

# *Headquarters USAF Warfare Center*

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*Testing - Tactics - Training*

## **Mixed Data Type Exponential Smoothing For Reliability Prediction**



**Lt Jeremy L. Thompson**

**49 TES**

**30 January 2012**

**This Briefing is:  
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# Overview

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- **Background**
- **Current Method**
- **Proposed Method**
- **Models**
- **Simulation**
- **Analysis**
- **Summary**



# Background

- **Currently Advanced Weapon Systems Analyst**
  - ALCM testing and analysis
  - Aircraft nuclear reliability/accuracy for USSTRATCOM





# Project Background

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- **ALCM analysis presents challenges**
  - Irregular testing schedule
  - Different types of testing
  - Annual projection required
  
- **Exponential Smoothing selected**
  - Accommodates irregular schedule with annual average
  - Allows projection



# Time Series

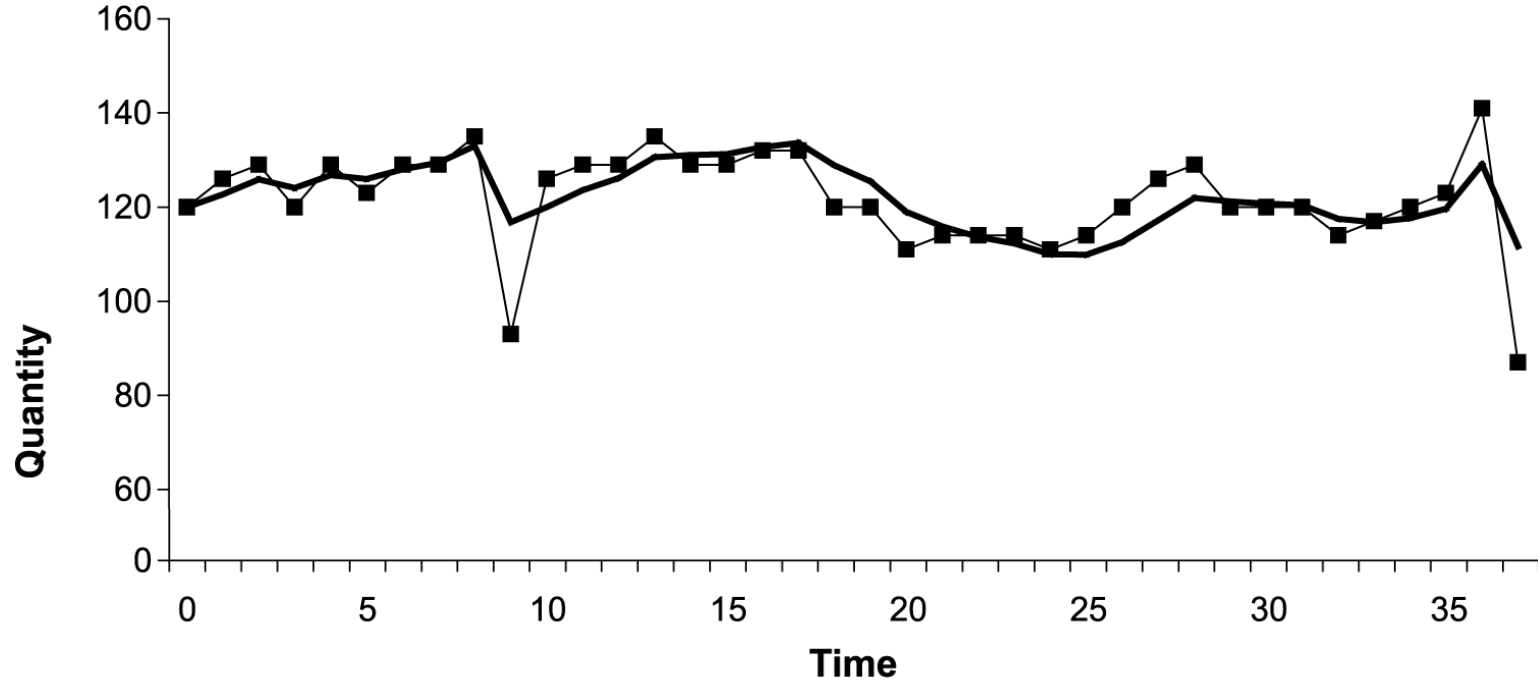
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- Data collected at uniform intervals, such as test successes over tests attempted, forms a time series
- Time series analysis consists of methods for analyzing time series data to extract meaningful statistics and other characteristics of the data



