

Polar alignment of a circumbinary disc around a brown dwarf binary

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ABSTRACT

Inspired by recent observations suggesting that the retrograde precession of the brown dwarf binary 2M1510 AB is consistent with induction by a polar circumbinary planet, we investigate the formation of such planets by studying the evolution of a primordial misaligned circumbinary disc around a brown dwarf binary. Analytical calculations show that a critical tilt angle of $i_{\text{crit}} \gtrsim 50^\circ$ for moderately eccentric binaries is needed for polar alignment of circumbinary discs in systems with low disc-to-binary angular momentum ratios. For higher ratios, this angle converges to the Kozai–Lidov instability threshold of $\sim 39^\circ$. We identify disc parameters, such as viscosity ($\alpha = 10^{-4}$) and aspect ratio ($H/r = 0.05$), that enable polar alignment within typical disc lifetimes. Notably, a circumbinary disc around a low-mass binary, such as a brown dwarf binary, will require more time to achieve polar alignment compared to higher mass systems. A hydrodynamical simulation confirms that an initially inclined disc around a brown dwarf evolves towards a polar state, creating favourable conditions for polar planet formation. Using these results, we finish by placing 2M1510 AB into a wider context and speculate why such a polar circumbinary configuration has not been identified before.

Key words: hydrodynamics – methods: analytical – planets and satellites: formation – protoplanetary discs – binaries: general – brown dwarfs.

1 INTRODUCTION

Binary brown dwarf systems are critical laboratories for studying the formation, evolution, and physical properties of substellar objects (Stassun, Mathieu & Valenti 2006; Triaud et al. 2020). Brown dwarfs occupy the mass range between stars and planets, typically between ~ 13 and 75 Jupiter masses, where core hydrogen fusion is insufficient to sustain long-term energy production (Kumar 1963; Chabrier et al. 2000; Burrows et al. 2001; Phillips et al. 2020). The intermediate mass range makes them a unique bridge in our understanding of stellar and planetary formation processes. Furthermore, despite there being very few exoplanets known to orbit brown dwarfs, understanding their properties is a chance to investigate how the outcomes of planet formation scale with central mass.

Investigating brown dwarf multiplicity provides critical insights into their formation mechanisms by constraining key parameters such as orbital separation (a_b) and mass ratio ($q = M_B/M_A$). For stellar binaries, orbital separations follow a lognormal distribution, with higher mass stars exhibiting larger peak separations (e.g. Raghavan et al. 2010; De Rosa et al. 2014; Winters et al. 2019), while mass ratio distributions vary with primary mass, transitioning from bottom-heavy in O-stars to top-heavy in brown dwarfs (Duchêne & Kraus 2013; Offner et al. 2022). Brown dwarf binaries typically form tighter orbital configurations, with early studies of T0 to T8 spectral types (Burgasser et al. 2003, 2006) identifying systems with separations under 5 au and mass ratios above 0.8. However, these

studies were limited in sensitivity, primarily detecting systems with $q > 0.5$. Recent work has expanded the observed parameter space, revealing a broader range of mass ratios ($q = 0.2$ –1) and separations ($a = 0.1$ –1000 au; e.g. Fontanive et al. 2018).

Recently, Triaud et al. (2020) reported the discovery of a fully substellar, hierarchical triple system in which the inner pair – 2MASSW J1510478–281817 AB (hereafter 2M1510 AB), constitutes a double-lined, eclipsing brown-dwarf binary, while the outer tertiary, 2MASS J15104761–2818234 (hereafter 2M1510 C), resides at a projected separation of ~ 250 au. Both components were originally catalogued in the Two Micron All-Sky Survey (Skrutskie et al. 2003). High-precision photometric and spectroscopic monitoring of 2M1510 AB yields an orbital period $P_{\text{orb}} \approx 20$ d and an orbital eccentricity $e = 0.36 \pm 0.02$ (Triaud et al. 2020). At a mean *Gaia* DR2 parallax distance of 36.6 ± 0.3 pc (Bailer-Jones et al. 2018), the tertiary’s projected separation corresponds to ~ 250 au. Kinematic analysis – combining *Gaia* proper motions, parallax, and ground-based radial velocities – confirms that all three substellar components are co-moving members of the $\sim 45 \pm 5$ Myr Argus moving group (Gagné et al. 2015; Zuckerman 2019; Triaud et al. 2020). Complementary observations by Calissendorff et al. (2019) uncovered a fourth, non-eclipsing companion (designated 2M1510 D) with an inferred mass of $M_D \approx 17.7^{+4.2}_{-2.1} M_{\text{Jup}}$ on a ~ 4.4 au orbit about the AB pair. However, 2M1510 D was observed at only a single epoch. Constraining its proper motion requires observations at two or more epochs, and future observations are needed to confirm whether this companion candidate is gravitationally bound to the 2M1510 system.

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Baycroft et al. (2025) performed radial velocity measurements of the brown dwarf components, revealing that the binary undergoes retrograde apsidal precession. They propose this precession is induced by the gravitational influence of a circumbinary planet in a near polar orbital configuration. For such a polar planet to exist around 2M1510 AB, a primordial disc would need to have formed initially misaligned with the binary orbital plane and subsequently evolved into a near-polar configuration before planet formation commenced (as in Smallwood et al. 2020). Polar-aligned circumbinary discs are thought to provide a favourable environment for the formation of polar planets (Smallwood et al. 2024a, b). Despite the lack of confirmed polar circumbinary planets, there is indirect evidence of such an orbital configuration in the post-asymptotic giant branch star binary AC Her (Martin et al. 2023).

Besides the claimed highly inclined planet 2M1510 (AB)b, there is also one example of a potentially misaligned planetary-mass object orbiting a brown dwarf binary. VHS 1256 (AB)b is a directly imaged planetary-mass companion orbiting a binary brown dwarf, VHS 1256 AB (total mass $\sim 0.141 \pm 0.008 M_{\odot}$; Dupuy et al. 2023), in a hierarchical triple system (Gauza et al. 2015; Rich et al. 2016; Stone et al. 2016). VHS 1256 (AB)b has an estimated mass of $12.0 \pm 0.1 M_{\text{jup}}$ or $16.0 \pm 1 M_{\text{jup}}$ (Dupuy et al. 2023; Miles et al. 2023), with a separation of $\sim 350^{+110}_{-150}$ au and an estimated orbital eccentricity of 0.7 ± 0.1 . The separation and eccentricity of the binary are 1.96 ± 0.03 au and 0.883 ± 0.003 , respectively, with a system age of 140 ± 20 Myr. Poon et al. (2024) estimated the inclination of VHS 1256 (AB)b with respect to the binary orbital plane to be $\sim 115 \pm 14^\circ$ (Dupuy et al. 2023), which is close to a polar orbit.

