אלגברה ליניארית 1
סמסטר א' תשפ"ד
תרגילים: מערכות משוואות לינאריות

פתרו את המערכות הבאות:

שאלה 1

$$\begin{aligned} x+y-2z &= 7 \\ 2x-y+z &= 0 \\ x+y-z &= 6 \end{aligned}$$

שאלה 2

$$\begin{aligned} y+z &= 3 \\ 3x+5y+9z &= -2 \\ x+2y+3z &= 3 \end{aligned}$$

שאלה 3

$$\begin{aligned} 2x+2y+2z &= 0 \\ -2x+5y+2z &= 1 \\ 8x+y+4z &= -1 \end{aligned}$$

שאלה 4

$$\begin{aligned} y+5z &= -4 \\ x+4y+z &= -2 \\ 2x+7y+z &= -1 \end{aligned}$$

שאלה 5

$$\begin{aligned} x+2y+3z &= 3 \\ 2x+3y+8z &= 4 \\ 3x+2y+17z &= 1 \end{aligned}$$

שאלה 6

$$\begin{aligned} 2x-3y+5z &= 8 \\ 2x+4y-6z &= -5 \\ x+2y-3z &= -1 \end{aligned}$$

שאלה 7

$$\begin{aligned} x + 2y + z &= 2 \\ 3x + y - 2z &= 1 \\ 4x + 3y - z &= 3 \\ 2x + 4y + 2z &= 4 \end{aligned}$$

שאלה 8

$$\begin{aligned} 3x + y + z + w &= 0 \\ 5x - y + z - w &= 0 \end{aligned}$$

שאלה 9

$$\begin{aligned} 2x + y + 3z &= 0 \\ x + 2y &= 0 \\ -x + z &= 0 \end{aligned}$$

שאלה 10

$$\begin{aligned} 2x - y - 3z &= 0 \\ -x + 2y - 3z &= 0 \\ x + y + 4z &= 0 \end{aligned}$$

שאלה 11

$$\begin{aligned} 3x - y + 2z &= 0 \\ y + z &= 0 \\ 2x - z &= 0 \\ x - y + 3z &= 0 \end{aligned}$$

פתרונות

שאלה 1

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 2 & -1 & 1 & 0 \\ 1 & 1 & -1 & 6 \end{array}\right) \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 0 & -3 & 5 & -14 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\xrightarrow{R_2 \to -\frac{1}{3} \cdot R_2} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 0 & 1 & -\frac{5}{3} & \frac{14}{3} \\ 0 & 0 & 1 & -1 \end{array}\right) \xrightarrow{R_2 \to R_2 + \frac{5}{3} \cdot R_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - R_2 + 2R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$(x, y, z) = (2, 3, -1) \ .$$

שאלה 2

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & 3 \\ 3 & 5 & 9 & -2 \\ 1 & 2 & 3 & 3 \end{array}\right) \xrightarrow{R_3 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 3 & 5 & 9 & -2 \\ 0 & 1 & 1 & 3 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 3R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & 0 & -11 \\ 0 & 1 & 1 & 3 \end{array}\right)$$

$$\xrightarrow{R_2 \to (-1) \cdot R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & 0 & 11 \\ 0 & 1 & 1 & 3 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & -8 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 2R_2 - 3R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & -8 \end{array}\right)$$

$$(x, y, z) = (5, 11, -8) \ .$$

שאלה 3

$$\left(\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right) \xrightarrow{R_1 \to \frac{1}{2}\cdot R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right)$$

$$\xrightarrow[R_3 \to R_3 - 8R_1]{R_2 \to R_2 + 2R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{array}\right)$$

$$\xrightarrow{R_2 \to \frac{1}{7}\cdot R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & -7 & -4 & -1 \end{array}\right) \xrightarrow{R_3 \to R_3 + 7R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & \frac{3}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left.\begin{array}{rl} x + \frac{3}{7}z &= -\frac{1}{7} \\ y + \frac{4}{7} &= \frac{1}{7} \end{array}\right\} \quad \Rightarrow \quad \left.\begin{array}{rl} x &= -\frac{1}{7} - \frac{3}{7}z \\ y &= \frac{1}{7} - \frac{4}{7}z \end{array}\right\} \quad \Rightarrow \quad \left.\begin{array}{rl} x &= \frac{-1-3z}{7} \\ y &= \frac{1-4z}{7}, z \end{array}\right\}$$

$$(x, y, z) = \left(\frac{-1-3z}{7}, \frac{1-4z}{7}, z\right) \ , \qquad z \in \mathbb{R} \ .$$

