Lecture: RANSAC and feature detectors

Juan Carlos Niebles and Ranjay Krishna

Stanford Vision and Learning Lab

CS 131 Roadmap

Pixels	Segments	Images	Videos	Web
Convolutions	Resizing	Recognition	Motion	Neural networks
Edges	Segmentation	Detection	Tracking	Convolutional
Features	Clustering	Machine learning		neural networks

What we will learn today?

- A model fitting method for line detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector

What we will learn today

- A model fitting method for line detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector

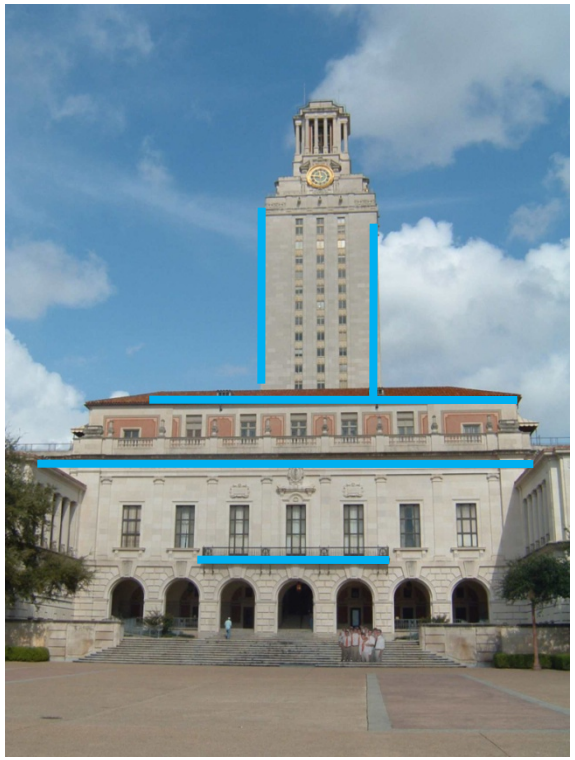
Fitting as Search in Parametric Space

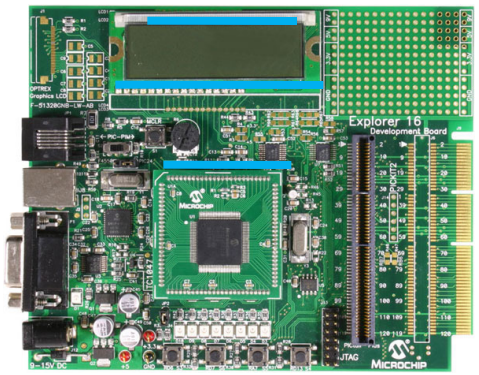
- Choose a parametric model to represent a set of features
- Membership criterion is not local
  - Can't tell whether a point belongs to a given model just by looking at that point.
- Three main questions:
  - What model represents this set of features best?
  - Which of several model instances gets which feature?
  - How many model instances are there?
- Computational complexity is important
  - It is infeasible to examine every possible set of parameters and every possible combination of features

Source: L. Lazebnik

Example: Line Fitting

- Why fit lines?
  Many objects characterized by presence of straight lines

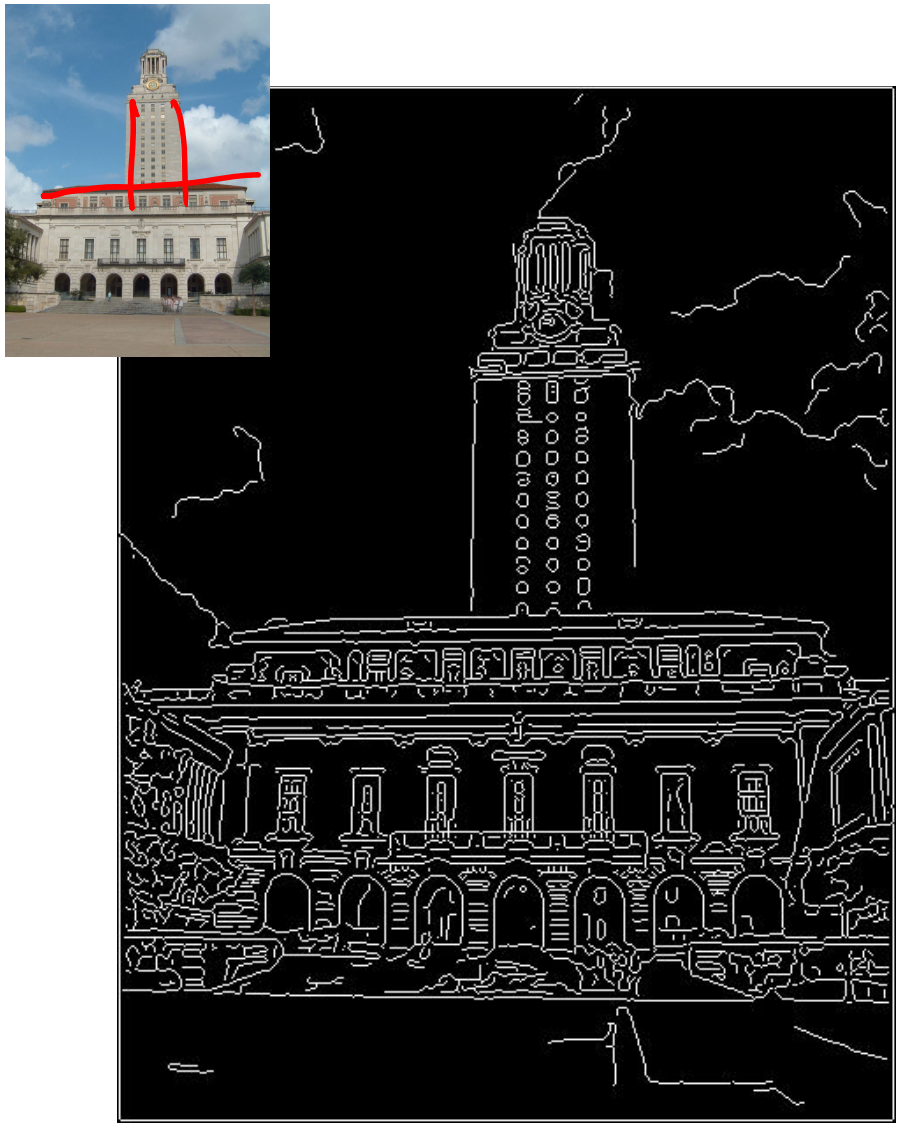




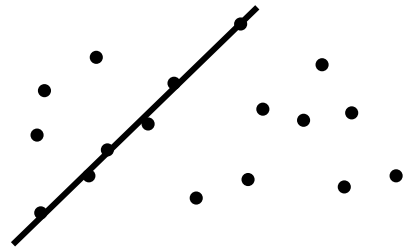
- Wait, why aren't we done just by running edge detection?

Slide credit: Kristen Grauman

Difficulty of Line Fitting



- Extra edge points (clutter), multiple models:
  - Which points go with which line, if any?
- Only some parts of each line detected, and some parts are missing:
  - How to find a line that bridges missing evidence?
- Noise in measured edge points, orientations:
  - How to detect true underlying parameters?



Slide credit: Kristen Grauman

Voting

- It's not feasible to check all combinations of features by fitting a model to each possible subset.
- Voting is a general technique where we let the features vote for all models that are compatible with it.
  - Cycle through features, cast votes for model parameters.
  - Look for model parameters that receive a lot of votes.
- Noise & clutter features will cast votes too, *but* typically their votes should be inconsistent with the majority of "good" features.
- Ok if some features not observed, as model can span multiple fragments.

Slide credit: Kristen Grauman

RANSAC [Fischler & Bolles 1981]

- RANdom SAmple Consensus

- Approach: we want to avoid the impact of outliers, so let's look for "inliers", and use only those.

- Intuition: if an outlier is chosen to compute the current fit, then the resulting line won't have much support from rest of the points.

Slide credit: Kristen Grauman

RANSAC [Fischler & Bolles 1981]

RANSAC loop:

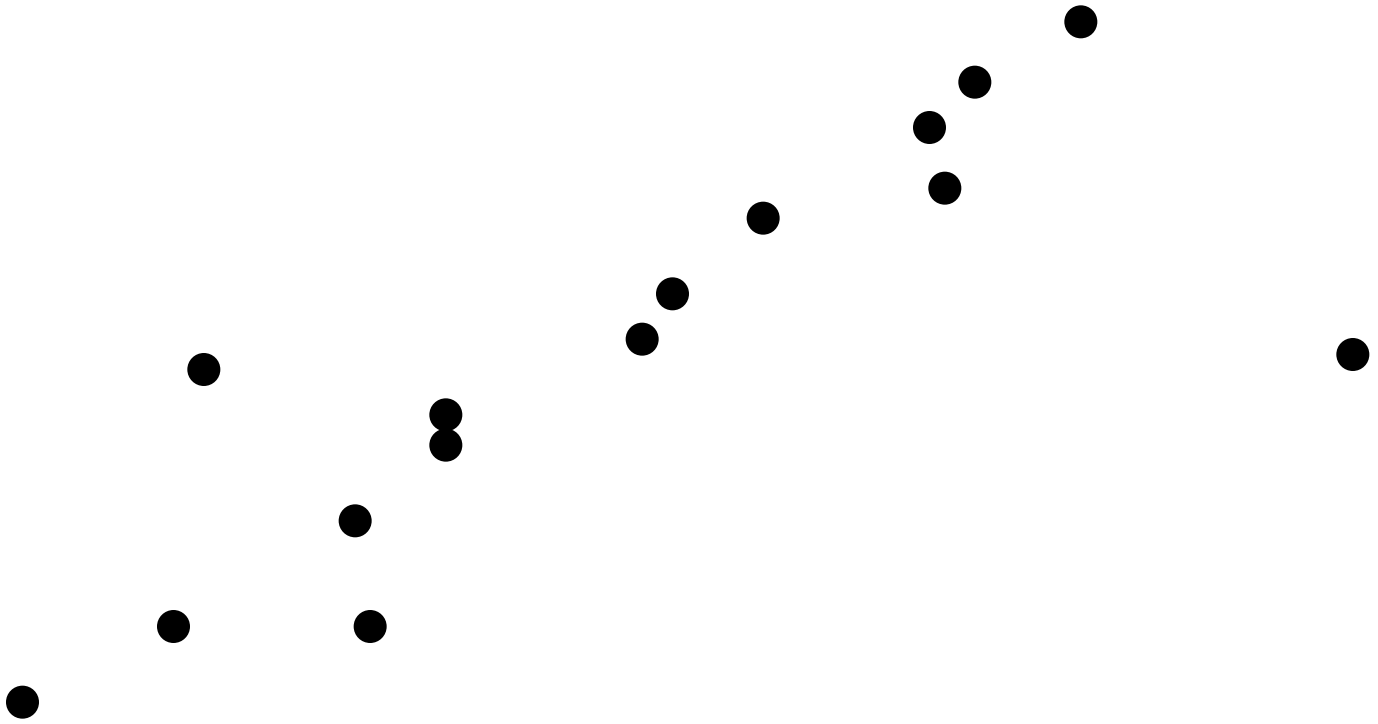
1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers

- Keep the transformation with the largest number of inliers

Slide credit: Kristen Grauman

RANSAC Line Fitting Example

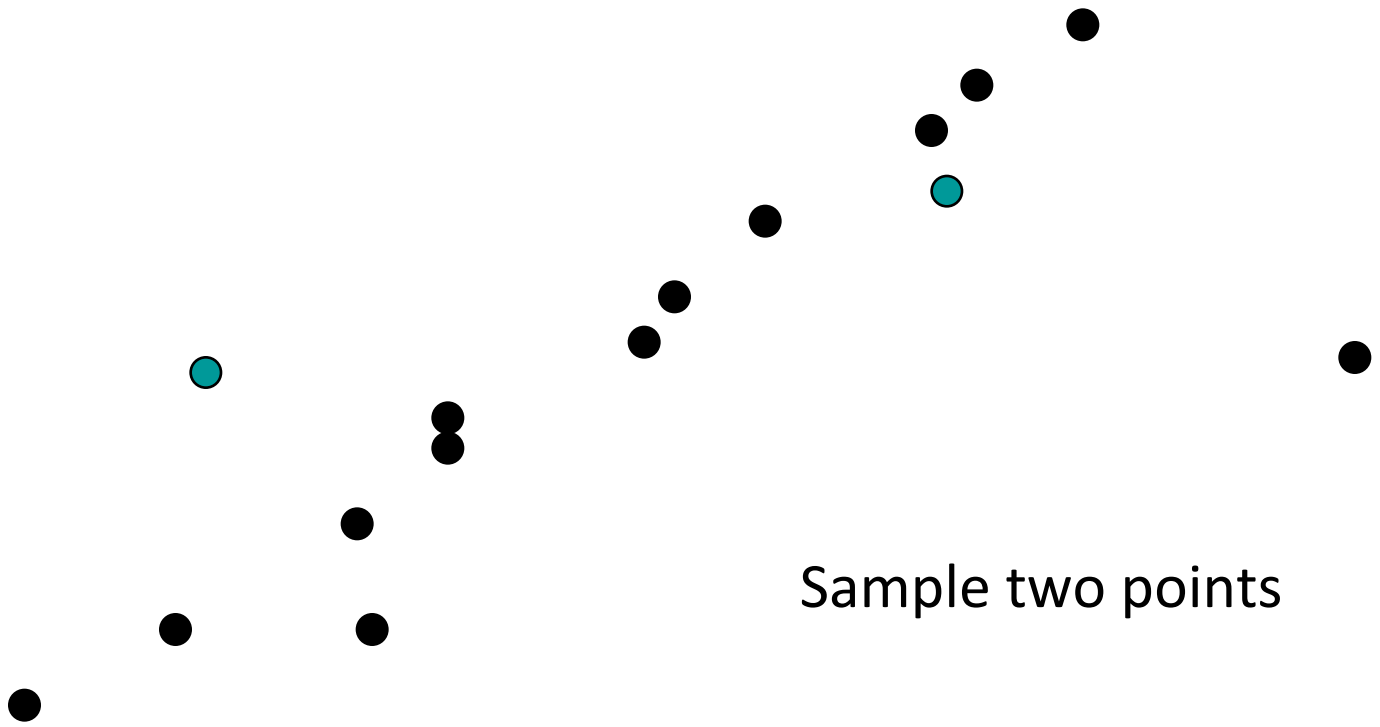
- Task: Estimate the best line
  - *How many points do we need to estimate the line?*