While no circumbinary discs around brown dwarf binaries have been directly observed, simulations suggest they can form and accrete material under certain conditions (e.g. Bate, Bonnell & Bromm 2003; Dittmann & Ryan 2024). Circumbinary discs are observed at various stages of stellar evolution (e.g. Czekala et al. 2019), implying that such discs around brown dwarf binaries are plausible and likely share properties with those around single brown dwarfs. Brown dwarf discs, initially detected in near- and mid-infrared surveys (Comeron et al. 1998; Muench et al. 2001; Natta & Testi 2001; Natta et al. 2002; Jayawardhana et al. 2003), are now studied across infrared, submillimetre, and millimetre wavelengths (Daemgen et al. 2016; Testi et al. 2016). These discs resemble those around classical T Tauri stars, exhibiting similar diversity in geometry and dust properties (Mohanty et al. 2004). Evidence of millimetre-sized dust grains in brown dwarf discs has been identified (e.g. Apai et al. 2005; Scholz et al. 2007), with high-resolution ALMA observations confirming substantial dust grain growth (Ricci et al. 2012, 2014).

In this letter, we investigate for the first time how the polar alignment of a circumbinary disc around a brown dwarf binary differs from that around a solar-type binary. We apply our findings to the system 2M1510 AB, which shows potential signatures of a polar circumbinary planet. If such a planet exists, it would likely have formed from a primordial polar-aligned circumbinary disc around the system. We constrain the initial disc properties required for polar alignment to occur. The outline of the letter is as follows: in Section 2, we discuss the criteria and time-scales for polar alignment around a brown dwarf binary. In Section 3, we detail the methods used for the hydrodynamical simulations. In Section 3.2, we present the results of the hydrodynamical simulations of a tilted circumbinary disc around a brown dwarf binary. In Section 4, we discuss the presence of the outer companion 2M1510 C, the efficiency of planet formation around brown dwarfs, and place the 2M1510 AB system into the wider context of circumbinary

exoplanet detections. Finally, we summarize our conclusions in Section 5.

2 CIRCUMBINARY DISC ALIGNMENT TIME-SCALES

Disc misalignment can occur at various stages of stellar evolution and arise from several processes. In star-forming regions, misaligned discs may result from the chaotic accretion of turbulent molecular clouds, introducing random angular momentum directions (Offner et al. 2010; Bate 2012; Tokuda et al. 2014). Post-formation accretion of misaligned gas can further alter the orientation of a circumbinary disc relative to the binary plane (Bate, Lodato & Pringle 2010; Bate 2018). Additionally, if a binary forms in an elongated cloud, its orbital plane can be misaligned with the cloud’s rotation axis due to anisotropic collapse or dynamical interactions (Bonnell & Bastien 1992). A misaligned circumbinary disc will undergo alignment process evolving towards a coplanar or polar configuration (e.g. Papaloizou & Lin 1995; Lubow & Ogilvie 2000; Aly et al. 2015; Martin & Lubow 2017; Zanazzi & Lai 2018).

Within the linear warp propagation theory, the warp is dissipated and a initially tilted circumbinary disc polar aligns on a time-scale

$$\tau_{\text{polar}} = \frac{1}{\alpha} \left(\frac{H}{r} \right)^2 \frac{\Omega_b}{\Omega_d^2}, \quad (1)$$

where α is the disc viscosity, H/r is the disc aspect ratio, and $\Omega_b = \sqrt{G(M_1 + M_2)/a^3}$ is the binary angular frequency. Ω_d is the global disc precession frequency, which differs depending on coplanar or polar alignment, and is given by

$$\Omega_d = \frac{3\sqrt{5}}{4} e_b \sqrt{1 + 4e_b^2} \frac{M_1 M_2}{M^2} \left\langle \left(\frac{a}{r} \right)^{7/2} \right\rangle \Omega_b, \quad (2)$$

(Lubow & Martin 2018; Smallwood et al. 2019), where

$$\left\langle \left(\frac{a}{r} \right)^{7/2} \right\rangle = \frac{\int_{r_{\text{in}}}^{r_{\text{out}}} \Sigma r^3 \Omega (a/r)^{7/2} dr}{\int_{r_{\text{in}}}^{r_{\text{out}}} \Sigma r^3 \Omega dr}. \quad (3)$$

Note that if the disc breaks, the constituent discs will precess at different rates and equation (3) cannot be applied.

Fig. 1 demonstrates how the polar alignment time-scale (τ_{polar}) from equation (1) varies with total binary mass (M), semimajor axis (a_b), and eccentricity (e_b). For these calculations, we assume an equal-mass binary ($M_1 = M_2$), $\alpha = 10^{-4}$, and $H/r = 0.05$, $r_{\text{in}} = 2a_b$. We show the results for $r_{\text{out}} = 50a_b$ (left plot), $r_{\text{out}} = 100a_b$ (middle plot), and $r_{\text{out}} = 200a_b$ (right plot). Across all panels, the alignment time-scale decreases with increasing binary mass, indicating that discs around more massive systems align more quickly. Similarly, alignment times increase with larger semimajor axis, suggesting that discs around wide binary systems take longer to align, which is consistent with observations (e.g. Czekala et al. 2019). Also, a wider disc will take longer to align than a more truncated disc. The four panels compare binary eccentricities $e_b = 0.36$, $e_b = 0.5$, $e_b = 0.7$, and $e_b = 0.88$, revealing that higher binary eccentricities correspond to shorter disc alignment times across the parameter space. This indicates that more eccentric binary systems promote faster alignment, while lower eccentricities result in slower polar alignment.

In Fig. 1, we also overlay the estimated lifetime of a protoplanetary disc, ranging from 10^6 yr (solid line) to 10^7 yr (dashed line). The star marks the parameters for 2M1510 AB, indicating that a circumbinary disc around a low-mass binary, such as a brown dwarf binary, will require more time to achieve polar alignment compared to

