Forecasting Lightweight Vehicle Sales

Sifan Wu, Jeremy Xu, Kairan Zhong, Felix Wang, Jun Tan

Time Series Analysis and Forecasting

May 24, 2023

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Outline

Introduction

Data Selection

Experimental results and Analysis

Model Selection

Conclusion and Future work

Introduction

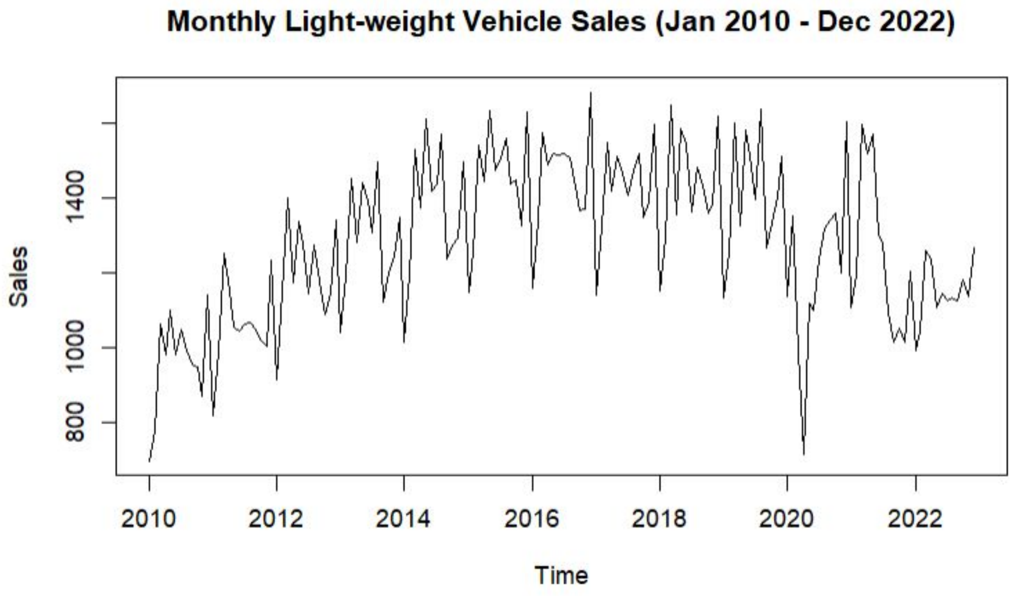
- Analyze the monthly lightweight vehicle sales data from Jan 2010 to Dec 2022 (https://fred.stlouisfed.org/series/LTOTALNSA & https://fred.stlouisfed.org/series/DSPIC96).

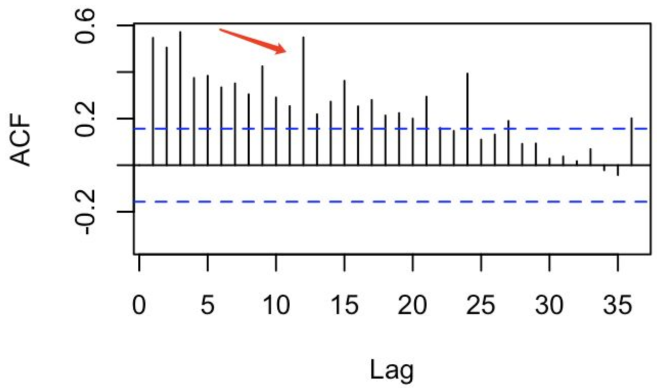
- To forecast the demand for lightweight vehicles over the next 24 months.
- To see if fuel-efficient and environmentally friendly vehicles will have a growing market
  - If so, help manufacturers prepare in advance for the increasing demand in the future
  - If not, encourage policymakers to make better policies of promoting the lightweight vehicles

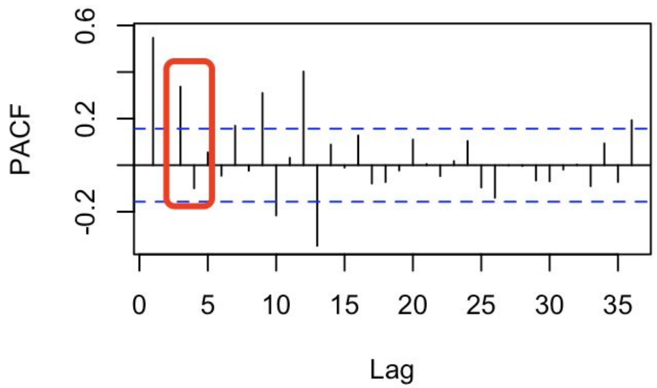
- Model used:
  - Linear regression - Sifan Wu
  - Exponential smoothing - Jeremy Xu
  - ARIMA - Kairan Zhong
  - Regression with ARMA Errors - Felix Wang
  - Intervention Analysis - Jun Tan

Data Selection

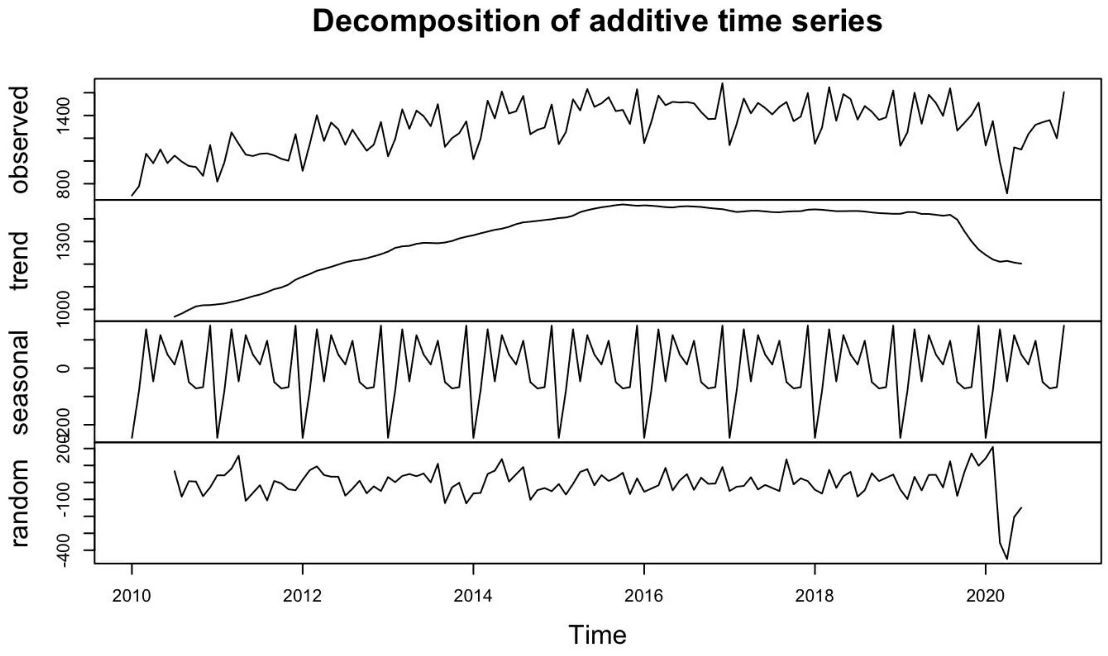
- Two variables
  - Light-weight Vehicle Sales
  - US Disposable Income
- The original data set
  - has seasonality with an upward trend.
  - is Non-stationary.
- Stationarize data
  - seasonal differencing -> first order differencing
- The sample ACF and the sample PACF seem to indicate that an $AR(3)$ model should be specified.
- Train set: 2020/01 - 2020/12   Test set: 2021/01 - 2022/12

