

Asteroid belt survival through stellar evolution: dependence on the stellar mass

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ABSTRACT

Polluted white dwarfs are generally accreting terrestrial-like material that may originate from a debris belt like the asteroid belt in the Solar system. The fraction of white dwarfs that are polluted drops off significantly for white dwarfs with masses $M_{\text{WD}} \gtrsim 0.8 M_{\odot}$. This implies that asteroid belts and planetary systems around main-sequence (MS) stars with mass $M_{\text{MS}} \gtrsim 3 M_{\odot}$ may not form because of the intense radiation from the star. This is in agreement with current debris disc and exoplanet observations. The fraction of white dwarfs that show pollution also drops off significantly for low-mass white dwarfs ($M_{\text{WD}} \lesssim 0.55 M_{\odot}$). However, the low-mass white dwarfs that do show pollution are not currently accreting but have accreted in the past. We suggest that asteroid belts around MS stars with masses $M_{\text{MS}} \lesssim 2 M_{\odot}$ are not likely to survive the stellar evolution process. The destruction likely occurs during the AGB phase and could be the result of interactions of the asteroids with the stellar wind, the high radiation, or, for the lowest mass stars that have an unusually close-in asteroid belt, scattering during the tidal orbital decay of the inner planetary system.

Key words: minor planets, asteroids: general – planets and satellites: dynamical evolution and stability – stars: AGB and post-AGB – white dwarfs.

1 INTRODUCTION

White dwarfs are about 10^5 times more dense than the Earth and so gravitational settling of heavy elements in the atmosphere is fast, less than around a few tens of Myr (e.g. Paquette et al. 1986; Wyatt et al. 2014). If the cooling age is older than this, but less than about 500 Myr, the atmosphere consists of hydrogen and/or helium only. Thus, the detection of metals in the atmosphere suggests accretion of material on to the white dwarf (e.g. Veras 2016). Observations show that at least 27 per cent of white dwarfs with cooling ages of 20–200 Myr are currently accreting debris and an additional 29 per cent have accreted material in the past (Koester, Gänsicke & Farihi 2014).

The composition of the white dwarf polluting material is similar to that of the bulk Earth and Solar system meteorites (e.g. Gänsicke et al. 2012; Jura & Young 2014; Xu et al. 2014; Farihi 2016; Harrison, Bonsor & Madhusudhan 2018; Hollands, Gänsicke & Koester 2018; Doyle et al. 2019; Swan, Farihi & Wilson 2019; Bonsor et al. 2020). Therefore, it must have formed inside of the snow line radius, the radius outside of which water is found in the form of ice that occurs at temperatures ~ 170 K in the protoplanetary disc (e.g. Podolak & Zucker 2004; Lecar et al. 2006; Kennedy & Kenyon 2008; Min et al. 2011; Martin & Livio 2012, 2013b). The

material may be delivered to the white dwarf from a planetesimal belt similar to the asteroid belt in the Solar system rather than a Kuiper belt equivalent. Only in rare cases is volatile rich material accreted (e.g. Xu et al. 2017). Other suggested sources include delivery by moons (Payne et al. 2016, 2017) or fragments of broken up terrestrial planets (or moons) (Malamud & Perets 2020a, b). Given the observed white dwarf accretion rates, it is expected that most polluted white dwarfs have a reservoir of mass at least comparable to the mass in the asteroid belt in the Solar system (Zuckerman et al. 2010).

Asteroidal material is delivered to the white dwarf through a debris disc close to the white dwarf that forms through tidal disruptions (e.g. Jura 2003; Debes, Walsh & Stark 2012; Veras et al. 2014a, 2015b; Xu et al. 2018; Malamud & Perets 2020a,b). Asteroids may be perturbed into highly eccentric orbits through interactions with undetected planets (e.g. Debes et al. 2012; Frewen & Hansen 2014; Bonsor & Veras 2015; Smallwood et al. 2018). We suggest that for a white dwarf to be polluted over long time-scales there are two requirements. First, an asteroid belt must form around the main-sequence (MS) star. Secondly, the asteroid belt must survive the stellar evolution process. In Section 2, we examine the properties of MS stars that host debris discs and planetary systems. We further examine observational evidence for planetary systems and debris discs around evolved stars including polluted white dwarfs. In Section 3, we propose that asteroid belts around low-mass MS stars (those with mass less than about $2 M_{\odot}$) are destroyed during stellar

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evolution leading to a lack of polluting material around low-mass white dwarfs (those with mass less than about $0.55 M_{\odot}$). We draw our conclusions in Section 4.

