Dominios y gráficos

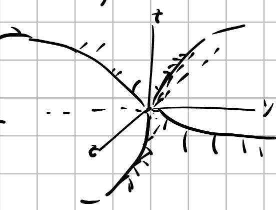
(1) Determinar el dominio $D \subseteq \mathbb{R}^2$ de las siguientes funciones y graficarlo.

- (a) $f(x,y) = \dfrac{x+y}{x-y}$
- (b) $f(x,y) = \sqrt{x\,y}$
- (c) $f(x,y) = \dfrac{x\,y}{x^2-y^2}$
- (d) $f(x,y) = \sqrt{4\,x^2+9\,y^2-36}$

a) Dom: $\{(x,y) \in \mathbb{R}^2 / x-y \neq 0\}$ $\Rightarrow$ $x \neq y$, es el plano completo a excepción de la recta $\boxed{x=y}$

b) $f(x,y) = \sqrt{xy}$ $\quad$ $\text{Dom}(f(x,y)) = \{(x,y) \in \mathbb{R}^2 / x,y \geq 0\} \cup \{(x,y) \in \mathbb{R}^2 / x,y \leq 0\}$

claramente se observa que el dominio es $[0,\infty)$.

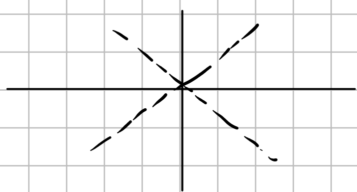


c) $f(x,y) = \dfrac{xy}{x^2-y^2}$ $\quad$ $\text{Dom}(f(x,y)) = \{(x,y) \in \mathbb{R}^2 / x^2-y^2 \neq 0\}$ $\Rightarrow x^2-y^2=0$

$(x+y)(x-y)=0$

$\Rightarrow \boxed{x \neq -y \ \wedge \ x \neq y}$

$\Rightarrow$ El Dominio es todo el plano excepto las dos rectas $\boxed{y=x}$ $\wedge$ $\boxed{y=-x}$



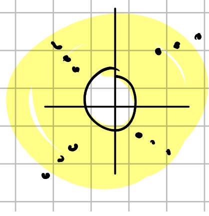
visto desde arriba.
esas dos rectas no pertenecen.

d) $f(x,y) = \sqrt{4x^2+9y^2-36} = \text{Dom}(f(x,y)) = \{(x,y) \in \mathbb{R}^2 / 4x^2+9y^2-36 \neq 0\}$

$\Rightarrow$ la imagen equivale a todos los puntos $\qquad \Rightarrow 4x^2+9y^2-36 \geq 0$

fuera de la circunferencia

$\dfrac{4x^2}{36} + \dfrac{9y^2}{36} \geq 1 \Rightarrow \dfrac{x^2}{9} + \dfrac{y^2}{4} \geq 1.$



(2) *Bosquejar* la gráfica de las siguientes funciones.

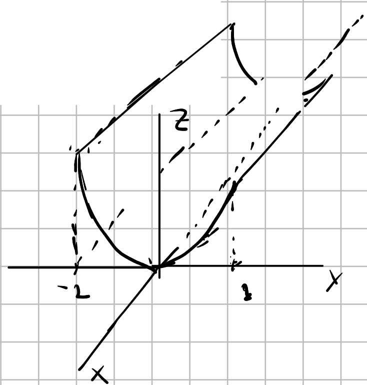
(a) $f(x,y) = y^2$, donde $-1 \leq x \leq 1$, $-1 \leq y \leq 1$

(b) $f(x,y) = x^2+y^2$ (paraboloide)

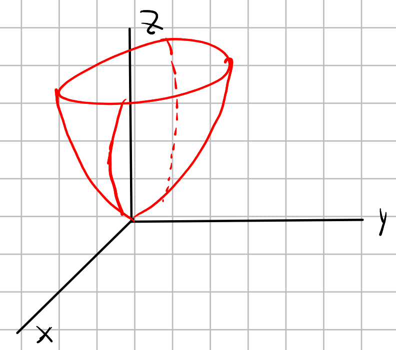
(c) $f(x,y) = x^2-y^2$ (silla de montar)

(d) $f(x,y) = \sqrt{x^2+y^2}$ (cono)

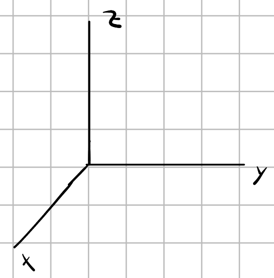
a) $f(x,y) = y^2$, donde $-1 \leq x \leq 1$, $1 \leq y \leq 1$