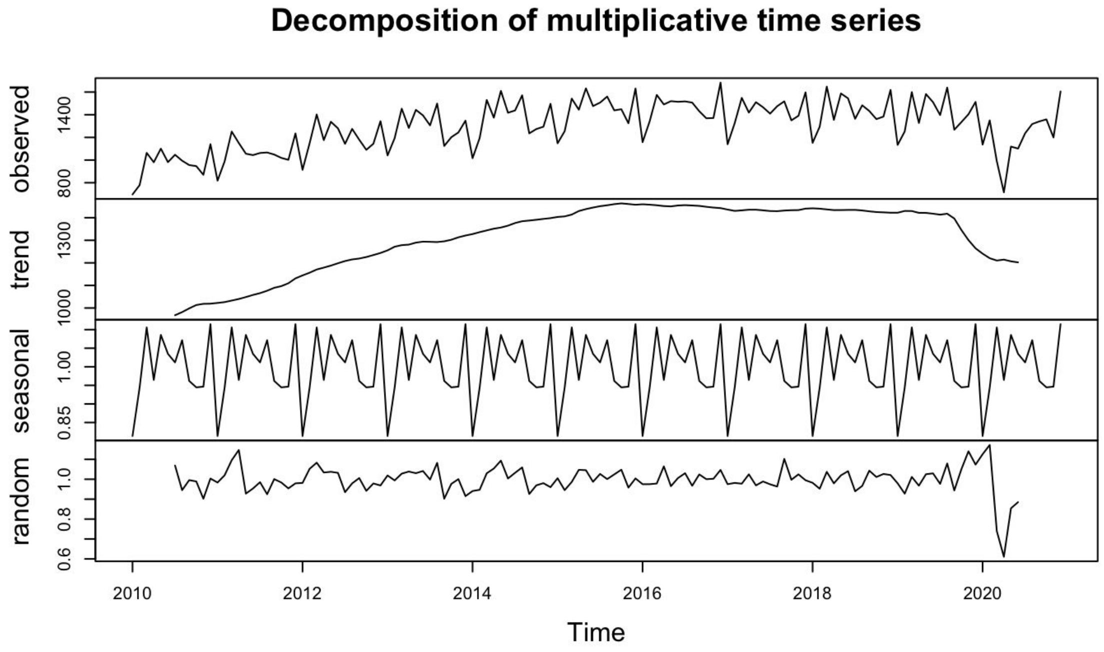






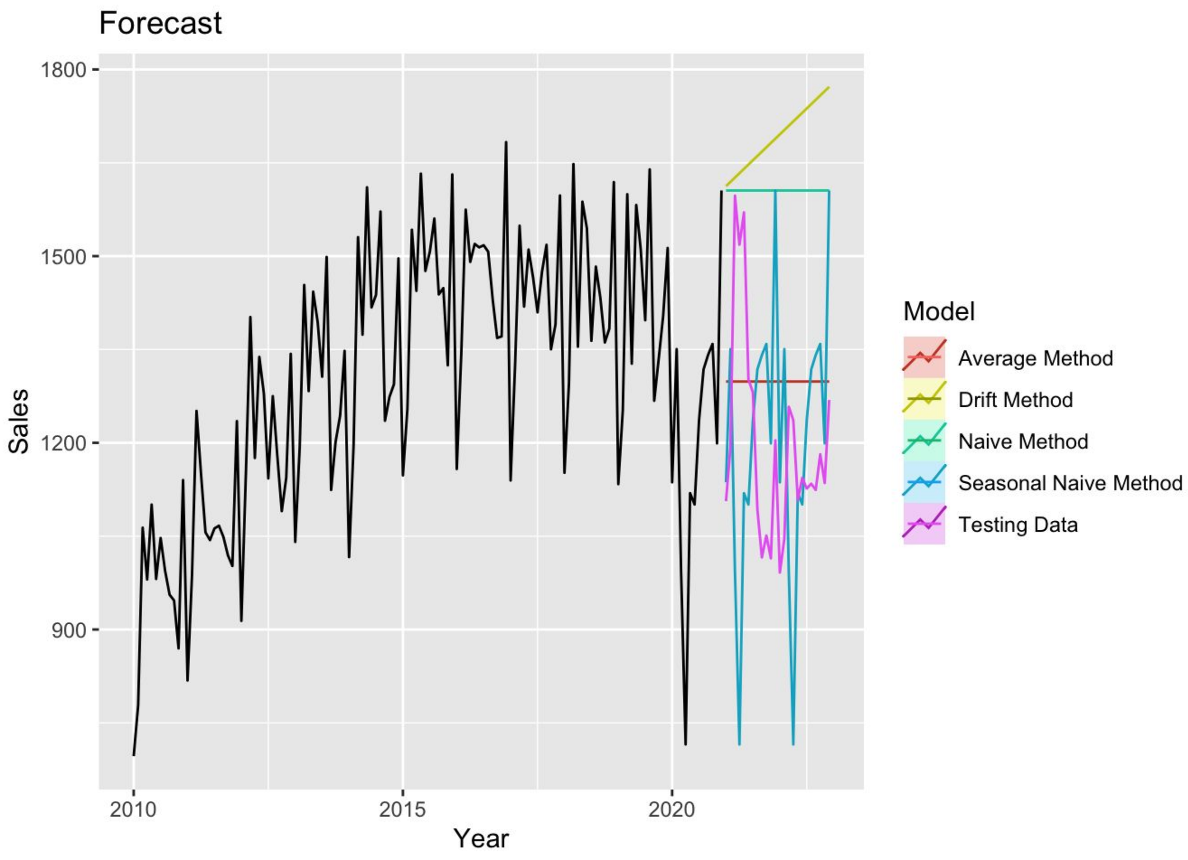
Time Series Decomposition





The remainder component for multiplicative decomposition looks like a white noise, so multiplicative decomposition makes more sense.

Baseline Approaches in Forecasting



	RMSE	MAE	Ljung-Box Test (p-value)
Average Method	192.0058	168.8056	< 0.05
Drift Method	529.8307	496.3407	< 0.05
Naive Method	440.5821	409.6111	< 0.05
Seasonal Naive Method	318.3891	254.9943	< 0.05

Linear Regression

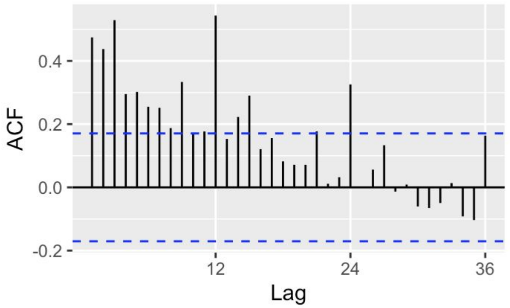
Linear Regression

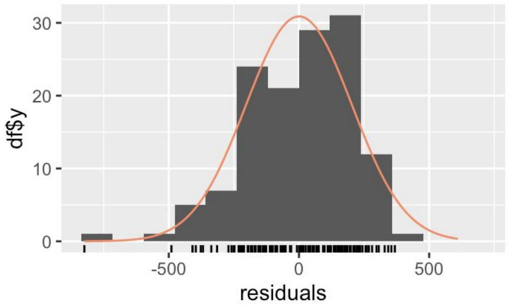
- **Model Fitting**

Light Weight Vehicle Sales ~ US Disposable Income

- **Model Diagnostics**

Residuals are autocorrelated



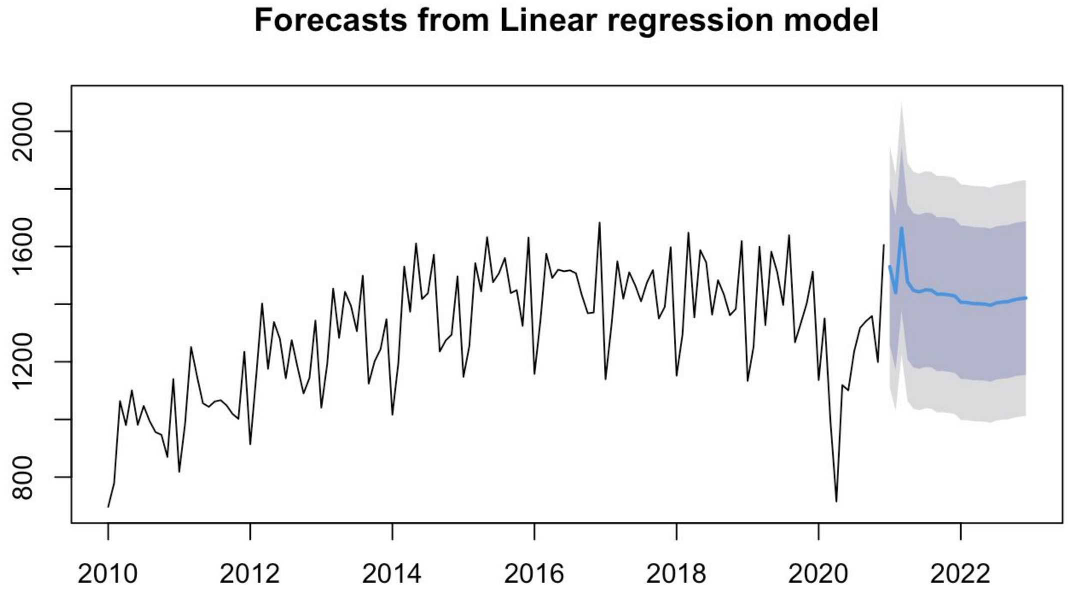


- **Model Evaluation**

Method: Train-test Split

RMSE: 279.3836

MAE: 255.9479




```
data:  Residuals from Linear regression model
LM test = 93.853, df = 24, p-value = 3.29e-10
```

Exponential Smoothing

Holt-Winters Seasonal Method

```r
## Linear trend with additive seasonality
m_add <- hw(ts_train, h=24, seasonal="additive", damped=FALSE)
m_add$model$aicc
```

```
## [1] 1887.252
```

```r
## Linear trend with additive seasonality and damping
m_damped_add <- hw(ts_train, h=24, seasonal="additive", damped=TRUE)
m_damped_add$model$aicc
```