שאלה 4

$$\left(\begin{array}{ccc|c} 0 & 1 & 5 & -4 \\ 1 & 4 & 1 & -2 \\ 2 & 7 & 1 & -1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 4 & 1 & -2 \\ 0 & 1 & 5 & -4 \\ 2 & 7 & 1 & -1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - 2R_1} \left(\begin{array}{ccc|c} 1 & 4 & 1 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & -1 & -1 & 3 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 4 & 1 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 4 & -1 \end{array}\right) \xrightarrow{R_3 \to \frac{1}{4}\cdot R_3} \left(\begin{array}{ccc|c} 1 & 4 & 1 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 1 & -\frac{1}{4} \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 5\cdot R_2} \left(\begin{array}{ccc|c} 1 & 4 & 1 & -2 \\ 0 & 1 & 0 & -\frac{11}{4} \\ 0 & 0 & 1 & -\frac{1}{4} \end{array}\right) \xrightarrow{R_1 \to R_1 - 4\cdot R_2 - R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{37}{4} \\ 0 & 1 & 0 & -\frac{11}{4} \\ 0 & 0 & 1 & -\frac{1}{4} \end{array}\right)$$

$$(x, y, z) = \left(\frac{37}{4}, \frac{-11}{4}, \frac{-1}{4}\right) \ .$$

שאלה 5

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 3 & 8 & 4 \\ 3 & 2 & 17 & 1 \end{array}\right) \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & 2 & -2 \\ 0 & -4 & 8 & -8 \end{array}\right)$$

$$\xrightarrow{R_2 \to (-1)\cdot R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & -4 & 8 & -8 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + 4R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & 7 & -1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left.\begin{array}{rl} x + 7z &= -1 \\ y - 2z &= 2 \end{array}\right\} \quad \Rightarrow \quad \left.\begin{array}{rl} x &= -1 - 7z \\ y &= 2 + 2z \end{array}\right\}$$

$$(x, y, z) = (-1 - 7z, 2 + 2z, z) \ , \qquad z \in \mathbb{R} \ .$$

שאלה 6

$$\left(\begin{array}{ccc|c} 2 & -3 & 5 & 8 \\ 2 & 4 & -6 & -5 \\ 1 & 2 & -3 & -1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 2 & -3 & 5 & 8 \\ 2 & 4 & -6 & -5 \end{array}\right)$$

$$\xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 10 \\ 0 & 0 & 0 & 3 \end{array}\right)$$

קיבלנו שורת סתירה לפיכך למערכת אין פתרון.

שאלה 7

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 1 & -2 & 1 \\ 4 & 3 & -1 & 3 \\ 2 & 4 & 2 & 4 \end{array}\right) \xrightarrow[R_4 \to R_4 - 2\cdot R_1]{\substack{R_2 \to R_2 - 3\cdot R_1 \\ R_3 \to R_3 - 4\cdot R_1}} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -5 & -5 & -5 \\ 0 & -5 & -5 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_2 \to -\frac{1}{5}\cdot R_2} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -5 & -5 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + 5\cdot R_2} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1 \to R_1 - 2\cdot R_2} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left.\begin{array}{rl} x - z &= 0 \\ y + z &= 1 \end{array}\right\} \quad \Rightarrow \quad \left.\begin{array}{rl} x &= z \\ y &= 1 - z \end{array}\right\}$$

$$(x, y, z) = (z, 1 - z, z) \ , \qquad z \in \mathbb{R} \ .$$

שאלה 8

$$\left(\begin{array}{cccc|c} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{array}\right) \xrightarrow{R_1 \to 2\cdot R_1 - R_2} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 3 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 5\cdot R_1} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 3 & 0 \\ 0 & -16 & -4 & -16 & 0 \end{array}\right)$$

$$\xrightarrow{R_2 \to -\frac{1}{16}\cdot R_2} \left(\begin{array}{cccc|c} 1 & 3 & 1 & 3 & 0 \\ 0 & 1 & \frac{1}{4} & 1 & 0 \end{array}\right) \xrightarrow{R_1 \to R_1 - 3\cdot R_2} \left(\begin{array}{cccc|c} 1 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & \frac{1}{4} & 1 & 0 \end{array}\right)$$

$$\left.\begin{array}{rl} x + \frac{1}{4}z &= 0 \\ y + \frac{1}{4}z + w &= 0 \end{array}\right\} \quad \Rightarrow \quad \left.\begin{array}{rl} x &= -\frac{1}{4}z \\ y &= -\frac{1}{4}z - w \end{array}\right\}$$

$$(x, y, z) = \left(-\frac{z}{4}\ ,\ -\frac{z}{4} - w\ ,\ z\ ,\ w\right)\ , \qquad z, w \in \mathbb{R}\ .$$

שאלה 9

$$(x, y, z) = (0, 0, 0)\ .$$

שאלה 10

$$(x, y, z) = (0, 0, 0)\ .$$

שאלה 11

$$(x, y, z) = (0, 0, 0)\ .$$