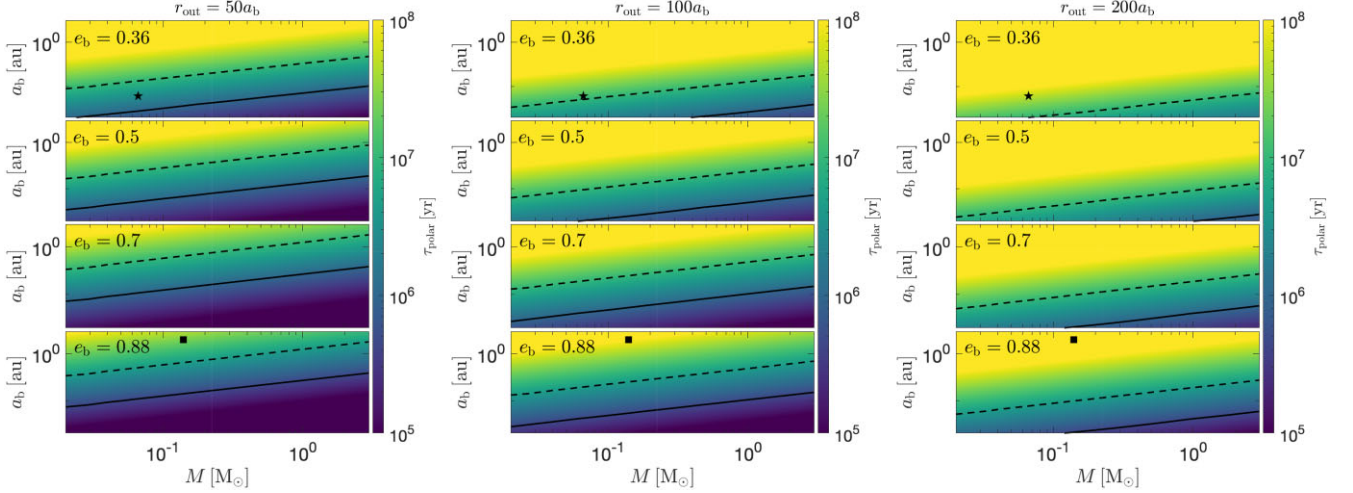


Figure 1. Polar alignment time-scale (τ_{polar}) from equation (1) as a function of binary mass (M) and semimajor axis (a_b) for binary eccentricities $e_b = 0.36$, $e_b = 0.5$, $e_b = 0.7$, and $e_b = 0.88$ (from top to bottom), for an outer disc radius $r_{\text{out}} = 50a_b$ (left plot), $r_{\text{out}} = 100a_b$ (middle plot), and $r_{\text{out}} = 200a_b$ (right plot). Higher masses and smaller separations lead to faster alignment, with shorter time-scales at higher eccentricities. We overlay the estimated lifetime of a protoplanetary disc, ranging from 10^6 yr (solid line) to 10^7 yr (dashed line). The star and square marks the approximate parameters for 2M1510 AB and VHS 1256 AB, respectively.

higher mass systems. Nevertheless, for typical protoplanetary disc parameters, the disc is expected to align within a typical 10 Myr lifetime given that the radial disc extent is not too large.

The square represents the parameters for VHS 1256 AB (which orbits at approximately $200a_b$), indicating that the alignment time-scale exceeds the disc lifetime for both $r_{\text{out}} = 50a_b$, $r_{\text{out}} = 100a_b$, and $r_{\text{out}} = 200a_b$, despite the high binary eccentricity ($e_b = 0.88$). This implies the nearly polar planetary-mass companion, VHS 1256 b, is qualitatively different from 2M1510(AB)b, and either formed from a primordially near-polar disc, or more likely that it acquired its highly inclined orbit through a scattering event, which is supported by its significant eccentricity ($e \approx 0.7$) whereas a polar circumbinary disc is expected to have less eccentricity growth (Smallwood, Lubow & Martin 2022).

We now examine the conditions necessary for a polar-aligned circumbinary disc around 2M1510 AB. Fig. 2 illustrates the polar alignment time-scale, τ_{polar} , as a function of the circumbinary disc outer radius, r_{out} , expressed in units of the binary semimajor axis, a_b . The figure presents a family of curves representing different values of disc viscosity (α) and disc aspect ratio (H/r). The black curves show the effect of varying α for a fixed $H/r = 0.05$, while the blue curves demonstrate the influence of changing H/r for a constant $\alpha = 10^{-4}$. For a fixed disc aspect ratio, increasing the disc viscosity results in a faster polar alignment time-scale. This implies that more viscous discs are more efficient at aligning perpendicular with the binary. For a fixed viscosity, increasing the disc aspect ratio leads to a longer polar alignment time-scale. The horizontal red line represents the estimated upper limit for the age of a protoplanetary disc, which ranges from 1 to 10 Myr. Brown dwarf inner disc lifetimes do not appear to be vastly different from those of discs around T Tauri stars (e.g. Jayawardhana et al. 2003). This line provides a reference point to evaluate whether the disc in this system has had enough time to reach a polar state before the disc disperses. However, circumbinary discs may have longer lifetimes than discs around single stars. For example, the discs around V4046 Sgr and AK Sco have ages of 23 ± 3 Myr (Mamajek & Bell 2014) and 18 ± 1 Myr (Czekala et al. 2015), respectively. The horizontal grey line represents the estimated age of 2M1510 AB (~ 45 Myr).

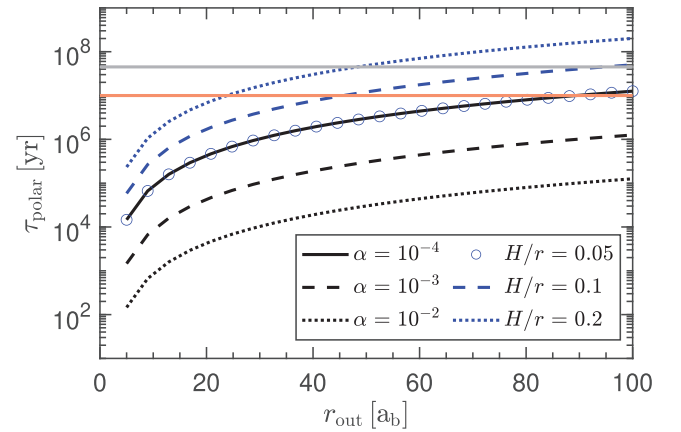


Figure 2. The polar alignment time-scale, τ_{polar} , is shown as a function of the circumbinary disc radius, r_{out} , expressed in units of the binary semimajor axis, a_b , for different values of disc viscosity (black) with disc aspect ratio $H/r = 0.05$, and different values of H/r (blue) with $\alpha = 10^{-4}$. The horizontal red line represents the estimated upper limit for the age of a protoplanetary disc, which ranges from 1 to 10 Myr. The horizontal grey line indicates the estimated age of 2M1510 AB (~ 45 Myr).