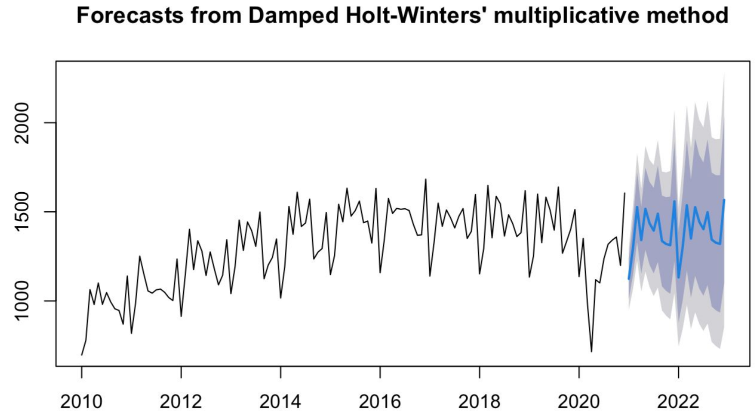
```
## [1] 1887.81
```

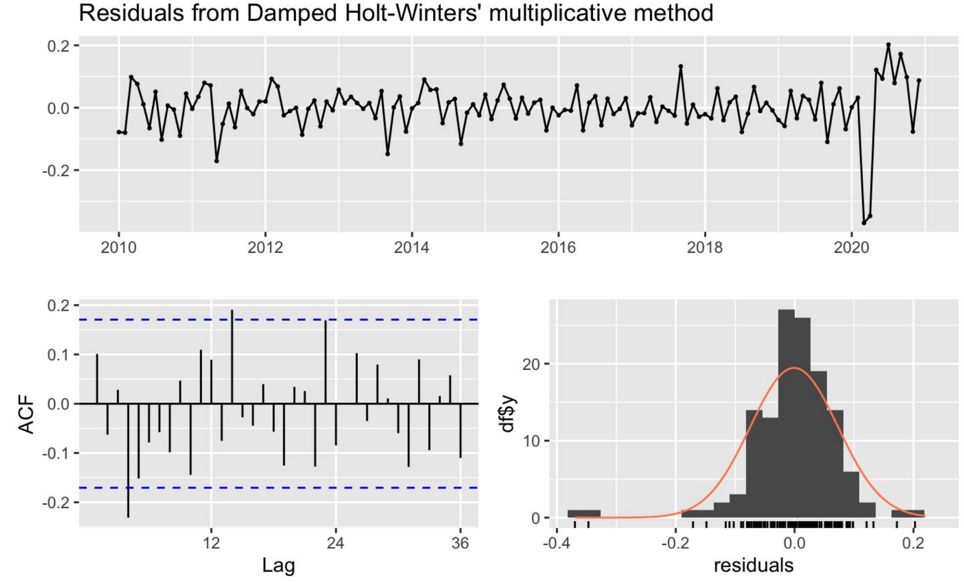
```r
## Linear trend with multiplicative seasonality
m_multi <- hw(ts_train, h=24, seasonal="multiplicative", damped=FALSE)
m_multi$model$aicc
```

```
## [1] 1899.315
```

```r
## Linear trend with multiplicative seasonality and damping
m_damped_multi <- hw(ts_train, h=24, seasonal="multiplicative", damped=TRUE)
m_damped_multi$model$aicc ## best model
```

```
## [1] 1886.972
```

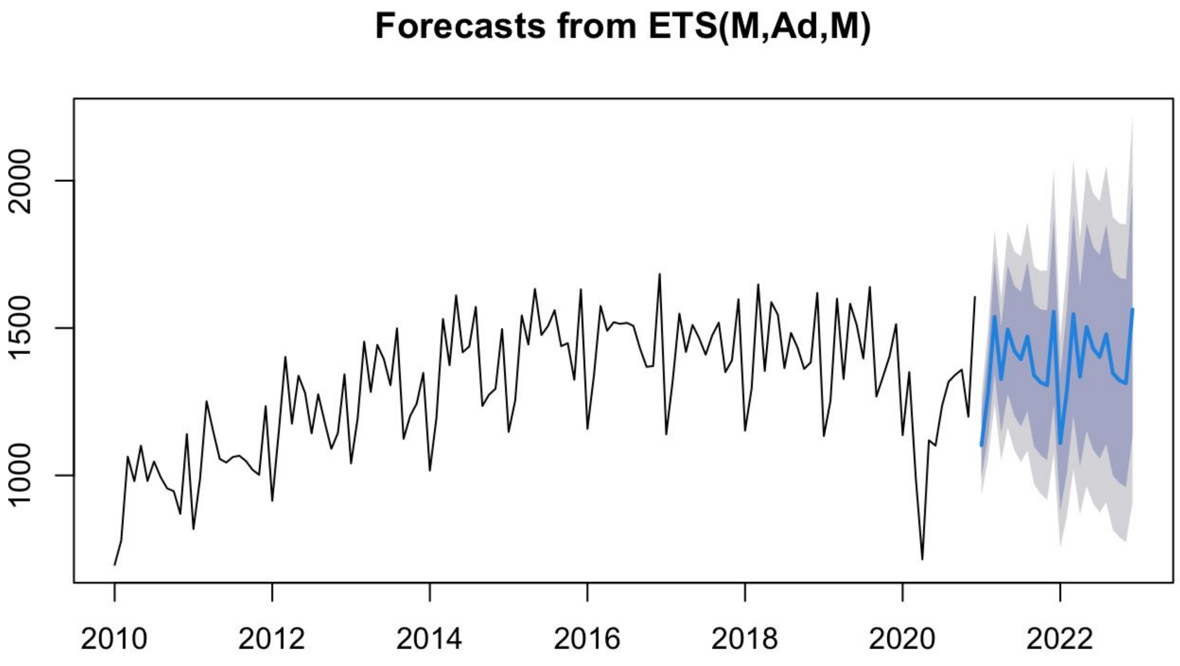


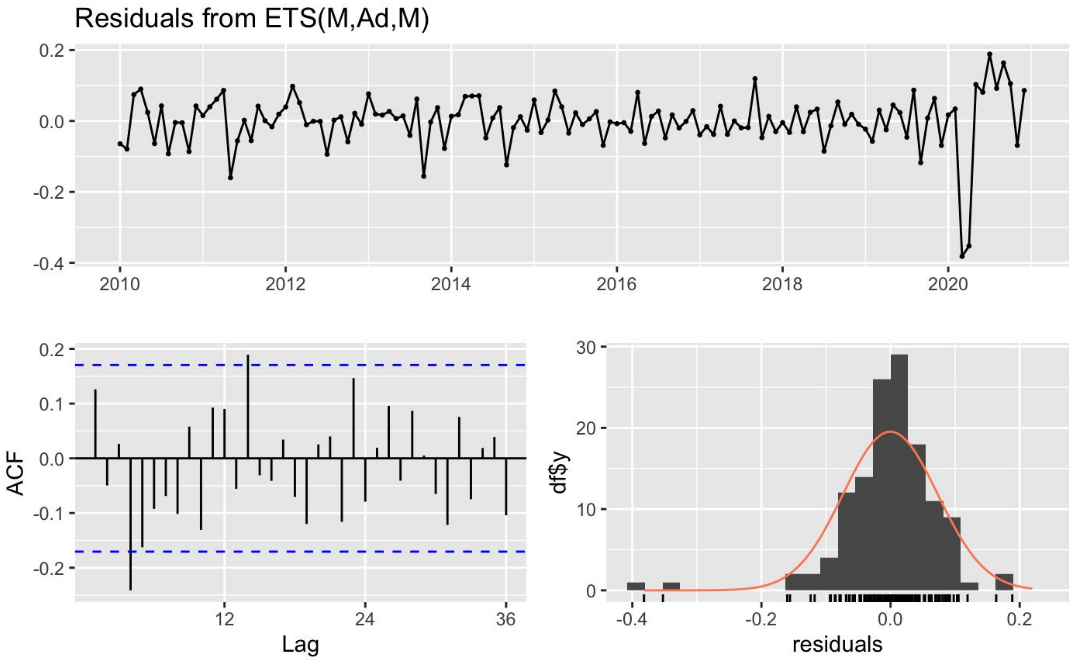



```
##
##  Ljung-Box test
##
## data:  Residuals from Damped Holt-Winters' multiplicative method
## Q* = 40.243, df = 24, p-value = 0.02013
##
## Model df: 0.   Total lags used: 24
```

Exponential Smoothing State Space Model

ETS (Error, Trend, Seasonal)