2 OBSERVATIONS OF DEBRIS DISCS AND PLANETARY SYSTEMS

In this section, we first examine observational evidence for the formation of asteroid belts around MS stars. The detection of asteroid belts themselves is difficult, but stars that host planetary systems may have undetected asteroid belts. We consider here the range of masses of MS stars that host planets and debris discs. We then explore the evidence for planetary systems around giant stars. Finally, we investigate observational evidence that suggests that asteroid belts that form around stars with mass greater than about $2 M_{\odot}$ can survive through the stellar evolution process.

2.1 Main-sequence star systems

Most stars in the Milky Way host planetary systems (Cassan et al. 2012). Nearly all observed exoplanets have been found around stars with masses $M_{\text{MS}} \lesssim 3 M_{\odot}$. There are only a few exceptions that have higher stellar mass. The highest mass star with a well-determined mass that hosts a planet is UMa that has mass $3.09 \pm 0.07 M_{\odot}$ (Sato et al. 2012). This upper mass limit is not sharp transition but a tail where the number of planets discovered decreases with host star mass (e.g. Reffert et al. 2015; Ghezzi, Montet & Johnson 2018). Observing planets around O-type and B-type stars is difficult and so the limit is a combination of detection limitations and where planets can form and survive around more massive stars (Kennedy & Kenyon 2008; Veras et al. 2020).

Debris discs are detected around about 25 per cent of MS stars (Hughes, Duchêne & Matthews 2018). Discs are observed around stars with masses $\lesssim 2.4 M_{\odot}$ (Koenig & Allen 2011). Discs around more massive stars may be photoevaporated on time-scales which are too short to be observed due to intense radiation from the host star.

Debris discs are observed through the thermal emission of the dust and may be characterized by the infrared excess observed in their SED. The excess may generally be modelled with one or two blackbody components (e.g. Su et al. 2009, 2013). The cold components have temperatures < 130 K while the warm components have temperatures ~ 190 K (Morales et al. 2011; Ballering et al. 2013; Chen et al. 2014). The warm and cold components come from different radial locations with different temperatures (Kennedy & Wyatt 2014). Debris disc observations show that two-component structures, like the asteroid belt and the Kuiper belt in the Solar system, are common (e.g. Kennedy & Wyatt 2014; Rebollido et al. 2018). The cause for the gap between the belts is likely the formation of planets in the gap that remove all the planetesimals from the region. Geiler & Krivov (2017) found that that 98 per cent of observed debris disc systems can be explained with a two-component structure rather than a one-component structure. The few sources for which warm dust in the systems cannot be explained by this structure must be a result of cometary sources or a recent major collision or planetary system instability.

Giant planets are thought to form outside of the snow line radius, since there is a higher density of solid material there (e.g. Pollack et al. 1996). Thus, asteroid belts may coincide with the location of the snow line radius (Martin & Livio 2013a). Ballering et al. (2017) found that the warm dust components in single-component systems (those without a cold component) are aligned with the

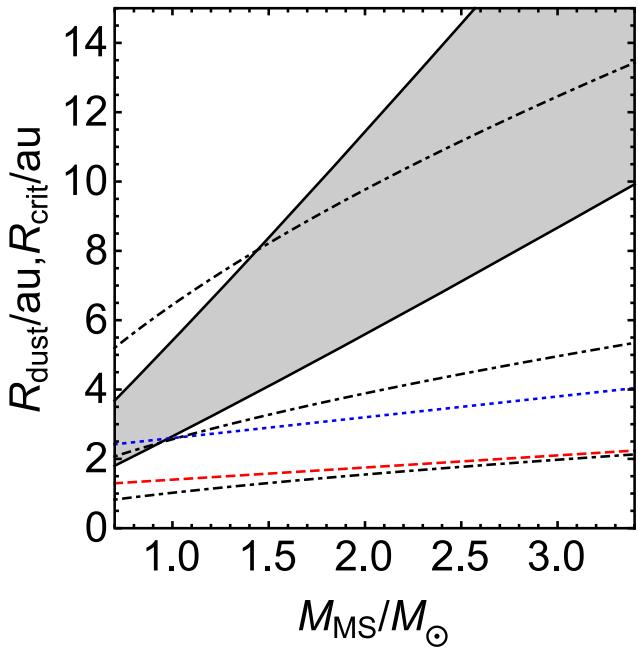


Figure 1. The shaded region shows the observed best-fitting region for the location of warm dust belts (those with temperature ~ 190 K) around MS stars (data from Ballering et al. 2017). The dotted blue line shows the critical initial semimajor axis above which a jovian planet survives the asymptotic giant branch (AGB) phase. The dashed red line shows the critical initial semimajor axis above which a terrestrial planet survives the AGB phase. These theoretical lines are approximated from the stellar evolution models of Mustill & Villaver (2012). The dot-dashed lines show theoretical survival radii for 100 m (upper), 1 km (middle), and 10 km sized asteroids approximated by equation (1) (Dong et al. 2010).