# Simple Exponential Smoothing

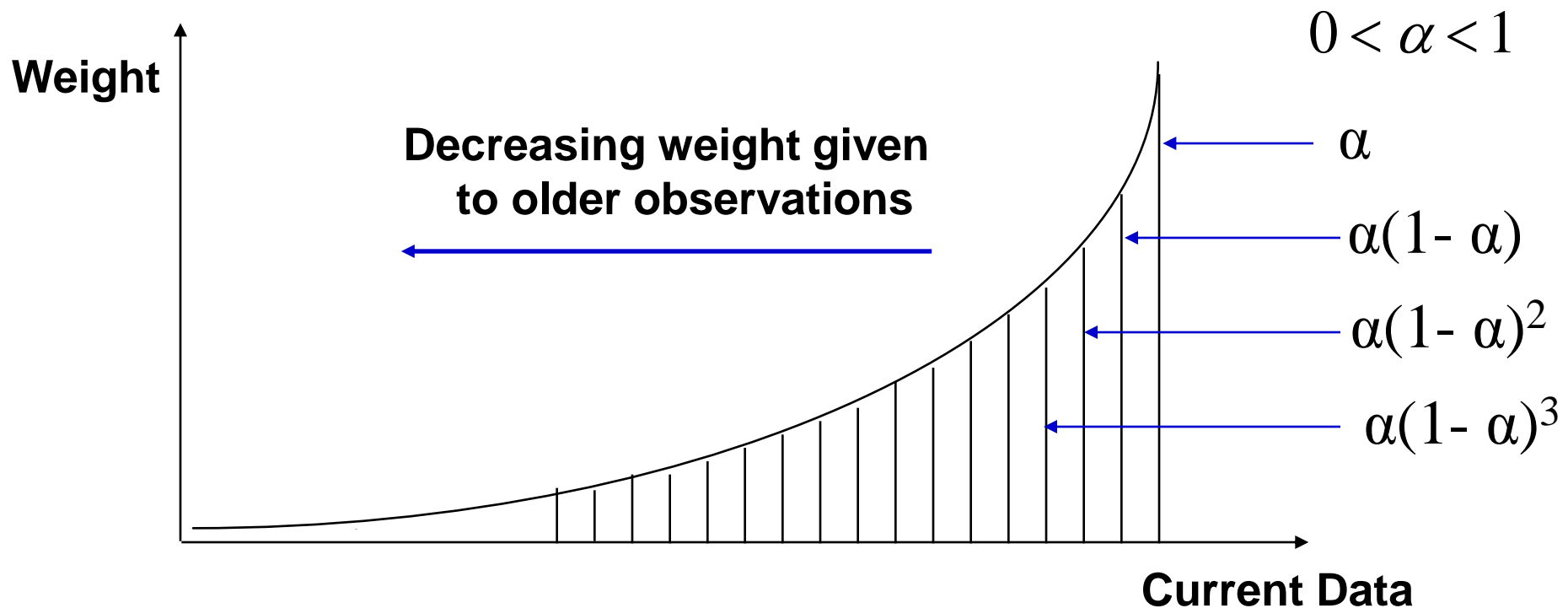
- Simple Exponential Smoothing works well with data that is stationary, or “moving sideways”





# Simple Exponential Smoothing

- New forecast is the weighted sum of old forecast and new test results
- Weighting factor (smoothing constant),  $\alpha$  is chosen to minimize error





# Assumptions of Models

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## ■ Assumptions of Time Series Models

- There is information about the past
- This information can be quantified in the form of data
- The pattern of the past will continue into the future

## ■ Assumptions of Simple Exponential Smoothing

- Error is randomly distributed
- Data is stationary, does not exhibit a long term trend





# Advantages/Drawbacks

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## ■ Advantages:

- Good accuracy for short-term
- Simple model
- Not highly sensitive to small sample size
- Accounts for changes/corrections to the system
  - Recent data given more weight than old data