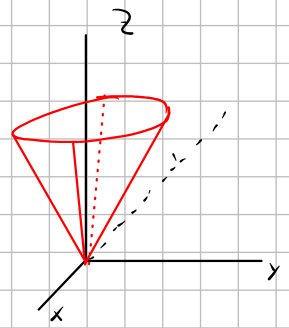
b) $f(x,y) = x^2 + y^2$ $\quad Im = [0, \infty)$



c) $f(x,y) = x^2 - y^2$



d) $f(x,y) = \sqrt{x^2+y^2}$



Derivadas parciales

✓(3) Calcular las derivadas parciales de las siguientes funciones y evaluarlas en el punto dado.

- (a) $f(x,y) = x - y$, $\quad p = (3,2)$
- (b) $f(x,y,z) = \dfrac{xz}{y+z}$, $\quad p = (1,1,1)$
- (c) $f(x,y) = xy + x^2$, $\quad p = (2,0)$
- (d) $w = e^{y \ln z}$, $\quad p = (e,2,e)$
- ✓(e) $f(x,y,z) = x^3y^4z^5$, $\quad p = (0,-1,-1)$
- (f) $w = \ln(1 + e^{xyz})$, $\quad p = (2,0,-1)$

a) $f(x,y) = x - y$ $\quad \bullet f_x(x,y) = 1$ $\quad \bullet f_y(x,y) = -1$ $\quad - f_x(3,2) = 1$
$- f_y(3,2) = 1$.

b) $f(x,y,z) = \dfrac{xz}{y+z}$ $\quad \bullet f_x(x,y,z) = \dfrac{z}{y+z}$ $\quad \bullet f_y(x,y,z) = -\dfrac{xz}{(y+z)^2}$ $\quad \bullet f_z(x,y,z) = \dfrac{xx}{(y+z)^2}$

$- f_x(1,1,1) = \boxed{\frac{1}{2}}$ $\quad - f_y(1,1,1) = \boxed{-\frac{1}{4}}$ $\quad f_z(x,y,z) = \boxed{\frac{1}{4}}$

c) $f(x,y) = xy + x^2$ $\quad \bullet f_x(x,y) = y + 2x$ $\quad \bullet f_y(x,y) = x$ $\quad - f_x(2,0) = \boxed{4}$ $\quad - f_y(2,0) = \boxed{2}$

d) $w = e^{y\ln(z)}$ $\quad \cdot \dfrac{\partial w}{\partial x}(x,y,z) = 0$ $\quad \bullet \dfrac{\partial w}{\partial y}(x,y,z) = \ln(z)\, e^{y\ln(z)}$ $\quad \cdot \dfrac{\partial w}{\partial z} = \dfrac{y}{z} e^{y\ln(z)}$

$- \dfrac{\partial w}{\partial x}(e,2,e) = \boxed{0}$ $\quad - \dfrac{\partial w}{\partial y}(e,2,e) = \cancel{\ln}(e)\, e^{2\cancel{\ln}(e)} = \boxed{e^2}$ $\quad - \dfrac{\partial w}{\partial z} = \dfrac{2}{e} e^{2\ln(e)} = \boxed{2e}$

e) $f(x,y,z) = x^3y^4z^5$

• $\frac{\partial f}{\partial x}(x,y,z) = 3x^2y^4z^5$ • $\frac{\partial f}{\partial y}(x,y,z) = 4y^3x^3z^5$ • $\frac{\partial f}{\partial z}(x,y,z) = 5z^4x^3y^4$

• $\frac{\partial f}{\partial x}(0,-1,-1) = \boxed{0}$ $\quad \frac{\partial f}{\partial y} = \boxed{0}$ $\quad \frac{\partial f}{\partial z} = \boxed{0}$

f) $w = \ln(1+e^{xyz})$ • $\frac{\partial w}{\partial x}(x,y,z) = \frac{yze^{xyz}}{1+e^{xyz}}$ • $\frac{\partial w}{\partial y}(x,y,z) = \frac{xze^{xyz}}{1+e^{xyz}}$

• $\frac{\partial w}{\partial z}(x,y,z) = \frac{yx.e^{xyz}}{1+e^{xyz}}$ $\quad - \frac{\partial w}{\partial x}(2,0,-1) = \boxed{0}$ $\quad - \frac{\partial w}{\partial y}(2,0,-1) = \boxed{-1}$ $\quad - \frac{\partial w}{\partial z}(2,0,-1) = \boxed{0}$

(4) Obtener las ecuaciones de la recta normal al plano tangente y del plano tangente al gráfico de las siguientes funciones en los puntos dados.