The critical tilt, i_{crit} , for a circumbinary disc's angular momentum vector to align with the binary angular momentum (coplanar alignment) or to the binary eccentricity vector (polar alignment) depends on the binary eccentricity, e_b , and the ratio between the initial disc angular momentum to the initial binary angular momentum, $j_0 = J_{d0}/J_{b0}$. The analytical approximate for i_{crit} is estimated given two regimes shown by

$$i_{\text{crit}} = \begin{cases} \arccos \left[\frac{\sqrt{5}e_b \sqrt{4e_b^2 - 4j_0^2(1-e_b^2)} + 1 - 2j_0(1-e_b^2)}{1+4e_b^2} \right], & \text{low } j_0, \\ \arccos \left[\frac{\sqrt{(1-e_b^2)(1+4e_b^2) + 60(1-e_b^2)j_0^2} - (1-e_b^2)}{10(1-e_b^2)j_0} \right], & \text{high } j_0, \end{cases} \quad (4)$$

(Martin & Lubow 2019). The initial angular momentum of the binary is given by

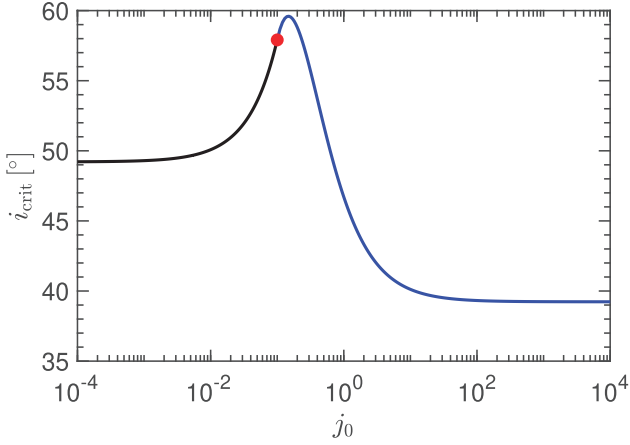


Figure 3. The critical tilt separating coplanar versus polar alignment from equation (4) as a function of disc angular momentum to binary angular momentum ratio, j_0 . The black and blue curves represent to low j_0 and high j_0 approximations, respectively. The red dot is the transition point between these two regimes.

$$J_{b0} = \mu \sqrt{G(M_1 + M_2)a_{b0}(1 - e_{b0}^2)}, \quad (5)$$

where μ is the reduced mass and the initial angular momentum of the disc is given by

$$J_{d0} = \int_{r_{in}}^{r_{out}} 2\pi r^3 \Sigma_0(r) \Omega dr, \quad (6)$$

where $\Omega = \sqrt{GM/r^3}$ is the Keplerian angular velocity. We explore a range of j_0 from 10^{-4} to 10^4 using the observed binary eccentricity of 2M1510 AB, $e_b = 0.36$. Fig. 3 shows the critical tilt angle, i_{crit} , from equation (4) as a function of j_0 . For low values of j_0 , the critical tilt is approximately 50° , while for high j_0 , i_{crit} approaches 40° . In the limit of large j_0 , the critical tilt approaches the critical angle for Kozai–Lidov (KL) oscillations (Martin & Lubow 2019). The transition between these regimes occurs at $j_0 = 0.1$. The peak in i_{crit} occurs at $j_0 \sim 0.14$ with $i_{crit} \sim 60^\circ$.

3 HYDRODYNAMICAL SIMULATIONS

3.1 Set-up

As a proof-of-concept, we use the smoothed particle hydrodynamics (SPH) code PHANTOM (Price et al. 2018) to simulate an initially inclined circumbinary disc around the brown dwarf binary 2M1510 AB. The discs within our hydrodynamical model are in the bending wave regime, meaning that the disc aspect ratio, H/r , is larger than the Shakura & Sunyaev (1973) α -viscosity parameter, which is appropriate for protoplanetary discs (e.g. Hueso & Guillot 2005; Rafikov 2016; Ansdell et al. 2018). In this regime, warps induced in the disc by the brown dwarf binary torque propagate as bending waves with a vertical averaged speed $c_s/2$ (Papaloizou & Lin 1995), where c_s is the sound speed.

We use the observed binary parameters of 2M1510 AB from Triaud et al. (2020) and Baycroft et al. (2025), with $a_b = 0.06$ au (semimajor axis), $e_b = 0.36$ (eccentricity), $M_1 = 0.033104 M_\odot$ (primary mass), and $M_2 = 0.033219 M_\odot$ (secondary mass). The binary separation and eccentricity are allowed to evolve in time. We employ a Cartesian coordinate system (x, y, z) , where the x -axis aligns with the direction of the binary eccentricity vector, and the z -axis aligns with the direction of the binary angular momentum vector. The binary

accretion radii are set to $r_{acc,1} = r_{acc,2} = 0.03$ au, in order to increase computational efficiency. The sink accretion radius is considered a hard boundary, where the accreted particles' mass and angular momentum are added to the sink (Bate, Bonnell & Price 1995).

Observations of brown dwarfs in the (sub)-mm and millimetre wavelengths indicate that disc masses are roughly proportional to stellar mass, typically around 1 per cent of the stellar mass, although with considerable scatter (Klein et al. 2003; Scholz, Jayawardhana & Wood 2006; Andrews et al. 2013; Mohanty et al. 2013). In our models, the circumbinary disc is represented by 1×10^6 equal-mass SPH particles, with a total disc mass of 0.1 per cent of the total binary mass. This simplification ensures that the disc mass does not significantly affect the orbit of the binary. A more massive circumbinary disc will align to a stationary tilt with respect to the binary (Martin & Lubow 2019). The particles are radially distributed from the inner disc radius, $r_{out} = 0.12$ au ($\sim 2a_b$), to the outer disc radius, $r_{out} = 0.6$ au ($\sim 10a_b$). We adopt a narrow, rigidly precessing disc configuration to avoid dynamical instabilities such as disc breaking or tearing, which arise when the radial communication time-scale is longer than the precession time-scale (Nixon 2012; Facchini, Lodato & Price 2013). The circumbinary disc is initially misaligned by 75° , causing it to evolve toward a polar state (see Fig. 3).

The gas surface density profile is initially a power-law distribution given by $\Sigma(r) = \Sigma_0(r/r_{in})^{-p}$, where $\Sigma_0 = 1.62 \times 10^3$ g/cm² is the density normalization (defined by the total mass), p is the power-law index, and r is the spherical radius. We set $p = 3/2$. We adopt the locally isothermal equation of state. The disc thickness is scaled with radius as $H = c_s/\Omega \propto r^{3/2-q}$, where $\Omega = \sqrt{GM/r^3}$ and $q = 3/4$. We set an initial gas disc aspect ratio of $H/r = 0.05$ at $r = r_{in}$. The Shakura & Sunyaev (1973) viscosity, α_{SS} , prescription is given by $\nu = \alpha_{SS} c_s H$, where ν is the kinematic viscosity. In order to simulate α_{SS} , we use the artificial viscosity α^{av} prescription in Lodato & Price (2010) given as $\alpha_{SS} \approx (\alpha_{AV}/10)(\langle h \rangle/H)$. We take the Shakura & Sunyaev (1973) α_{SS} parameter to be 0.005. The circumbinary disc is initially resolved with average smoothing length per scale height $\langle h \rangle/H = 0.34$. With the above prescription, $\langle h \rangle/H$ and α_{SS} are constant over the radial extent of the disc (Lodato & Pringle 2007). Note, that $\langle h \rangle/H$ can vary due to the formation of warps in the disc (Fairbairn & Ogilvie 2021; Deng & Ogilvie 2022).