## ■ Drawbacks

- Assumes performance will be like recent history
- Not sensitive to dramatic positive/negative changes in data
- Inaccurate recent year data can cause significant prediction error

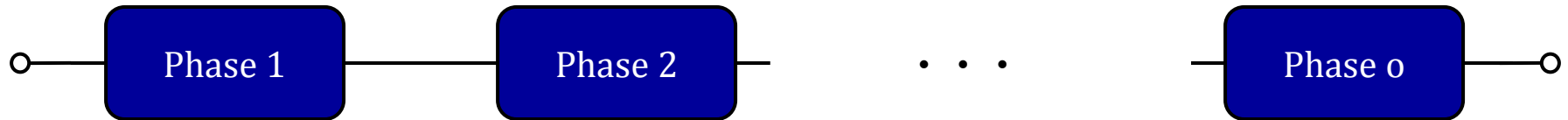


# Overall Model

Phase 1

Phase 2

Phase o



- System considered as a whole
- Multiple phases of operation



# Current Method

- Estimates annual reliability with test success rate
- Form time series from annual reliability estimates
- Simple exponential smoothing to project future reliability

$$P_t = \alpha \bar{R}_{t-1} + (1 - \alpha)P_{t-1}$$
$$t > 1$$

**Exponential Smoothing Equation**

$$\bar{R}_{t-1} = \frac{S_{t-1}}{S_{t-1} + F_{t-1}}$$

**Annual Reliability Estimate Equation**

$P_t$  - Projection at time  $t$

$R_t$  - Reliability at time  $t$

$S_t$  - Flight Test successes at time  $t$

$F_t$  - Flight test failures at time  $t$

$t$  - Time index

$\alpha$  - Alpha value



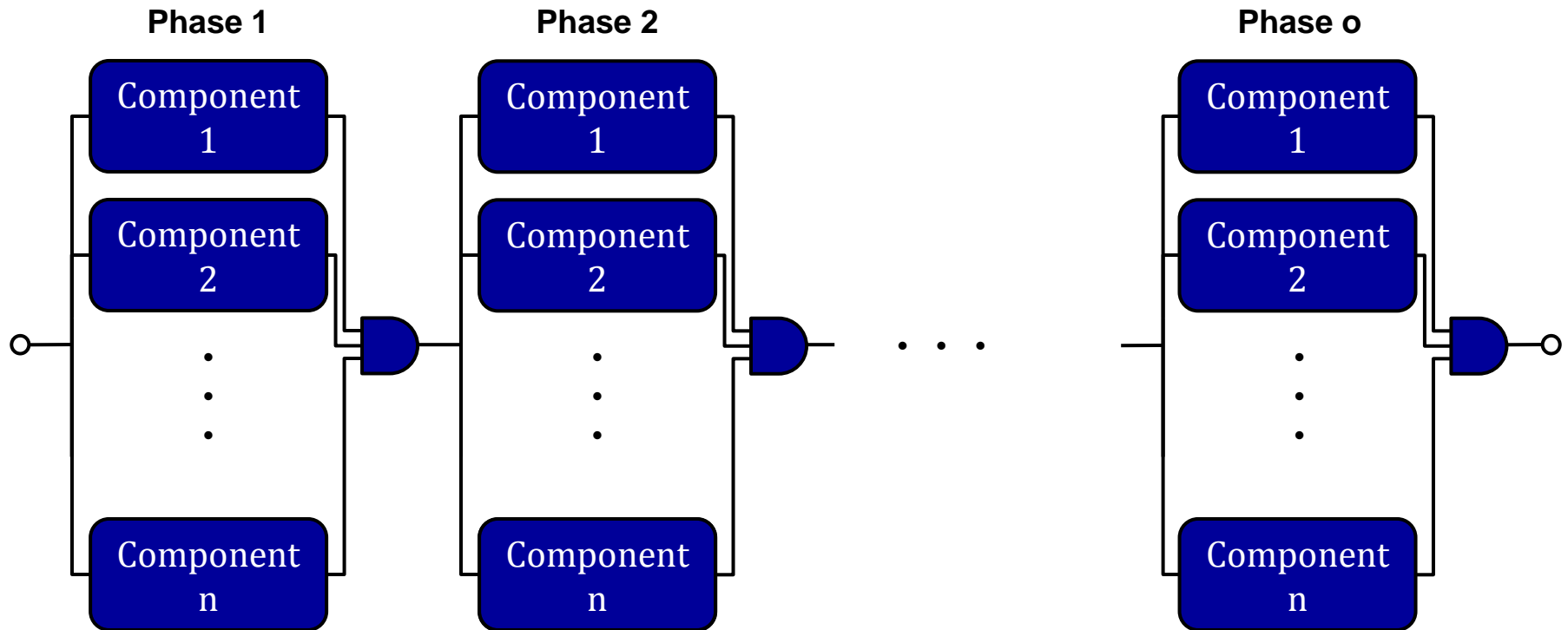
# Mixed Data Types

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- Live fire testing is cost prohibitive
- Other testing stresses different system components more or less than live fire testing
- Need to model reliability at component level to incorporate different types of testing



# Overall Model



- Multiple serial components
- Multiple phases of operation



# Proposed Method

- Estimates annual reliability with product of component test success rates
- Incorporates mixed types of data

$$P_t = \alpha \bar{R}_{t-1} + (1 - \alpha)P_{t-1}$$
$$t > 1$$

**Exponential Smoothing Equation**

$$\bar{R}_{t-1} = \prod_{i=1}^n \prod_{j=1}^o R_{i,j}$$

**Annual Reliability Estimate Equation**

$P_t$  - Projection at time  $t$

$R_t$  - Reliability at time  $t$

$t$  - Time index