Slide credit: Kristen Grauman

RANSAC Line Fitting Example

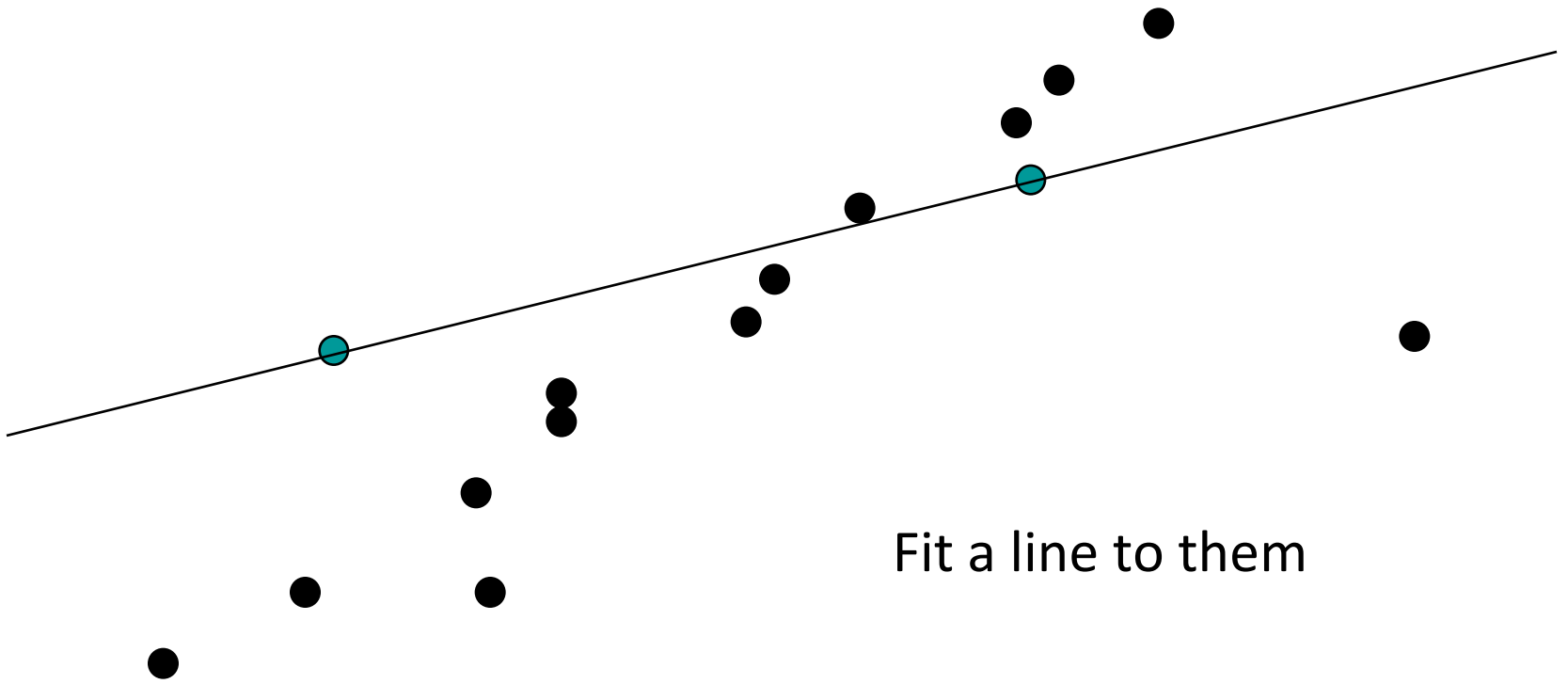
- Task: Estimate the best line



Slide credit: Kristen Grauman

RANSAC Line Fitting Example

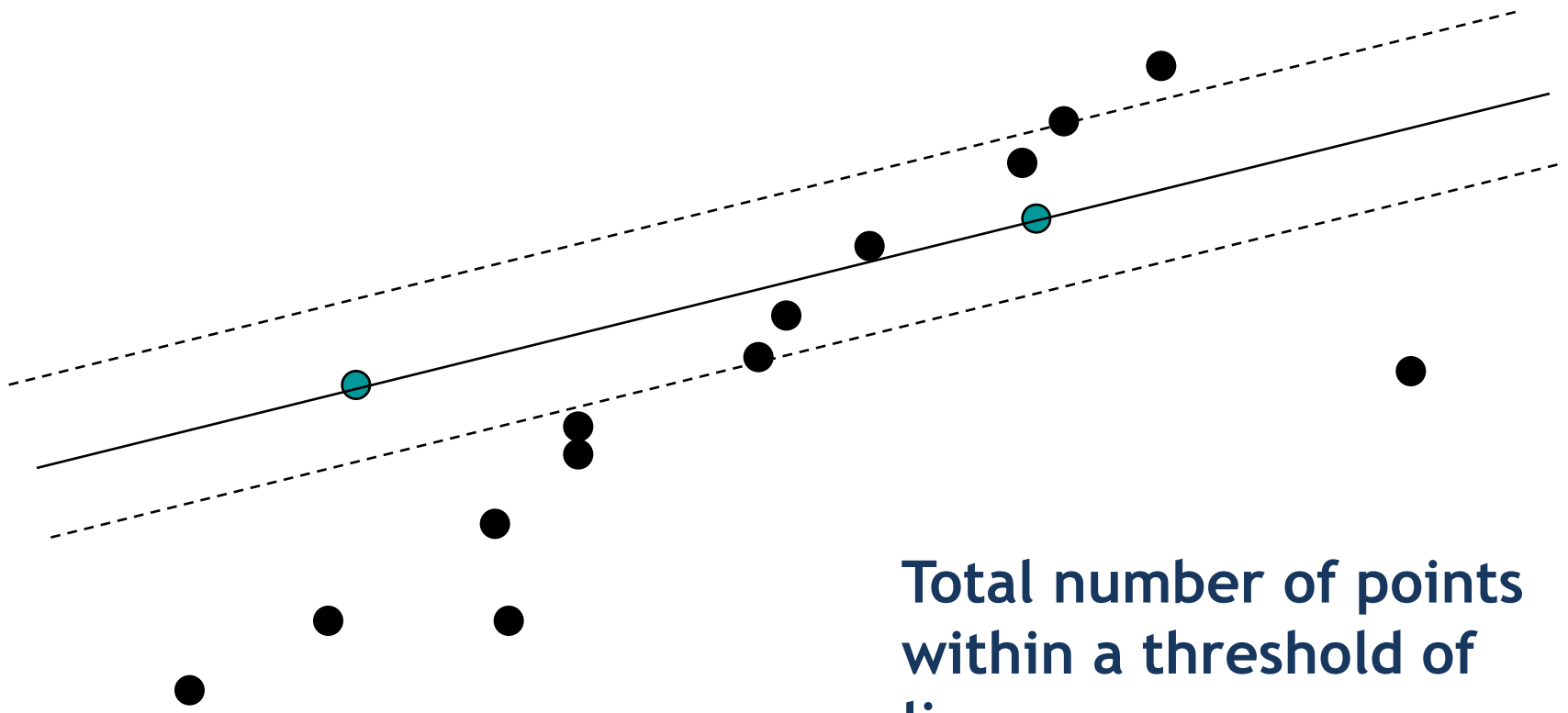
- Task: Estimate the best line



Slide credit: Kristen Grauman

RANSAC Line Fitting Example

- Task: Estimate the best line

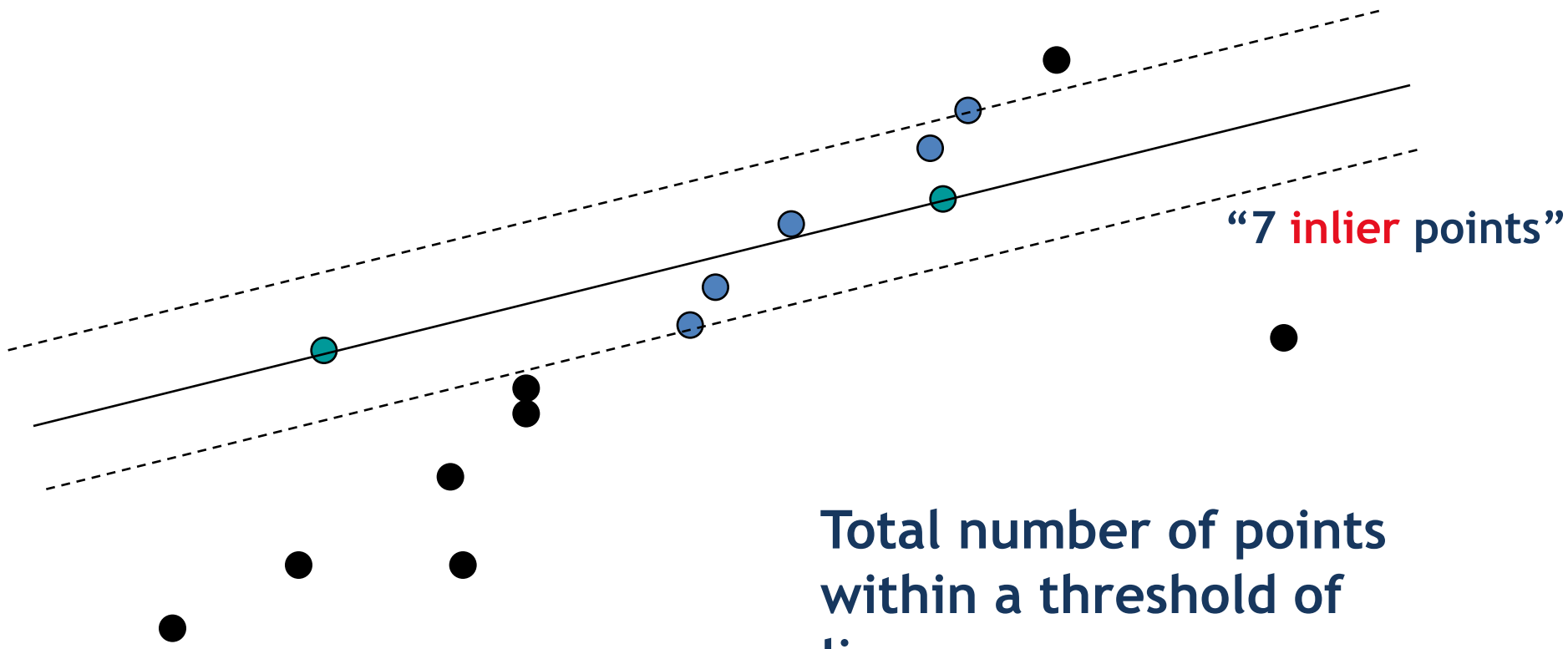


Total number of points within a threshold of line.