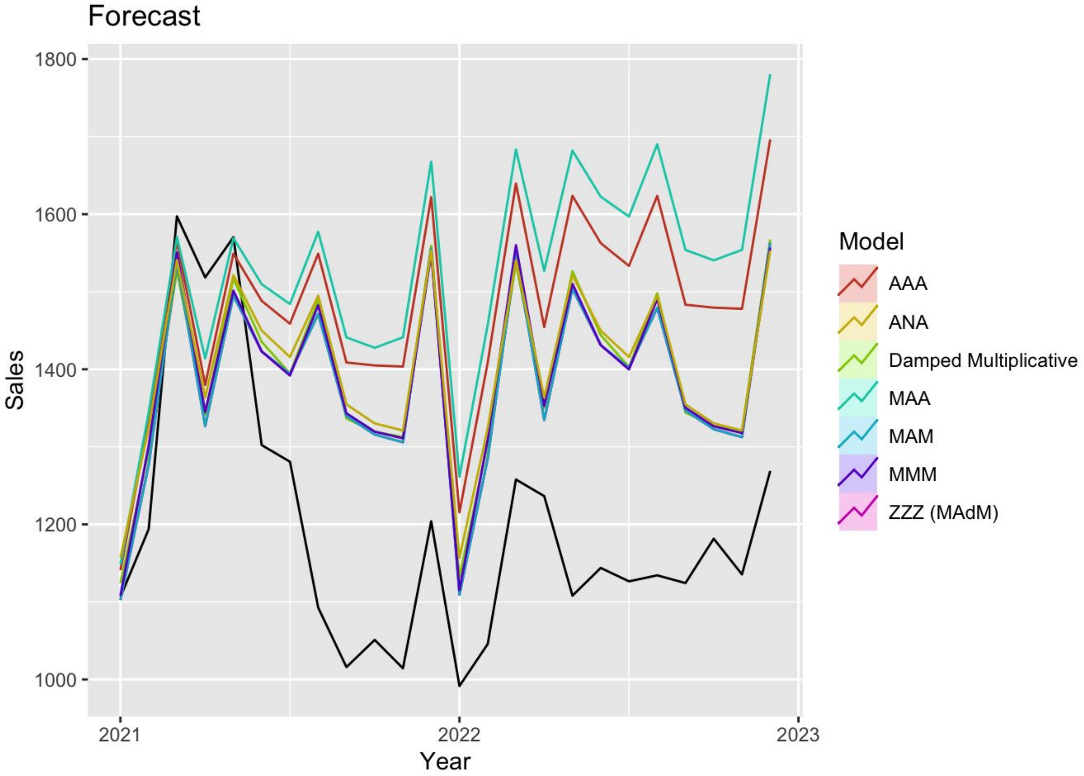




Model with *multiplicative errors, additive damped trend, and multiplicative seasonality*.

```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(M,Ad,M)
## Q* = 39.554, df = 24, p-value = 0.02387
##
## Model df: 0.   Total lags used: 24
```

Comparing Exponential Smoothing Models



The ETS (M, Ad, M) model has the best accuracy.

```
##                  ME     RMSE      MAE       MPE     MAPE      ACF1 Theil's U
## Test set -186.6456 240.8748 214.1647 -16.87944 18.67013 0.5718254  1.840553
```

```
## ETS(M,Ad,M)
##
## Call:
##  ets(y = ts_train, model = "ZZZ")
##
##   Smoothing parameters:
##     alpha = 0.5253
##     beta  = 1e-04
##     gamma = 2e-04
##     phi   = 0.9789
##
##   Initial states:
##     l = 914.3187
##     b = 14.1728
##     s = 1.125 0.945 0.9527 0.9705 1.0663 1.0109
##            1.0325 1.0852 0.9637 1.1176 0.9285 0.8021
##
##   sigma:  0.0782
##
##      AIC     AICc      BIC
## 1878.922 1884.975 1930.813
```

ARIMA

ARIMA

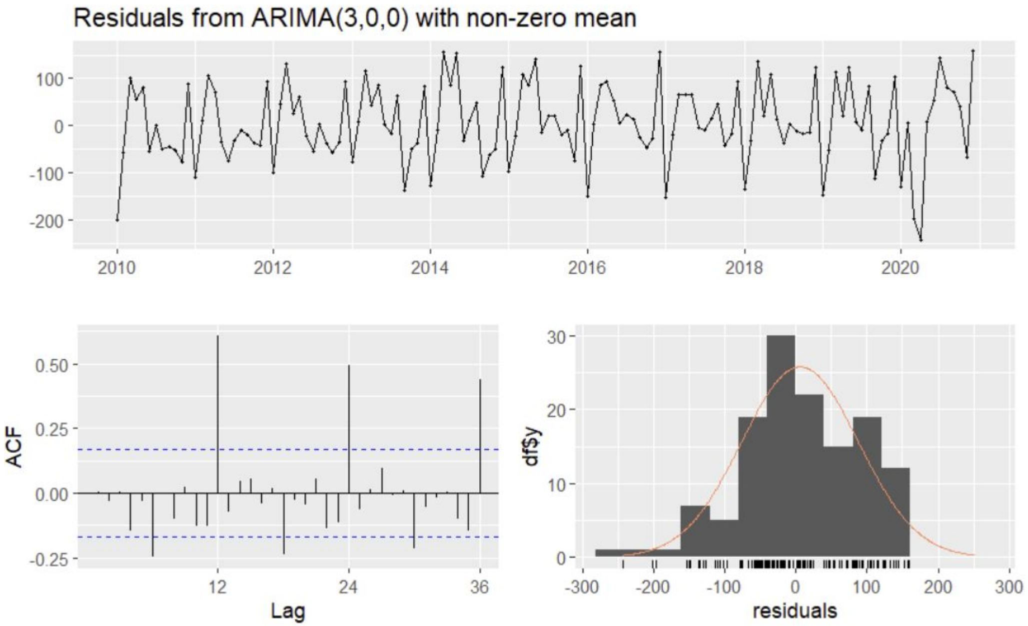
ARIMA(3,0,3) with no seasonality

```
Series: train_ts
ARIMA(3,0,0) with non-zero mean
Box Cox transformation: lambda= 0.910537

Coefficients:
         ar1     ar2     ar3      mean
      0.2451  0.2065  0.4112  731.5146
s.e.  0.0812  0.0832  0.0826   48.1274

sigma^2 = 6927:  log likelihood = -769.6
AIC=1549.2   AICc=1549.68   BIC=1563.62

Training set error measures:
                      ME     RMSE      MAE        MPE     MAPE     MASE         ACF1
Training set 11.9926 155.0954 122.8004 -0.5851951 10.01626 1.277135 0.0004511321
```



AICc = 1549.68 BIC = 1563.62

Significant spikes at lag 12, 24… indicates the residual is not white noise and has strong seasonality

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SARIMA

Model1: ARIMA(0,1,1)(2,1,2)[12]

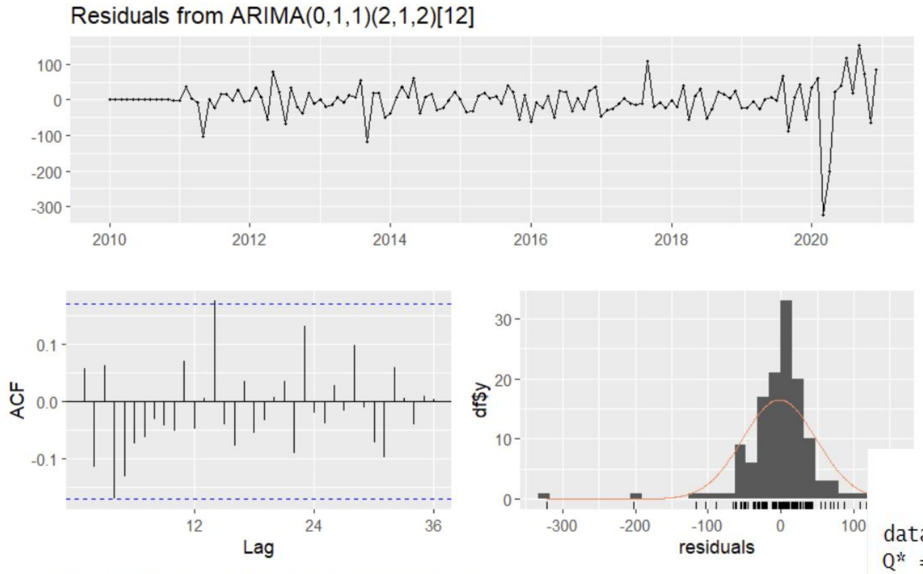
```
Series: train_ts
ARIMA(0,1,1)(2,1,2)[12]
Box Cox transformation: lambda= 0.9105539

Coefficients:
          ma1     sar1      sar2      sma1   sma2
      -0.4653  0.7983  -0.6690  -1.1568  0.659
s.e.   0.1139  0.2330   0.2045   0.2650  0.301

sigma^2 = 2947:  log likelihood = -646.64
AIC=1305.27   AICc=1306.02   BIC=1321.95

Training set error measures:
                    ME     RMSE      MAE        MPE     MAPE      MASE       ACF1
Training set -4.134603 95.38499 57.09142 -0.7680598 4.632912 0.5937556 0.04946321
```

