Models Describing the Response of Skeletal Muscle

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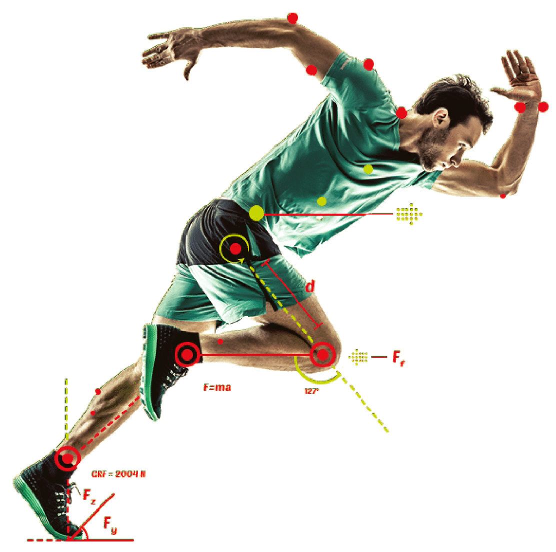
Abstract

Biomechanics is *the science of biological systems* and therefore studies their structures and functions. Thanks to biomechanics, it is now possible to model muscle of the human body. Biomechanics uses *engineering sciences* in order to analyze biological systems. Some *mechanical models* can supply correct approximations to the mechanics of many biological systems even if usually, they are much more complex than man-built systems. The aim of this article is to study the different models of the skeletal muscle from activation dynamics to contraction dynamics through linear and non-linear effects.



Introduction

Even if many researches in biomechanics are done in order to find the ultimate model that could predict and explain every aspect of the skeletal muscle, there are plenty of models that can characterize a muscle. The most popular one, and also the oldest one, is Archibald Vivian Hill's model. **But are the different models describing the response of skeletal muscle really accurate or too far from reality?**

Main Objectives

1. Describe models of activation and contraction dynamics.
2. Compare the linear and non-linear effects from the different models of the skeletal muscle.
3. Study the particularities of some models describing the muscle.
4. Discuss the different models and identify the best ones.

Activation Dynamics

This section presents two models :

- **Hill**'s mechanical model that treats the muscle as **a black box** and tried to find the input and output characteristics of a muscle during an effort.
- **Hammerstein**'s physiological model that describes **an electrically stimulated muscle**. It models the muscle movement by a *non-linear static system* combine with a *linear dynamic system*.

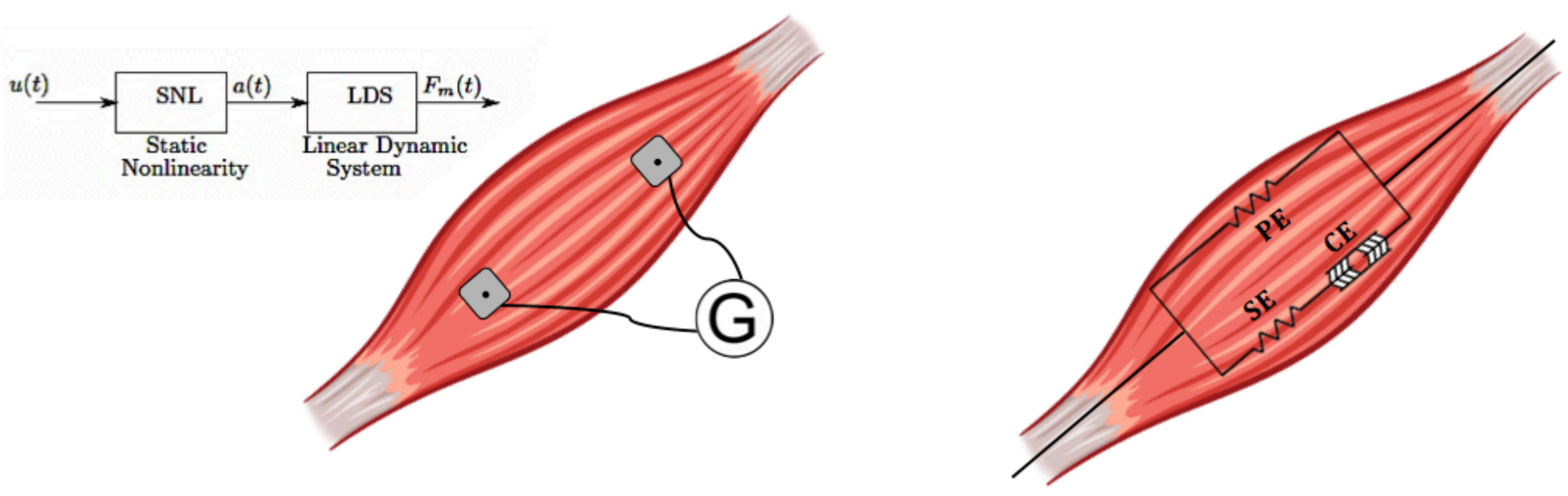


Figure 1: *(left)* Hammerstein's model. ($u(t)$: neutral stimulus, $a(t)$: activation, $F_m(t)$: muscle force). *(right)* Hill's muscle model. (PE : parallel element, SE : series element, CE : contractile element).