$\alpha$  - Alpha value

$i$  - Component

$j$  - Phase



# Test Weighting

- Relative weights for different types of testing, phases of operation, and components

Test Phase	1					2					...	o				
Component	Test Type					Test Type					...	Test Type				
	FT	1	2	...	m	FT	1	2	...	m	...	FT	1	2	...	m
1	1.0	1.0	1.3	...	0.9	1.0	1.0	0.8	...	0.0	...	1.0	1.0	0.8	...	0.0
2	1.0	0.7	1.0	...	1.0	1.0	0.9	1.5	...	0.5	...	1.0	0.9	1.5	...	0.5
3	0.0	0.0	0.0	...	0.0	1.0	0.2	0.7	...	1.0	...	1.0	0.2	0.7	...	1.0
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
n	1.0	0.9	0.8	...	1.0	1.0	0.9	1.0	...	1.5	...	1.0	0.9	1.0	...	1.5



# Model 1

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$$R_{i,j} = \frac{S_{i,j,FT}}{S_{i,j,FT} + F_{i,j,FT}}$$

- Flight Testing only
- Control model

$R_t$  - Reliability at time  $t$   
 $S_{i,j,FT}$  - Flight Test successes for  
component  $i$ , phase  $j$

$F_{i,j,FT}$  - Flight test failures for  
component  $i$ , phase  $j$   
 $t$  - Time index





# Model 2

$$R_{i,j} = \frac{S_{i,j,FT} + S_{i,j,T1} + \dots + S_{i,j,Tm}}{S_{i,j,FT} + F_{i,j,FT} + S_{i,j,T1} + F_{i,j,T1} + \dots + S_{i,j,Tm} + F_{i,j,Tm}}$$

- Simple average
- Control model

$R_t$  - Reliability at time  $t$

$S_{i,j,FT}$  - Flight Test successes for component  $i$ , phase  $j$

$S_{i,j,Tm}$  - Test type  $m$  successes for component  $i$ , phase  $j$

$F_{i,j,FT}$  - Flight test failures for component  $i$ , phase  $j$

$F_{i,j,Tm}$  - Test type  $m$  failures for component  $i$ , phase  $j$

$t$  - Time index



# Model 3

$$R_{i,j} = \frac{S_{i,j,FT} + S_{i,j,T1} \cdot W_{i,j,T1} + \dots + S_{i,j,Tm} \cdot W_{i,j,Tm}}{S_{i,j,FT} + F_{i,j,FT} + S_{i,j,T1} \cdot W_{i,j,T1} + F_{i,j,T1} + \dots + S_{i,j,Tm} \cdot W_{i,j,Tm} + F_{i,j,Tm}}$$