AICc = 1306.02 BIC = 1321.95

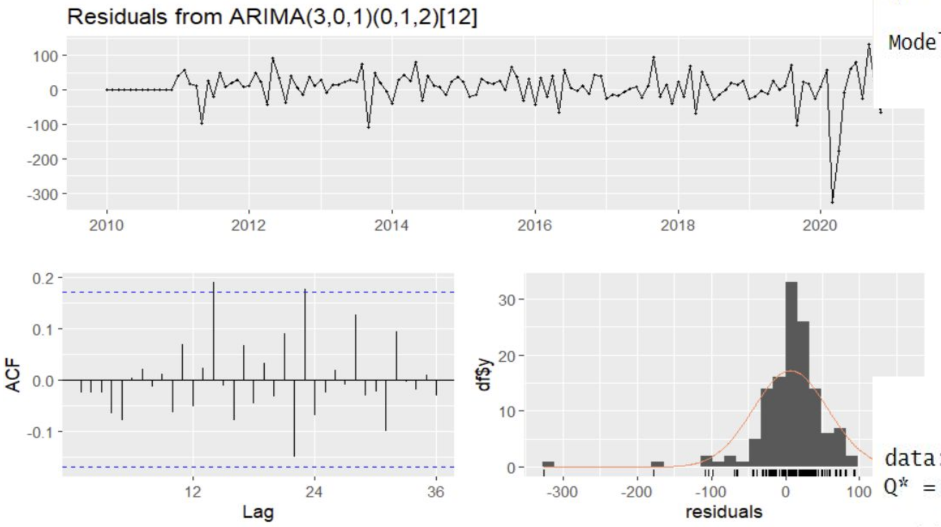
Model2: ARIMA(3,0,1)(0,1,2)[12] (with d =0)

```
Series: train_ts
ARIMA(3,0,1)(0,1,2)[12]
Box Cox transformation: lambda= 0.9105539

Coefficients:
          ar1     ar2     ar3     ma1     sma1     sma2
      -0.1314  0.5076  0.3503  0.7102  -0.3332  -0.2898
s.e.   0.3431  0.2625  0.0897  0.3726   0.1633   0.1453

sigma^2 = 2934:  log likelihood = -649.32
AIC=1312.63   AICc=1313.63   BIC=1332.14

Training set error measures:
                   ME     RMSE      MAE      MPE     MAPE      MASE        ACF1
Training set 12.80432 95.27845 61.44751 0.580297 4.963525 0.6390592 -0.03469308
```

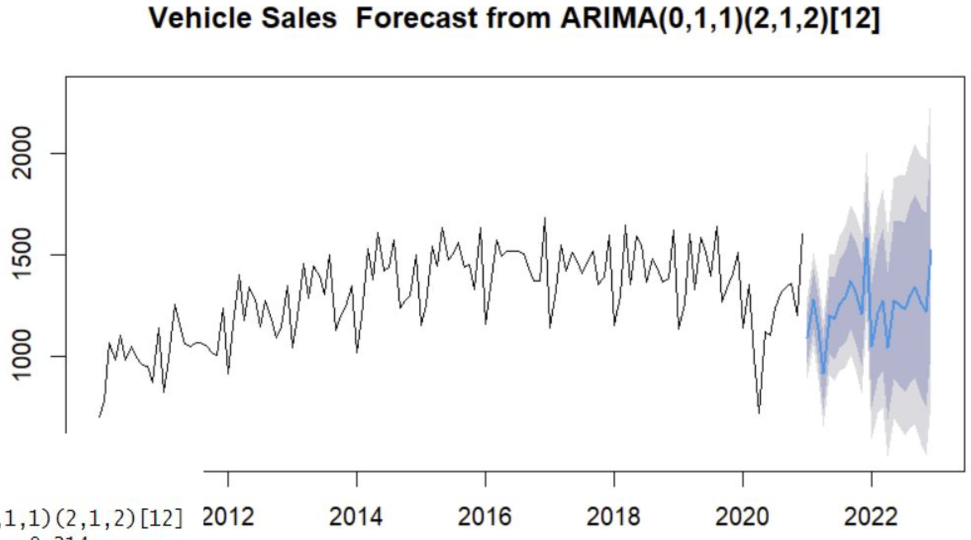
AICc = 1313.63 BIC = 1332.14

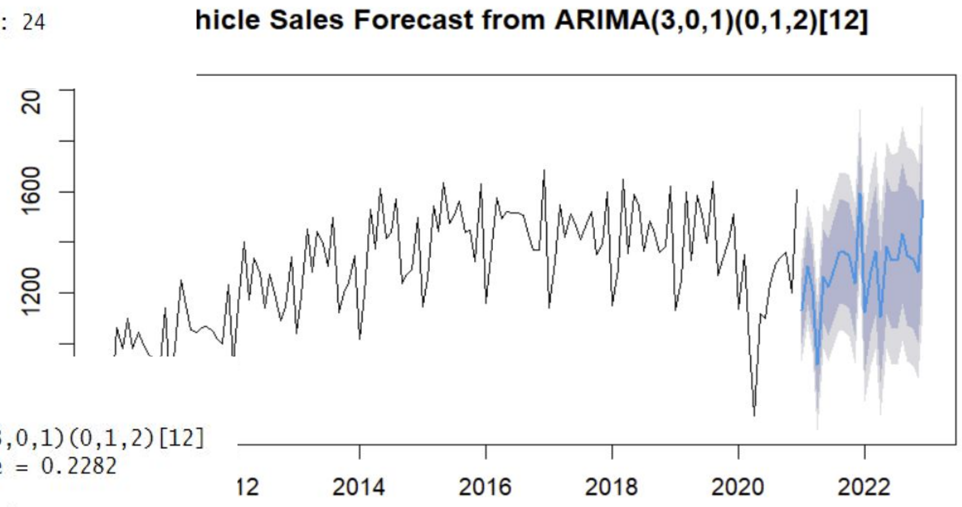



```
        Ljung-Box test

data:  Residuals from ARIMA(0,1,1)(2,1,2)[12]
Q* = 23.548, df = 19, p-value = 0.214

Model df: 5.   Total lags used: 24
```




```
        Ljung-Box test

data:  Residuals from ARIMA(3,0,1)(0,1,2)[12]
Q* = 22.087, df = 18, p-value = 0.2282

Model df: 6.   Total lags used: 24
```





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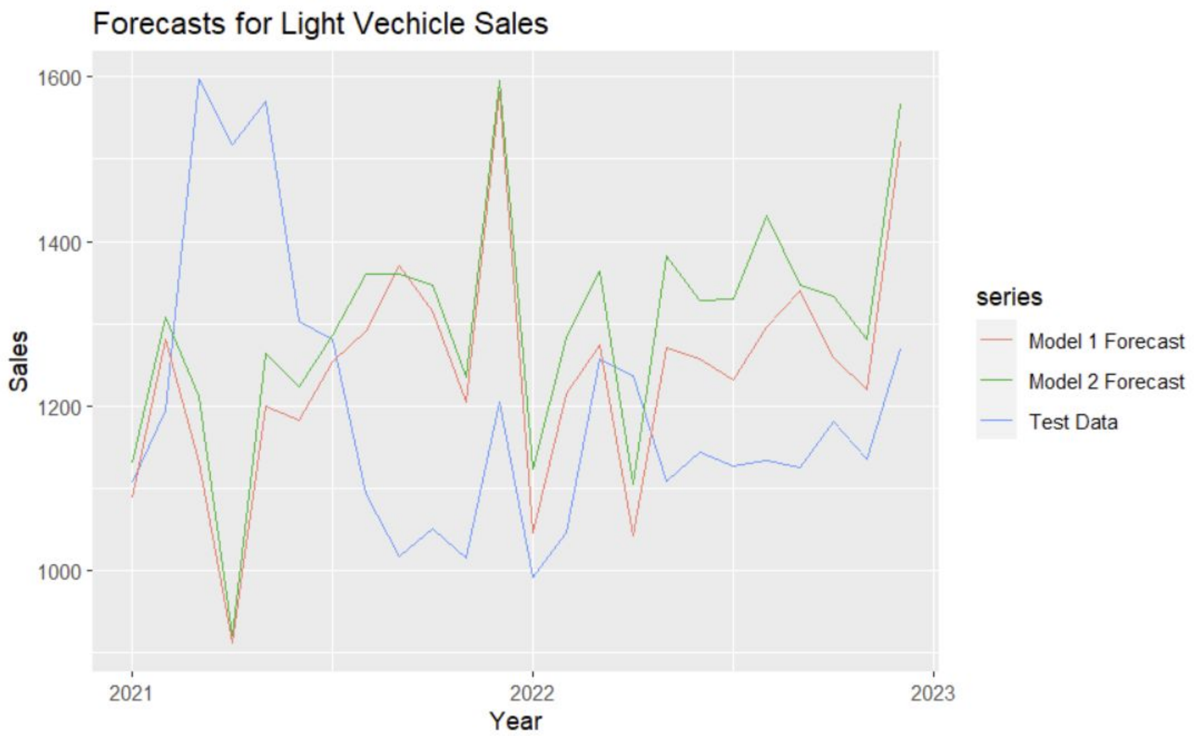
SARIMA



