⟨ Slides → Forward | Inverse models ⟩

Efference copy, empirical examples: eye movement through muscles vs through finger; can't tickle yourself.

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Internal Models (forward & inverse), Cerebellum

u_t : Motor command at t
x_t : state / trajectory

$$x_{t+1} = f(x_t, u_t) \quad \text{traj / dynamics eqn}$$

Aim of controller: issue appropriate u_t given x_{t+1} (desired state at t+1)

$$u_t = g(x^*_{t+1}, x_t)$$

- g not necessarily unique (may not even exist, but still ok to look for 'g' if it does)

Such that:

$$x^*_{t+1} = f(x_t, g(x^*_{t+1}, x_t))$$

But dynamics can change over time & with context, use a context signal c_t ← context at time t.

$$x_{t+1} = f(x_t, u_t, c_t)$$

⇒ Same state, control pair, but different outcome depending on context.

Aim of controller is to learn the control system under different & unknown contexts.

Wolpert & colleagues: modular control systems: a bunch of fwd. models

$\hat{x}^1_{t+1}, \hat{x}^2_{t+1} \ldots \hat{x}^n_{t+1}$ → { n fwd models producing some prediction x^i_{t+1} for time t+1

$$x^i_{t+1} = \Phi(w^i_t, x_t, u_t)$$

x^i_{t+1} → i^{th} module

w^i_t → Parameters of function approximator Φ
eg: wts of a neural network to model fwd dynamics.

Responsibility Signal (relative error)

$$\lambda^i_t = \frac{e^{-|x_t - \hat{x}^i_t|^2/\sigma^2}}{\sum_j e^{-|x_t - \hat{x}^j_t|^2/\sigma^2}} \quad \text{Softmax function}$$

The closer $\hat{x}^i_t$ to x_t, the larger λ^i_t

Total fwd model prediction

$$\hat{x}_{t+1} = \sum \lambda_t^i x_{t+1}^i = \sum \lambda_t^i \phi(w_t^i, x_t, u_t)$$

Learning of fwd models is wtd. by responsibility:

$$\Delta w_t^i = \varepsilon \lambda_t^i \underbrace{\frac{d\phi_i}{dw_t^i}}_{\phi_i'} (x_t - \hat{x}_t^i) = \varepsilon \frac{d\hat{x}_t^i}{dw_t^i} \lambda_t^i (x_t - \hat{x}_t^i)$$

fwd models divides up the system dynamics experienced, reflecting which model best captures the current behaviour of the system

$$u_t = \sum \lambda_t^i u_t^i$$

λ_t^i ↓ responsibility signal of module 'i'.

u_t^i → control signal from inv. model of module i

$$= \underbrace{\sum \lambda_t^i \psi(\alpha_t^i, x_t, x_{t+1}^*)}_{\text{inv. model of module } i.}$$

$$\Delta \alpha_t^i = \varepsilon \lambda_t^i \frac{d\psi}{d\alpha_t^i} (u_t^* - u_t^i)$$

u_t^* ↑ total desired motor command is known