- (a) $f(x,y) = \cos\left(\frac{x}{y}\right)$, en $p = (\pi, 4)$. ✓(b) $f(x,y) = \frac{x}{x^2+y^2}$, en $p = (1,2)$.

a) $f(x,y) = \cos\left(\frac{x}{y}\right)$, $P = (\pi, 4)$

Sabemos que la ec del Plano tangente $\xrightarrow{\text{en } (a,b,f(a,b))}$ tiene la forma $z = (x-a)f_x(a,b) + (y-b)f_y(a,b) + f(a,b)$

$\Rightarrow$ evalúo en el Punto $f(x,y) = \cos\left(\frac{x}{y}\right) \longrightarrow f(\pi,4) = \cos\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$

• $\frac{\partial f}{\partial x}(x,y) = -\operatorname{sen}\left(\frac{x}{y}\right) \cdot \frac{1}{y} \longrightarrow \frac{\partial f}{\partial x}(\pi,4) = -\operatorname{sen}\left(\frac{\pi}{4}\right) \cdot \frac{1}{4} = \boxed{-\frac{\sqrt{2}}{8}}$

• $\frac{\partial f}{\partial y}(x,y) = \operatorname{sen}\left(\frac{x}{y}\right) \frac{x}{y^2} \longrightarrow \frac{\partial f}{\partial y}(\pi,4) = \operatorname{sen}\left(\frac{\pi}{4}\right) \frac{\pi}{16} = \boxed{\frac{\pi\sqrt{2}}{32}}$

$\Rightarrow$ la Ec del Plano tangente es de la forma $z = -\frac{\sqrt{2}}{8}(x-\pi) + \frac{\pi\sqrt{2}}{32}(y-4) + \frac{\sqrt{2}}{2}$
que Pasa Por el Punto

$\Rightarrow z = -\frac{\sqrt{2}}{8}x + \cancel{\frac{\sqrt{2}}{8}\pi} + \frac{\pi\sqrt{2}}{32}y - \cancel{\frac{\pi\sqrt{2}}{8}} + \frac{\sqrt{2}}{2}$ $\quad \therefore \boxed{z = -\frac{\sqrt{2}}{8}x + \frac{\pi\sqrt{2}}{32}y + \frac{\sqrt{2}}{2}}$

• La recta normal al Plano que Pasa Por el Punto → Perpendicular a la superficie

el vector normal es $\left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right) = \vec{n}\left(\frac{\sqrt{2}}{8}, -\frac{\pi\sqrt{2}}{32}, 1\right)$

$\Rightarrow \boxed{\vec{r}(t) = \left(\pi, 4, \frac{\sqrt{2}}{2}\right) + t\left(\frac{\sqrt{2}}{8}, -\frac{\pi\sqrt{2}}{32}, 1\right), \ t \in \mathbb{R}}$

b) $f(x,y) = \frac{x}{x^2+y^2}$ $\quad P = (1,2)$

$\Rightarrow f(1,2) = \boxed{\frac{1}{5}}$

• $\frac{\partial f}{\partial x}(x,y) = \boxed{\frac{y^2 - x^2}{(x^2+y^2)^2}} \longrightarrow \frac{\partial f}{\partial x}(1,2) = \boxed{\frac{3}{25}}$

• $\frac{\partial f}{\partial y}(x,y) = \boxed{\frac{-2xy}{(x^2+y^2)^2}} \longrightarrow \frac{\partial f}{\partial y}(1,2) = \boxed{\frac{-4}{25}}$

$\Rightarrow$ La Ec del Plano tangente es $z = \frac{3}{25}(x-1) - \frac{4}{25}(y-2) + \frac{1}{5} \Rightarrow z = \frac{3}{25}x - \frac{3}{25} - \frac{4y}{25} + \frac{8}{25} + \frac{1}{5}$

$\therefore \boxed{z = \frac{3}{25}x - \frac{4}{25}y + \frac{26}{5}}$

• La Ec Normal al Plano tangente que Pasa Por el Punto.

$\vec{n} = \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right) \Rightarrow \vec{n} = \left(\frac{-3}{25}, \frac{4}{25}, 1\right)$

$\Rightarrow \boxed{r(t) = \left(1, 2, \frac{1}{5}\right) + t\left(\frac{-3}{25}, \frac{4}{25}, 1\right)}$

(5) Para las siguientes funciones $f(x,y)$ encontrar:

(a) El gradiente en el punto p indicado.