## ■ Weighted Successes Model

$R_t$  - Reliability at time  $t$

$S_{i,j,FT}$  - Flight Test successes for component  $i$ , phase  $j$

$S_{i,j,Tm}$  - Test type  $m$  successes for component  $i$ , phase  $j$

$F_{i,j,FT}$  - Flight test failures for component  $i$ , phase  $j$

$F_{i,j,Tm}$  - Test type  $m$  failures for component  $i$ , phase  $j$

$W_{i,j,Tm}$  - Weight for test type  $m$ , component  $i$ , phase  $j$

$t$  - Time index



# Model 4

$$R_{i,j} = \frac{S_{i,j,FT} + S_{i,j,T1} + \dots + S_{i,j,Tm}}{S_{i,j,FT} + F_{i,j,FT} + S_{i,j,T1} + F_{i,j,T1} / W_{i,j,T1} + \dots + S_{i,j,Tm} + F_{i,j,Tm} / W_{i,j,Tm}}$$

## ■ Weighted Failures Model

$R_t$  - Reliability at time  $t$

$S_{i,j,FT}$  - Flight Test successes for component  $i$ , phase  $j$

$S_{i,j,Tm}$  - Test type  $m$  successes for component  $i$ , phase  $j$

$F_{i,j,FT}$  - Flight test failures for component  $i$ , phase  $j$

$F_{i,j,Tm}$  - Test type  $m$  failures for component  $i$ , phase  $j$

$W_{i,j,Tm}$  - Weight for test type  $m$ , component  $i$ , phase  $j$

$t$  - Time index



# Simulation

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- **Coded in Fortran 90**
  - **Input**
    - **Set of component reliabilities for each phase**
- **Number of each type of test**
- **Simulates 100,000 test years**
- **Compares estimated system reliability to input system reliability for each model**
- **Output**
  - **Mean error for estimate from input reliability for each model**
  - **Standard deviation of errors for each model**



# Simulation Parameters

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## ■ Several adjustable parameters for the simulation

■ Number of types of tests	2 - 5
■ Number of tests of a type	1 - 10
■ Number of components	30 - 45
■ Number of phases	1 - 3
■ Test weights	Varies
■ True component reliabilities	Varies
■ True system reliability	~0.5 - ~0.9



# Simulation

```
!-----
! Mixed Data Type Exponential Smoothing
! For Reliability Prediction - Model Selection

! Author: Jeremy L. Thompson

! 04 November 2011

! This algorithm compares the average performance of 4 potential models for integrating mixed data types into exponential smoothing
! for reliability prediction.

!-----
! Models:

! Model 1:  $[S_{T1}] / [S_{T1} + F_{T1}]$ 

! Model 2:  $[S_{T1} + S_{T1} + \dots + S_{Tm}] / [S_{T1} + F_{T1} + S_{T1} + F_{T1} + \dots + S_{Tm} + F_{Tm}]$ 

! Model 3:  $[S_{T1} + S_{T2} * W_{T2} + \dots + S_{Tm} * W_{Tm}] / [S_{T1} + F_{T1} + S_{T2} * W_{T2} + F_{T2} + \dots + S_{Tm} * W_{Tm} + F_{Tm}]$ 

! Model 4:  $[S_{T1} + S_{T2} + \dots + S_{Tm}] / [S_{T1} + F_{T1} + S_{T2} + F_{T2} / W_{T2} + \dots + S_{Tm} + F_{Tm} / W_{Tm}]$ 

! Notes:
!   Models 1 and 2 are control models
!   Models 3 and 4 are candidate models
!   If a weight factor is 0, then the success or failure of that test is not included

!-----

program modelselect

!-----
! Part 0: Setup
!-----

!-----
! 0.1 Define variables
!-----
implicit none
real (kind = 8) :: random, reld, wgtf, wgts
```



# Tested Parameter Combinations

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■ Extreme combinations	Min	Max
■ Number of types of tests	2	5
■ Number of tests of a type	1	10
■ Number of components	30	45
■ Number of phases	1	3
■ Test weights	Varies	
■ True component reliabilities	Varies	
■ True system reliability	~0.5	~0.9