Slide credit: Kristen Grauman

RANSAC Line Fitting Example

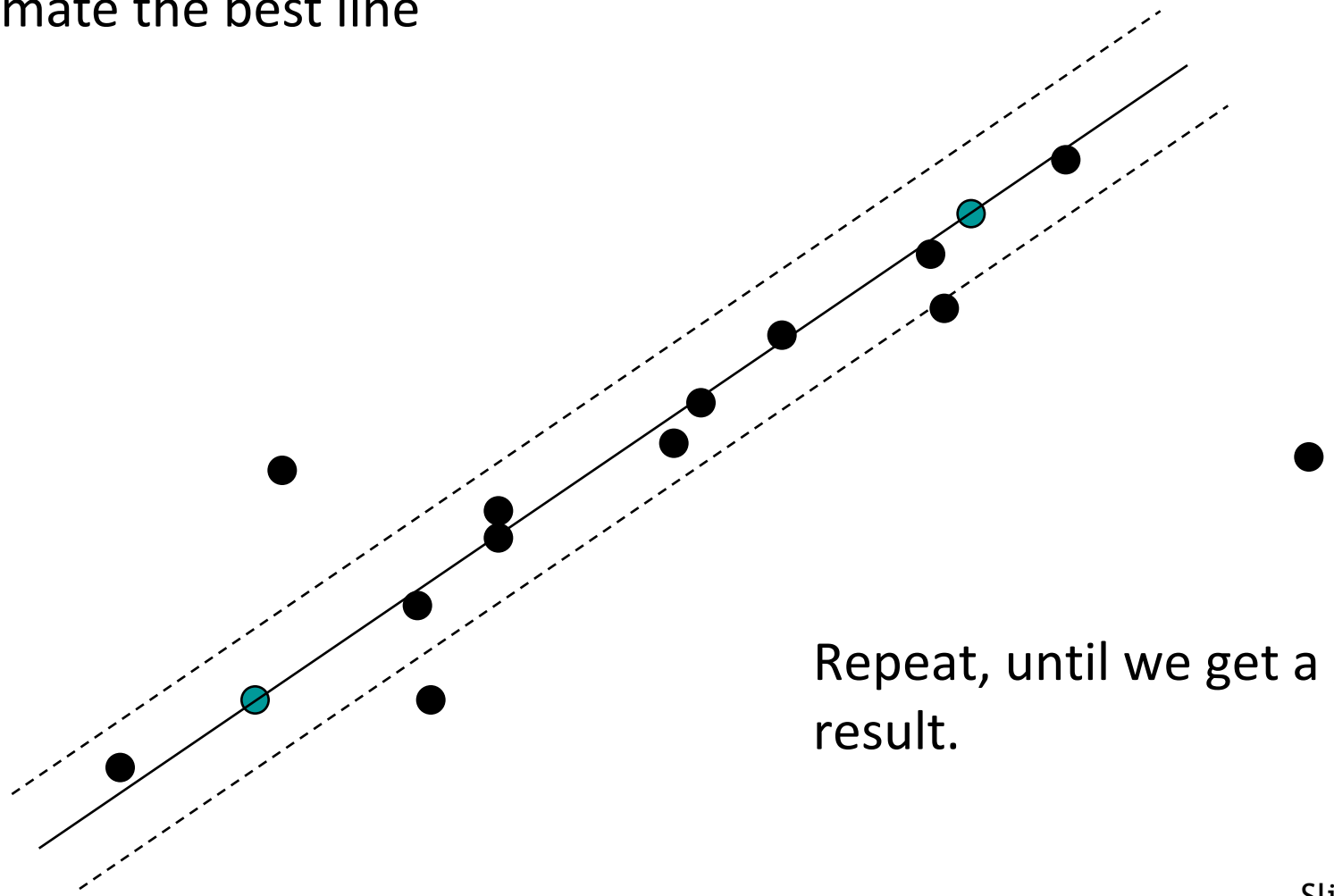
- Task: Estimate the best line



Slide credit: Kristen Grauman

RANSAC Line Fitting Example

- Task: Estimate the best line

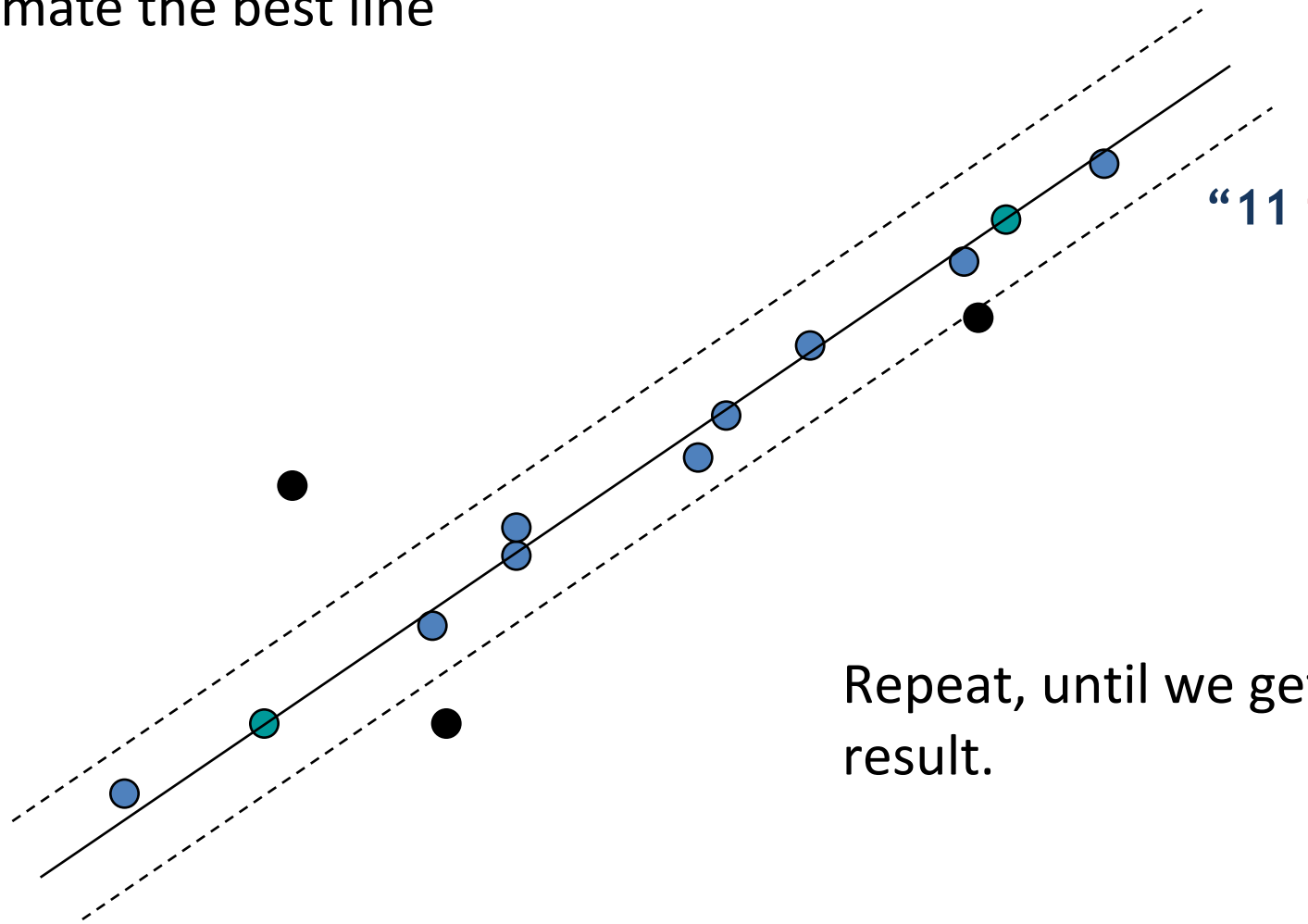


Repeat, until we get a good result.

Slide credit: Kristen Grauman

RANSAC Line Fitting Example

- Task: Estimate the best line



"11 inlier points"

Repeat, until we get a good result.

Slide credit: Kristen Grauman

Algorithm 15.4: RANSAC: fitting lines using random sample consensus

Determine:
    n — the smallest number of points required
    k — the number of iterations required
    t — the threshold used to identify a point that fits well
    d — the number of nearby points required
      to assert a model fits well
Until k iterations have occurred
    Draw a sample of n points from the data
      uniformly and at random
    Fit to that set of n points
    For each data point outside the sample
      Test the distance from the point to the line
        against t; if the distance from the point to the line
        is less than t, the point is close
    end
    If there are d or more points close to the line
      then there is a good fit. Refit the line using all
      these points.
end
Use the best fit from this collection, using the
  fitting error as a criterion

RANSAC: How many samples?

- How many samples are needed?
  - Suppose w is fraction of inliers (points from line).
  - n points needed to define hypothesis (2 for lines)
  - k samples chosen.
- Prob. that a single sample of n points is correct: w^n
- Prob. that all k samples fail is: $(1-w^n)^k$

$\Rightarrow$ Choose k high enough to keep this below desired failure rate.

Slide credit: David Lowe

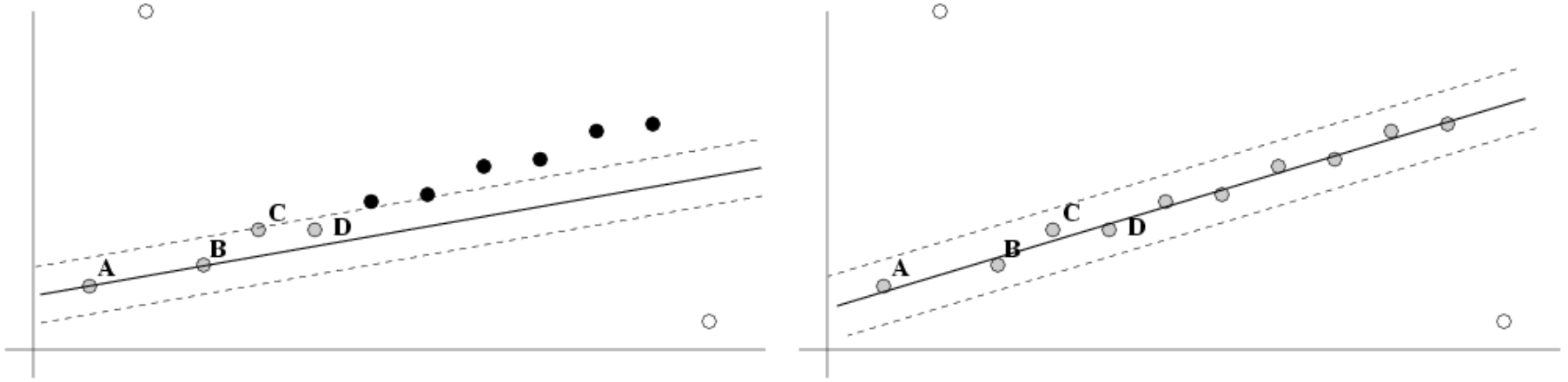
RANSAC: Computed k (p=0.99)

Sample size n	Proportion of outliers						
	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Slide credit: David Lowe

After RANSAC

- RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers.
- Improve this initial estimate with estimation over all inliers (e.g. with standard least-squares minimization).
- But this may change inliers, so alternate fitting with re-classification as inlier/outlier.



Slide credit: David Lowe

RANSAC: Pros and Cons

- **Pros**:
  - General method suited for a wide range of model fitting problems
  - Easy to implement and easy to calculate its failure rate
- **Cons**:
  - Only handles a moderate percentage of outliers without cost blowing up
  - Many real problems have high rate of outliers (but sometimes selective choice of random subsets can help)
- A voting strategy, The Hough transform, can handle high percentage of outliers

What we will learn today?

- A model fitting method for edge detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector

Some background reading:
Rick Szeliski, Chapter 4.1.1; David Lowe, IJCV 2004

Image matching: a challenging problem

Image matching: a challenging problem

by Diva Sian

by swashford

Slide credit: Steve Seitz

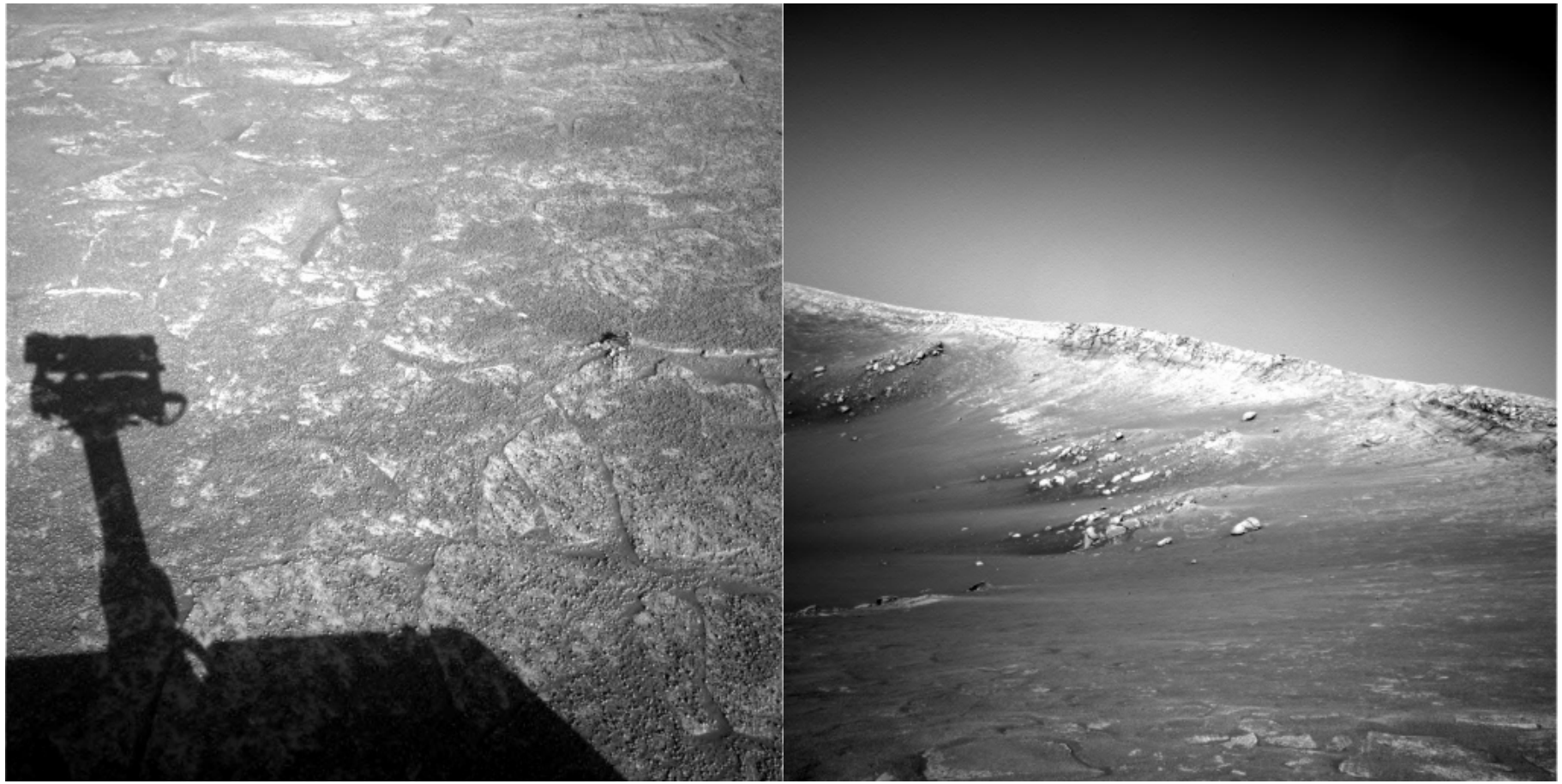
Harder Case

by Diva Sian

by scgbt

Slide credit: Steve Seitz

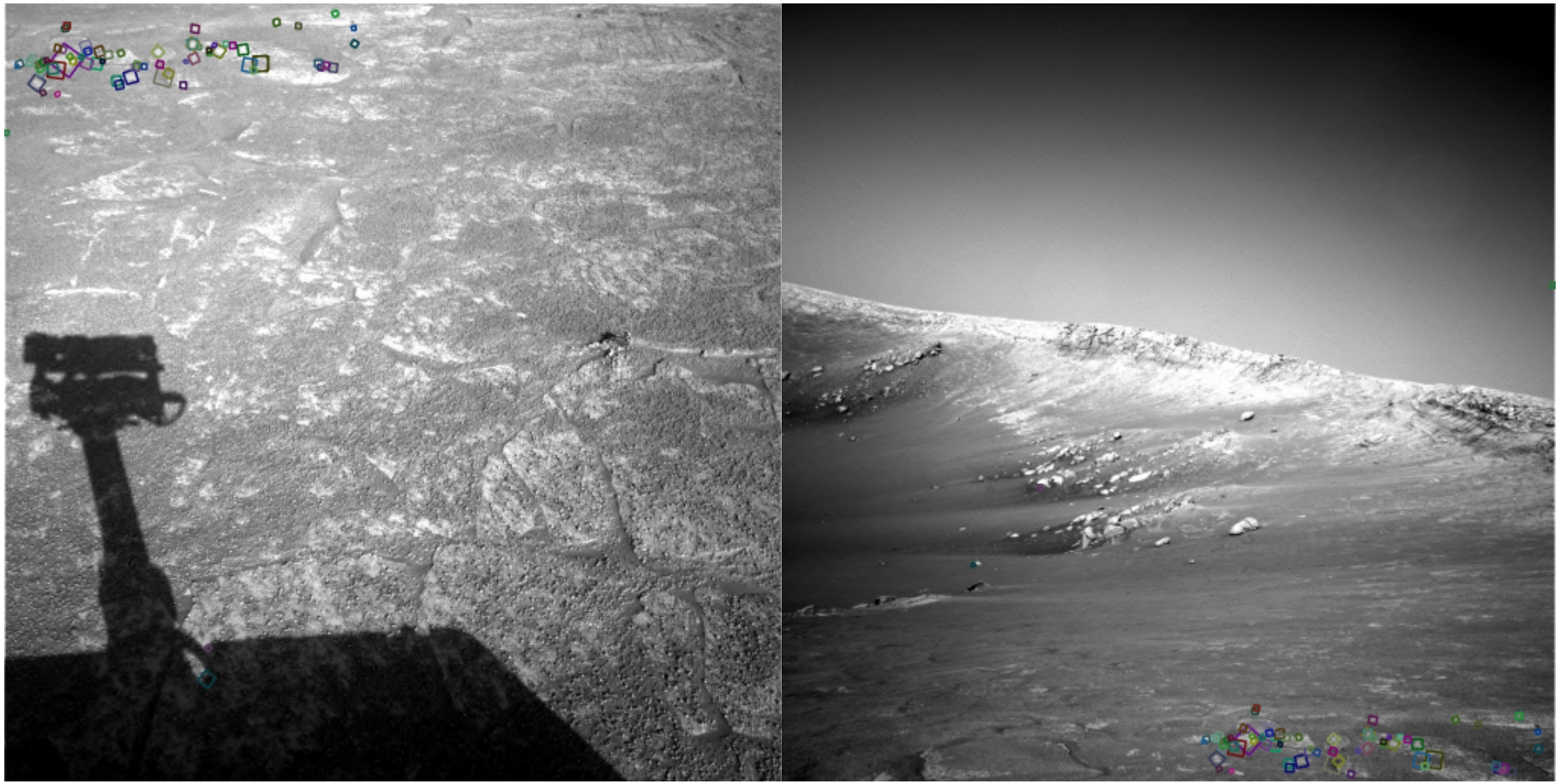
Harder Still?



NASA Mars Rover images

Slide credit: Steve Seitz

Answer Below (Look for tiny colored squares)



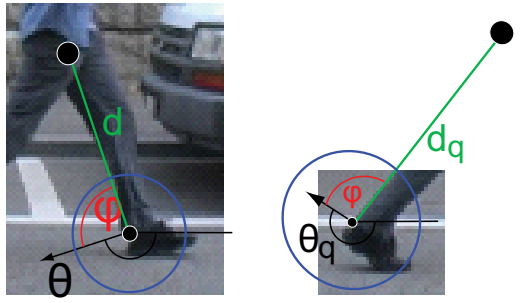
NASA Mars Rover images with SIFT feature matches
(Figure by Noah Snavely)

Slide credit: Steve Seitz

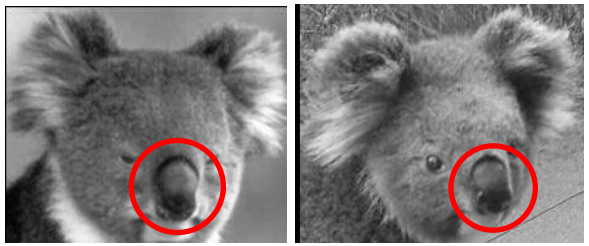
Motivation for using local features

- Global representations have major limitations
- Instead, describe and match only local regions
- Increased robustness to
  - Occlusions

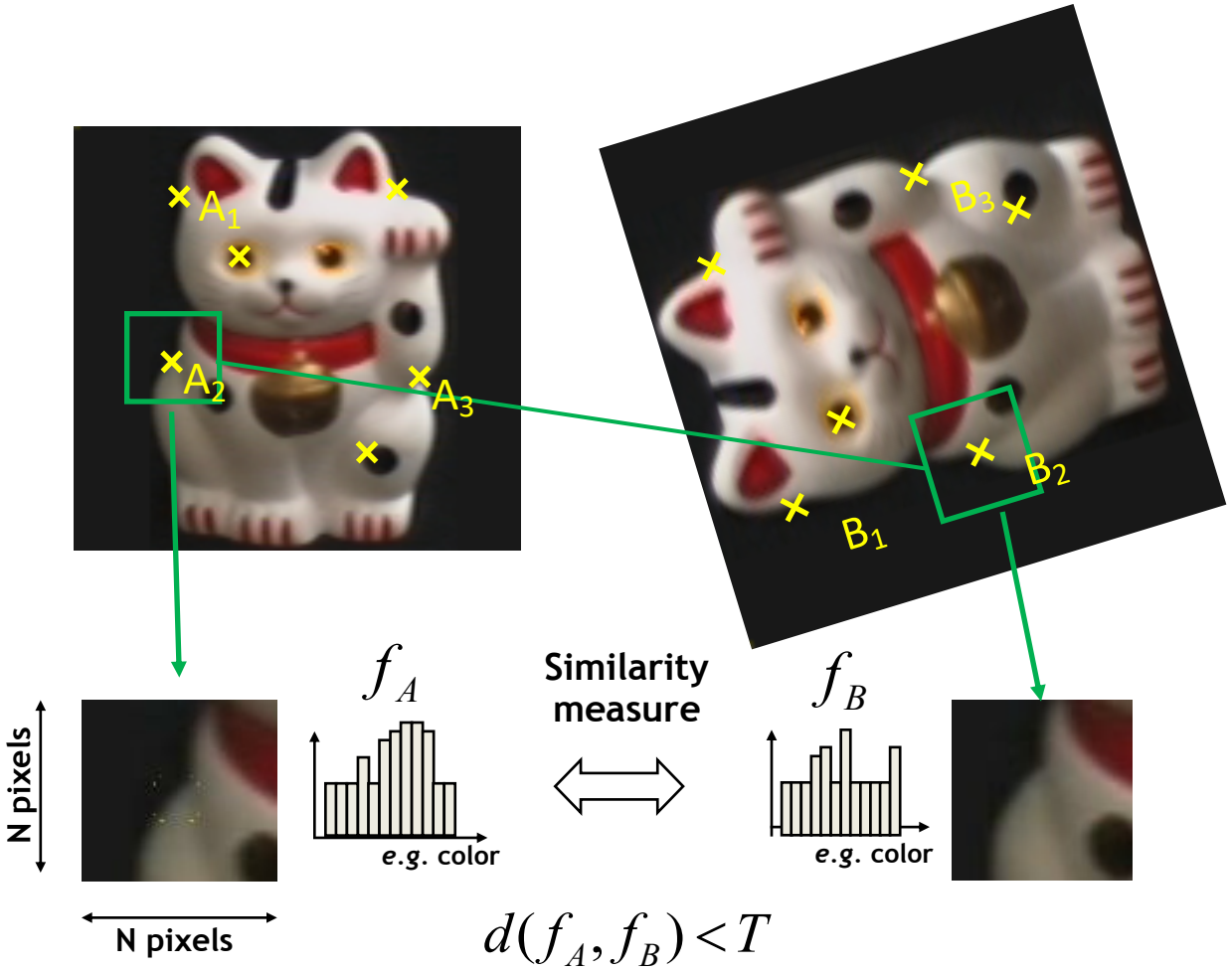
  - Articulation



  - Intra-category variations



General Approach



1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Slide credit: Bastian Leibe

Common Requirements

- Problem 1:
  - Detect the same point *independently* in both images

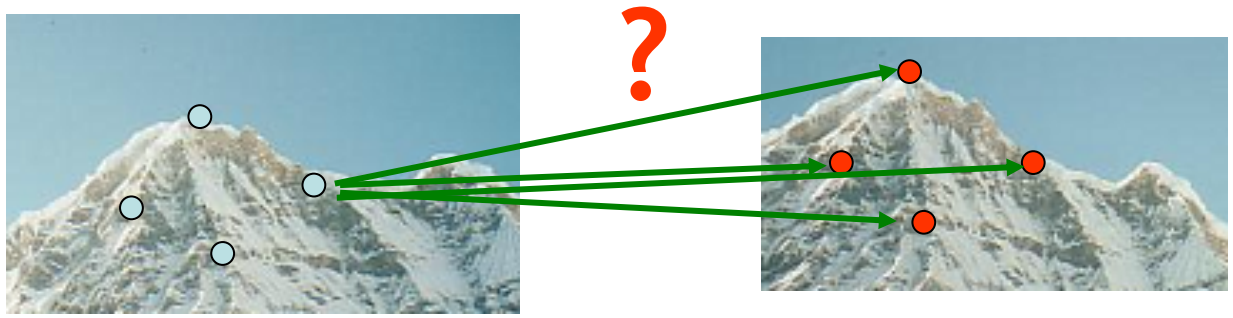
No chance to match!

We need a repeatable detector!

Slide credit: Darya Frolova, Denis Simakov

Common Requirements

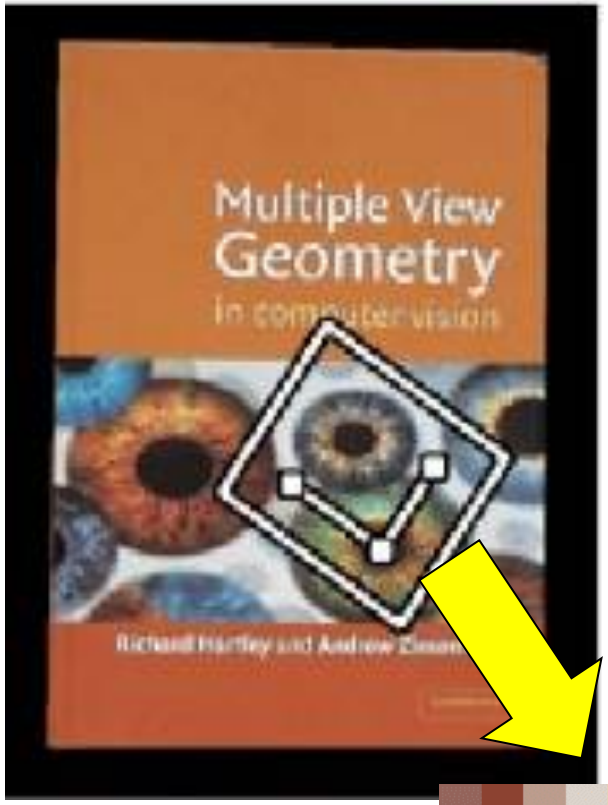
- Problem 1:
  - Detect the same point *independently* in both images
- Problem 2:
  - For each point correctly recognize the corresponding one

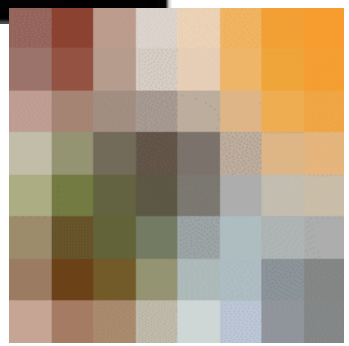


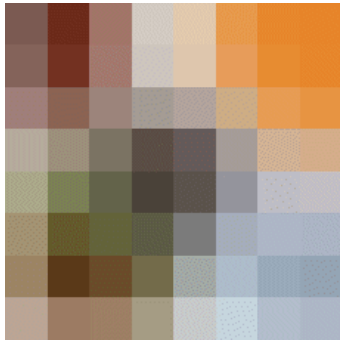
We need a reliable and distinctive descriptor!