(b) Una ecuación del plano tangente al gráfico de f en el punto dado.

✓ (i) $f(x,y) = \frac{x-y}{x+y}$, en $p = (1,1)$. ✓ (ii) $f(x,y) = \frac{2xy}{x^2+y^2}$, en $p = (0,2)$.

$P = (1,1)$

i) $f(x,y) = \frac{x-y}{x+y} \longrightarrow f(1,1) = \boxed{0}$

b) obtengo sus derivadas Parciales

• $\frac{\partial f}{\partial x}(x,y) = \frac{2y}{(x+y)^2} \longrightarrow \frac{\partial f}{\partial x}(1,1) = \boxed{\frac{1}{2}}$

• $\frac{\partial f}{\partial y}(x,y) = \frac{-2x}{(x+y)^2} \longrightarrow \frac{\partial f}{\partial y}(1,1) = \boxed{-\frac{1}{2}}$

$\Rightarrow$ La Ec. del Plano tangente al gráfico es $z = \frac{1}{2}(x-1) - \frac{1}{2}(y-1) \Rightarrow \boxed{z = \frac{1}{2}x - \frac{1}{2}y}$

a) El gradiente es de la forma $\nabla f(1,1) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \boxed{\nabla f(1,1) = \left(\frac{1}{2}, -\frac{1}{2}\right)}$

$P = (0,2)$

ii) $f(x,y) = \frac{2xy}{x^2+y^2} \longrightarrow f(0,2) = \boxed{0}$

b) • $\frac{\partial f}{\partial x}(x,y) = \frac{2y(\cancel{x^2}+y^2-2\cancel{x^2})}{(x^2+y^2)^2} \longrightarrow \frac{\partial f}{\partial x}(0,2) = \boxed{1}$

• $\frac{\partial f}{\partial y}(x,y) = \frac{2x(x^2+y^2) - 4xy^2}{(x^2+y^2)^2} \longrightarrow \frac{\partial f}{\partial y}(0,2) = \boxed{0}$

$\Rightarrow$ La Ec que de la forma $\boxed{z = x}$

a) El gradiente $\boxed{\nabla f(0,2) = (1,0)}$

(6) Calcular la derivada direccional de f en el punto P y en la dirección del vector $\vec{u}$ dado.

✓(a) $f(x,y) = xe^{2y}$, $P = (2,0)$, $\vec{u} = (\frac{1}{2}, \frac{\sqrt{3}}{2})$.

✓(b) $f(x,y) = \ln(x^2+y^2+z^2)$, $P = (1,3,2)$, $\vec{u} = (\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$.

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a) $f(x,y) = xe^{2y}$ $P = (2,0)$, $\vec{u} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

• Corroboro que el vector sea unita. $\|\vec{u}\| = \sqrt{\frac{1}{4} + \frac{3}{4}} = \boxed{1}$.

• Calculo sus derivadas parciales.

• $\frac{\partial f}{\partial x}(x,y) = e^{2y} \longrightarrow \frac{\partial f}{\partial x}(2,0) = \boxed{1}$

• $\frac{\partial f}{\partial y}(x,y) = 2xe^{2y} \longrightarrow \frac{\partial f}{\partial x}(2,0) = \boxed{4}$

$\Rightarrow$ El gradiente $\nabla f(2,0) = (1,4)$

$D_{\vec{u}} f(2,0) = \langle \nabla f(2,0), \vec{u} \rangle$

∴ Por teorema, la derivada direccional es $\langle \nabla f(2,0), \vec{u} \rangle = \left\langle (1,4), \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \right\rangle = \boxed{\frac{1}{2} + 2\sqrt{3}}$

b) $f(x,y) = \ln(x^2+y^2+z^2)$, $P = (1,3,2)$, $\vec{u} = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$.

• Verifico que el vector sea unitario $\|\vec{u}\| = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \boxed{1}$ ✓

• Calculo sus derivadas parciales.

• $\frac{\partial f}{\partial x}(x,y,z) = \frac{2x}{(x^2+y^2+z^2)^2} \longrightarrow \frac{\partial f}{\partial x}(1,3,2) = \boxed{\frac{1}{7}}$

• $\frac{\partial f}{\partial y}(x,y,z) = \frac{2y}{(x^2+y^2+z^2)^2} \longrightarrow \frac{\partial f}{\partial x}(1,3,2) = \boxed{\frac{3}{7}}$

• $\frac{\partial f}{\partial z}(x,y,z) = \frac{2z}{(x^2+y^2+z^2)^2} \longrightarrow \frac{\partial f}{\partial z}(1,3,2) = \boxed{\frac{2}{7}}$

$\Rightarrow \nabla f(1,3,2) = \left(\frac{1}{7}, \frac{3}{7}, \frac{2}{7}\right)$.