# Tested Parameter Combinations

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■ Center combinations	Min	Max
■ Number of types of tests	3	4
■ Number of tests of a type	5	5
■ Number of components	37	38
■ Number of phases	2	2
■ Test weights	Varies	
■ True component reliabilities	Varies	
■ True system reliability	~0.7	~0.7





# Analysis Parameters

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## ■ Parameters were consolidated for analysis

■ Number of types of tests	2 - 5
■ Number of tests	2 - 50
■ Ratio of live fire tests to total tests	$0.\overline{09} - 0.\overline{90}$
■ Number of components	30 - 45
■ Number of phases	1 - 3
■ True system reliability	0.5 - 0.9



# Analysis

- MANOVA indicates that all parameters except number of phases affect model selection
- Optimal model choice depends upon system under test

```
> summary(model)
```

	Df	Pillai	approx F	num Df	den Df	Pr(>F)
data\$X.Types.of.Tests	1	0.05368	9.104	4	642	3.707e-07 ***
data\$System.Reliability	1	0.64649	293.512	4	642	< 2.2e-16 ***
data\$X.Tests	1	0.30713	71.145	4	642	< 2.2e-16 ***
data\$Ratio.Live.Total	1	0.38682	101.251	4	642	< 2.2e-16 ***
Residuals	645					

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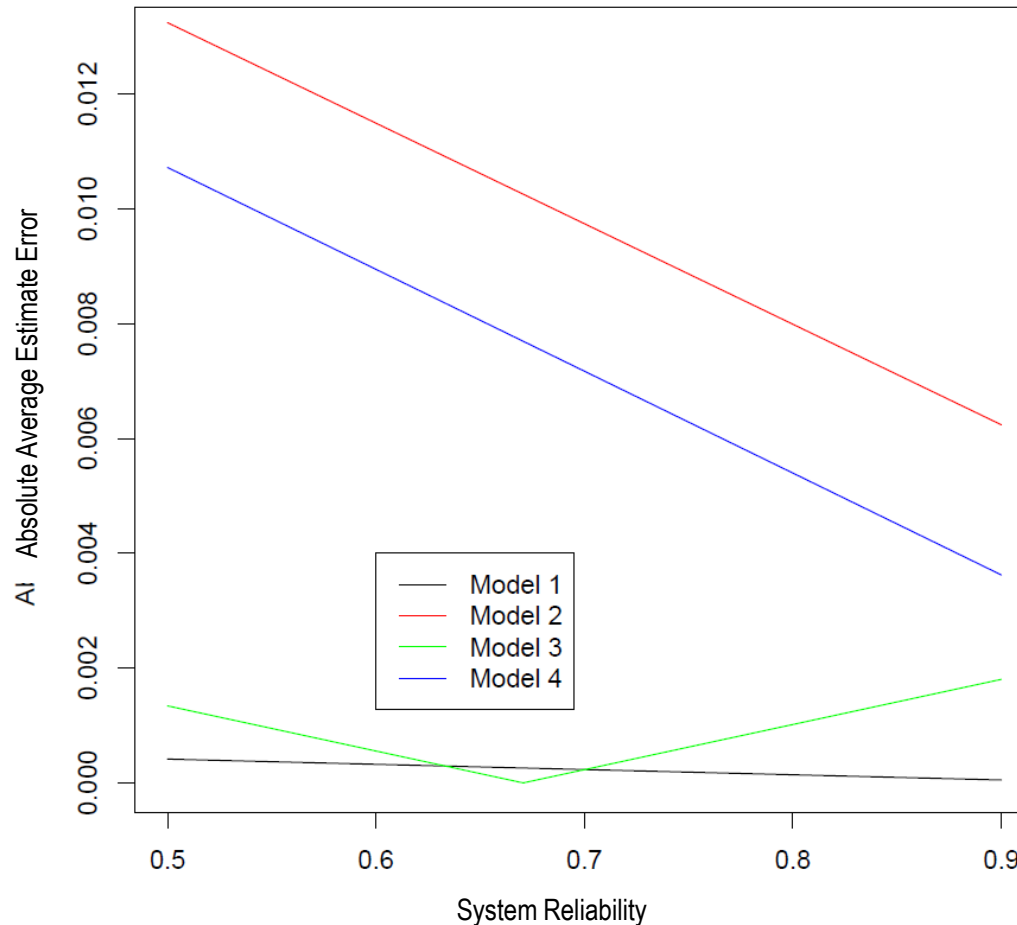
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1