Slide credit: Darya Frolova, Denis Simakov

Invariance: Geometric Transformations

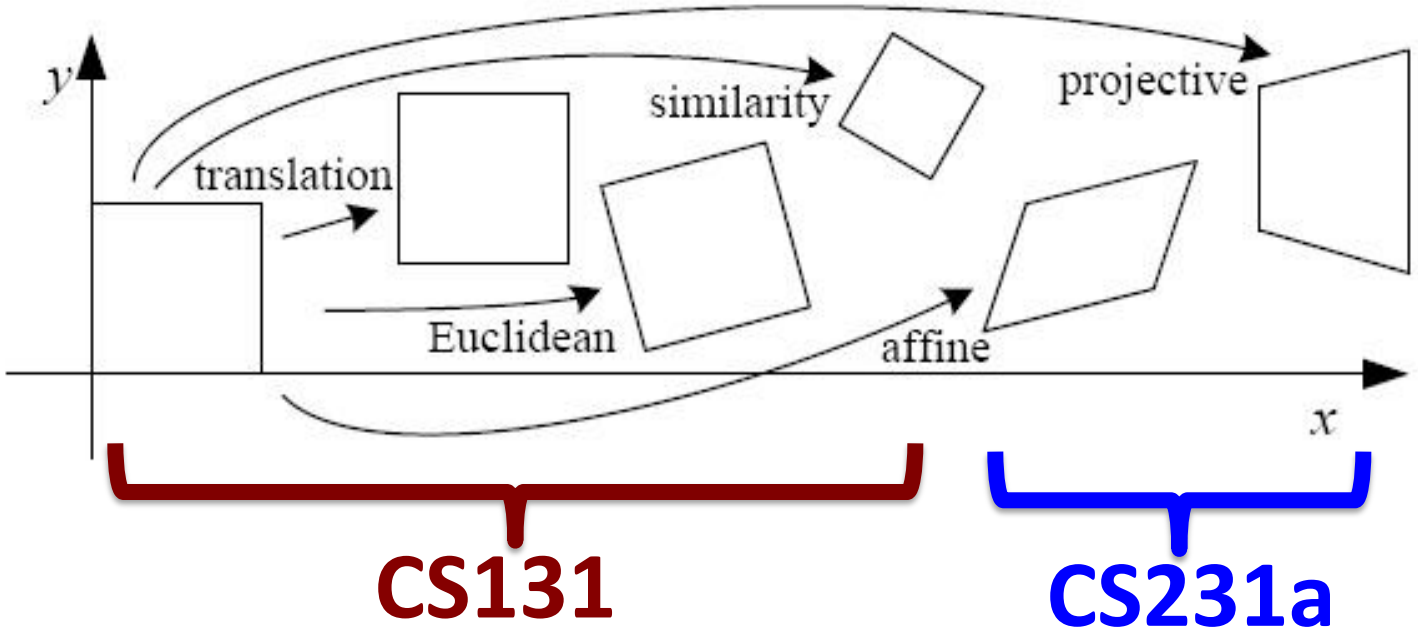






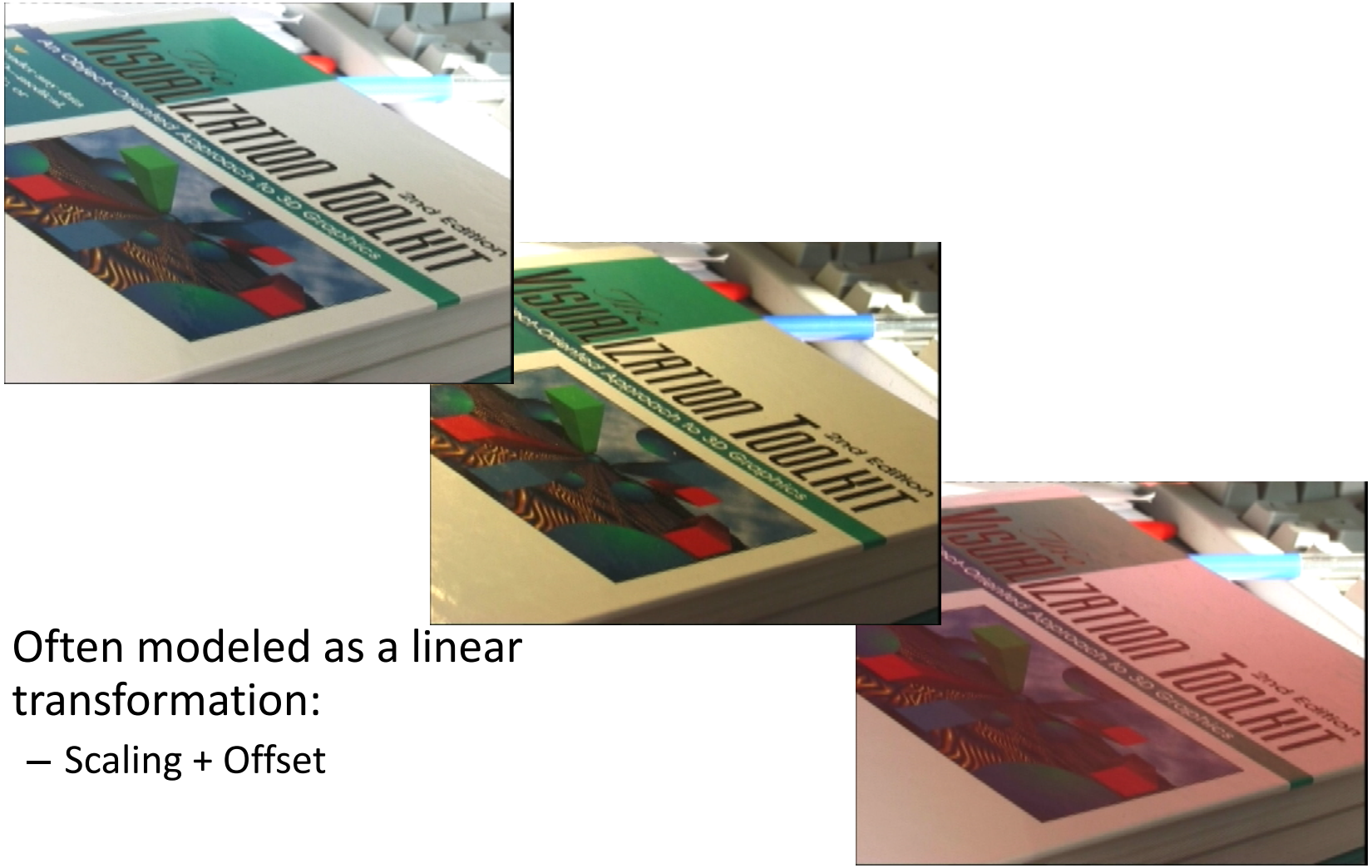
Slide credit: Steve Seitz

Levels of Geometric Invariance



Slide credit: Bastian Leibe

Invariance: Photometric Transformations



- Often modeled as a linear transformation:
  - Scaling + Offset

Slide credit: Tinne Tuytelaars

Requirements

- Region extraction needs to be repeatable and accurate
  - Invariant to translation, rotation, scale changes
  - Robust or covariant to out-of-plane (≈affine) transformations
  - Robust to lighting variations, noise, blur, quantization
- Locality: Features are local, therefore robust to occlusion and clutter.
- Quantity: We need a sufficient number of regions to cover the object.
- Distinctivenes : The regions should contain “interesting” structure.
- Efficiency: Close to real-time performance.

Slide credit: Bastian Leibe

Many Existing Detectors Available

- Hessian & Harris [Beaudet ‘78], [Harris ‘88]
- Laplacian, DoG [Lindeberg ‘98], [Lowe ‘99]
- Harris-/Hessian-Laplace [Mikolajczyk & Schmid ‘01]
- Harris-/Hessian-Affine [Mikolajczyk & Schmid ‘04]
- EBR and IBR [Tuytelaars & Van Gool ‘04]
- MSER [Matas ‘02]
- Salient Regions [Kadir & Brady ‘01]
- Others…

- *Those detectors have become a basic building block for many applications in Computer Vision.*

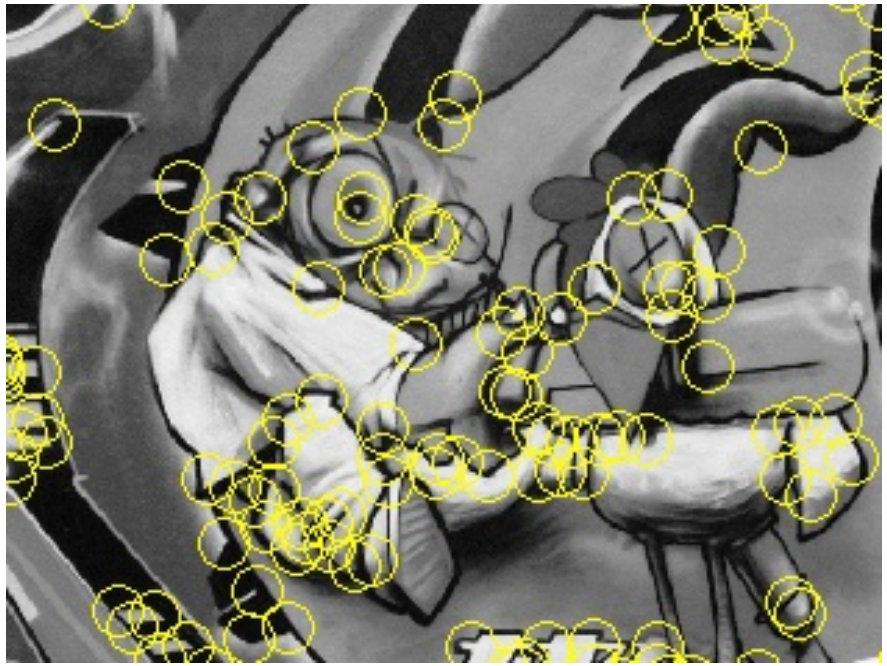
Slide credit: Bastian Leibe

What we will learn today?

- A model fitting method for edge detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector

Some background reading:
Rick Szeliski, Chapter 4.1.1; David Lowe, IJCV 2004

Keypoint Localization

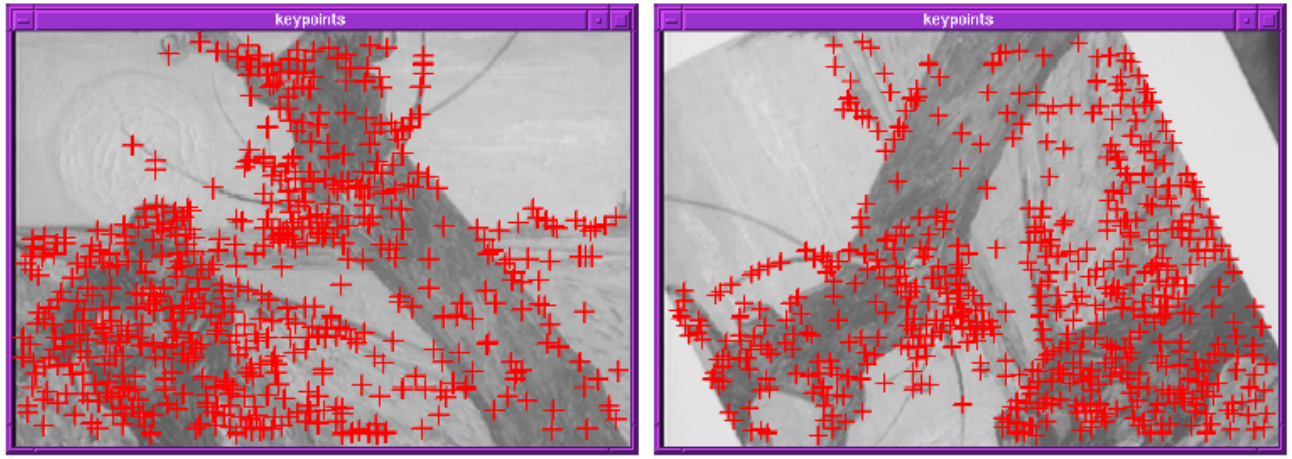


- Goals:
  - Repeatable detection
  - Precise localization
  - Interesting content

$\Rightarrow$ *Look for two-dimensional signal changes*

Slide credit: Bastian Leibe

Finding Corners



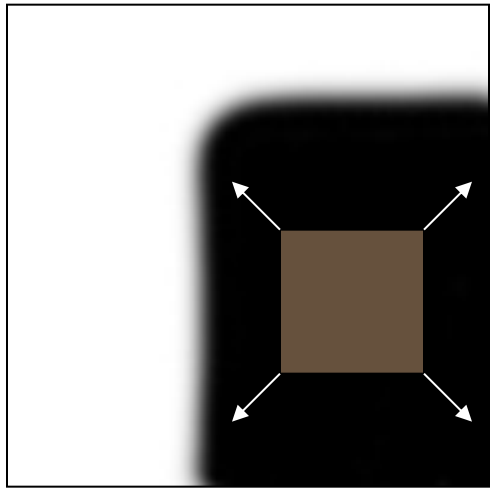
- Key property:
  - In the region around a corner, image gradient has two or more dominant directions
- Corners are *repeatable* and *distinctive*

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference, 1988.

