Input : Array of n positive integers $\{a_1, a_2, \ldots, a_n\}$
Key to be searched for - k

Tape : $0^{a_1}10^{a_2}10^{a_3}1\ldots10^{a_n}110^{k}1$

Logic : Compare array elements with the key to be searched for, one at a time, until all the elements are compared with the key (or) key is found in the array.

There are 3 cases that arise during the comparison :

Case 1 : Element = Key

Element Key Position

B0001...11000010...0

↓

BB001...11X0010...0

↓

BBB01...11XX010...0

↓

BBBB1...11XXX10...0

We encounter a 1 on the element side. So we check the key (present after 11). Since there are only Xs followed by a 1, we get a match. We append a 0 at the end (to denote the position) and replace all the array elements

with B (blank) and stop.

BBBB1...11XXX10...0

↓

BBBBB...11XXX10...00 — Append 0 to the position

↓

BBBBBXXX...XXXXXX0...00 — Replace everything other than position with X (upto B)

↓

BBBBB...BBBBB0...00 — Replace X with B, Go to final state

The tape will finally contain the position of the key

Case 2 : Element < Key

001...11000010...0

↓ after consuming entire element

BBB...11XX0010...0

After consuming the entire element and reaching 11, we start checking if key is entirely replaced with X. It is not here. So we simply append a 0 to index ~~convert~~ replace the Xs back with 0 (to check with other array elements) and start over ~~for~~ our comparison for the next array element.

BBB...11XX0010...0

↓

BBB...11XX0010...00 — Append 0 to position

↓

BBB...11000010...00 — Regenerate the key and start comparing the key with next element of the array

We go back to start state.

Case 3 : Element > Key

000D1...1100010...0

↓ after consuming entire key

BB0D1..11XX10...0

↓ we read a 0 in element and start to find a 0 in key

BBBD1...11XX10...0

Now we skip over all Xs to find a 0 for the corresponding 0 in the element. But we find a 1 indicating key is entirely consumed already. So we append a 0 to position, reconvert all Xs to 0 and skip over the element under consideration (as we already know it greater than the key).

BBB01...11XX10...0

↓

BBB01...11XX10...00 Append 0 to position

↓

BBBBB...1100 10...00

② ①

① Regenerate key

② Skip the current element

We go back to the start state and start comparing the next element with key

When key is not found in the array

When key is not found we read a 1 directly instead of 0 in the start state (because we use 11 to as a delimiter for array and key). But we basically consider it as end of the element that was being considered (elements are separated by 1).

So we try to find 11 to or check if key is completely consumed. Since there is no 11 in the tape, our state will read a B (after skipping over the key and position.)

We simply put a 1 at the place of B, to indicate that element is not found and replace every non-blank character before it with X and replace all those

X with B.

000...0110010...0

last element of array being compared with key

↓ after comparing with key

BB0...011XX10...0

↓

BBB...BB100 10...00

Now we go back to start state and it reads a 1 directly. there is no 11 but.

BBB...BB10010...00

↓

BBB...BBBXXXX...XX1

↓

Append 1 and replace everything else with X.

BBB...BBBBBBB...BB1

Finally we go to a final state. The tape contains only 1 indicating key is not present in the array

	0	1	X	B
q_0	(q_1, B, R)	(q_5, B, R)	-	-
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	-	-
q_2	$(q_1, 0, R)$	$(q_3, 1, R)$	-	-
q_3	(q_4, X, L)	$(q_{14}, 1, R)$	(q_3, X, R)	-
q_4	$(q_4, 0, L)$	$(q_4, 1, L)$	(q_4, X, L)	(q_0, B, R)
q_5	$(q_6, 0, R)$	$(q_8, 1, R)$	-	-
q_6	$(q_6, 0, R)$	$(q_7, 1, R)$	-	$(q_{17}, 1, L)$
q_7	$(q_6, 0, R)$	$(q_8, 1, R)$	-	-
q_8	$(q_9, 0, R)$	$(q_{11}, 1, R)$	(q_8, X, R)	-
q_9	$(q_9, 0, R)$	$(q_9, 1, R)$	-	$(q_{10}, 0, L)$
q_{10}	$(q_{10}, 0, L)$	$(q_{10}, 1, L)$	$(q_{10}, 0, L)$	(q_0, B, R)
q_{11}	$(q_{11}, 0, R)$	-	-	$(q_{12}, 0, L)$
q_{12}	$(q_{12}, 0, L)$	(q_{13}, X, L)	-	-
q_{13}	(q_{13}, X, L)	(q_{13}, X, L)	(q_{13}, X, L)	(q_{18}, B, R)
q_{14}	$(q_{14}, 0, R)$	-	-	$(q_{15}, 0, L)$
q_{15}	$(q_{15}, 0, L)$	$(q_{15}, 1, L)$	$(q_{15}, 0, L)$	(q_{16}, B, R)
q_{16}	(q_{16}, B, R)	(q_0, B, R)	-	-
q_{17}	(q_{17}, X, L)	(q_{17}, X, L)	-	(q_{18}, B, R)
q_{18}	$(q_{19}, 0, L)$	$(q_{19}, 1, L)$	(q_{18}, B, R)	-
q_{19}	-	-	-	-