{n=-\infty}^{-1} 0.9^{-n} e^{-jwn}$$

$$= \frac{1}{1-0.9e^{-jw}} + \frac{0.9e^{jw}}{1-0.9e^{jw}}$$

$$= \frac{1-0.9^2}{1-2(0.9)\cos(w)+0.9^2} \Rightarrow \frac{1-a^2}{1-2a\cos w+a^2}$$

$$= \frac{0.19}{1.81-1.8\cos(w)}$$

3. $x(n) = 2\delta(n-2) + 3u(n-3)$

using table and properties

$\Downarrow$

$2z^{-2} + 3z^{-3}u(n)$

$$= 2z^{-2} + \frac{3z^{-3}}{1-z^{-1}}$$

$$= \frac{2z^{-2}(1-z^{-1})+3z^{-3}}{1-z^{-1}} = \frac{2z^{-2}-2z^{-3}+3z^{-3}}{1-z^{-1}}$$

$$= \frac{2z^{-2}+z^{-3}}{1-z^{-1}}, \text{ROC } |z|>1$$

From $2\delta(n-2)$, ROC: All z

From $3u(n-3)$

ROC $|z|>1$

So ROC $|z|>1$

4.

$$H(z) = \frac{z+1}{z-0.5} = \frac{1+z^{-1}}{1-0.5z^{-1}} = \frac{1}{1-0.5z^{-1}} + \frac{z^{-1}}{1-0.5z^{-1}}$$

$$= |z| > 0.5$$

i. impulse response

$$h[n] = 0.5^n u[n] + 0.5^{n-1} u[n-1]$$

ii. Difference equation

$$H(z) = \frac{1+z^{-1}}{1-0.5z^{-1}} = \frac{Y(z)}{X(z)} \Rightarrow Y(z)(1-0.5z^{-1}) = X(z)(1+z^{-1})$$

$$: Y(z) - 0.5\,Y(z)z^{-1} = X(z) + X(z)z^{-1}$$

$$= y(n) - 0.5y[n-1] = x[n] + x[n-1]$$

$$= Y[n] = X[n] + X[n-1] + 0.5Y[n-1]$$

iii. Pole zero plot given with the figures

iv. $x[n] = 3\cos\left[\frac{\pi n}{3}\right] u[n]$

$$= 3\left[\frac{1-\cos(\frac{\pi}{3})z^{-1}}{1-2\cos(\frac{\pi}{3})z^{-1}+z^{-2}}\right] \quad |z|>1$$

$$= 3\left[\frac{1-0.5z^{-1}}{1-z^{-1}+z^{-2}}\right]\left[\frac{1+z^{-1}}{1-0.5z^{-1}}\right]$$

$$= 3\left[\frac{\cancel{1-0.5z^{-1}}(1+z^{-1})}{1-z^{-1}+z^{-2}(\cancel{1-0.5z^{-1}})}\right] = 3\left[\frac{1+z^{-1}}{1-z^{-1}+z^{-2}}\right]$$

$$= 3\left[\frac{1-0.5z^{-1}+z^{-1}}{1-z^{-1}+z^{-2}}\right] = 3\,\frac{1-0.5z^{-1}}{1-z^{-1}+z^{-2}} + 3\sqrt{3}\,\frac{\frac{\sqrt{3}}{2}z^{-1}}{1-z^{-1}+z^{-2}}$$

$$Y(n) = 3\cos\left(\frac{\pi n}{3}\right)u(n) + 3\sqrt{3}\sin\left(\frac{\pi n}{3}\right)u(n)$$