Final model:

ARIMA(0,1,1)(2,1,2)[12]

Model <chr>	MAPE <dbl>	RMSE <dbl>
Model 1	15.88123	243.9537
Model 2	18.79237	260.0702

2 rows

Regression with ARMA Errors

Regression with ARMA Errors

- **Model specification and Estimation: ARIMA(4,1,1)(0,1,1)[12] errors**

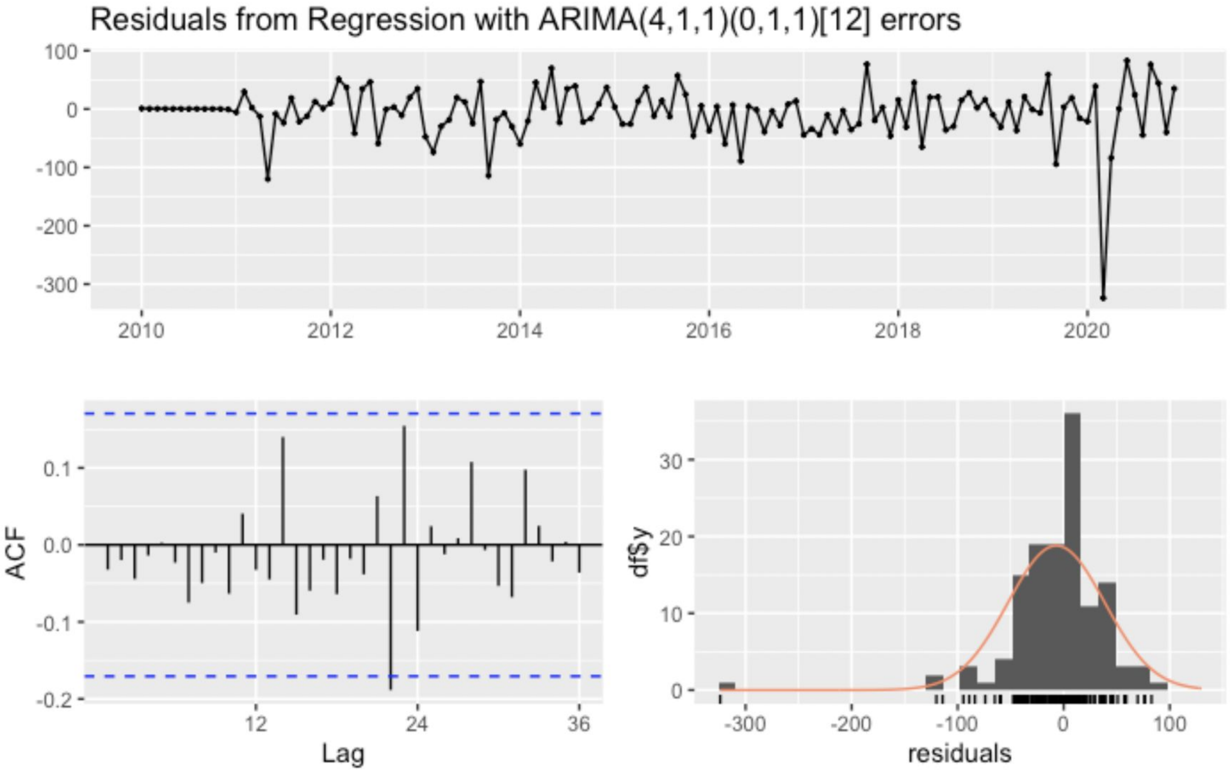
```
Series: train_vehicle_sales
Regression with ARIMA(4,1,1)(0,1,1)[12] errors
Box Cox transformation: lambda= 0.910537

Coefficients:
         ar1     ar2     ar3      ar4      ma1     sma1     xreg
      0.1426  0.1300  0.2447  -0.3146  -0.8344  -0.6794  -0.0978
s.e.  0.1199  0.1048  0.0978   0.0920   0.0924   0.1198   0.0219

sigma^2 = 2486:  log likelihood = -634.95
AIC=1285.91   AICc=1287.22   BIC=1308.14

Training set error measures:
                    ME     RMSE      MAE       MPE   MAPE      MASE
Training set -13.00368 87.19715 55.40375 -1.345904 4.4077 0.5762037 -0
```

- **Model Diagnostics: checkresiduals()**



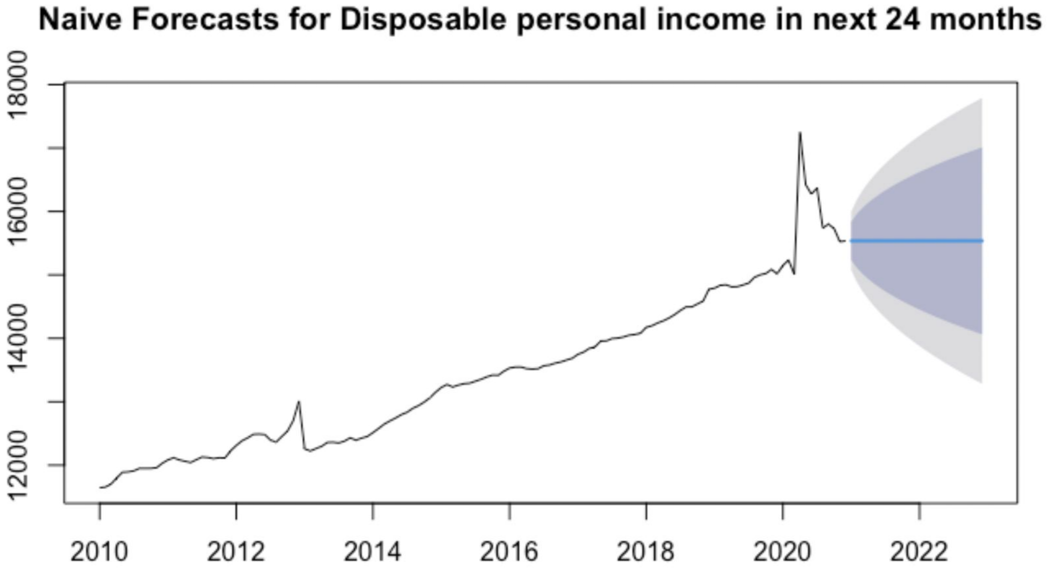

```
        Ljung-Box test

data:  Residuals from Regression with ARIMA(4,1,1)(0,1,1)[12] errors
Q* = 20.993, df = 18, p-value = 0.2797

Model df: 6.   Total lags used: 24
```