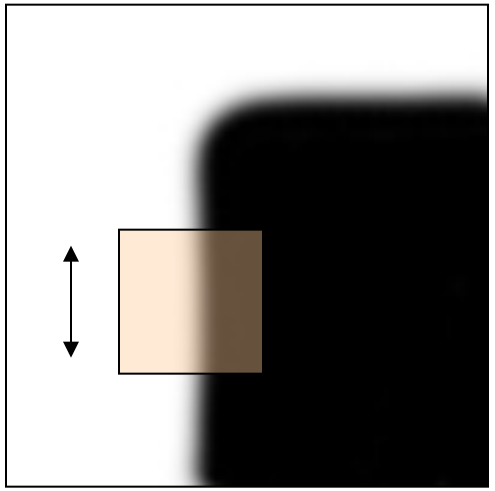
Slide credit: Svetlana Lazebnik

Corners as Distinctive Interest Points

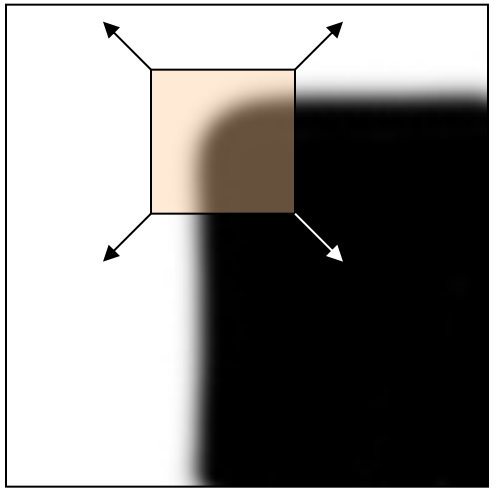
- Design criteria
  - We should easily recognize the point by looking through a small window (*locality*)
  - Shifting the window in *any direction* should give *a large change* in intensity (*good localization*)



“flat” region: no change in all directions



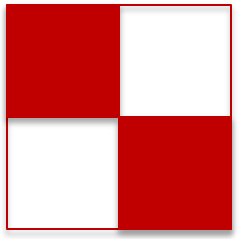
“edge”: no change along the edge direction



“corner”: significant change in all directions

Slide credit: Alyosha Efros

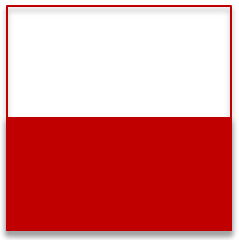
Corners versus edges



$\sum I_x^2$ → Large

$\sum I_y^2$ → Large

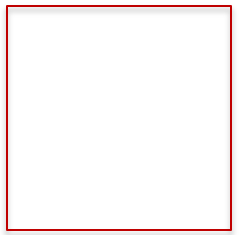
Corner



$\sum I_x^2$ → Small

$\sum I_y^2$ → Large

Edge

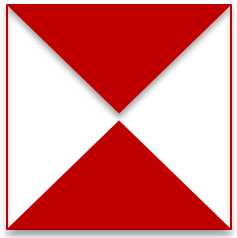


$\sum I_x^2$ → Small

$\sum I_y^2$ → Small

Nothing

Corners versus edges

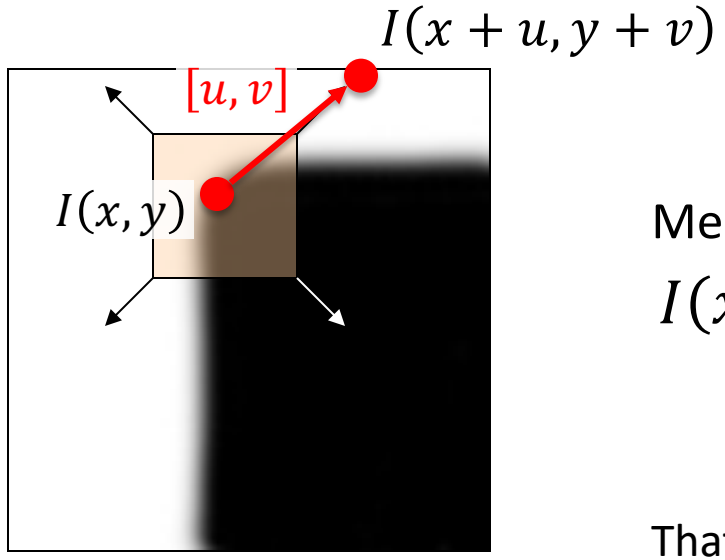


$$\sum I_x^2 \longrightarrow ??$$

$$\sum I_y^2 \longrightarrow ??$$

Corner

Measure change in all directions [u, v]



“corner”: significant change in all directions

Measure change as intensity difference:

$$I(x+u, y+v) - I(x, y)$$

That’s for a single point, but we have to accumulate over a ”small window” around that point…

Harris Detector Formulation

- Change of intensity for the shift [u,v]:

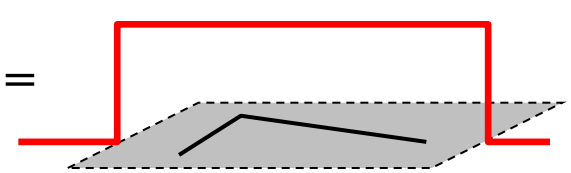
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function — $w(x,y)$

Shifted intensity — $I(x+u,y+v)$

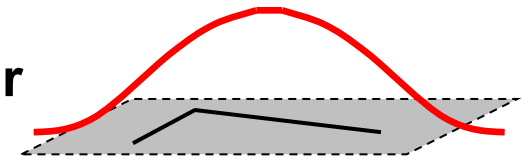
Intensity — $I(x,y)$

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Slide credit: Rick Szeliski

Harris Detector Formulation

- This measure of change can be approximated by (Taylor expansion):

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

Sum over image region – the area we are checking for corner

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \;\; I_y]$$

Slide credit: Rick Szeliski

Harris Detector Formulation

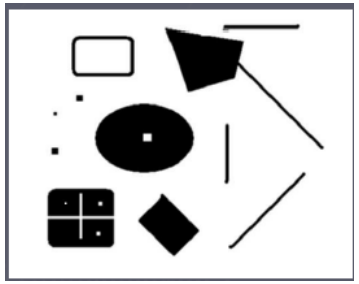
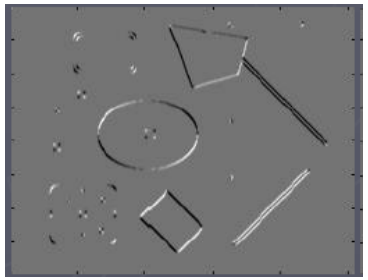


Image I

I_x

I_y



$I_x I_y$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

Sum over image region – the area we are checking for corner

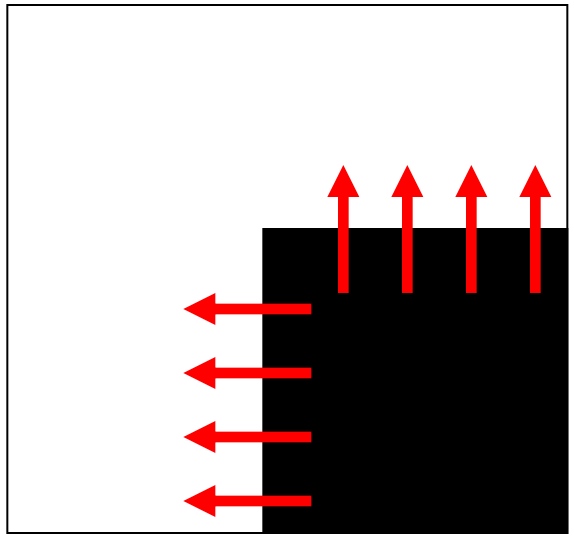
$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \; I_y]$$

Slide credit: Rick Szeliski

What Does This Matrix Reveal?

- First, let's consider an axis-aligned corner:

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

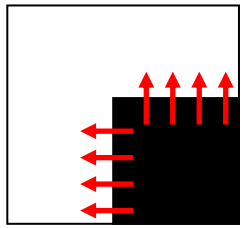


Slide credit: David Jacobs

What Does This Matrix Reveal?

- First, let's consider an axis-aligned corner:

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$



- This means:
  - Dominant gradient directions align with x or y axis
  - If either λ is close to 0, then this is not a corner, so look for locations where both are large.
- What if we have a corner that is not aligned with the image axes?

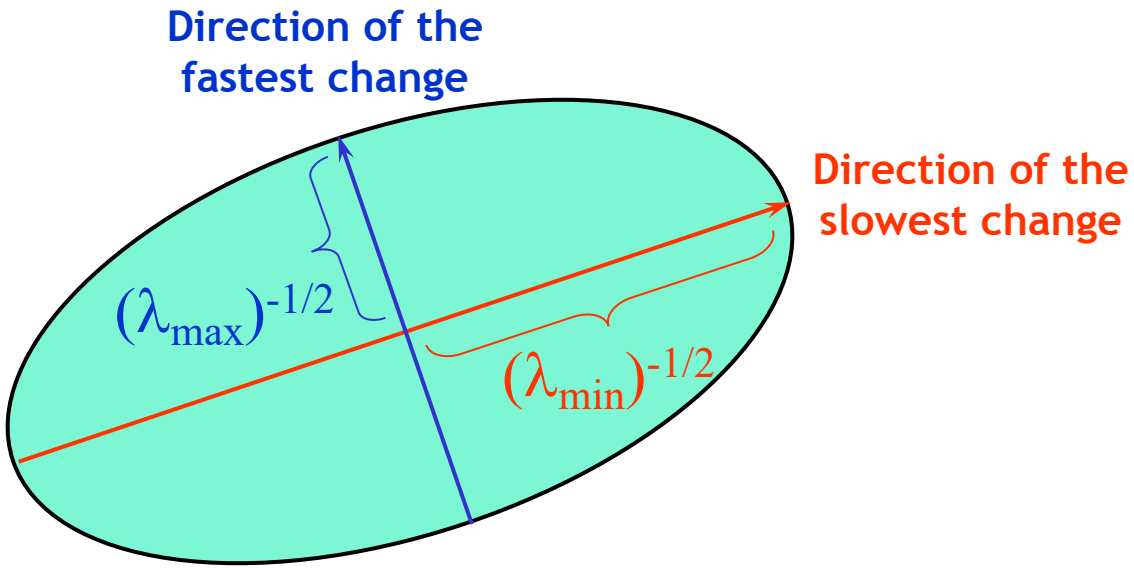
Slide credit: David Jacobs

General Case

- Since M is symmetric, we have $$M = R^{-1}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

(Eigenvalue decomposition)

