5 Appendix 1: Quadratic Spatial Utility

I now treat the case where the the prime minister has quadratic spatial utility, $u_p(x) = -(1-x)^2$. To get the PM's expected utility from a vote of confidence proposal in the presence of uncertainty, we substitute this and Eq. 9 into Eq. 6 to obtain:

$$\underline{U_P}(\bar{x}) = -(1-x)^2 - \frac{\bar{x} - \hat{x} + \Delta/2}{\Delta}\left[-(1-x)^2 + (1-x_0)^2 + c^P\right] - e \tag{48}$$

Combining terms, we obtain:

$$\begin{aligned}\underline{U_P}(\bar{x}) = \quad & x^3 * (1/\Delta) \\ & + x^2 * \left(-1/2 - \frac{2+\hat{x}}{\Delta}\right) \\ & + x * \left(1 + \frac{2x_0 - x_0^2 - c^P + 2\hat{x}}{\Delta}\right) \\ & + 1 * \left(-1 - e + x_0 + \frac{-2x_0\hat{x} + x_0^2\hat{x} + c^P\hat{x}}{\Delta} - \frac{x_0^2 + c^P}{2}\right)\end{aligned} \tag{49}$$

Taking the first derivative with respect to x, we have:

$$\underline{U_P}'(\bar{x}) = \quad x^2 * (3/\Delta) + x * 2\left(-1/2 - \frac{2+\hat{x}}{\Delta}\right) + 1 * \left(1 + \frac{2x_0 - x_0^2 - c^P + 2\hat{x}}{\Delta}\right) \tag{50}$$

Setting the first derivative equal to zero, and applying the quadratic formula, we have the first-order condition:

$$x = \frac{\Delta}{6}\left(2\left(\frac{1}{2} + \frac{2+\hat{x}}{\Delta}\right) \pm \sqrt{\left(-2\left(\frac{1}{2} + \frac{2+\hat{x}}{\Delta}\right)\right)^2 - \frac{12}{\Delta}\left(1 + \frac{2x_0 - x_0^2 - c^P + 2\hat{x}}{\Delta}\right)}\right) \tag{51}$$

Taking the second derivative of Eq. 49, we find:

$$\underline{U_P}''(\bar{x}) = \frac{6x}{\Delta} - 2\left(\frac{1}{2} + \frac{2+\hat{x}}{\Delta}\right) \tag{52}$$

Because the second derivative is increasing in x, we know that only the lesser solution in Eq. 51 can be a maximum, which we denote as $\bar{x}^*$.

Meanwhile, the PM's utility from a vote of confidence in the absence of uncertainty, with quadratic spatial utility, is:

$$U_p(\hat{x}) = -(1-\hat{x})^2 - e \tag{53}$$

Thus we arrive at the condition for the PM to benefit from the presence of uncertainty, in the case of quadratic spatial utility:

$$-(1-\hat{x})^2 - e < -(1-\bar{x}^*)^2 - \frac{\bar{x}^* - \hat{x} + \Delta/2}{\Delta}\left[-(1-\bar{x}^*)^2 + (1-x_0)^2 + c^P\right] - e \quad (54)$$

where $\bar{x}^*$ is given by Eq. 51. Figure 4 shows the PM's increase in expected utility from a confidence vote in the presence of uncertainty. The parameter values are the same as in Fig. 2, i.e. $x_0 = 0.49$, $\hat{x} = 0.50$, and $c^P = 0.05$. This shows that, as in the case of linear utilities, the PM can under some conditions obtain higher utility in the presence of political uncertainty than in its absence. The assumption of a quadratic spatial utility function reduces the parameter space somewhat compared to linear utility, but does not eliminate it.

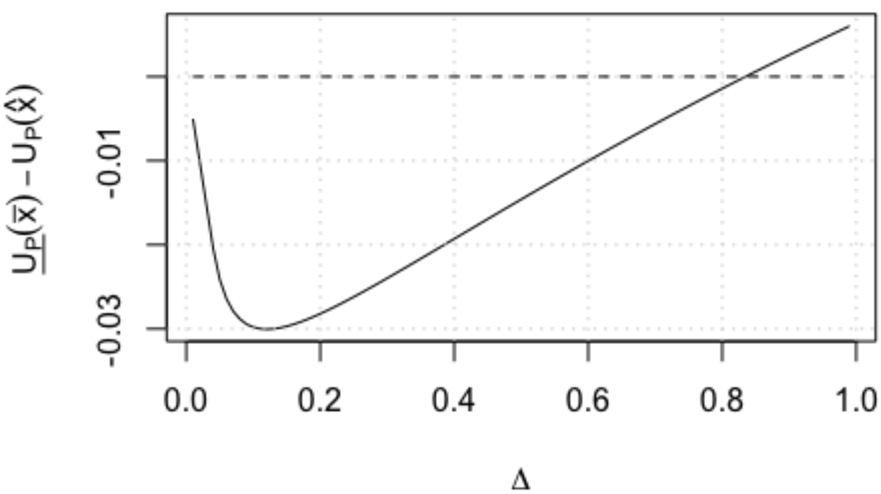

Figure 4: Increased utility to PM from using confidence vote in presence of uncertainty, with quadratic spatial utility.