$D_{\vec{u}} f(1,3,2) = \left\langle \left(\frac{1}{7}, \frac{3}{7}, \frac{2}{7}\right), \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right) \right\rangle = \frac{1}{7\sqrt{3}} - \frac{3}{7\sqrt{3}} - \frac{2}{7\sqrt{3}} = \boxed{\frac{-4}{7\sqrt{3}}}$

(7) ¿En qué dirección debemos movernos, partiendo de $(1,1)$, para obtener la más alta y la más baja tasa de crecimiento de la función $f(x,y) = (x+y-2)^2 + (3x-y-6)^2$?

$f(x,y) = (x+y-2)^2 + (3x-y-6)^2$

Calculo sus derivadas parciales

- $\frac{\partial f}{\partial x}(x,y) = 2(x+y-2) + 6(3x-y-6) \longrightarrow \frac{\partial f}{\partial x}(1,1) = \boxed{-24}$
- $\frac{\partial f}{\partial y}(x,y) = 2(x+y-2) - 2(3x-y-6) \longrightarrow \frac{\partial f}{\partial y}(1,1) = \boxed{8}$

El gradiente $\nabla f(1,1) = (-24, 8)$

- Normalizo el vector y lo hago unitario

$\|\nabla f(1,1)\| = \sqrt{576+64} = \sqrt{640} = \boxed{8\sqrt{10}}$

- $\vec{u} = \frac{\nabla f(1,1)}{\|\nabla f(1,1)\|} = \left(\frac{-24}{8\sqrt{10}}, \frac{8}{8\sqrt{10}}\right) = \boxed{\vec{u} = \left(\frac{-3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)}$ $\Rightarrow$ Dirección de máximo crecimiento

$\boxed{-\vec{u} = \left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right) -}$ $\Rightarrow$ Dirección de mínimo crecimiento

Regla de la cadena

(8) Calcular las derivadas parciales segundas de las siguientes funciones.

(a) $z = x^2(1+y^2)$ (b) $w = x^3y^3z^3$

a) $z = x^2(1+y^2)$

- $\frac{\partial z}{\partial x}(x,y) = 2x(1+x^2)$
- $\frac{\partial^2 f}{\partial x^2}(x,y) = 2(1+y^2)$
- $\frac{\partial z}{\partial y}(x,y) = \boxed{2yx^2}$
- $\frac{\partial^2 z}{\partial y^2}(x,y) = \boxed{2x^2}$
- $\frac{\partial z}{\partial y \cdot \partial x}(x,y) = \boxed{4xy}$

b) $w = x^3y^3z^3$

- $\frac{\partial w}{\partial x}(x,y,z) = 3x^2y^3z^3$
- $\frac{\partial^2 w}{\partial x^2}(x,y,z) = \boxed{6xy^3z^3}$
- $\frac{\partial w}{\partial y}(x,y,z) = 3y^2x^3z^3$
- $\frac{\partial^2 w}{\partial y^2}(x,y,z) = \boxed{6yx^3z^3}$
- $\frac{\partial w}{\partial z}(x,y,z) = 3z^2x^3y^3$
- $\frac{\partial^2 w}{\partial z^2}(x,y,z) = \boxed{6zx^3y^3}$
- $\frac{\partial w}{\partial x \partial y \partial z}(x,y,z) = 3x^2 3y^2 z^3 = \boxed{3x^2 3y^2 3z^2}$

(9) Aplique la regla de la cadena para hallar dz/dt

(a) $z = x^2 + y^2 + xy$, $x = \sin t$, $y = e^t$
(b) $z = \cos(x+4y)$, $x = 5t^4$, $y = 1/t$
(c) $z = \sqrt{1+x^2+y^2}$, $x = \ln t$, $y = \cos t$
(d) $\arctan(y/x)$, $x = e^t$, $y = 1 - e^{-t}$

a) $x'(t) = x^2 e^t y^2, + x y'(t) = e^{-t}$ $x = \sin(t)$, $y = e^t$

$$\frac{\partial z}{\partial t}(x(t),y(t)) = \frac{\partial z}{\partial x}(x(t),y(t))\cdot x'(t) + \frac{\partial z}{\partial y}(x(t),y(t))\, y'(t)$$