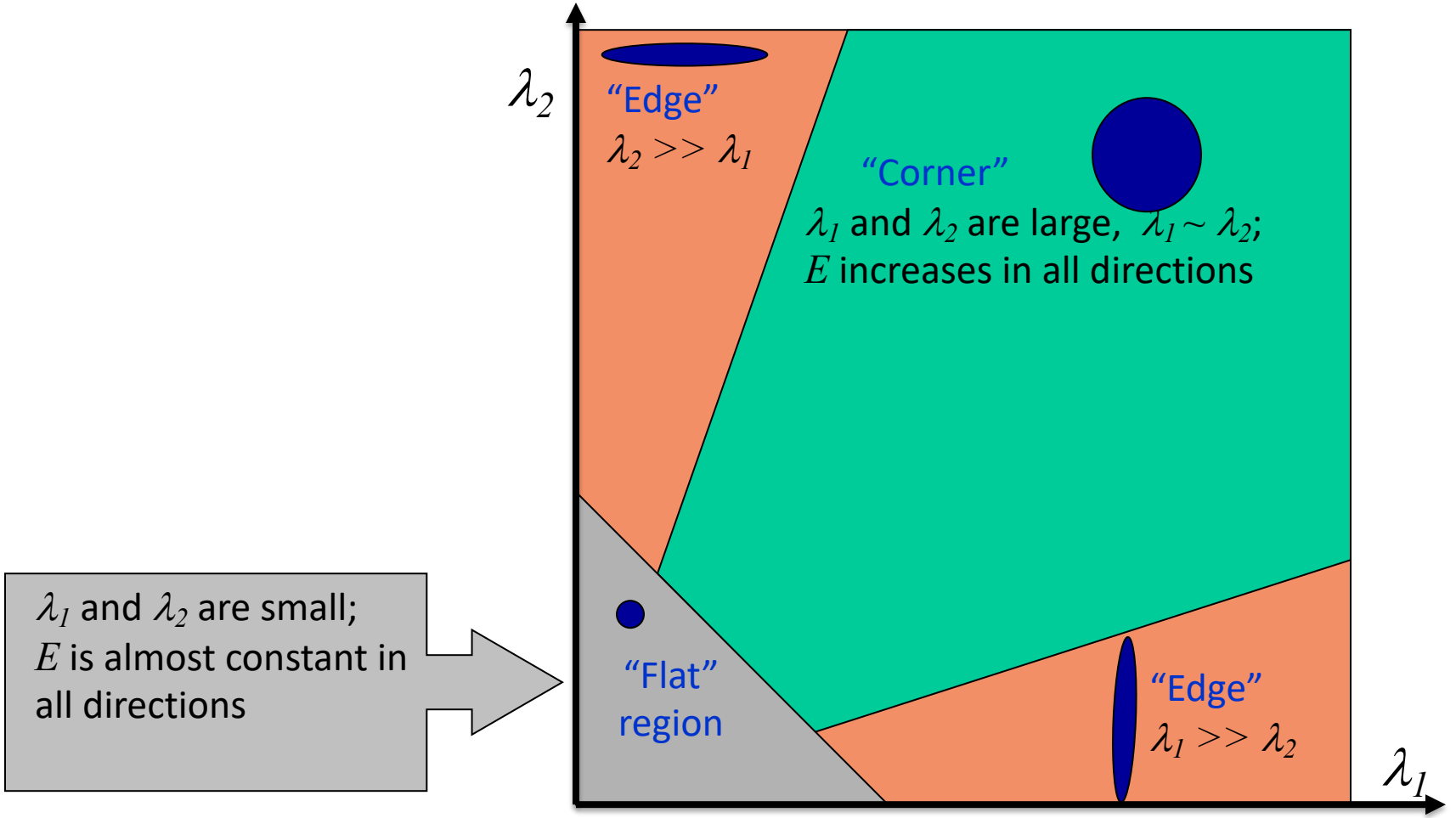
- We can think of M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R



adapted from Darya Frolova, Denis Simakov

Interpreting the Eigenvalues

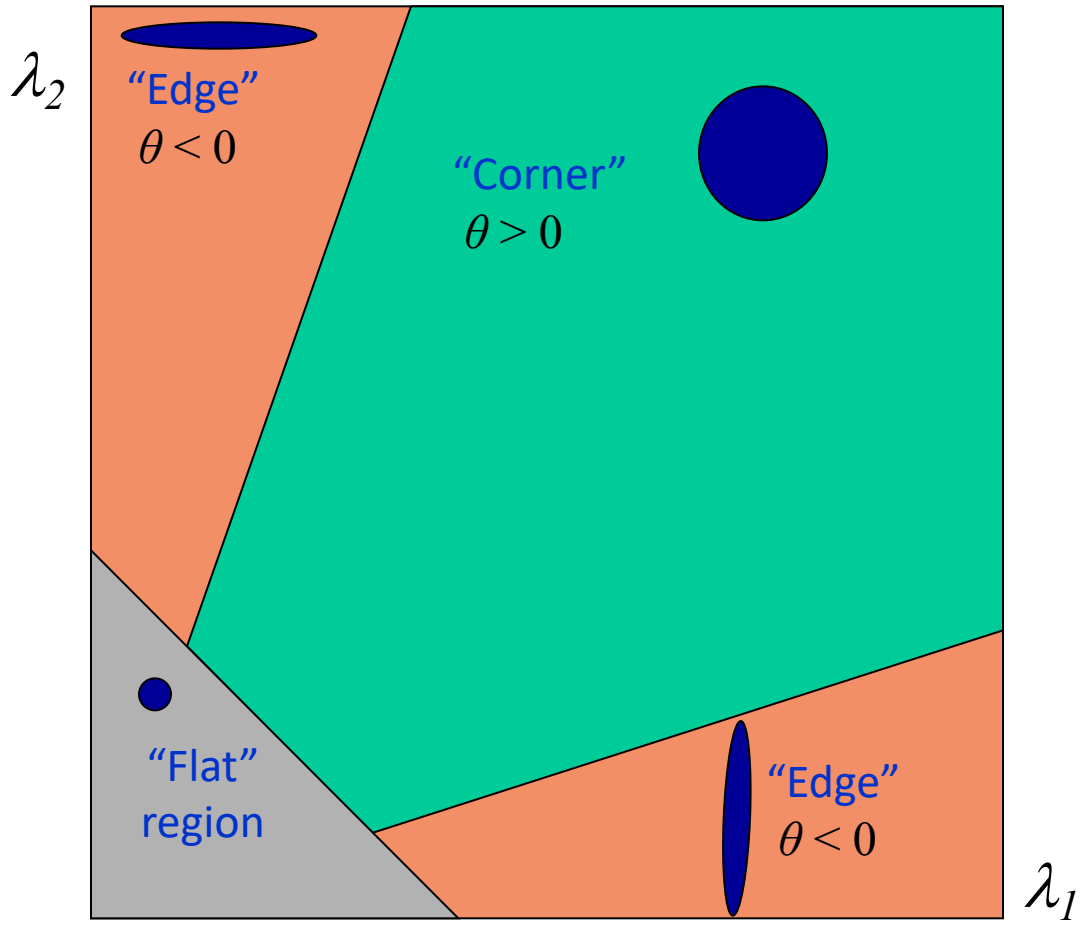
- Classification of image points using eigenvalues of M:



Slide credit: Kristen Grauman

Corner Response Function

$$\theta = \det(M) - \alpha \,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$



- Fast approximation
  - Avoid computing the eigenvalues
  - α: constant (0.04 to 0.06)

Slide credit: Kristen Grauman

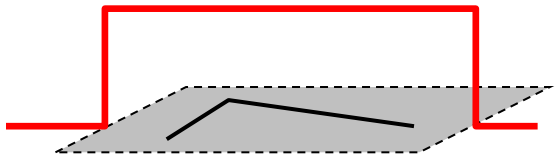
Window Function $w(x,y)$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Option 1: uniform window
  - Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

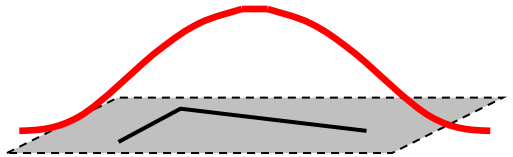
  - Problem: not rotation invariant



1 in window, 0 outside

- Option 2: Smooth with Gaussian
  - Gaussian already performs weighted sum

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Gaussian

  - Result is rotation invariant

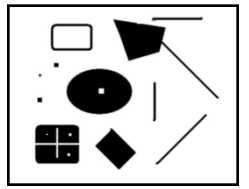
Slide credit: Bastian Leibe

Summary: Harris Detector [Harris88]

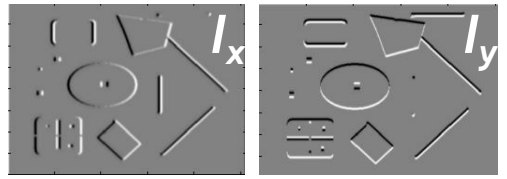
- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I,\sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

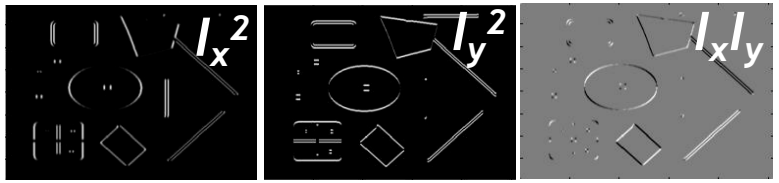
σ_D: for Gaussian in the derivative calculation
σ_I: for Gaussian in the windowing function



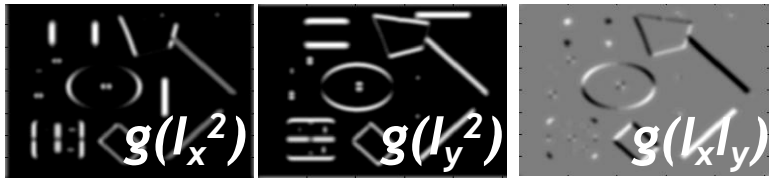
1. Image derivatives



2. Square of derivatives



3. Gaussian filter $g(\sigma_I)$



4. Cornerness function – two strong eigenvalues

$$\theta = \det[M(\sigma_I,\sigma_D)] - \alpha[\text{trace}(M(\sigma_I,\sigma_D))]^2$$
$$= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2$$