primordial snow line, meaning the snow line in the protoplanetary disc. However, in two-component systems, the location is more diverse. The belts, at least in the one-component system, may be formed of terrestrial material. The location of the warm dust belts in one-component models has a best-fitting $R_{\text{dust}}/\text{au} = 3.68(M/M_{\odot})^{1.08}$ (Ballering et al. 2017). The shaded region in Fig. 1 shows the 1σ scatter around the best-fitting line to the radius of warm dust belts (Ballering et al. 2017). In the two-component models, the warm dust components show little correlation with stellar mass and are scattered in the approximate range 0.5–30 au. We discuss this figure in more detail in Section 3.

2.2 Giant star systems

To date, 112 substellar companions¹ around 102 G and K giant stars have been found (e.g. Reichert et al. 2019). Grunblatt et al. (2019) investigated 2476 low-luminosity red giant branch stars observed by the K2 mission (Howell et al. 2014). They found a higher occurrence rate of planets with size greater than Jupiter in orbital periods less than 10 d compared to around dwarf Sun-like stars. This suggests that the effects of stellar evolution on the occurrence of close-in planets that are larger than Jupiter are not significant until the star moves significantly up the red giant branch. Debris discs have also been observed around giants suggesting that they can also survive the stellar evolution (e.g. Bonsor et al. 2013, 2014). Debris

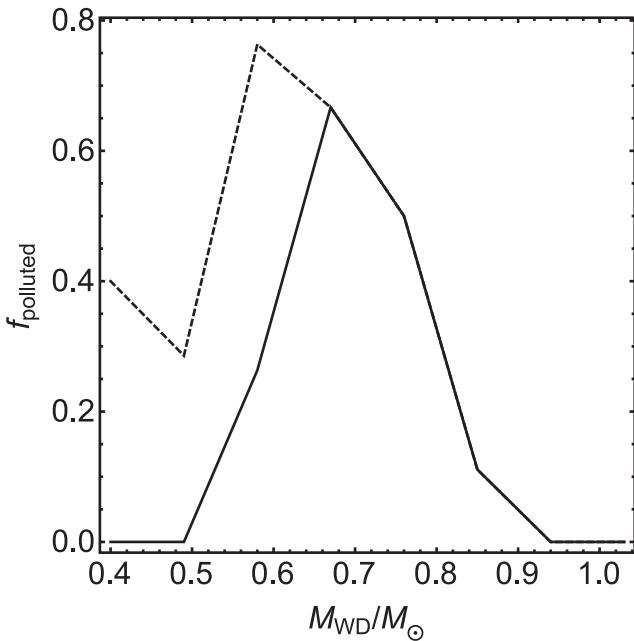


Figure 2. The fraction of the observed white dwarfs that show evidence for accretion (dashed line) and the fraction of currently accreting white dwarfs (solid line) as a function of the white dwarf mass. The data are taken from Koester et al. (2014).

discs around giant stars are more difficult to detect than around MS stars because radiation pressure removes small-particle dust around higher luminosity stars (Bonsor & Wyatt 2010).

2.3 White dwarf systems

The fraction of white dwarfs that host debris discs are somewhere between a few per cent up to 100 per cent, but most discs are too faint to detect (e.g. Barber et al. 2012; Veras 2016; Bonsor et al. 2017; Swan et al. 2019). The current observational limit is reached for discs around white dwarfs with cooling ages $t_{\text{cool}} > 0.5$ Myr (Bergfors et al. 2014). Compact debris discs are thought to be formed by the tidal disruption of small bodies around the white dwarf (Jura 2003; Farihi, Jura & Zuckerman 2009; Veras et al. 2014a). Atomic emission lines suggest the existence of gaseous discs co-located with the compact circumstellar dust (Gänsicke et al. 2006; Guo et al. 2015). There is one exception to the compactness of gaseous discs, the disc around WD J0914+1914 is thought to be formed from an evaporating giant planet on a close-in orbit around the white dwarf (Gänsicke et al. 2019).