We separate the disc into 300 bins in spherical radius. Within each bin, we calculate the azimuthally averaged tilt and longitude of ascending node.

3.2 Results

Fig. 4 illustrates the evolution of a circumbinary disc tilted initially by 75° relative to the orbital plane of a brown dwarf binary. The top left panel depicts the initial structure of the disc, where the colour scale represents the surface density, with yellow regions indicating values approximately two orders of magnitude higher than the blue regions. The binary components are shown as red dots, and the system is viewed in the x - z plane. The top right panel shows the disc's structure at $t = 1500 P_{orb}$, where the disc continues the precess about the eccentricity vector of the binary and is transitioning toward polar alignment.

The bottom panels display the time evolution of the disc's tilt, i , and the longitude of the ascending node, ϕ , at three distinct radii: 0.15 au (black), 0.35 au (blue), and 0.55 au (red). These radii correspond to the inner, middle, and outer regions of the disc, respectively. The disc tilt, i , begins at 75° and evolves over time, with tilt oscillations driven by the eccentricity of the binary (e.g.

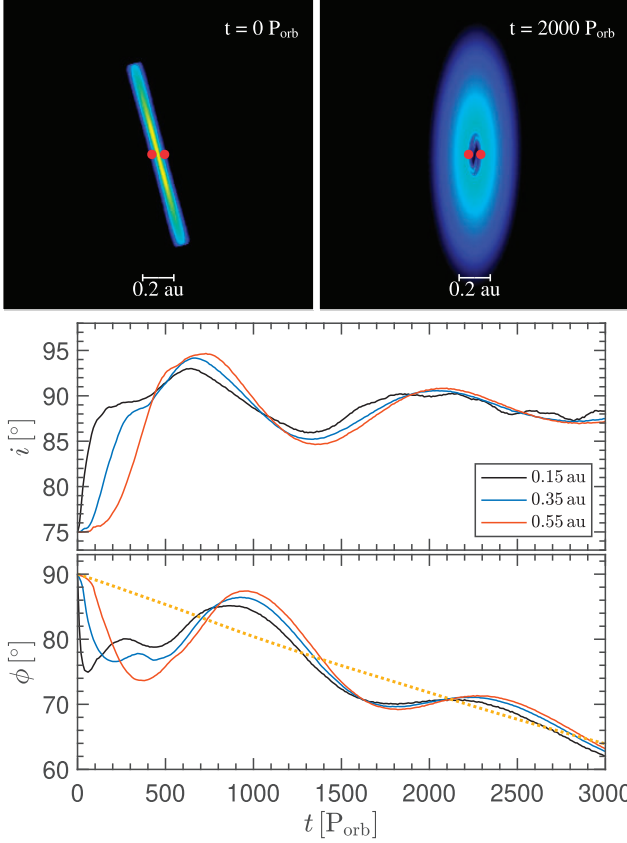


Figure 4. Top left panel: initial disc structure tilted by 75° , with the colour indicating surface density – yellow corresponds to values approximately two orders of magnitude greater than blue. The brown dwarf binary components are represented by red dots, and the disc is viewed in the x - z plane. Top right panel: disc structure at $t = 1500 P_{\text{orb}}$. Bottom panel: evolution of the disc tilt, i , and longitude of the ascending node, ϕ , over time. The disc is probed at three different radii: 0.15 au (black), 0.35 au (blue), and 0.55 au (red), corresponding to the inner, middle, and outer regions of the disc. The dotted yellow line represents the azimuthal angle of the binary’s eccentricity vector. The circumbinary disc is undergoing alignment to a polar configuration relative to the brown dwarf binary.

Smallwood et al. 2019). Since we model a moderately extended disc, the inner disc region (black curve) aligns more rapidly compared to the middle (blue) and outer (red) regions, highlighting differential alignment across the disc.

The evolution of ϕ indicates the disc precession. The dotted yellow line in the bottom panel represents the azimuthal angle of the binary’s eccentricity vector. The disc’s longitude of the ascending node, ϕ , converges toward the binary’s azimuthal angle, further confirming the polar alignment process. Over time, the entire circumbinary disc transitions toward a polar configuration relative to the binary’s orbital plane.

The hydrodynamical simulation demonstrates that despite its overall low mass, a binary brown dwarf system, such as 2M1510 AB, remains highly efficient in polar-aligning a circumbinary disc. The results indicate that the time-scale for polar alignment is shorter than the time-scale required for substantial grain growth within the disc (e.g. Smallwood et al. 2024a, b). This implies that the dust will remain well coupled to the gas as the disc aligns to a polar orientation relative to the binary’s orbital plane. The efficient polar disc alignment has important implications for planet formation.

A polar-aligned disc provides the conditions necessary for the formation of polar circumbinary planets, as the dust and gas dynamics facilitate the accumulation and growth of planetesimals in a stable polar configuration (Smallwood et al. 2024b). Such a process could explain the potential existence of a polar-aligned planet around 2M1510 AB, offering a unique environment for planetary system formation compared to the more common coplanar circumbinary systems.

4 DISCUSSION

4.1 The presence of 2M1510 C

Based on Martin et al. (2022), a circumbinary disc can still evolve into a polar alignment despite the presence of an outer companion. The companion may excite von Zeipel–KL (vZKL) oscillations (von Zeipel 1910; Kozai 1962; Lidov 1962), which could reduce the disc misalignment if the companion’s torque dominates. Conversely, if the inner binary’s torque dominates, the disc will align to a polar orientation. An example is HD 98 800 BaBb, which hosts a polar-aligned circumbinary disc (Kennedy et al. 2019). This system belongs to a hierarchical quadruple star system, where the A-B semimajor axis is only 54 au (~ 54 times the separation of the B binary). The circumbinary disc maintains its polar alignment despite the proximity of the outer binary companion and remains stable over long time-scales without being susceptible to KL oscillations (e.g. Martin et al. 2022; Smallwood et al. 2022). Given that the separation of 2M1510 C is ~ 250 au (~ 4200 times the separation of the AB binary), the impact of 2M1510 C on the polar alignment of the disc around 2M1510 AB will therefore be negligible.