Because the activation and contraction dynamics are assumed to be uncoupled, a distinction between these two states has to be made.

Contraction Dynamics

The muscle contraction dynamics describe how a muscle activation is converted into a muscle force. Sometimes, both dynamics use the same model such as Hill's model.

This section goes further with Hill's model and presents one new model :

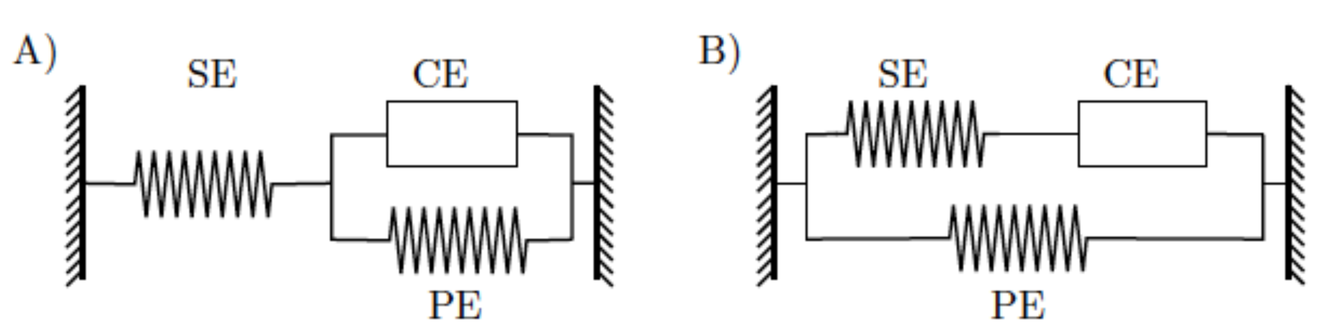


Figure 2: Popular functional arrangements of Hill-type lumped component models. A) Hill model with PE component in parallel with the CE. B) Hill model with PE component in parallel with both the SE and CE. A & B are two popular functional arrangements of the element but it's only conceptual.
SE represent tendons, CE represent muscle fibres and PE represent the sarcolemmma.

Hill-Type model, there is no relation between biological and mechanical behaviour. Hill's model represents fibres behaviour on a *macroscopic* scale.

Huxley-Type model describes a relationship between metabolic and mechanical behaviour. Huxley's model represents sarcomere on a *microscopic* scale.

	Advantages	Drawbacks
Hill-Type	- Mathematical simplicity - Response to a wide range of inputs	- Contractile properties in CE - Division of forces between CE & PE - Division of elongation between SE & CE
Huxley-Type	- Describe the sarcomere behaviour	- More complex than Hill's model

In the following sections both linear and non-linear models stand mainly for activation dynamics.

Linear Model

We will focus on an empirical model given by :

$$\theta_3 F^{(3)}(t) + \theta_2 F^{(2)}(t) + \theta_1 F^{(1)}(t) + F(t) = \theta_0 u(t)$$

where θ_3, θ_2, θ_1, θ_0 are parameters and $F(t)$ represents the muscle force response. The aim of this model is to determine the values of θ_0, θ_1, θ_2 and θ_3. $u(t)$ is the electrical signal input provided by the muscle.
Linear models do not accurate properly the reality because many parameters are overlooked. Actually the muscle is a complex non-linear model.

Non-Linear Models

Bobet and Stein Model

Bobet and Stein model is a five dimensions non-linear model with parameters such as: the signal input, the force response output, the amount of Ca^{2+}, the muscle displacement. The model shown below represents the muscle force depending on the time during the pulse trains. This experience was made by electrical stimulations applied to cat's *soleus muscle*. According to the results, the measures reflect the model expectations.