$x'(t) = \cos(t)$, • $y'(t) = e^t$

$\frac{\partial z}{\partial x}(x,y) = 2x + y$, • $\frac{\partial z}{\partial y}(x,y) = 2y + x$

$$= \frac{x(t)}{x^2(t)+y^2(t)}\cdot e^t - \frac{y(t)}{x^2(t)+y^2(t)}\cdot e^{-t}$$

$$\frac{\partial z}{\partial t}(x(t),y(t)) = \frac{\partial z}{\partial x}(x(t),y(t))\cdot x'(t) + \frac{\partial z}{\partial y}(x(t),y(t))\, y'(t)$$

$$= \left(2x(t) + y(t)\right)\cos(t) + \left(2y(t) + x(t)\right)\cdot e^t$$

$$= \boxed{(2\,\text{sen}(t) + e^t)\cos(t) + (2e^t + \text{sen}(t))\,e^t}$$

b) $z = \cos(x+4y)$, $x = 5t^4$, $y = \frac{1}{t}$

$\frac{\partial z}{\partial x} = -\text{sen}(x+4y)$, $\frac{\partial z}{\partial y} = -4\,\text{sen}(x+4y)$

• $x'(t) = 20t^3$ • $y'(t) = \frac{-1}{t^2}$

$$\frac{\partial z}{\partial t}(x(t),y(t)) = \frac{\partial z}{\partial x}(x(t),y(t))\, x'(t) + \frac{\partial z}{\partial y}(x(t),y(t))\, y'(t)$$

$$= -\text{sen}(x(t)+4y(t))\cdot 20t^4 + \frac{4\,\text{sen}(x(t)+4y(t))}{t}$$

$$= \boxed{-20t^4\,\text{sen}\left(5t^4+\frac{4}{t}\right) + \frac{4\,\text{sen}\left(5t^4+\frac{4}{t}\right)}{t}}$$

c) $z = \sqrt{1+x^2+y^2}$, $x = \ln(t)$, $y = \cos(t)$

• $\frac{\partial z}{\partial x}(x,y) = \frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2+y^2}}$ • $\frac{\partial z}{\partial y}(x,y) = \frac{y}{\sqrt{1+x^2+y^2}}$ • $x'(t) = \frac{1}{t}$ • $y'(t) = -\text{sen}(t)$

$$\frac{\partial z}{\partial t}(x(t),y(t)) = \frac{\partial z}{\partial x}(x(t),y(t))\cdot x'(t) + \frac{\partial z}{\partial y}(x(t),y(t))\, y'(t)$$

$$= \frac{x(t)}{t\sqrt{1+x^2(t)+y^2(t)}} + \frac{-y(t)\,\text{sen}(t)}{\sqrt{1+x^2(t)+y^2(t)}}$$

$$= \boxed{\frac{\ln(t)}{t\sqrt{1+\ln^2(t)+\cos^2(t)}} - \frac{\cos(t)\,\text{sen}(t)}{\sqrt{1+\ln^2(t)+\cos^2(t)}}}$$

d) $f = \arctan\left(\frac{y}{x}\right)$, $x = e^t$, $y = (1-e^{-t})$

$\frac{1}{x}$

• $\frac{\partial f}{\partial x}(x,y) = \frac{-\frac{y}{x^2}}{1+\left(\frac{y}{x}\right)^2} = \frac{-y}{x^2+y^2}$ • $\frac{\partial f}{\partial y}(x,y) = \frac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2} = \frac{x}{x^2+y^2}$

• $x'(t) = e^{t}$ • $y'(t) = e^{-t}$

$$\frac{\partial z}{\partial t}(x(t),y(t)) = \frac{\partial z}{\partial x}(x(t),y(t))\cdot x'(t) + \frac{\partial z}{\partial y}(x(t),y(t))\, y'(t)$$

$$= \frac{x(t)\cdot e^{t}}{x^2(t)+y^2(t)} - \frac{y(t)\cdot e^{-t}}{x^2(t)+y^2(t)}$$

$$= \boxed{\frac{e^{t^2}}{e^{t^2}+(1-e^{-t})} - \frac{(1-e^{-t})e^{-t}}{e^{t^2}+(1-e^{-t})}}$$

✓ (10) Sea $u = \sqrt{x^2+y^2}$ donde $x = e^{st}$, $y = 1 + s^2\cos t$. Calcular $\dfrac{\partial u}{\partial t}$ usando la regla de la cadena y comparar con el resultado que se obtiene reemplazando x e y en u y luego derivar.