The vZKL oscillation of a circumbinary disc can be quenched by gas-pressure-induced precession when the dimensionless stability parameter S falls below unity (Zanazzi & Lai 2017):

$$S = 0.36 \left(\frac{a_{\text{AB-C}}}{3R_{\text{out}}} \right)^3 \frac{M_{\text{AB}}}{M_{\text{C}}} \left(\frac{H(R_{\text{out}})}{0.1R_{\text{out}}} \right)^2 \lesssim 1, \quad (7)$$

where $a_{\text{AB-C}}$ is the separation of the AB-C binary, $M_{\text{AB}}/M_{\text{C}}$ is the mass ratio of the central binary with the outer companion, $H(R_{\text{out}})$ is the disc scale height at the outer disc edge. Physically, S^{-1} measures the ratio of the external companion’s tidal torque on the disc to the internal torque arising from gas pressure; when gas-pressure torques dominate ($S \ll 1$), vZKL oscillations are suppressed (Zanazzi & Lai 2017; Nealon et al. 2025). For a given choice of R_{out} , we compute H using the irradiated, flared-disc prescription of Chiang & Goldreich (1997), adopting the known luminosity of the central AB binary (Triaud et al. 2020).

Fig. 5 maps the stability parameter S for triggering the vZKL instability as a function of the binary semimajor axis $a_{\text{AB-C}}$ and the gaseous disc’s outer radius R_{out} . The red-shaded region delineates the $(a_{\text{AB-C}}, R_{\text{out}})$ combinations that permit vZKL oscillations. Given the low mass of the outer companion, the radial extent of the disc must be sufficiently large compared to the AB binary separation for the instability to occur. For an assumed AB–C separation of $a_{\text{AB-C}} = 250$ au, we infer a minimum outer edge of $R_{\text{out}} \approx 75$ au. We emphasize that the observed projected separation of 250 au likely underestimates the true semimajor axis; any increase in correspondingly raise the required R_{out} , since vZKL torques must overcome gas-pressure-induced precession to remain operative.

Given that our analysis depends critically on the radial extent of the disc around 2M1510 AB, we estimate the size of its primordial disc from the total mass of the central binary. Observations indicate that discs around very low-mass stars and brown dwarfs flatten relative

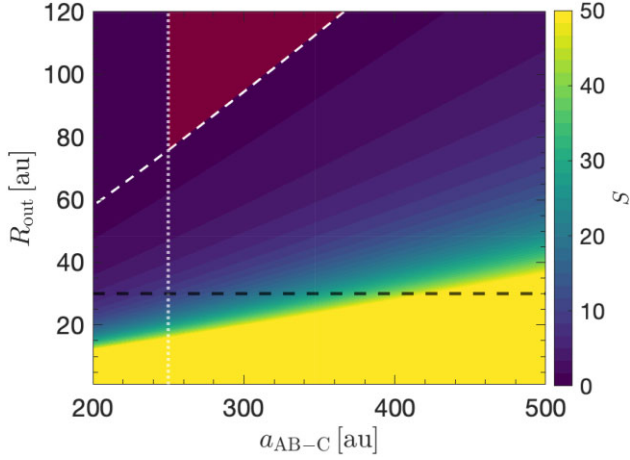


Figure 5. The S -parameter from equation (7) is shown as a function of the binary separation, a_{AB-C} , and the outer edge of the gaseous disc, R_{out} . The dashed curve represents the points where $S \lesssim 1$. The dotted line corresponds to the lower limit of the projected separation of 2M1510 AB, 250 au. The red-shaded region indicates the combinations of a_b and R_{out} for which the disc is susceptible to the vZLK instability.

to a simple power-law scaling at low masses: Hendler et al. (2017) show that these discs are smaller than expected from a linear size-mass relation, and Mohanty et al. (2013) find typical outer radii of only ~ 20 au for brown-dwarf discs. The canonical empirical relation between stellar mass M_* and dust-disc outer radius R_{out} is

$$R_{out} \approx R_0 \left(\frac{M_*}{M_\odot} \right)^\alpha, \quad (8)$$

with $R_0 \approx 100$ au and $\alpha = 0.5$ (Andrews et al. 2013, 2018). To capture the observed flattening at very low masses, we adopt the modified form

$$R_{out} \approx R_0 \left(\frac{M_*}{M_\odot} \right)^\alpha \left[1 + \left(\frac{M_{break}}{M_*} \right)^\beta \right]^{-\gamma}, \quad (9)$$

where $M_{break} \approx 0.1 M_\odot$ denotes the characteristic stellar mass at which the relation flattens (Testi et al. 2016; Hendler et al. 2017), and β, γ parameterize the transition's sharpness and degree of flattening. In Fig. 6, we overplot both equations (8) and (9) on a compilation of observed brown dwarf disc radii; several systems fall below the simple power law but align with the modified relation. Applying equation (9) to 2M1510 AB ($M_* = M_A + M_B = 0.0663 M_\odot$) yields $R_{out} \approx 15$ au. Accounting for the fact that gas discs typically extend to several times the dust radius (Ansdell et al. 2018; Sanchis et al. 2021), we infer a gas-disc outer edge of ~ 30 au, marked by the dashed red line in Fig. 5. Such a compact primordial disc would be too small to undergo vZKL oscillations during polar alignment.

However, once the disc disperses, a planet may be susceptible to vZKL oscillations for certain values of the planet separation and eccentricity of 2M1510 C. We estimate the vZKL oscillation time-scale for a circumbinary particle perturbed by the outer companion, 2M1510 C, defined as (Kiseleva, Eggleton & Mikkola 1998; Ford, Kozinsky & Rasio 2000):

$$\tau_{vZKL} = \frac{2\pi}{3} \left(\frac{P_3^2}{P} \right) \left(\frac{M_1 + M_2 + M_3}{M_3} \right) (1 - e_3^2)^{\frac{3}{2}}, \quad (10)$$

where P_3 and P are the orbital periods of 2M1510 C and a circumbinary test particle, respectively; e_3 is the eccentricity of 2M1510 C's orbit; and M_1, M_2 , and M_3 are the masses of 2M1510 A, 2M1510 B, and 2M1510 C, respectively. For the mass of the

