

# Extreme value theory

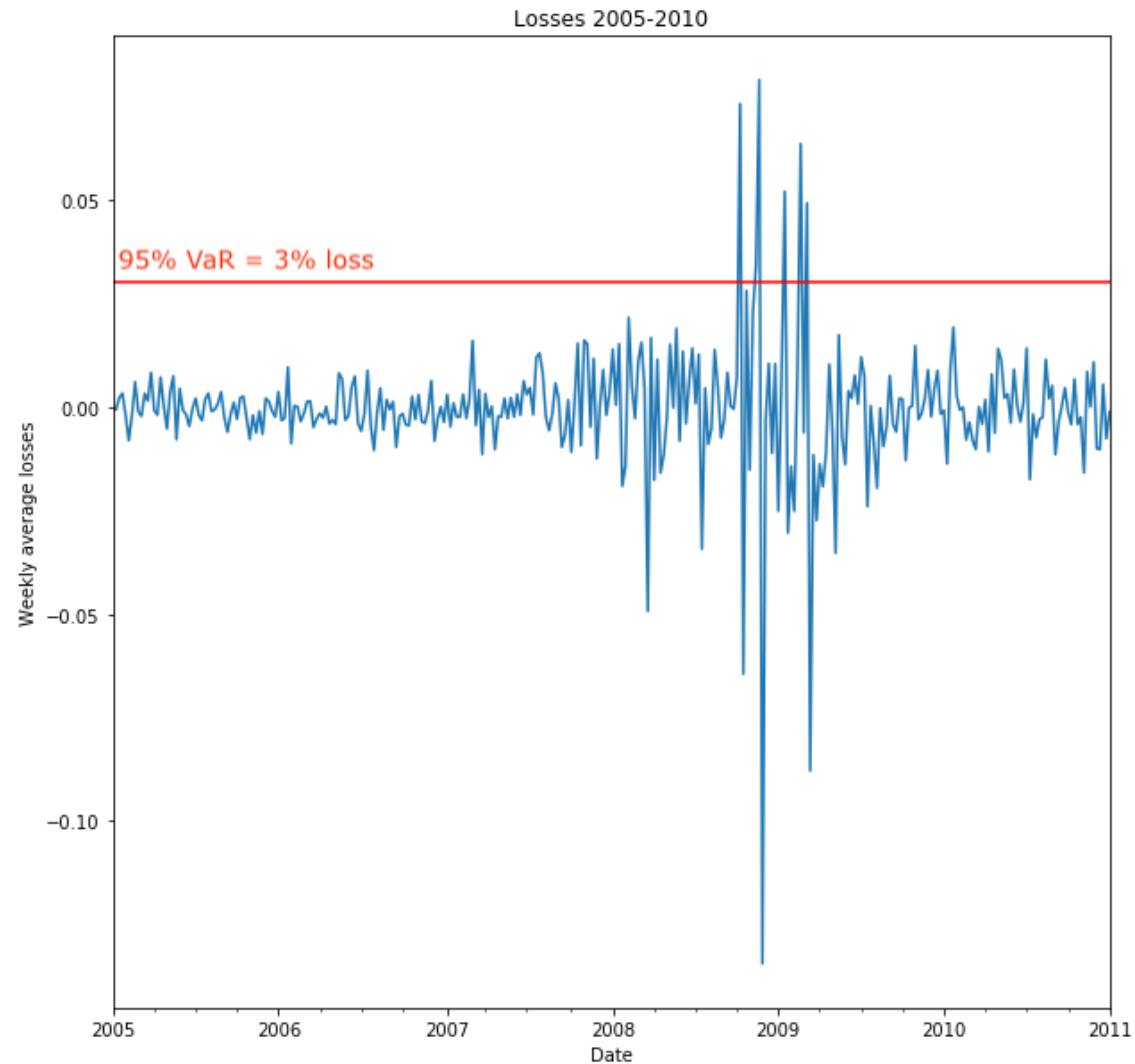
QUANTITATIVE RISK MANAGEMENT IN PYTHON



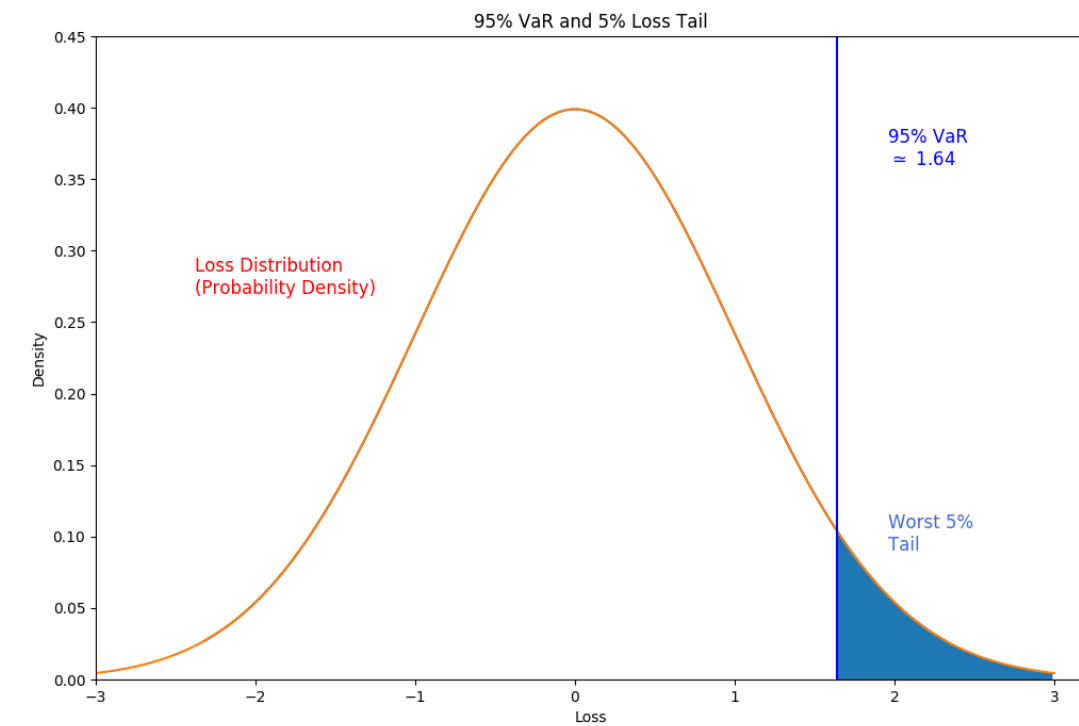
**Jamsheed Shorish**  
Computational Economist

# Extreme values

- Portfolio losses: extreme values



- Extreme values: from tail of distribution
  - Tail losses: losses exceeding some value
  - Model tail losses => better risk management