• $\dfrac{\partial u}{\partial x}(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}$ • $\dfrac{\partial u}{\partial y}(x,y) = \dfrac{y}{\sqrt{x^2+y^2}}$

• $\dfrac{\partial x}{\partial t}(s,t) = s e^{st}$

• $\dfrac{\partial y}{\partial t}(s,t) = -\mathrm{sen}(t)s^2$

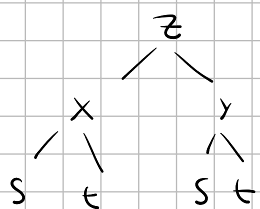
• $$\frac{\partial u}{\partial t}(s,t) = \frac{x(s,t)\cdot s e^{st}}{\sqrt{x^2(s,t)+y^2(s,t)}} - \frac{y(s,t)\cdot \mathrm{sen}(t)s^2}{\sqrt{x^2(s,t)+y^2(s,t)}}$$

$$= \boxed{\frac{s^2 e^{(st)^2}}{e(st)^2+(1+s^2\cos(t))^2} - \frac{(1+s^2\cos(t))\,\mathrm{sen}(t)s^2}{e(st)^2+(1+s^2\cos(t))^2}}$$

(11) Sea $z = f(x,y)$, $x = 2s+3t$, $y = 3s-2t$. Calcular:

(a) $\dfrac{\partial^2 z}{\partial s^2}$ (b) $\dfrac{\partial^2 z}{\partial s\partial t}$ (c) $\dfrac{\partial^2 z}{\partial t^2}$

a) $z = f(x,y)$



• $\dfrac{\partial x}{\partial s}(s,t) = 2$ • $\dfrac{\partial x}{\partial t} = 3$

• $\dfrac{\partial y}{\partial s} = 3$ • $\dfrac{\partial y}{\partial t} = -2$

$$\Rightarrow \frac{\partial z}{\partial s}(x(s,t),y(s,t)) = \frac{\partial f}{\partial x}\cdot 2 + \frac{\partial f}{\partial y}\cdot 3$$

• $\dfrac{\partial^2 z}{\partial s^2}(x(s,t),y(s,t)) = 2\dfrac{\partial f_x}{\partial s} + 3\dfrac{\partial f_y}{\partial s}$ • $\dfrac{\partial^2 f}{\partial x^2}$, $\dfrac{\partial f_1}{\partial y\partial x}$

• Como $\frac{\partial f}{\partial x}(x(s,t), y(s,t)) = \frac{\partial f_x}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f_x}{\partial y} \cdot \frac{\partial y}{\partial s} = \frac{\partial^2 f}{\partial x^2} \cdot 2 + \frac{\partial f}{\partial y \partial x} \cdot 3$

(Analogo respecto a y.

$$\Rightarrow \frac{\partial^2 z}{\partial s^2} = 2\left(\frac{\partial^2 f}{\partial x^2} \cdot 2 + \frac{\partial f}{\partial y \partial x} \cdot 3\right) + 3\left(\frac{\partial^2 f}{\partial y^2} \cdot 3 + \frac{\partial f}{\partial x \partial x} \cdot 2\right) = \boxed{4\frac{\partial^2 f}{\partial x^2} + 12\frac{\partial f}{\partial x \partial z} + 9\frac{\partial^2 f}{\partial y^2}}$$

Superficies y curvas de nivel

(17) Para las funciones del Ejercicio (5), hallar una ecuación de la recta tangente a la curva de nivel que pasa por el punto dado.

(i) $f(x,y) = \dfrac{x-y}{x+y}$, en $p = (1,1)$. (ii) $f(x,y) = \dfrac{2xy}{x^2+y^2}$, en $p = (0,2)$.

17).

i) $f(x,y) = \dfrac{x-y}{x+y}$, $P = (1,1)$.

• Calculo sus derivadas parciales

• $\dfrac{\partial f}{\partial x}(x,y) = \dfrac{2y}{(x+y)^2} \longrightarrow \dfrac{\partial f}{\partial x}(1,1) = \boxed{\dfrac{1}{2}}$

• $\dfrac{\partial f}{\partial y}(x,y) = \dfrac{-2x}{(x+y)^2} \longrightarrow \dfrac{\partial f}{\partial y}(1,1) = \boxed{-\dfrac{1}{2}}$

$\Rightarrow$ el gradiente en el punto P es $\nabla f(1,1) = \left(\frac{1}{2}, -\frac{1}{2}\right)$.

