Deep Learning HW 2.

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1. $y = \frac{1}{1+e^{-x}}$ let $a = 1+e^{-x}$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial a} \cdot \frac{\partial a}{\partial x}$$

$$= \frac{\partial}{\partial a} a^{-1} \cdot \frac{\partial}{\partial x} 1+e^{-x}$$

$$= -a^{-2} \cdot -e^{-x}$$

$$= \frac{1}{(1+e^{-x})^2} \cdot e^{-x}$$

$$= \frac{e^{-x}}{1+e^{-x}} \cdot \frac{1}{1+e^{-x}}$$

$$= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}+1-1}{1+e^{-x}}$$

$$= y(1-y)$$

2. $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Quotient Rule: $\frac{d}{dx}\left[\frac{F(x)}{g(x)}\right] = \left(\frac{d}{dx} F(x) \cdot g(x) - F(x) \cdot \frac{d}{dx} g(x)\right) \cdot \frac{1}{(g(x))^2}$

$$\frac{\partial y}{\partial x} = \frac{\frac{\partial}{\partial x}(e^x - e^{-x})(e^x + e^{-x}) - (e^x - e^{-x}) \cdot \frac{\partial}{\partial x}(e^x + e^{-x})}{(g(x))^2}$$

$$= \frac{(e^x + e^{-x}) \cdot (e^x + e^{-x})}{(e^x + e^{-x})^2} - \frac{(e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})}$$

$$= 1 - \frac{(e^x - e^{-x})^2}{(e^x + e^{-x})^2}$$

$$= 1 - \tanh(x)^2$$

3. $y = \log(1+e^x)$ $\quad a = 1+e^x$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial a} \cdot \frac{\partial a}{\partial x}$$

$$= \frac{\partial}{\partial a} \log(a) \cdot \frac{\partial}{\partial x} 1+e^x$$

$$= \frac{1}{a \ln(e)} \cdot e^x$$

$$= \frac{e^x}{1+e^x} \quad \frac{\frac{1}{e^x}}{\frac{1}{e^x}}$$

$$= \frac{1}{1+e^{-x}}$$

4a. $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$$= \frac{e^x + e^{-x} - 2e^{-x}}{e^x + e^{-x}}$$

$$= \frac{e^x + e^{-x}}{e^x + e^{-x}} - \frac{2e^{-x}}{e^x + e^{-x}}$$

$$= 1 - \frac{2e^{-x}}{e^x + e^{-x}} \quad \frac{\frac{1}{e^{-x}}}{\frac{1}{e^{-x}}}$$

$$= 1 - \frac{2}{e^{2x}+1} \qquad \text{but } \sigma(x) = \frac{1}{1+e^{-x}}$$

$$= 1 - 2\sigma(-2x)$$

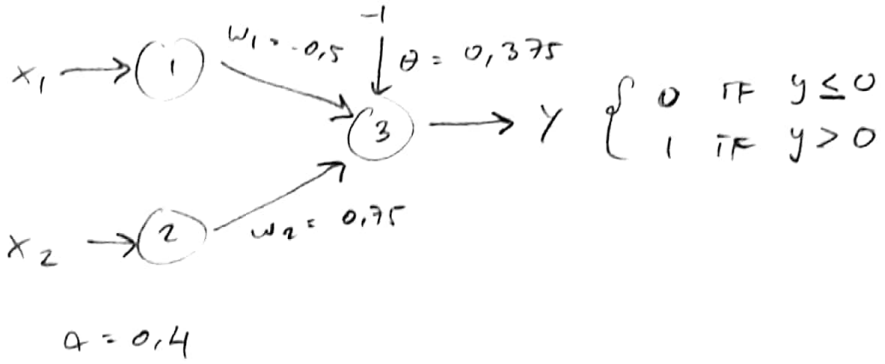
$$= 1 - 2(1 - \sigma(2x))$$

$$= 1 - 2 + 2\sigma(2x)$$

$$= 2\sigma(2x) - 1$$

5. OR with table

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1



$\alpha = 0,4$

Δ epoch 1
batch 1

- calculate the Forward propagation ($x_1 = 0$ $x_2 = 0$)

$$\hat{y} = x_1 . w_1 + x_2 . w_2 - \theta$$
$$= 0.(-0,5) + 0.0,75 - 0,375$$
$$= -0,375$$
$$= 0.$$

- calculate error

$$error = y - \hat{y}$$
$$= 0 - 0$$
$$= 0$$

batch 2

- calculate the Forward propagation ($x_1 = 0$, $x_2 = 1$).

$$\hat{y} = x_1 w_1 + x_2 w_2 - \theta$$
$$= 0.(-0,5) + 1.0,75 - 0,375$$
$$= 0,375$$
$$= 1.$$

- calculate error

$$error = y - \hat{y}$$
$$= 1 - 1$$
$$= 0$$

batch 3

- calculate the forward propagation ($x_1 = 1$, $x_2 = 0$)

$\hat{y} = x_1 w_1 + x_2 w_2 - \theta$

$= 1.(0,5) + 0.0,75 - 0,375$

$= -0,875$

$= 0$

- calculate error

error $= y - \hat{y}$

$= 1 - 0$

$= 1$ → error occured, update the weight

- update weight

$w_1 = w_1 + \eta . x_1 . error$

$= -0,5 + 0,4 . 1 . 1$

$= -0,1$

$w_2 = w_2 + \eta . x_2 . error$

$= 0,75 + 0,4 . 0 . 1$

$= 0,75$

batch 4

- calculate the forward propagation ($x_1 = 1$, $x_2 = 1$).

$\hat{y} = x_1 w_1 + x_2 w_2 - \theta$

$= 1.(-0,1) + 1.0,75 - 0,375$

$= 0,65 - 0,375$

$= 0,275$

$= 1$

- calculate error

error $= y - \hat{y}$

$= 1 - 1$

$= 0.$

* epoch 1 updated weight :

$w_1 = -0,1$

$w_2 = 0,75$

repeat for several epoch until the error is always zero

6a $F(net) = \frac{2a}{1+e^{-bnet}} - a \qquad z = 1+e^{-bnet}$

$$\frac{\partial F}{\partial net} = \frac{\partial F}{\partial z} \cdot \frac{\partial z}{\partial net}$$

$$= \frac{\partial}{\partial z} 2a z^{-1} - a \frac{\partial}{\partial net} 1+e^{-bnet}$$

$$= -2a z^{-2} - bnet^{-bnet}$$

$$= \frac{2a}{(1+e^{-bnet})^2} \cdot be^{-bnet}$$

$$= \frac{2a}{1+e^{-bnet}} \cdot \frac{be^{-bnet}}{1+e^{-bnet}}$$

$$= (F(net) - a) \cdot (1 - (F(net) - a))$$

7. $E = \frac{1}{2}\sigma^2 - y_d w + \frac{1}{2} y_x w^2$

a. $\frac{\partial E}{\partial w} = \frac{\partial}{\partial w} \frac{1}{2}\sigma^2 - y_d w + \frac{1}{2} y_x w^2$

$= -y_d + y_x w$

b. $w_{k+1} = w_k - \alpha(-y_d + y_x w_k)$

c. $\frac{\partial E}{\partial w} = 0$

$-y_d + y_x w = 0$

$w = \frac{y_d}{y_x}$