5. Perform non-maximum suppression

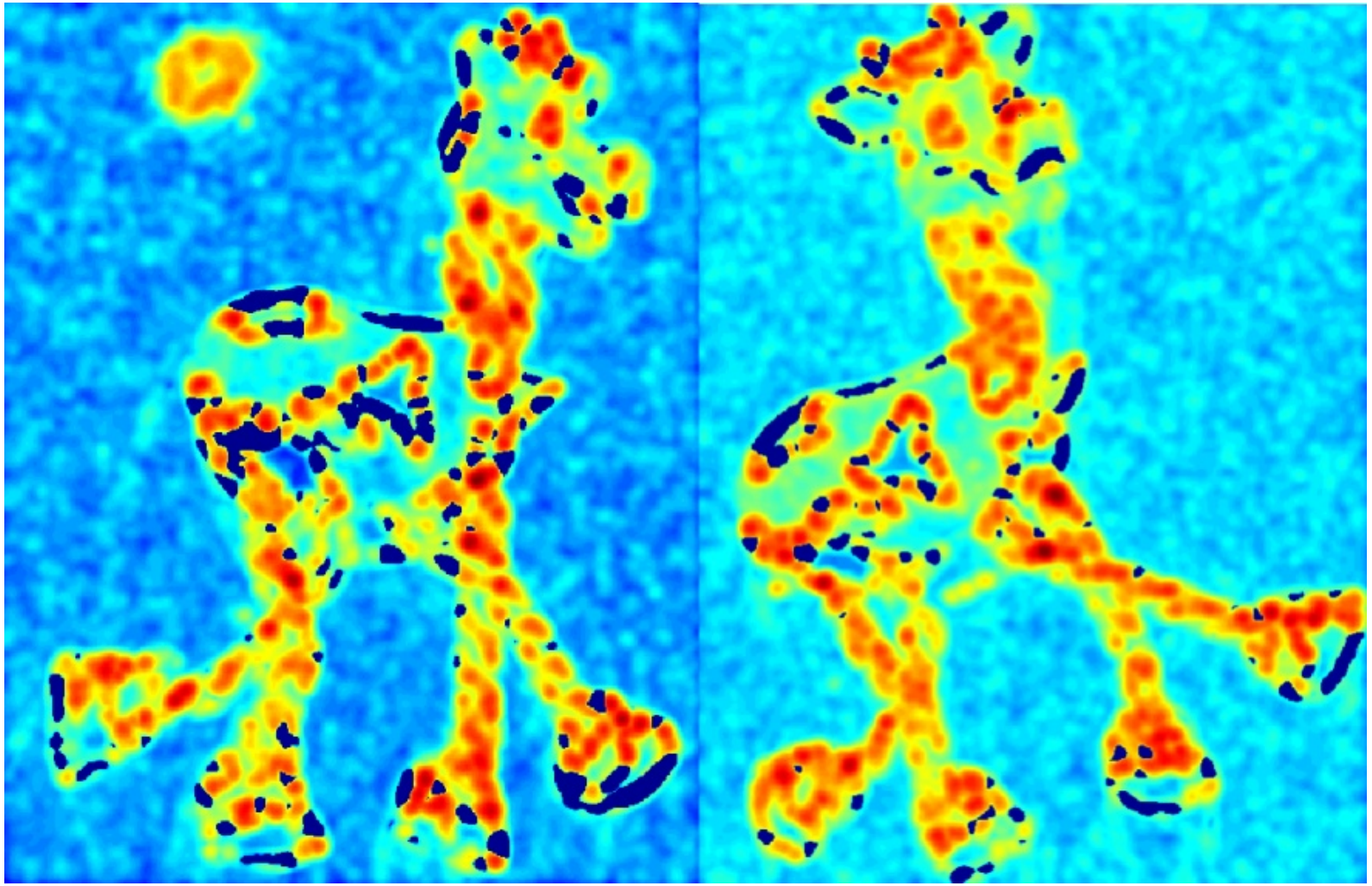
Slide credit: Krystian Mikolajczyk

Harris Detector: Workflow

Slide adapted from Darya Frolova, Denis Simakov

Harris Detector: Workflow

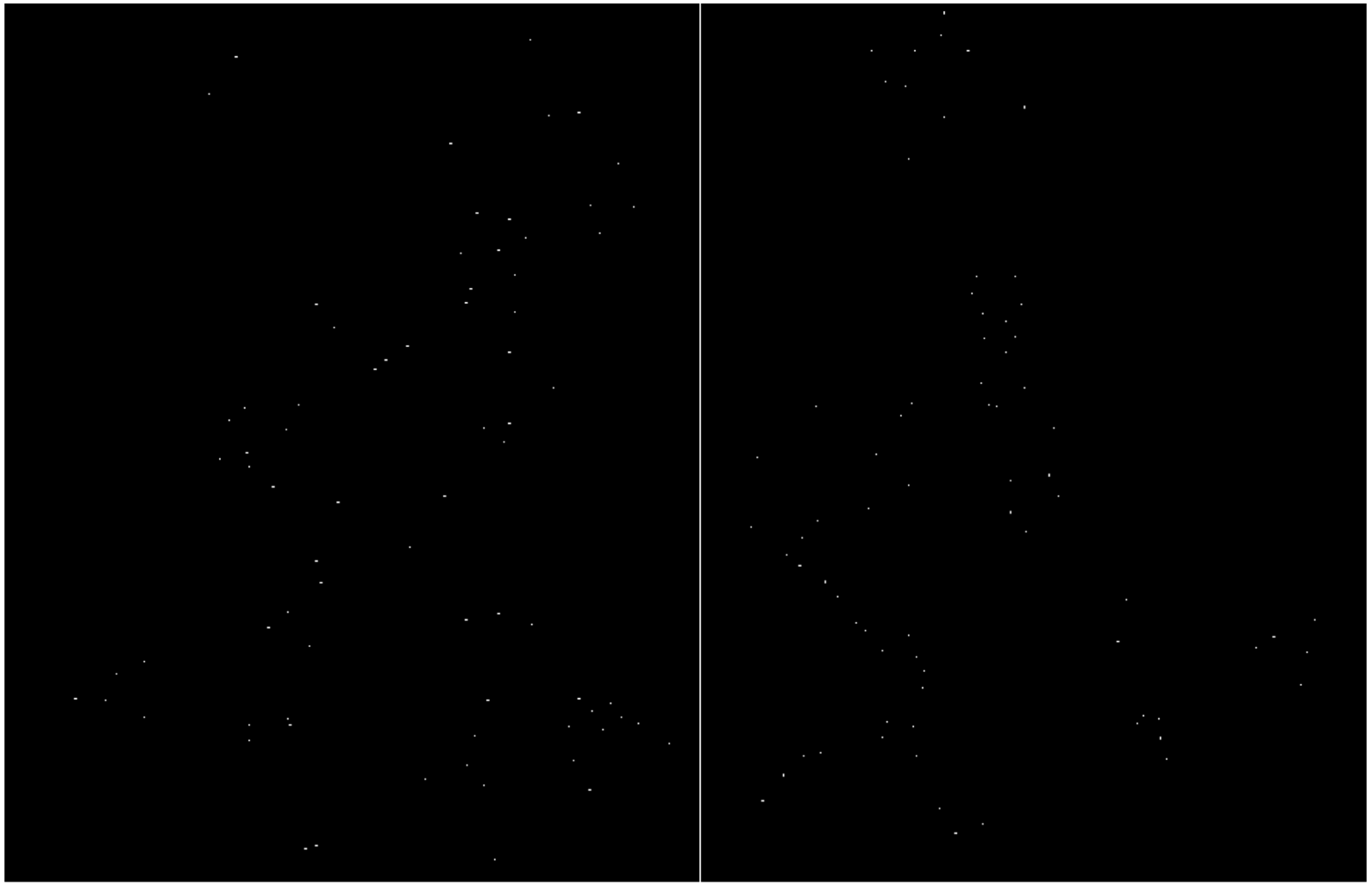
- computer corner responses θ



Slide adapted from Darya Frolova, Denis Simakov

Harris Detector: Workflow

- Take only the local maxima of θ, where θ>threshold



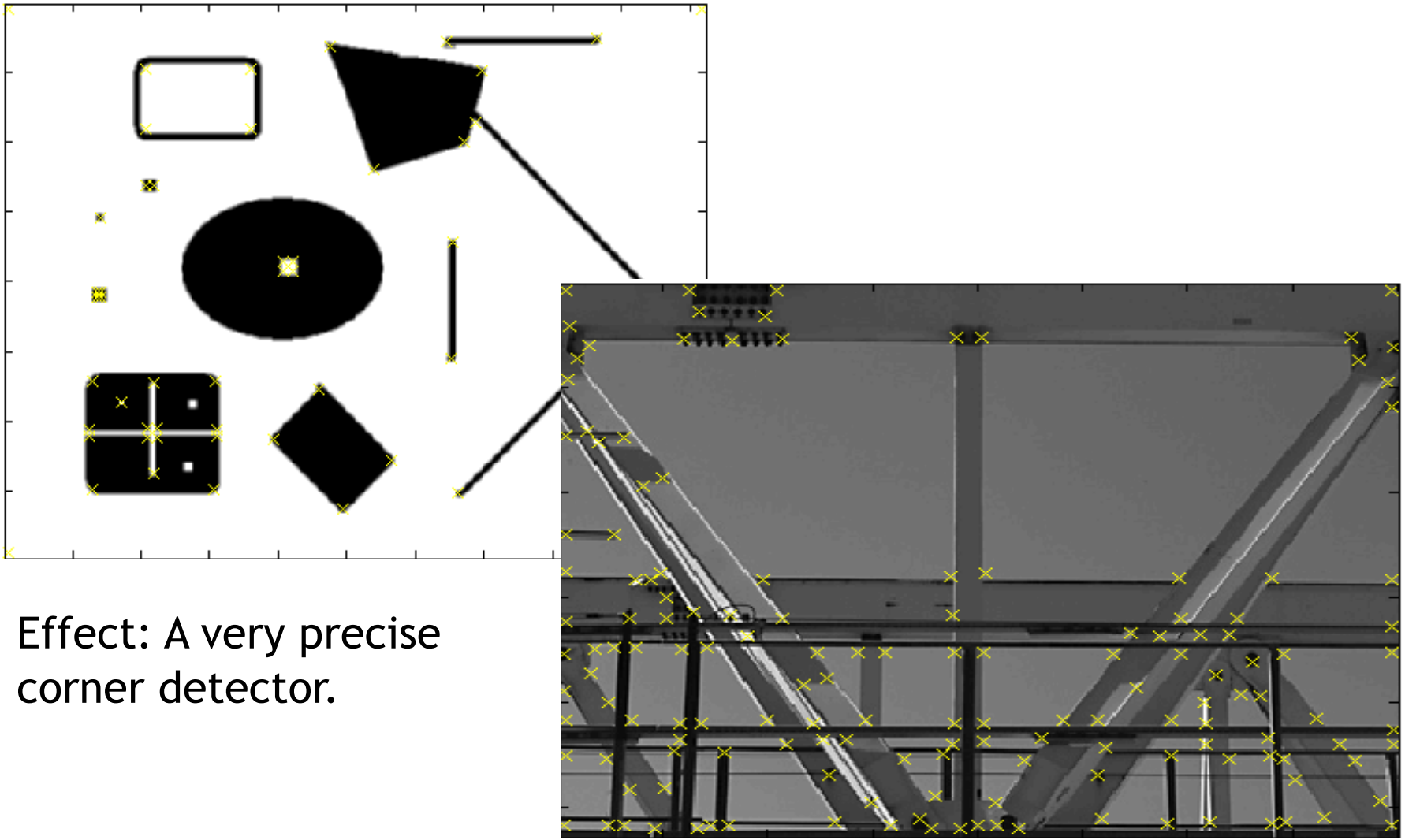
Slide adapted from Darya Frolova, Denis Simakov

Harris Detector: Workflow

- Resulting Harris points

Slide adapted from Darya Frolova, Denis Simakov

Harris Detector – Responses [Harris88]



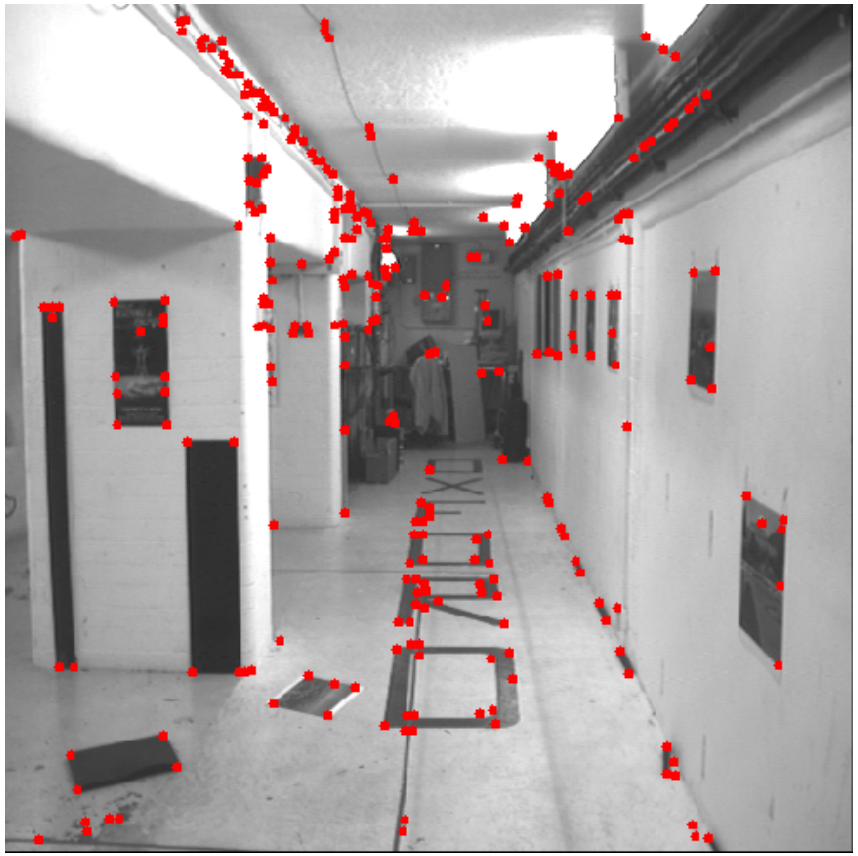
Effect: A very precise corner detector.

Slide credit: Krystian Mikolajczyk

Harris Detector – Responses [Harris88]

Slide credit: Krystian Mikolajczyk

Harris Detector – Responses [Harris88]



- Results are well suited for finding stereo correspondences

Slide credit: Kristen Grauman

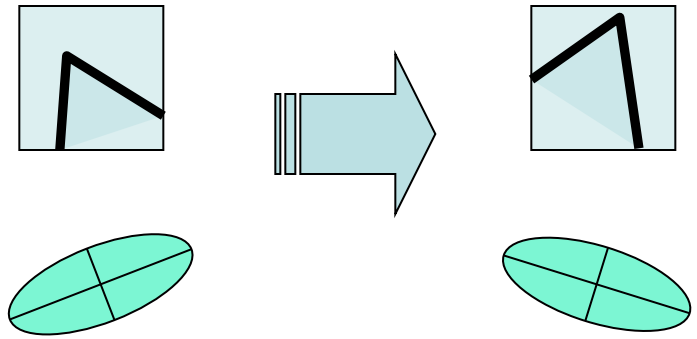
Harris Detector: Properties

- Translation invariance?

Slide credit: Kristen Grauman

Harris Detector: Properties

- Translation invariance
- Rotation invariance?



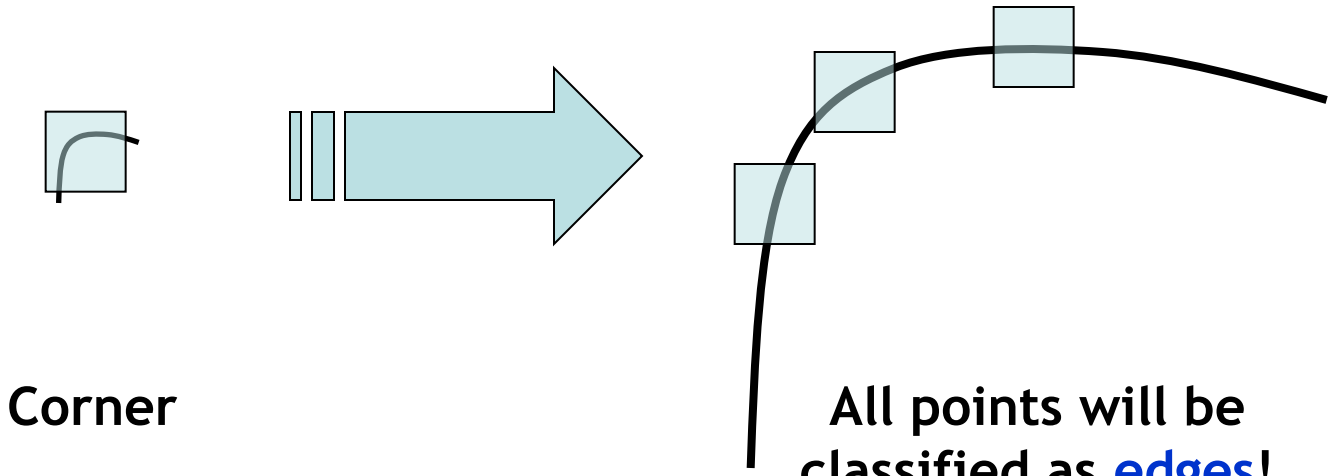
Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response θ is invariant to image rotation

Slide credit: Kristen Grauman

Harris Detector: Properties

- Translation invariance
- Rotation invariance
- Scale invariance?



Not invariant to image scale!

Slide credit: Kristen Grauman

What we are learned today?

- A model fitting method for edge detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector