Review on the Thesis by Jérémy Rouot

Méthodes géométriques et numériques en contrôle optimal et applications au transfert orbital à poussée faible et à la nage à faible nombre de Reynolds

by Ugo Boscain

The present thesis deals with problems of optimal swimming at low Reynolds numbers and of minimum time orbital transfer. The common point is the use of techniques of geometric optimal control including suitable numerical methods.

The thesis is divided in 3 parts. The first part (Chapters 1,2,3,4) is devoted to introductory material mainly related to swimmers. The second part (Chapters 5,6) is research material and contains two preprints about optimal swimming. The third part (Chapters 7,8,9) is devoted to the problem of optimal orbital transfer. It contains discussions about averaging techniques and their applications to the Pontryagin Maximum Principle and certain numerical results. In the following I briefly describe the different chapters.

Chapter 1 contains the description of certain models of swimmers: the Copepod swimmer and the three-link Purcell swimmers. The problem of swimming is then formulated as an optimal control problem.

Chapter 2 is devoted to basic material in optimal control: Pontryagin maximum Principle, second order optimality conditions, and sufficient conditions.

In Chapter 3 the author introduces sub-Riemannian geometry as a class of optimal control problems. The Chow theorem is presented together with equations for extremals (normal and abnormal). He describes the nilpotent approximation and discusses the appearance of the cut and of the conjugate locus.

In Chapter 4 numerical methods for optimal control are introduced: direct methods, indirect methods, and homotopic methods. Certain softwares are also discussed as the Bocop and the HamPath softwares.

Chapter 5 contains material of a preprint. The three-link Purcell swimmers is presented. The corresponding sub-Riemannian problem is not simple: it has a grow vector of type $(2, 3, 5)$ and probably it is not Liouville integrable. The authors study the nilpotent approximation, thereby obtaining an analysis describing the behaviour of the three-link Purcell swimmers for small deformations up to higher order errors. The nilpotent approximation is the famous Cartan model, and the authors find explicit expression of geodesics in terms of elliptic functions. The authors study conjugate points, abnormal extremals and get some interesting numerical analysis of the trajectories using different numerical methods. They are particularly interested to periodic geodesics.

Chapter 6 contains material of a second preprint. In the first part, it contains the study of optimal strokes for the Copepod swimmers and the comparison with same solutions presented in the literature. In the second part, a further analysis of the three-link Purcell swimmer is pursued using numerical continuation methods and sufficient conditions for optimality.

In Chapter 7, the author introduces the two-body problem with the perturbations introduced by the earth and by the moon. The orbital transfer problem is treated as an optimal control problem. After discussing the difficulty in solving such a problem, even numerically, the author introduces averaging techniques.

Chapter 8 is devoted to find approximated solutions of the Pontryagin Maximum Principle via averaging techniques. Different types of averaging techniques are used depending whether the perturbation given by the moon is take into account or not. Convergence results are presented.

Chapters 9 present numerical experiments and conjectures.

The thesis contains several original and interesting results. It is well-written and shows a uncommon ability of treating problems coming from applications with different techniques, from differential geometry up to sophisticated numerical methods. The results are useful and will be certainly published on high level journals.

Concluding, for the quantity and the quality of the results, I give an "avis très favorable à la soutenance de cette thèse".

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