$f(1,1) = \boxed{0}$

Dado que tengo el punto P y un vector director tangente a la curva de nivel

La Ec de la recta es $\boxed{\bar{r}(t) = (1,1) + t\left(\frac{1}{2}, -\frac{1}{2}\right)}$.

ii) $f(x,y) = \dfrac{x}{x^2+y^2}$ $P = (1,2)$

$\Rightarrow f(1,2) = \boxed{\dfrac{1}{5}}$

• $\dfrac{\partial f}{\partial x}(x,y) = \boxed{\dfrac{y^2 - x^2}{(x^2+y^2)^2}} \longrightarrow \dfrac{\partial f}{\partial x}(1,2) = \boxed{\dfrac{3}{25}}$

• $\dfrac{\partial f}{\partial y}(x,y) = \boxed{\dfrac{-2xy}{(x^2+y^2)^2}} \longrightarrow \dfrac{\partial f}{\partial y}(1,2) = \boxed{\dfrac{-4}{25}}$

$\Rightarrow \nabla f(1,2) = \left(\frac{3}{25}, \frac{-4}{25}\right)$.

$\therefore \boxed{\bar{r}(t) = (1,2) + t\left(\frac{3}{25}, \frac{-4}{25}\right)}$.

(18) Obtener la ecuación del plano tangente a la superficie de nivel de la función f que pasa por el punto dado.

(a) $f(x, y, z) = x^2y + y^2z + z^2x$, en $p = (1, -1, 1)$.

(b) $f(x, y, z) = \cos(x + 2y + 3z)$, en $p = (\pi/2, \pi, \pi)$.

18) a) $f(x,y,z) = x^2y + y^2z + z^2x$ $\qquad P = (1,-1,1)$.

Calculo sus derivadas Parciales

- $\frac{\partial f}{\partial x}(x,y,z) = 2xy + z^2$ $\qquad$ - $\frac{\partial f}{\partial y}(x,y,z) = x^2 + 2yz$ $\qquad$ - $\frac{\partial f}{\partial z}(x,y,z) = y^2 + 2zx$

$\frac{\partial f}{\partial x}(1,-1,1) = \boxed{-1}$ $\qquad$ $-\frac{\partial f}{\partial y}(1,-1,1) = \boxed{-1}$ $\qquad$ $-\frac{\partial f}{\partial z}(1,-1,1) = \boxed{3}$

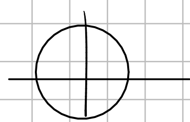
- $f(1,-1,1) = \boxed{1}$ $\qquad$ $\nabla f(1,-1,1) = (-1,-1,3)$

$\Rightarrow \langle (x,y,z) - (1,-1,1), \nabla f(-1,-1,1) \rangle = 0$

$= ((x-1), (y+1), (z-1)) \cdot (-1,-1,3) = 0$

$-(x-1) - (y+1) + 3(z-1) = 0$

$\boxed{-x-y+3z = 3}$



b) (b) $f(x, y, z) = \cos(x + 2y + 3z)$, en $p = (\pi/2, \pi, \pi)$.

- $\frac{\partial f}{\partial x}(x,y,z) = -\operatorname{sen}(x+2y+3z) \longrightarrow \frac{\partial f}{\partial x}(\pi/2, \pi, \pi) = -\operatorname{sen}(\overset{(-1)}{\frac{11\pi}{2}}) = \boxed{1}$

- $\frac{\partial f}{\partial y}(x,y,z) = -2\operatorname{sen}(x+2y+3z) \longrightarrow \frac{\partial f}{\partial x}(\pi/2, \pi, \pi) = -2\operatorname{sen}(\frac{11\pi}{2}) = \boxed{2}$

- $\frac{\partial f}{\partial z}(x,y,z) = -3\operatorname{sen}(x+2y+3z) \longrightarrow \frac{\partial f}{\partial x}(\pi/2, \pi, \pi) = -3\operatorname{sen}(\frac{11\pi}{2}) = \boxed{3}$

$\nabla f(\frac{\pi}{2}, \pi, \pi) = (1,2,3)$.

$\langle (x,y,z) - (\frac{\pi}{2}, \pi, \pi), (1,2,3) \rangle = (x - \frac{\pi}{2}) + 2(y-\pi) + 3(z-\pi) = 0$

$x + 2y + 3z = \frac{\pi}{2} + 2\pi + 3\pi$

$\boxed{x + 2y + 3z = \frac{11}{2}\pi}$