Geometric and numerical methods in optimal control and applications to low thrust orbital transfer and swimming at low Reynolds number

Abstract:

The first part of this work is devoted to the study of the swimming at low Reynolds number. We deal with two models of microswimmers: one is a symmetric 2-link swimmer introduced by D. Takagi to model the movement of an abundant variety of zooplankton called Copepod and the seminal model introduced by E.M. Purcell, called the Purcell Three-link swimmer, which consists of three rigid links and is more intricate. We propose a geometric and numerical approach with tools of optimal control theory assuming for example that the motion occurs minimizing the energy dissipated by the drag fluid forces in relation with the concept of efficiency of a stroke. The Maximum Principle is used to compute periodic controls considered as minimizing control using proper transversality conditions, in relation with periodicity, minimizing the energy dissipated for a fixed displacement or maximizing the efficiency of a stroke. These problems fall into the framework of sub-Riemannian geometry which provides efficient techniques to tackle these problems, in particular, the nilpotent approximation can be used to compute strokes with small amplitudes and they can be continued numerically to compute more general strokes. Second order optimality, necessary or sufficient, are presented and numerically used to select weak minimizers, again in the framework of periodic optimal controls. This leads to a complete solution of the optimal problems for the Copepod swimmer, that is to select on each energy level an optimal stroke producing a given displacement and finally using a further selection to compute the most efficient stroke. In the second part, we focus on the motion of a spacecraft in a central field with perturbations and control. We take into account the perturbation arising from the gravitational interaction of the Moon or the perturbation resulting from the oblateness of the Earth. They are treated as perturbations of the two-body problem composed of the Earth and the spacecraft. Our purpose is to study the time minimal orbital transfer problem with low thrust using both analytical and numerical tools. Due to the small control amplitude, the transfer may take several revolutions around the Earth and our approach is to average the extremal flow provided by Pontryagin maximum principle. Minimizing trajectories are projections of the state space of the flow which lives in an extended space of dimension twice the dimension on the state. The difficulty to apply standard averaging techniques is that there is a priori no obvious way to identify orders and weights of variables in this extended space. We define an averaged system and study the related approximations to the non averaged system. We provide proofs of convergence and give numerical results where we use the averaged system to solve the non averaged system using indirect method implemented in the software `HamPath` .