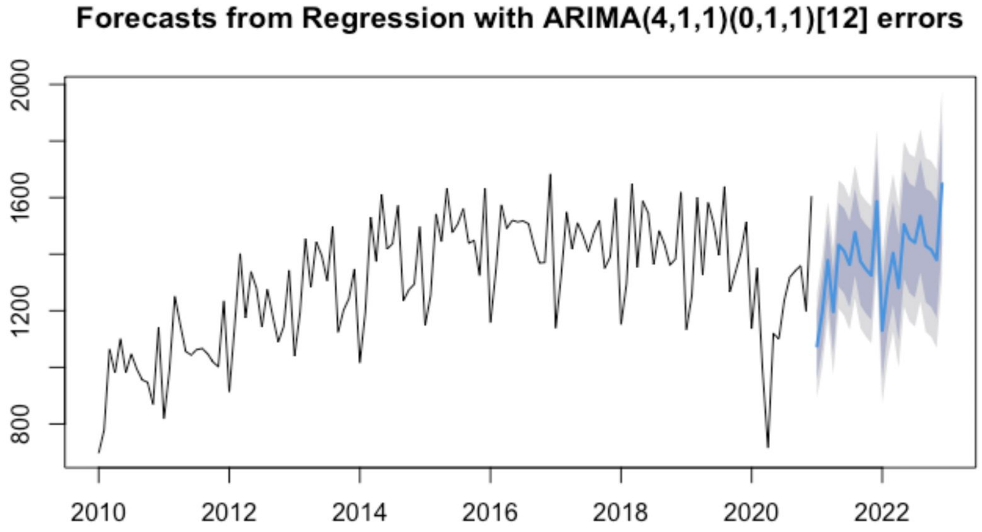
Regression with ARMA Errors

- **Forecast**



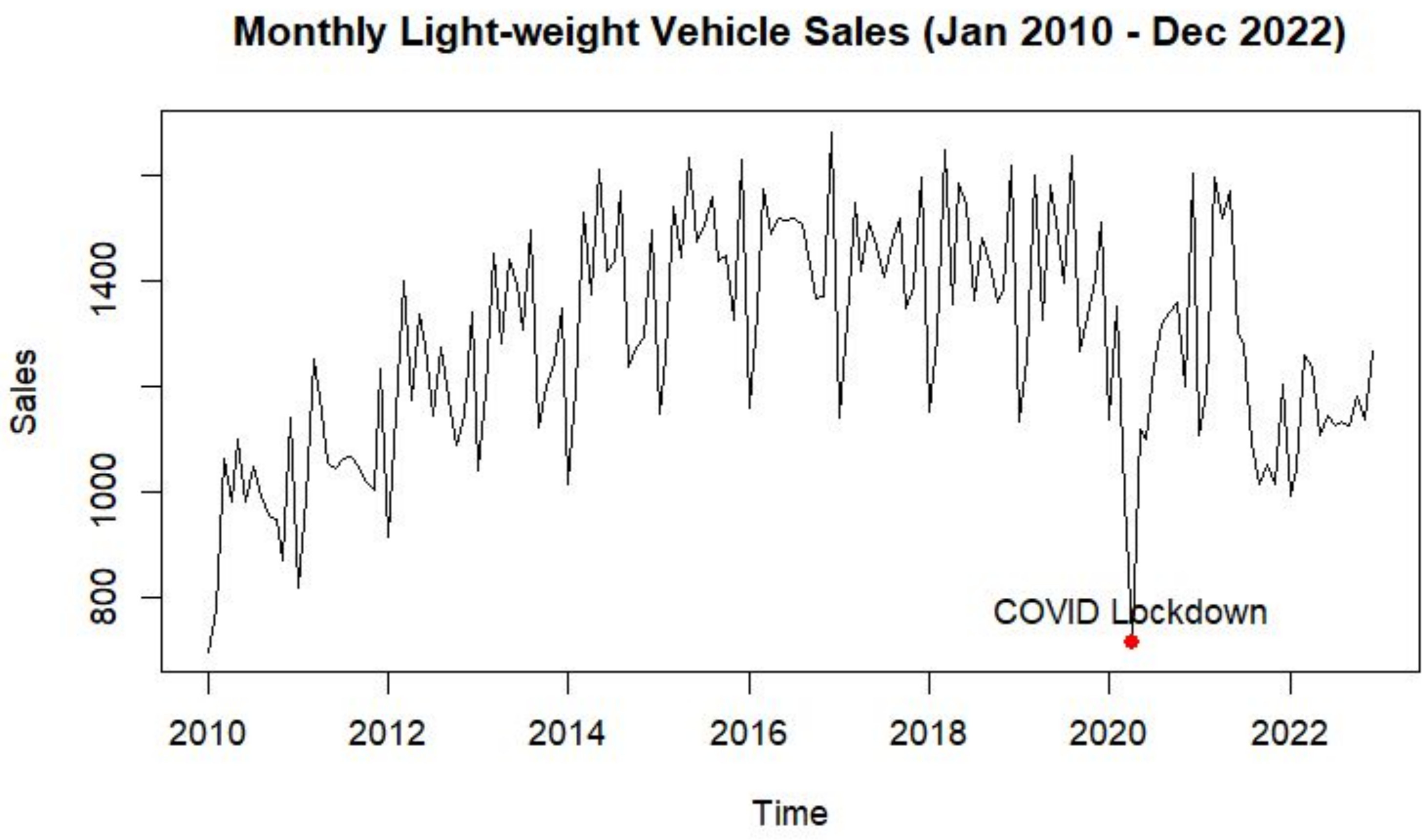
- **Model Evaluation:**
  - Train: 132 (2010.1-2020.12)
  - Test: 24 (2021.1-2022.12)
  - Around 80%, 20%

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	-13.00368	87.19715	55.40375	-1.345904	4.40770	0.5762037	-0.03867808	NA
Test set	-182.76525	271.45624	241.92899	-16.988012	20.86397	2.5160819	0.68527567	2.049608

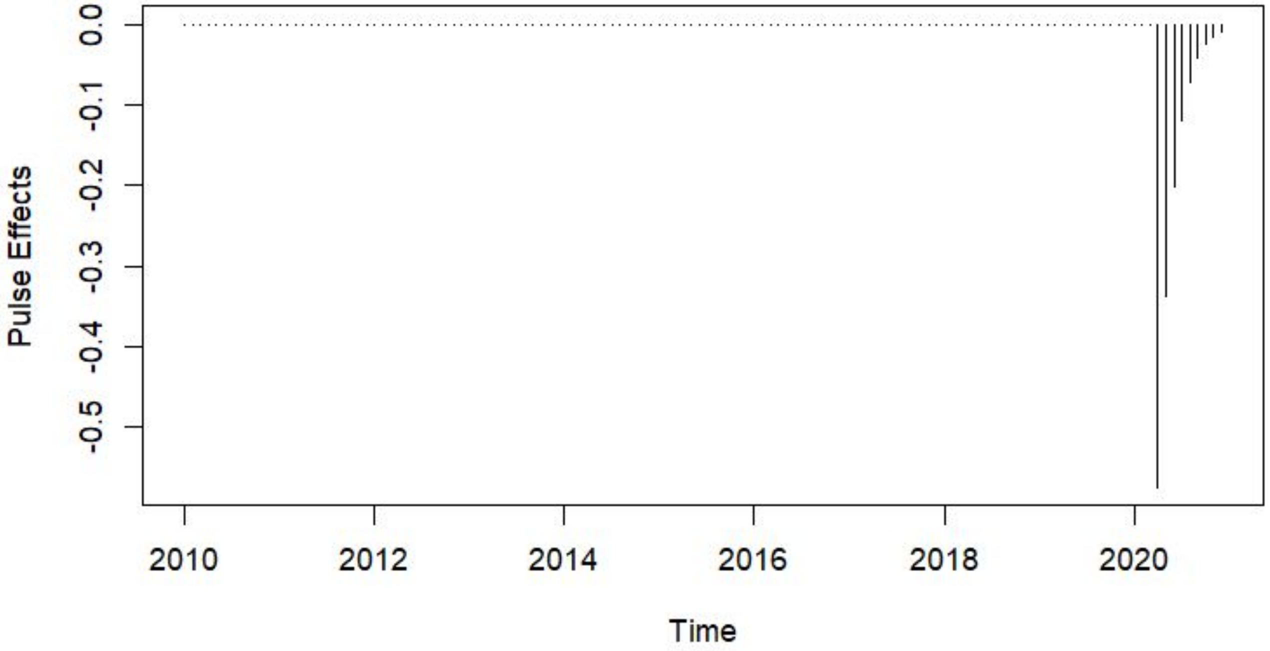


Intervention Analysis

Intervention Specification - Pulse Function



Intervention Analysis




```
Call:
arimax(x = log(train), order = c(0, 1, 1), seasonal = list(order = c(0, 1, 0),
    period = 12), method = "ML", xtransf = data.frame(I2020, I2020), transfer = list(c(0,
    0), c(1, 0)))

Coefficients:
          ma1  I2020-MA0  I2020.1-AR1  I2020.1-MA0
      -0.8897    -0.0086       0.5953      -0.5658
s.e.   0.0391     0.1810       0.1035       0.1663

sigma^2 estimated as 0.005411:  log likelihood = 140.91,  aic = -273.82

Training set error measures:
                      ME       RMSE        MAE        MPE     MAPE      MASE       ACF1
Training set -0.01220034 0.06987645 0.04486797 -0.1744702 0.627302 0.3579166 -0.1304569
```

- Based on the pre intervention data, an ARIMA(0, 1, 1) × (0, 1, 0)[12] model was tentatively specified for the unperturbed process.
- The fitted model estimates that the the COVID-related activity restrictions reduced light-weight vehicle sales by 57.4% and sales k months later was lowered by $(1 - \exp(-0.5658 * 0.5953^k)) * 100\%$.