Since white dwarfs are intrinsically faint, transit searches for debris and planets are difficult. However, the light curve of WD 1145+017 shows transit features thought to be produced by dust clouds released by planetesimals that orbit the white dwarf with an orbital period of about 4.5 h (Vanderburg et al. 2015; Gänsicke et al. 2016). There is also evidence for solid objects orbiting around white dwarfs SDSS J1228+1040 (Manser et al. 2019) and ZTF J0139+5245 (Vanderbosch et al. 2019). The discoveries made so far have arisen from ZTF, GTC, and SDSS. van Sluijs & Van Eylen (2018) examined a sample of 1148 white dwarfs observed by K2 and did not identify any substellar body transits with orbital separation < 0.5 au.

Fig. 2 shows the fraction of observed white dwarfs that are either currently accreting or show evidence for past accretion using the

data from Koester et al. (2014). The highest mass white dwarf with evidence for pollution has mass $M_{\text{WD}} = 0.91 M_{\odot}$ (Gentile Fusillo et al. 2019). This corresponds to a progenitor MS star mass of about $4 M_{\odot}$ (Koester et al. 2014). However, there is a transition where the fraction of white dwarfs that are polluted falls off significantly at a mass of around $M_{\text{WD}} = 0.8 M_{\odot}$. This corresponds to an MS star of around $M_{\text{MS}} = 3 M_{\odot}$. This suggests that asteroid belt formation or survival around high-mass stars (those with mass $\gtrsim 3 M_{\odot}$) is difficult. Stars with mass greater than about $3 M_{\odot}$ are too hot for the formation of a long-lived dusty disc. This is consistent with the observations of debris discs and planetary systems around MS stars discussed in Section 2.1.

Recently, Veras et al. (2020) explored the limits on the locations of planets that would be able to survive to the white dwarf phase around stars with masses in the range $6-8 M_{\odot}$. They found that a major planet must be located at orbital distance greater than about 3–6 au at the end of the MS lifetime in order to survive stellar evolution. The orbital radius outside of which minor planets survive is in the range 10–1000 au depending on planet size. Thus, if white dwarf pollution is to be observed around higher mass white dwarfs in the future it would come from already fragmented debris since the minor planets would likely not be still intact.

While the number of white dwarfs included in the data drops off at low masses, there does also appear to be a transition at small masses for which the fraction of white dwarfs that are polluted drops, at around $M_{\text{WD}} = 0.55 M_{\odot}$. The white dwarfs with these low masses tend to be younger and no longer accreting. We therefore suggest that asteroid belts around low-mass MS stars with mass less than $2 M_{\odot}$ may form, but they do not survive the stellar evolution process to the formation of the white dwarf. We discuss possible theoretical explanations for this scenario in the next section.

3 ASTEROID BELT DESTRUCTION AROUND LOW-MASS STARS

In this section, we examine theoretical models for the evolution of asteroid orbits through stellar evolution. Our goal is to explain why the asteroid belts around stars with mass less than about $2 M_{\odot}$ may not survive the process, while those around more massive stars (those with mass $2-3 M_{\odot}$) do.

3.1 Planet survival

Planets and debris that are close to the star during the MS will not survive stellar evolution to the white dwarf phase as they may be engulfed or evaporated by a giant star (e.g. Villaver & Livio 2007, 2009; Kunitomo et al. 2011). Bodies that become engulfed by the star are expected to be destroyed unless their mass is a Jupiter mass or more (e.g. Livio & Soker 1984; Mustill et al. 2018). The star is largest during the AGB phase and at that time its size in au is about equal to its initial MS masses in M_{\odot} for mass in the range $1-5 M_{\odot}$ (e.g. Mustill et al. 2018). For higher stellar mass, there is more mass-loss that occurs during the AGB phase. The mass-loss leads to the expansion of the orbits of substellar bodies and therefore allows them to survive even if they begin at radii such that the stellar radius subsequently expands beyond (e.g. Livio & Soker 1984; Mustill & Villaver 2012).

There are two competing effects that determine where the critical survival orbital radius is for a planet mass body. The tidal force pulls the object towards the expanded envelope while the effects of stellar mass-loss push the planet away (e.g. Mustill & Villaver 2012). Tidal forces are stronger for more massive planets and so

the survival radius increases with planet mass. The more massive the MS star, the farther out planets must be to survive engulfment. Fig. 1 shows approximate survival radii for initially circular orbit terrestrial planets (dashed red line) and Jupiter mass planets (dotted blue line) (Mustill & Villaver 2012). The survival radii are larger for eccentric planets (Villaver et al. 2014). The corresponding lines for the RGB would be much closer in (e.g. Kunitomo et al. 2011; Villaver et al. 2014).

