Robot Motion Planning (EN.530.663)
Homework 1

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1. Consider the set of all continuous, real-valued functions $f : [0, 1] \to \mathbb{R}$ that are the solutions to the following differential equation

$$\frac{d^2 f}{dx^2} + 4\pi^2 n^2 f = 0$$

with the boundary conditions $f(0) = 1$ and $\frac{df}{dx}|_{x=1} = 0$, where n denotes any non-negative integer.

(a) Explicitly write the elements of the set.

(b) Now, let us consider the set of the "nice" functions which are generated by the linear combination of the elements in the set from part (a). Show that the set forms a vector space over $\mathbb{R}$.

(c) What is the dimension of this vector space?

2. Given

$$A = \begin{pmatrix} 1 & 3 & 0 & 2 \\ 2 & 6 & 2 & 4 \\ 3 & 9 & 4 & 6 \end{pmatrix}$$

do the followings:

(a) Compute the null space $\mathcal{N}(A)$ by hand.

(b) What is the rank of A?

(c) Use Matlab to compute $\mathcal{N}(A)$ (find the command to do this) and the rank of A. Attach the Matlab session (screen shot for example) in your submission.

3. For $A, B \in \mathbb{R}^{n\times n}$ and a nonsingular matrix $P \in \mathbb{R}^{n\times n}$, if $A = PBP^{-1}$, then A and B are said to be *similar* to each other. Let the eigenvalues and eigenvectors of A are denoted as λ_i and $\mathbf{v}_i$, respectively ($i = 1, \cdots, n$). What are the eigenvalues and eigenvectors of B that is similar to A?

4. (a) Let $x = f(u, v)$ and $y = g(u, v)$, where x, y denotes the coordinates of a point in $\mathbb{R}^2$. The infinitesimal volume (in this case, area) near a point (x, y) is $dA = dxdy$ in the Cartesian coordinates. Show that

$$dA = |\det J(u, v)| dudv.$$

Note that the area A of the parallelogram made of two vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$ is computed as $A = \|\mathbf{u} \times \mathbf{v}\|$.

(b) Now we parameterize $\mathbb{R}^2$ with the polar coordinates, i.e., $u = r$ and $v = \theta$. Explicitly write the mapping between coordinates. Also what is dA in the polar coordinates?

(c) Continuing from (b), in order to make the mapping surjective, how to you want to define the domain of the mapping?

5. (a) Show that if a graph $G = (V, E)$ is simple, then

$$|E| \leq \binom{|V|}{2}$$

where $\binom{n}{a}$ denotes the binomial coefficient.

(b) Draw the diagram of the graph $G = (V, E)$ (note: not a digraph) where

$$V = \{1, 2, 3, 4, 5\}$$
$$E = \{(1, 2), (1, 3), (2, 4), (3, 4), (4, 5), (3, 1)\},$$

and the graph $G' = (V', E')$ where

$$V' = \{1, 2, 3, 4, 5\}$$
$$E' = \{(1, 4), (2, 4), (2, 5), (3, 4), (3, 5)\}.$$

(c) Are G and G' simple graphs? Are they isomorphic?

(d) Compute the adjacency matrices of G and G'.