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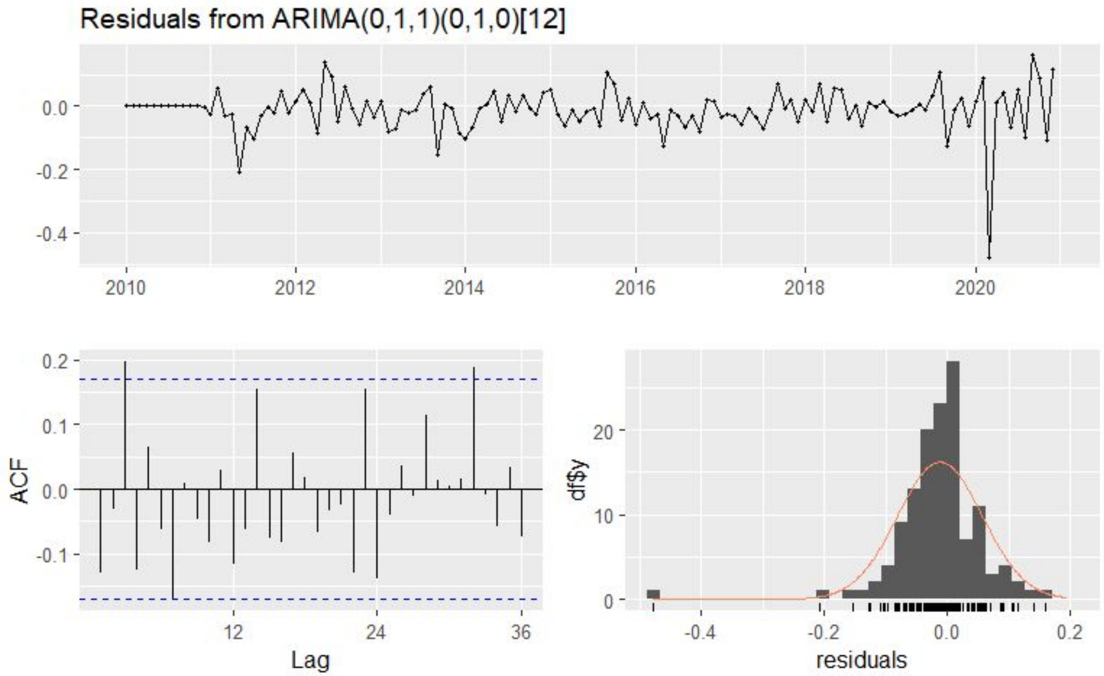
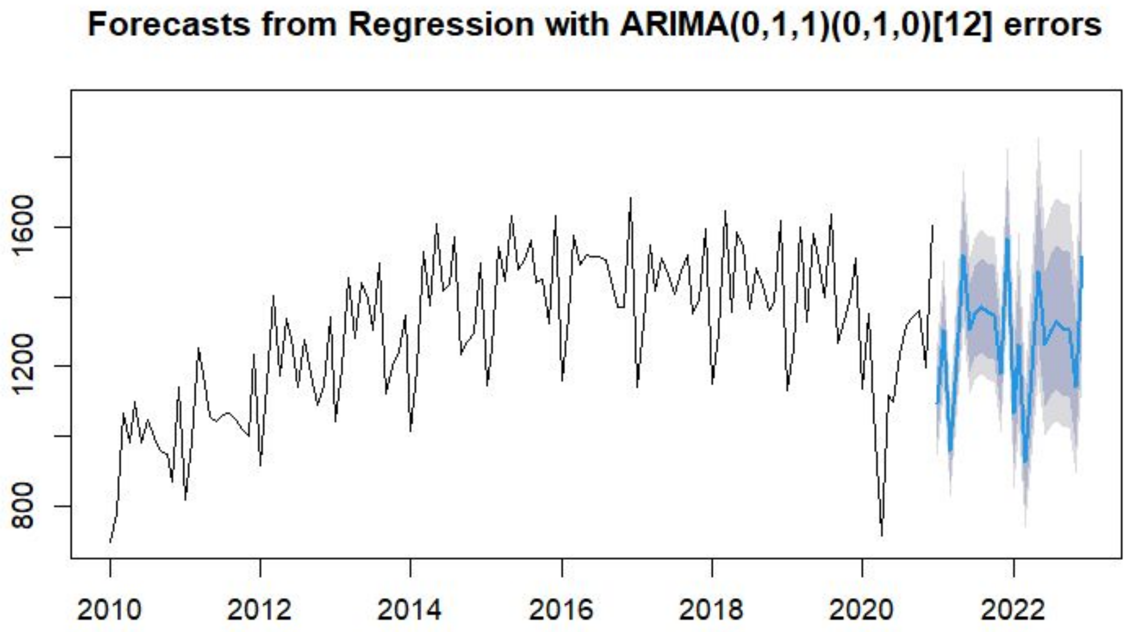
Intervention Analysis - Residuals and Forecasts






```
        Ljung-Box test

data:  Residuals from ARIMA(0,1,1)(0,1,0)[12]
Q* = 36.216, df = 23, p-value = 0.03921

Model df: 1.   Total lags used: 24
```

Conclusion and Future Work

Model Selection: Average Method

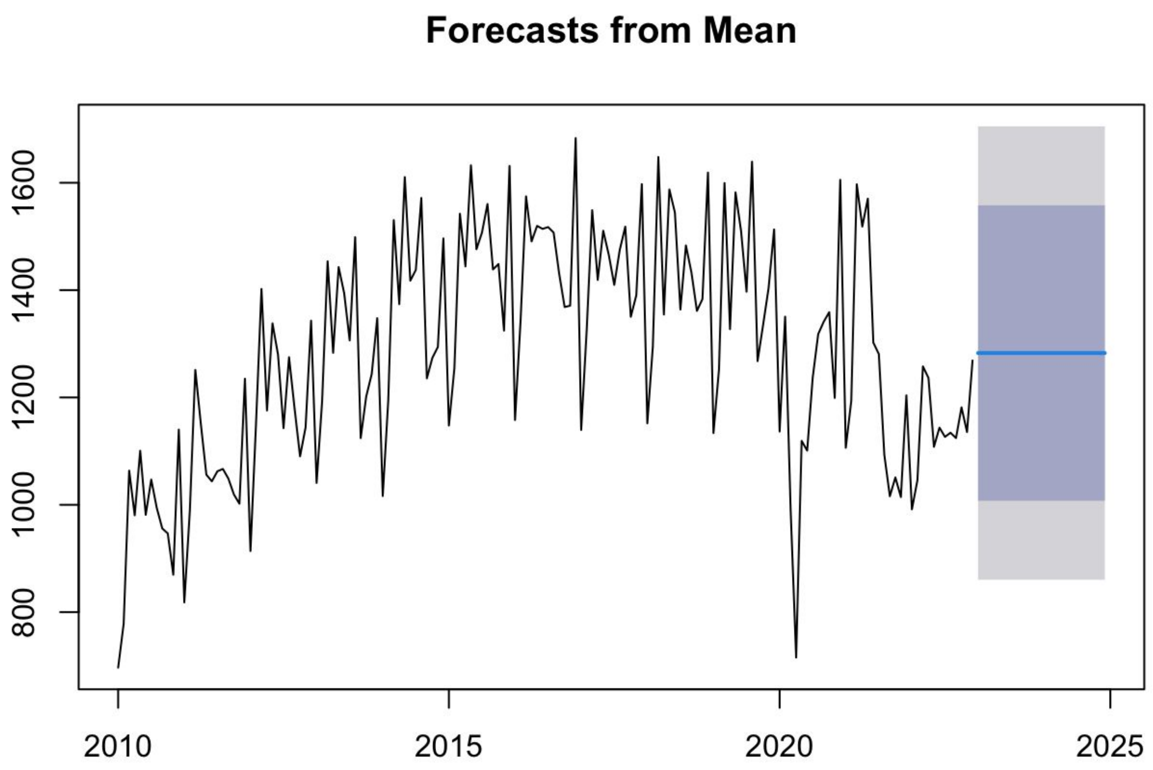
Baseline

	RMSE	MAE	Ljung-Box Test (p-value)
Average Method	192.0058	168.8056	< 0.05
Drift Method	529.8307	496.3407	< 0.05
Naive Method	440.5821	409.6111	< 0.05
Seasonal Naive Method	318.3891	254.9943	< 0.05

Our Models

	RMSE	MAE	Ljung-Box Test (p-value)
Linear Regression	279.3836	255.9479	< 0.05
Exponential Smoothing	240.8748	214.1647	< 0.05
ARIMA	243.95	195.2654	> 0.05
Regression with ARMA Errors	271.4562	241.9290	> 0.05
Intervention Analysis	245.6292	196.0529	< 0.05

Prediction with Average Method



- Forecasting Lightweight Vehicle Sales
- Forecast period: Jan 2023 to Dec 2024

##	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
## Jan 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Feb 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Mar 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Apr 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## May 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Jun 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Jul 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Aug 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Sep 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Oct 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Nov 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Dec 2023	1282.735	1007.528	1557.942	860.3377	1705.132
## Jan 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## Feb 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## Mar 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## Apr 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## May 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## Jun 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## Jul 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## Aug 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## Sep 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## Oct 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## Nov 2024	1282.735	1007.528	1557.942	860.3377	1705.132
## Dec 2024	1282.735	1007.528	1557.942	860.3377	1705.132