Terrestrial planets form inside of the snow line, and hence inside of the warm dust belt location (e.g. Raymond et al. 2009). The critical orbital radius for which terrestrial planets survive stellar evolution (the red dashed line in Fig. 1) is close to the location of the warm dust belts (the shaded region in Fig. 1). Consequently, terrestrial planets around stars with mass less than about $1 M_{\odot}$ may not survive until the white dwarf phase. However, around higher mass stars, there is a range of orbital radii between the location of the warm dust belt and the critical survival radius where terrestrial planets may survive.

Close-in giant planets also do not survive. If there is a giant planet that is engulfed, as it spirals in it may disrupt an interior asteroid belt. Thus, it would seem that for asteroid belt survival we require the giant planets to survive the engulfment process. However, as shown in Fig. 1, the observed asteroid belts are at radii larger than the critical survival radius for a Jupiter mass planet. Thus, scattering of asteroids from a planet undergoing tidal decay is unlikely, except perhaps for the lowest mass stars that have close-in asteroid belts.

3.2 Planetesimal survival

For a planetesimal to survive stellar evolution to the white dwarf phase, it must not be engulfed by the star itself. Considering only the effects of stellar mass-loss and tidal forces, this is a less stringent constraint than that which applies to the survival of giant planets, since the planetesimal exerts only a weak tidal torque. The orbital locations of the warm dust belts are much larger than the maximum size of an AGB star (see Fig. 1) and so engulfment is not likely unless there is an unusually strong gas drag in the stellar wind.

The adiabatic approximation for the expansion of the orbits of planet and asteroid objects may be employed within about 100 au (Veras et al. 2011, 2016). Orbital eccentricity is conserved and the relative semimajor axis increase scales with the relative stellar mass-loss. Additionally, asteroids may interact strongly with the radiation from the AGB star. The interaction is complex since it depends upon the shape, orientation, and albedo of each asteroid. Asteroids can be radiatively pushed by the Yarkovsky effect (Bottke et al. 2001, 2006; Veras, Higuchi & Ida 2019). The Yarkovsky drift may be several orders of magnitude larger than that from Poynting–Robertson and radiation pressure (Veras, Eggel & Gänsicke 2015a). Asymmetric asteroids can be spun up through the YORP effect (e.g. Rubincam 2000; Vokrouhlický & Čapek 2002). The YORP effect may destroy asteroids with sizes 100 m–10 km at orbital radii $\lesssim 7$ au (Veras, Jacobson & Gänsicke 2014b; Veras & Scheeres 2020). For such conditions, the YORP effect alone may be responsible for the destruction of asteroid belts around low-mass MS stars (those with mass less than about $2 M_{\odot}$), while those around more massive stars survive because they are at larger orbital radii.

For asteroids with sizes small enough that the Yarkovsky effect is not important, the drag force becomes dominant (Veras et al. 2015a). The survival of planetesimals during the AGB phase may be determined by balancing the expansion of the orbit due to the stellar

mass-loss and the gas resistance. The critical radius for survival is

$$R_{\text{crit}} = 2.57 \left(\frac{M_{\text{MS}}}{M_{\odot}} \right)^{3/5} \left(\frac{s}{0.1 \text{ km}} \right)^{-2/5} \text{ au} \quad (1)$$

(Dong et al. 2010), where s is the asteroid size and we assume a wind speed $v_{\text{wind}} = 10 \text{ km s}^{-1}$ and an asteroid density of $\rho = 3 \text{ g cm}^{-3}$. In Fig. 1, we show the critical survival radius for asteroids of size 100 m (upper dot-dashed line), 1 km (middle dot-dashed line), and 10 km (lower dot-dashed line). The smaller asteroids in most belts may not survive the wind loss, while larger asteroids can. Asteroid belts around low-mass stars (with mass less than about $1 M_{\odot}$) may be removed for sizes $\lesssim 1$ km. Thus, asteroid belts around low-mass stars may be severely depleted in mass through the interaction with the stellar wind.

4 CONCLUSIONS

There is strong observational and theoretical evidence that white dwarf pollution occurs from asteroid belt-like material. There are significant drop offs in the fraction of white dwarfs that are polluted at masses higher than about $0.8 M_{\odot}$ and lower than about $0.55 M_{\odot}$. We have therefore proposed that (i) asteroid belts (and planetary systems) do not form around stars more massive than about $3 M_{\odot}$ and (ii) asteroid belts around stars less massive than about $2 M_{\odot}$ do not survive stellar evolution to the white dwarf stage. There are several mechanisms that can contribute to asteroid belt destruction during the AGB phase. These include the interaction of asteroids with the stellar wind through gas drag and the YORP effect, both of which affect the close-in asteroid belts around lower mass MS stars. The orbital decay of a giant planet due to tides may scatter an inner asteroid belt for the very lowest mass stars.

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