Comparison Tests

MAKE CONCLUSIONS OR SUFFER

Integral test v.s. Limit comparison test: use comparison test whenever possible (it's easier)

EX) $\sum_{n=1}^{\infty} \frac{2^{\frac{1}{n}}}{n^2}$ Determine if it diverges or converges

Use LCT with $b_n = \frac{1}{n^2}$, $a_n = \frac{2^{\frac{1}{n}}}{n^2}$

Then $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{2^{\frac{1}{n}}}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} 2^{\frac{1}{n}} = 1$

Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series, p=2>1), the series $\sum_{n=1}^{\infty} \frac{2^{\frac{1}{n}}}{n^2}$ also converges by LCT

EX) $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

Use Comp Test: $n+\sqrt{n} < 2n$

$\frac{1}{n+\sqrt{n}} > \frac{1}{2n}$

Since $\frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

So does $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

By comp test

OR use LCT with $b_n = \frac{1}{n}$ and $a_n = \frac{1}{n+\sqrt{n}}$

then $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{n+\sqrt{n}} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{n}{n+\sqrt{n}} = 1$

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (p-series, p=1)

so does $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

EX) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}\tan^{-1}(n)}{n^3}$

Use LCT with $b_n = \frac{1}{n^2}$ and $a_n = \frac{\sqrt{n^2+1}\tan^{-1}(n)}{n^3}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\pi}{2}$

since $\sum_{\square}^{\square} \frac{1}{n^2}$ converges (p-series p=2>1)

So does $\sum_{\square}^{\square} \frac{\sqrt{n^2+1}\tan^{-1}(n)}{n^3}$

OR use Comp Test

$\frac{\sqrt{n^2+1}\tan^{-1}(n)}{n^3} < \frac{\pi}{2}\frac{\sqrt{n^2+n^2}}{n^3} = \frac{\pi}{\sqrt{2}}\frac{1}{n^2}$

EX) $\sum_{n=1}^{\infty} \frac{1}{n-\ln(n)}$

Use comp test

$n-\ln(n) < n$

$\rightarrow \frac{1}{n-\ln(n)} > \frac{1}{n}$

Note: The integral test and comparison tests only apply to positive series. Also, LCT doesn't require an inequality.