Triple Integrals

EX:

Let R denote the region in the first octant $(x, y, z \ >= \ 0)$ bounded between the planes $x + y + z = 1$ and $2x + 2y + z = 2$.

Solution:

- Sketch the region of R

- Determine the volume of R:

$$V = \int \int_R \int dV = \int_0^1 \int_0^{1-x} \int_{1-x-y}^{2(1-x-y)} dzdydx$$

$$= \int_0^1 \int_0^{1-x} z \int_{1-x-y}^{2(1-x-y)} dzdy = \int_0^1 \int_0^{1-x} (1-x-y)dydx$$

$= \int_0^1 [(1-x)y - \frac{1}{2}y^2]$ from 0 to $1 - x \ dx$

- Determine the average value of $f(x, y, z) = z$ over the region R:

Recall: $f_{average} = \frac{1}{volume(R)} \int \int_R \int (x, y, z)dV = \frac{1}{1/6} \int_0^1 \int_0^{1-x} \int_{1-x-y}^{2(1-x-y)} zdzdydx$

$= 6 \int_0^1 \int_0^{1-x} \frac{1}{2}z^2$ from $1 - x - y$ to $2(1 - x - y) \ dydx$

$$= 9 \int_0^1 \int_0^{1-x} \frac{1}{2}z^2(1-x-y)^2 dydx = ... = \frac{3}{4}$$

EX:

Compute the volume of the solid under the paraboloid $z = 9 - x^2 - y^2$, outside the cylinder $x^2 + y^2 = 1$, and above the xy plane.

Solution:

- Sketch the region R:

In the xy plane, there is an outer circle $x^2 + y^2 = 9$ and an inner circle $x^2 + y^2 = 1$. We are interested in the area between the inner and outer circles.

$V = \int\int_R \int dV = \int_0^{2\pi} \int_1^3 \int_0^{9-r^2} rdzdrd\theta = \int_0^{2\pi} d\theta \int_1^3 rz \int_0^{9-r^2} dr$

$= 2\pi \int_1^3 r(9-r^2)dr = 2\pi \int_1^3 [9r - r^3]dr = 2\pi[\frac{9}{2}r^2 - \frac{1}{4}r^4]$ from 1 to 3 $= .. = 32\pi$