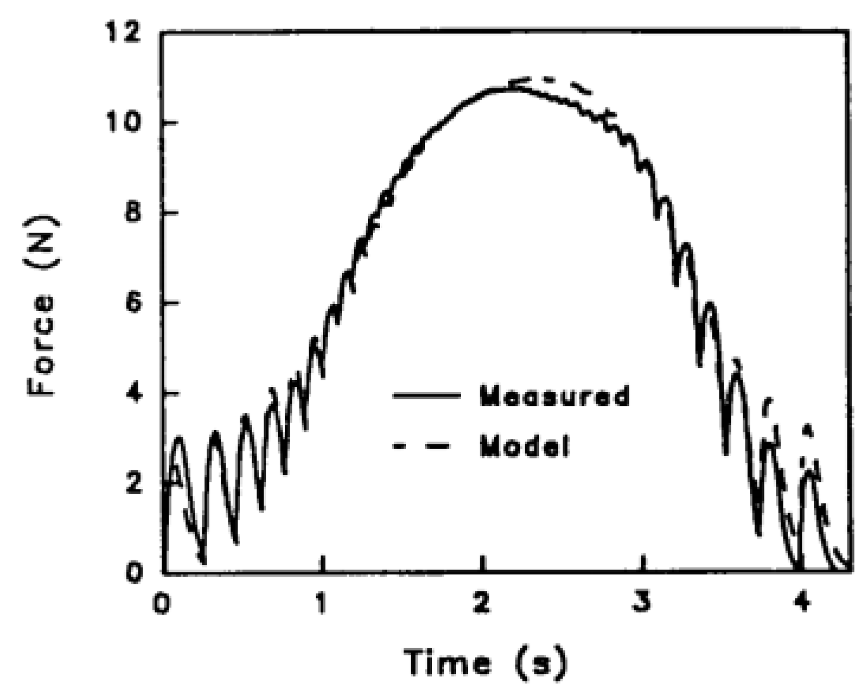


Figure 3: Graphic representation of the muscular response in the Bobet and Stein Model.

Ding and al Model

This model, based on a Hill-Type model, is represented by two differential equations :

$$\frac{dC_N}{dt} = \frac{1}{\tau_c}\sum_{i=1}^{n} R_i e^{-\frac{t-t_i}{\tau_c}} - \frac{C_N}{\tau_c} \qquad \text{with} \qquad R_i = 1 + (R_0 - 1)\, e^{-\frac{t-t_i}{\tau_c}}$$

$$\text{and} \qquad \frac{dF}{dt} = A\frac{C_N}{K_m + C_N} - \frac{F}{\tau_1 + \tau_2 \frac{C_N}{K_m + C_N}}.$$

Where C_N is the amount of Ca^{2+}-troponin complex, τ_c is the time constant controlling the rise and decay of C_N, F is the instantaneous force, K_m is the sensitivity, τ_1 and τ_2 are the time force of force decline.

The first one represents the formation of Ca^{2+} − troponin complex by the activation of the muscle and the second one represents the development of the mechanical force. The non-linearity comes from the factor $-\frac{t-t_i}{\tau_c}$ in R_i formula. This factor represents a physiological phase called tetany where the muscle is on a unintended long contraction state. The model has six parameters, τ_c, R_o, A, k, τ_1, τ_2 and uses several constant frequency, variable frequency and doublet-frequency trains.

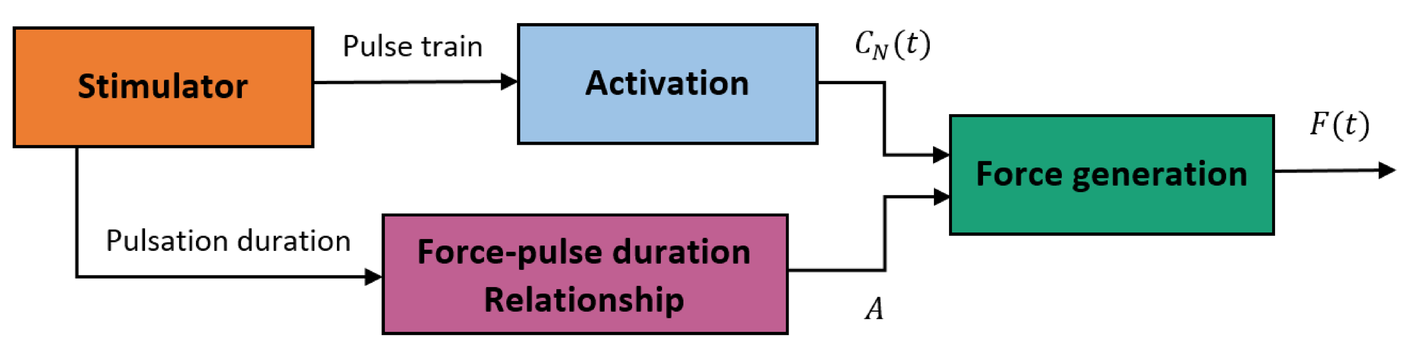


Figure 4: A representation of Ding and al model.

Conclusion and Forthcoming Research

Both activation and contraction dynamics are represented by models **quite close** to the reality. Although the scientific **majority** is using Hill's model that is proven to be the most **efficient** one. Today, there is a real interest on models and on finding mathematical equations that describe the **real physiological phenomena** of the muscle. Many researchers are still working on in order to find the **ultimate model** that could predict and explain the response of skeletal muscle.

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