

תרגילים: מערכות משוואות ליניאריות

שאלה 1

$$x + y - 2z = 7$$

$$2x - y + z = 0$$

$$x + y - z = 6$$

שאלה 2

$$y + z = 3$$

$$3x + 5y + 9z = -2$$

$$x + 2y + 3z = 3$$

שאלה 3

$$2x + 2y + 2z = 0$$

$$-2x + 5y + 2z = 1$$

$$8x + y + 4z = -1$$

שאלה 4

$$y + 5z = -4$$

$$x + 4y + z = -2$$

$$2x + 7y + z = -1$$

שאלה 5

$$x + 2y + 3z = 3$$

$$2x + 3y + 8z = 4$$

$$3x + 2y + 17z = 1$$

שאלה 6

$$2x - 3y + 5z = 8$$

$$2x + 4y - 6z = -5$$

$$x + 2y - 3z = -1$$

שאלה 7

$$x + 2y + z = 2$$

$$3x + y - 2z = 1$$

$$4x + 3y - z = 3$$

$$2x + 4y + 2z = 4$$

שאלה 8

$$3x + y + z + w = 0$$

$$5x - y + z - w = 0$$

שאלה 9

$$2x + y + 3z = 0$$

$$x + 2y = 0$$

$$-x + z = 0$$

שאלה 10

$$2x - y - 3z = 0$$

$$-x + 2y - 3z = 0$$

$$x + y + 4z = 0$$

שאלה 11

$$3x - y + 2z = 0$$

$$y + z = 0$$

$$2x - z = 0$$

$$x - y + 3z = 0$$

פתרונות

שאלה 1

$$\begin{aligned}
 & \left(\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 2 & -1 & 1 & 0 \\ 1 & 1 & -1 & 6 \end{array} \right) \xrightarrow[\substack{R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - R_1}]{} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 0 & -3 & 5 & -14 \\ 0 & 0 & 1 & -1 \end{array} \right) \\
 & \xrightarrow{R_2 \rightarrow -\frac{1}{3} \cdot R_2} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 0 & 1 & -\frac{5}{3} & \frac{14}{3} \\ 0 & 0 & 1 & -1 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 + \frac{5}{3} \cdot R_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right) \\
 & \xrightarrow{R_1 \rightarrow R_1 - R_2 + 2R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right) \\
 & (x, y, z) = (2, 3, -1) .
 \end{aligned}$$

שאלה 2

$$\begin{aligned}
 & \left(\begin{array}{ccc|c} 0 & 1 & 1 & 3 \\ 3 & 5 & 9 & -2 \\ 1 & 2 & 3 & 3 \end{array} \right) \xrightarrow{R_3 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 3 & 5 & 9 & -2 \\ 0 & 1 & 1 & 3 \end{array} \right) \\
 & \xrightarrow{R_2 \rightarrow R_2 - 3R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & 0 & -11 \\ 0 & 1 & 1 & 3 \end{array} \right) \\
 & \xrightarrow{R_2 \rightarrow (-1) \cdot R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & 0 & 11 \\ 0 & 1 & 1 & 3 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & -8 \end{array} \right) \\
 & \xrightarrow{R_1 \rightarrow R_1 - 2R_2 + 3R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & -8 \end{array} \right) \\
 & (x, y, z) = (5, 11, -8) .
 \end{aligned}$$

שאלה 3

$$\begin{aligned}
 & \left(\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array} \right) \xrightarrow{R_1 \rightarrow \frac{1}{2} \cdot R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array} \right) \\
 & \xrightarrow{\substack{R_2 \rightarrow R_2 + 2R_1 \\ R_3 \rightarrow R_3 - 8R_1}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{array} \right) \\
 & \xrightarrow{R_2 \rightarrow \frac{1}{7} \cdot R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & -7 & -4 & -1 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 + 7R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{array} \right) \\
 & \xrightarrow{R_1 \rightarrow R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & \frac{3}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{array} \right) \\
 & \left. \begin{array}{l} x + \frac{3}{7}z = -\frac{1}{7} \\ y + \frac{4}{7}z = \frac{1}{7} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = -\frac{1}{7} - \frac{3}{7}z \\ y = \frac{1}{7} - \frac{4}{7}z \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = \frac{-1-3z}{7} \\ y = \frac{1-4z}{7}, z \end{array} \right\} \\
 & (x, y, z) = \left(\frac{-1-3z}{7}, \frac{1-4z}{7}, z \right), \quad z \in \mathbb{R}.
 \end{aligned}$$