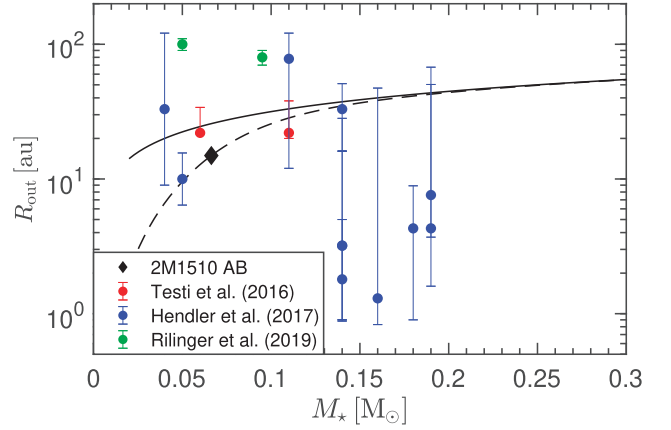


Figure 6. The observed outer disc edge, R_{out} , as a function of star mass of discs around brown dwarfs from Testi et al. (2016; red), Hendler et al. (2017; blue), and Rilinger, Espaillat & Macías (2019; green). The solid black curves denotes the scaling relation from equation (8) and the dashed black curve denotes the modified scaling relation from equation (9). The black diamond represent the estimated radial extent of a primordial disc around 2M1510 AB.

outer companion, we assume $M_3 = M_1$. The top panel in Fig. 7 illustrates that the vZKL time-scale, τ_{vZKL} , is strongly dependent on the particle's separation, r , and the eccentricity of the perturber, e_C . Specifically, τ_{vZKL} decreases significantly with increasing r and increasing e_C . This implies that particles at larger separations from the central binary or systems with a more eccentric outer perturber are susceptible to vZKL oscillations on shorter time-scales.

The bottom panel in Fig. 7 delineates the boundary between stable and vZKL unstable orbits by plotting the critical radius, r_{crit} , as a function of e_C . This critical radius, where τ_{vZKL} equals the system's age ($T_{age} \approx 45$ Myr), decreases with increasing eccentricity of the outer companion (black curve). The green shaded area signifies orbits where particles are considered stable against vZKL oscillations. This stability implies that for particles with semimajor axes $r < r_{crit}$, the time-scale for significant vZKL-induced perturbations exceeds the age of the system. Conversely, the pink shaded region positioned above the black curve denotes orbits that are unstable due to vZKL oscillations. For particles in this zone, where $r < r_{crit}$, the vZKL time-scale is shorter than the system's age. As a result, these particles are susceptible to large-amplitude oscillations in their eccentricity and inclination. Therefore, much of the range of possible planetary-mass companions should correspond to stable polar circumbinary orbits.

4.2 Planet formation in brown dwarf discs

Daemgen et al. (2016) found that approximately half of the brown dwarfs in their sample possess discs containing at least one Jupiter mass of material, confirming that many brown dwarfs have sufficient material to form Earth-mass to sub-Neptune planets. Grain growth and planetesimal formation via the streaming instability may be efficient in these discs (e.g. Liu et al. 2020). Payne & Lodato (2007) explored planet formation in brown dwarf discs by extending the sequential core accretion model of Ida & Lin (2004, 2005) to lower mass systems. Their findings indicate that discs with a total mass of a few Jupiter masses (M_{Jup}) around a $0.05 M_\odot$ brown dwarf cannot form giant planets, as rocky cores fail to accrete substantial gas envelopes. However, these discs can still produce rocky planets, with the maximum planet mass strongly dependent on the slope (p) of the dust's radial surface density profile. Steeper profiles ($p = 1.5$)

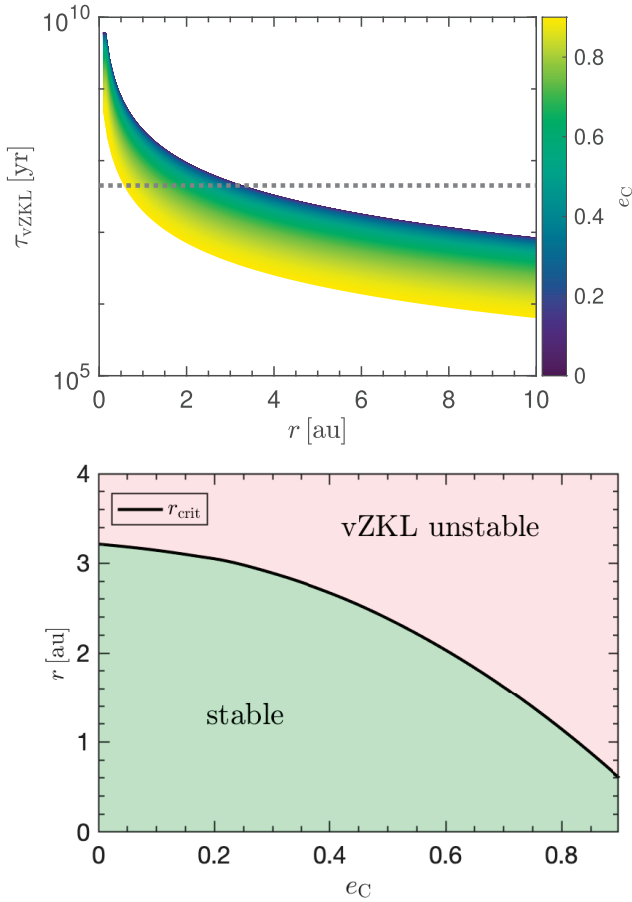


Figure 7. Stability of circumbinary particle orbits around 2M1510 AB under the influence of the tertiary component 2M1510 C. Top panel: the vZKL timescale, τ_{vZKL} (in years), as a function of the particle’s semimajor axis, r (in au), from the central binary. The colour bar indicates the eccentricity of the outer perturbing body, 2M1510 C, e_c . The dotted horizontal line represents the estimated age of the 2M1510 system, $T_{\text{age}} \approx 45$ Myr. Bottom panel: the critical radius, r_{crit} , (in au) for vZKL instability as a function of eccentricity of 2M1510 C, e_c . The critical radius is defined as the semimajor axis at which $\tau_{vZKL} = T_{\text{age}}$. The green shaded region below the black curve indicates orbits that are stable against vZKL oscillations ($r < r_{\text{crit}}$), while the pink shaded region above the curve indicates orbits that are unstable to vZKL oscillations ($r > r_{\text{crit}}$).

enhance surface densities in the inner disc, enabling the formation of rocky planets with masses of 1–5 Earth masses (M_{\oplus}) at orbital distances of 0.2–3 au. In contrast, shallower profiles ($p = 1.0$) reduce planet masses by a factor of 5–10 at similar distances (Payne & Lodato 2007). Consequently, any giant planetary-mass companions observed around very low-mass objects like brown dwarfs are likely to have formed through alternative mechanisms, such as gravitational instability, as exemplified by 2M1207b (Chauvin et al. 2005; Lodato, Delgado-Donate & Clarke 2005).