# 1 Phase, 2 Types of Test, 20 Tests

## 0.25 Ratio of Live Fire Testing to Total

Absolute Average Estimate Error vs System Reliability

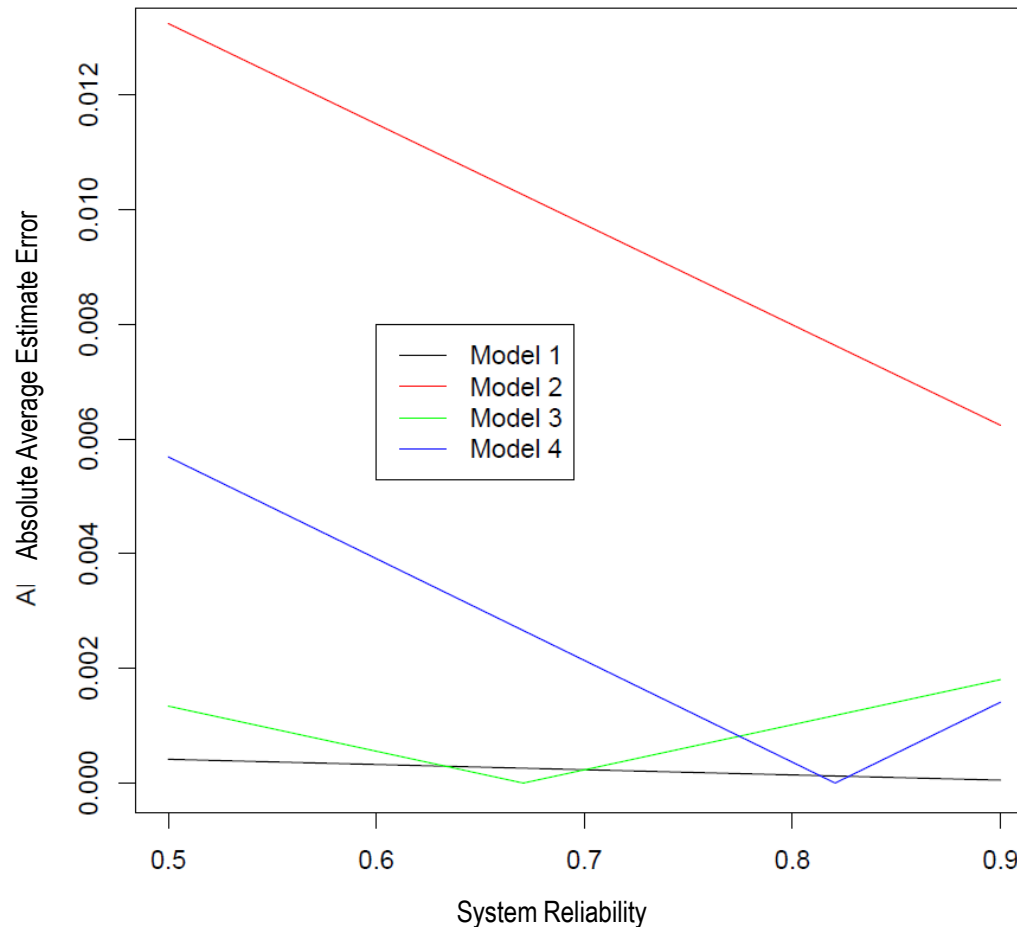




# 1 Phase, 2 Types of Test, 20 Tests

## 0.75 Ratio of Live Fire Testing to Total

Absolute Average Estimate Error vs System Reliability





# Summary

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- **Incorporating mixed data types can improve reliability estimates**
- **Model selection depends upon system under test**
  - **Simulation should be run for system under test**
  - **Simulation should be used for sensitivity analysis also**



# Questions

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## Questions?

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# Future Research

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- **Data transformation**
- **Confidence/prediction bands**
  - **Take into consideration varying confidence of annual reliability estimates**

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