שאלה 4

$$\begin{aligned}
 & \left(\begin{array}{ccc|c} 0 & 1 & 5 & -4 \\ 1 & 4 & 1 & -2 \\ 2 & 7 & 1 & -1 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 4 & 1 & -2 \\ 0 & 1 & 5 & -4 \\ 2 & 7 & 1 & -1 \end{array} \right) \\
 & \xrightarrow{R_3 \rightarrow R_3 - 2R_1} \left(\begin{array}{ccc|c} 1 & 4 & 1 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & -1 & -1 & 3 \end{array} \right) \\
 & \xrightarrow{R_3 \rightarrow R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 4 & 1 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 4 & -1 \end{array} \right) \xrightarrow{R_3 \rightarrow \frac{1}{4} \cdot R_3} \left(\begin{array}{ccc|c} 1 & 4 & 1 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 1 & -\frac{1}{4} \end{array} \right) \\
 & \xrightarrow{R_2 \rightarrow R_2 - 5 \cdot R_3} \left(\begin{array}{ccc|c} 1 & 4 & 1 & -2 \\ 0 & 1 & 0 & -\frac{11}{4} \\ 0 & 0 & 1 & -\frac{1}{4} \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 - 4 \cdot R_2 - R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{37}{4} \\ 0 & 1 & 0 & -\frac{11}{4} \\ 0 & 0 & 1 & -\frac{1}{4} \end{array} \right) \\
 & (x, y, z) = \left(\frac{37}{4}, \frac{-11}{4}, \frac{-1}{4} \right).
 \end{aligned}$$

שאלה 5

$$\begin{aligned}
 & \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 3 & 8 & 4 \\ 3 & 2 & 17 & 1 \end{array} \right) \xrightarrow[R_3 \rightarrow R_3 - 3R_1]{R_2 \rightarrow R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & 2 & -2 \\ 0 & -4 & 8 & -8 \end{array} \right) \\
 & \xrightarrow{R_2 \rightarrow (-1) \cdot R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & -4 & 8 & -8 \end{array} \right) \\
 & \xrightarrow{R_3 \rightarrow R_3 + 4R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & 7 & -1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right) \\
 & \left. \begin{array}{l} x + 7z = -1 \\ y - 2z = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = -1 - 7z \\ y = 2 + 2z \end{array} \right\} \\
 & (x, y, z) = (-1 - 7z, 2 + 2z, z) , \quad z \in \mathbb{R} .
 \end{aligned}$$

שאלה 6

$$\begin{aligned}
 & \left(\begin{array}{ccc|c} 2 & -3 & 5 & 8 \\ 2 & 4 & -6 & -5 \\ 1 & 2 & -3 & -1 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 2 & -3 & 5 & 8 \\ 2 & 4 & -6 & -5 \end{array} \right) \\
 & \xrightarrow[R_3 \rightarrow R_3 - 2R_1]{R_2 \rightarrow R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 10 \\ 0 & 0 & 0 & 3 \end{array} \right)
 \end{aligned}$$

קיבלנו שורת סתירה לפיכך למערכת אין פתרון.

שאלה 7

$$\begin{aligned}
 & \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 1 & -2 & 1 \\ 4 & 3 & -1 & 3 \\ 2 & 4 & 2 & 4 \end{array} \right) \xrightarrow[R_4 \rightarrow R_4 - 2 \cdot R_1]{R_2 \rightarrow R_2 - 3 \cdot R_1, R_3 \rightarrow R_3 - 4 \cdot R_1} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -5 & -5 & -5 \\ 0 & -5 & -5 & -5 \\ 0 & 0 & 0 & 0 \end{array} \right) \\
 & \xrightarrow{R_2 \rightarrow -\frac{1}{5} \cdot R_2} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -5 & -5 & -5 \\ 0 & 0 & 0 & 0 \end{array} \right) \\
 & \xrightarrow{R_3 \rightarrow R_3 + 5 \cdot R_2} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 - 2 \cdot R_2} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \\
 & \left. \begin{array}{l} x - z = 0 \\ y + z = 1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = z \\ y = 1 - z \end{array} \right\} \\
 & (x, y, z) = (z, 1 - z, z) , \quad z \in \mathbb{R} .
 \end{aligned}$$

שאלה 8

$$\begin{aligned}
 & \left(\begin{array}{cccc|c} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{array} \right) \xrightarrow{R_1 \rightarrow 2 \cdot R_1 - R_2} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 3 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{array} \right) \\
 & \xrightarrow{R_2 \rightarrow R_2 - 5 \cdot R_1} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 3 & 0 \\ 0 & -16 & -4 & -16 & 0 \end{array} \right) \\
 & \xrightarrow{R_2 \rightarrow -\frac{1}{16} \cdot R_2} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 3 & 0 \\ 0 & 1 & \frac{1}{4} & 1 & 0 \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 - 3 \cdot R_2} \left(\begin{array}{cccc|c} 1 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & \frac{1}{4} & 1 & 0 \end{array} \right) \\
 & \left. \begin{array}{l} x + \frac{1}{4}z = 0 \\ y + \frac{1}{4}z + w = 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = -\frac{1}{4}z \\ y = -\frac{1}{4}z - w \end{array} \right\} \\
 & (x, y, z) = \left(-\frac{z}{4}, -\frac{z}{4} - w, z, w \right), \quad z, w \in \mathbb{R}.
 \end{aligned}$$

שאלה 9

$$(x, y, z) = (0, 0, 0) .$$

שאלה 10

$$(x, y, z) = (0, 0, 0) .$$

שאלה 11

$$(x, y, z) = (0, 0, 0) .$$