As shown in Figs 1 and 2, a disc within $\sim 100 a_b$ around 2M1510 AB is expected to align polar within its lifetime, potentially facilitating the formation of polar planets. Numerical simulations suggest that polar terrestrial planets can form efficiently around eccentric binaries (Childs & Martin 2021), and studies of their long-term stability indicate that polar configurations are particularly stable around such systems (e.g. Cuello & Giuppone 2019; Chen, Lubow & Martin 2020). Additionally, circumbinary discs can be more massive

than circumstellar discs because the gravitational influence of two stars allows them to retain a larger reservoir of material (Artymowicz & Lubow 1994; Bate & Bonnell 1997). A more massive disc can be more efficient in forming larger mass planets by providing higher surface densities and a greater amount of material, which enhances core accretion and gravitational instability processes, leading to the formation of more massive planets (Lodato & Rice 2004; Andrews et al. 2018).

4.3 Placing 2M1510 AB and VHS 1256 AB in a wider context

The results shown in Fig. 1 give a picture of alignment timescales across different binary parameters. Alignment to a polar configuration is more efficient for more massive binaries, for more eccentric binaries, and for smaller binary separations. Therefore, 2M1510 AB would appear not ideal for efficient polar alignment, being low-mass and of moderate eccentricity and neither would VHS 1256 AB due its wider separation. Binary brown dwarfs are rare, and planet-hosting brown dwarfs are rare too (mainly due to detection difficulties; Blake, Charbonneau & White 2010; Gillon et al. 2013; Sahlmann et al. 2014). While there is enough time for a disc of small radial extent to reach polar alignment around both VHS 1256 AB and 2M1510 AB, there remains a question about why polar circumbinary configurations have been identified in such exotic systems instead of more suitable and more common binaries. We speculate about three reasons:

(i) Detecting misaligned or polar planets in transit is difficult due to there not necessarily being a transit at every conjunction (Schneider 1994; Martin & Triaud 2014; Chen & Kipping 2022). All of the known transiting circumbinary planets were found in eclipsing binaries, which favours finding coplanar transiting planets (Schneider 1994; Martin & Triaud 2015). A polar planet would only be expected to transit binaries if their line of apsides near the plane of the sky. With only 12 transiting circumbinary systems it is not surprising no polar transiting system has yet been found.

(ii) There have been radial velocity surveys for circumbinary planets (Konacki et al. 2009; Martin et al. 2019; Baycroft, Triaud & Lai 2024). So far only one discovery of a circumbinary planet has been published from these (Standing et al. 2023; Baycroft et al. 2024), and no dedicated search for retrograde apsidal precession has been performed to our knowledge. Most systems in the radial velocity samples have a low eccentricity ($e_b < 0.3$; Martin et al. 2019).

(iii) Based on theoretical arguments, Cuello & Giuppone (2019) predict the most likely polar circumbinary configuration ought to be found in equal-mass ($q \approx 1$) binaries with moderate orbital eccentricities ($e_b \approx 0.4$). They argue polar circumbinary planet formation capabilities are maximized for these parameters. 2M1510 AB falls very much in this category, unlike the radial velocity binaries from the BEBOP survey (Martin et al. 2019; Baycroft et al. 2024) which are mostly $q \lesssim 0.3$. VHS 1256 AB does not fall so clearly in this category, the eccentricity being very high.

With 2M1510 (AB)b and VHS 1256 (AB)b, there are now two planetary mass companions on close-to-polar orbits around low-mass binaries. Delorme-1 is a visual, very low-mass binary, with an imaged planetary mass companion (Delorme et al. 2013) which has an unconstrained mutual inclination at the moment (Bowler et al. 2023) but could join these in the future. Combined with the lack of known polar planets orbiting more massive main-sequence binaries, this indicates that there is a potential mechanism to form polar objects specific to low-mass binaries.

2M1510 (AB)b does not have mass and semimajor axis measurements yet. If it is comparable to VHS 1256 (AB)b, then they would both be long-period massive objects which would likely not have formed from a polar-aligning circumbinary disc (see Fig. 1). However 2M1510 (AB)b could well be a lower mass close-in planet and hence have formed in a distinct way to VHS 1256 (AB)b. More observations to constrain the parameters of 2M1510 (AB)b are vital to test whether it falls into the same category as VHS 1256 (AB)b.

5 CONCLUSIONS

Recent observations by Baycroft et al. (2025) suggest that the retrograde precession of the binary brown dwarf system 2M1510 AB is compatible with the presence of a polar circumbinary planet. If correct this would be the first detection of this type of orbital configuration. We investigated the evolution of a primordial circumbinary disc around the 2M1510 AB brown dwarf binary to assess the likelihood that the disc could have evolved into a polar configuration.

From analytical calculations, we determined that the time-scale for polar disc alignment decreases as the mass of the binary system increases. This suggested that discs surrounding more massive binary systems tend to align more rapidly compared to those around less massive systems. Furthermore, the efficient of alignment to a polar configuration is influenced by several key factors: it is enhanced for more massive binaries, for binaries with higher orbital eccentricity, and for systems with smaller binary separations. When applied to brown dwarf binaries, which are characterized by their relatively low binary masses, the alignment time-scales are expected to be significantly longer compared to more massive systems. However, within a specific parameter space of disc properties – such as disc size, viscosity, and initial misalignment – it is possible for the primordial disc around a brown dwarf binary, like 2M1510 AB, to achieve polar alignment within the disc’s lifetime. This suggests that while alignment processes are typically slower for low-mass binaries, certain conditions can still facilitate efficient polar alignment, even in systems like 2M1510 AB.

We found that the critical tilt angle for a circumbinary disc to evolve into a polar alignment around 2M1510 AB was determined to be $i_{\text{crit}} \gtrsim 50^\circ$ for a low ratio of the disc angular momentum to the binary angular momentum. For a higher angular momentum ratio, the critical tilt converges to the KL instability angle, $\sim 39^\circ$. Thus, a primordial disc with an initial misalignment of $\gtrsim 50^\circ$ would be required for the disc to evolve into a polar configuration around 2M1510 AB. We calculated a range of disc parameters, focusing on viscosity and aspect ratio, that facilitate the formation of a polar-aligned disc prior to its dispersal. For a disc viscosity of $\alpha = 10^{-4}$ and an aspect ratio of $H/r = 0.05$, a circumbinary disc with an outer edge extending to approximately $100a_b$ can achieve polar alignment within the disc’s lifetime. Using hydrodynamical simulations, we demonstrated that an initially inclined circumbinary disc evolves toward a polar configuration with respect to the orbit of 2M1510 AB. A polar-aligned circumbinary disc could provide a favourable environment for the formation of polar circumbinary planets (e.g. Smallwood et al. 2024a, b).

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DATA AVAILABILITY

The data supporting the plots within this article are available on reasonable request to the corresponding author. A public version of the PHANTOM and SPLASH codes are available at <https://github.com/danieljprice/phantom> and <http://users.monash.edu.au/~dprice/splash/download.html>.

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