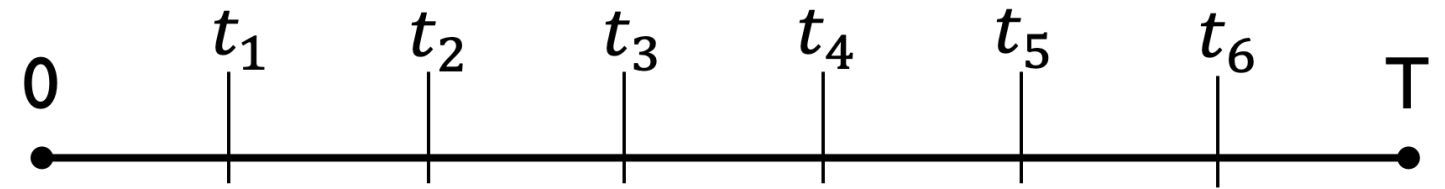
# Extreme value theory

- **Extreme value theory:** statistical distribution of extreme values
- **Block maxima**



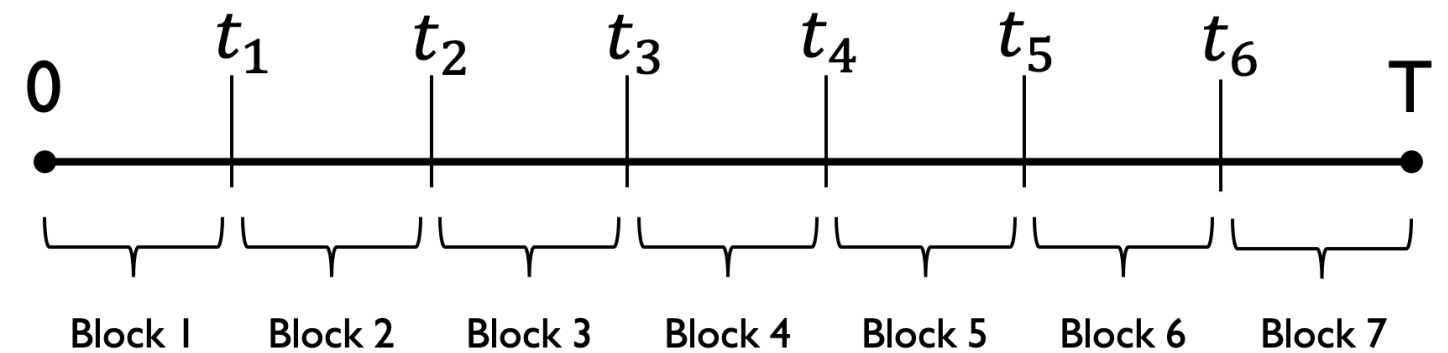
# Extreme value theory

- **Extreme value theory:** statistical distribution of extreme values
- **Block maxima:**
  - Break period into sub-periods



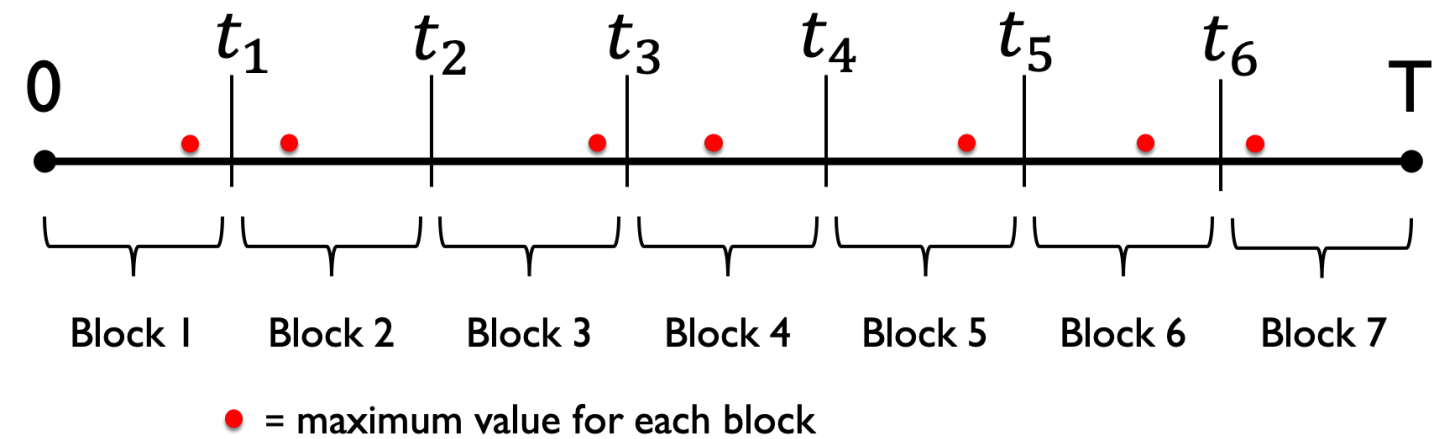
# Extreme value theory

- **Extreme value theory:** statistical distribution of extreme values
- **Block maxima:**
  - Break period into sub-periods
  - Form *block* from each sub-period



# Extreme value theory

- **Extreme value theory:** statistical distribution of extreme values
- **Block maxima:**
  - Break period into sub-periods
  - Form *blocks* from each sub-period
  - Set of block maxima = dataset
- **Peak over threshold (POT):**
  - Find all losses over given level
  - Set of such losses = dataset



# Generalized Extreme Value Distribution

- **Example:** Block maxima for 2007 - 2009
  - Resample losses with desired period (e.g. weekly)

```
maxima = losses.resample("W").max()
```

- **Generalized Extreme Value Distribution (GEV)**
  - Distribution of *maxima* of data
  - Example: parametric estimation using `scipy.stats.genextreme`

```
from scipy.stats import genextreme  
params = genextreme.fit(maxima)
```

# VaR and CVaR from GEV distribution

- **99% VaR** from GEV distribution
  - Use `.ppf()` percent point function to find 99% VaR
  - Requires `params` from fitted GEV distribution
  - Finds maximum loss over one week period at 99% confidence
- **99% CVaR** from GEV distribution
  - CVaR is conditional expectation of loss *given* VaR as minimum loss
  - Use `.expect()` method to find expected value

```
VaR_99 = genextreme.ppf(0.99, *params)
```

```
CVaR_99 = ( 1 / (1 - 0.99) ) * genextreme.expect(lambda x: x, *params, lb = VaR_99)
```



# Covering losses

- **Risk management:** *covering losses*
  - Regulatory requirement (banks, insurance)
  - Reserves must be available to cover losses
    - For a specified period (e.g. one week)
    - At a specified confidence level (e.g. 99%)
- **VaR from GEV distribution:**
  - estimates maximum loss
    - given period
    - given confidence level

# Covering losses

- **Example:** Initial portfolio value = \$1,000,000
- **One week reserve requirement** at 99% confidence
  - $VaR_{99}$  from GEV distribution: maximum loss over one week at 99% confidence
- **Reserve requirement:** Portfolio value  $\times VaR_{99}$ 
  - Suppose  $VaR_{99} = 0.10$ , i.e. 10% maximum loss
  - Reserve requirement = \$100,000
- Portfolio value *changes*  $\Rightarrow$  reserve requirement *changes*
- **Regulation** sets frequency of reserve requirement updating

# Let's practice!

QUANTITATIVE RISK MANAGEMENT IN PYTHON

# Kernel density estimation

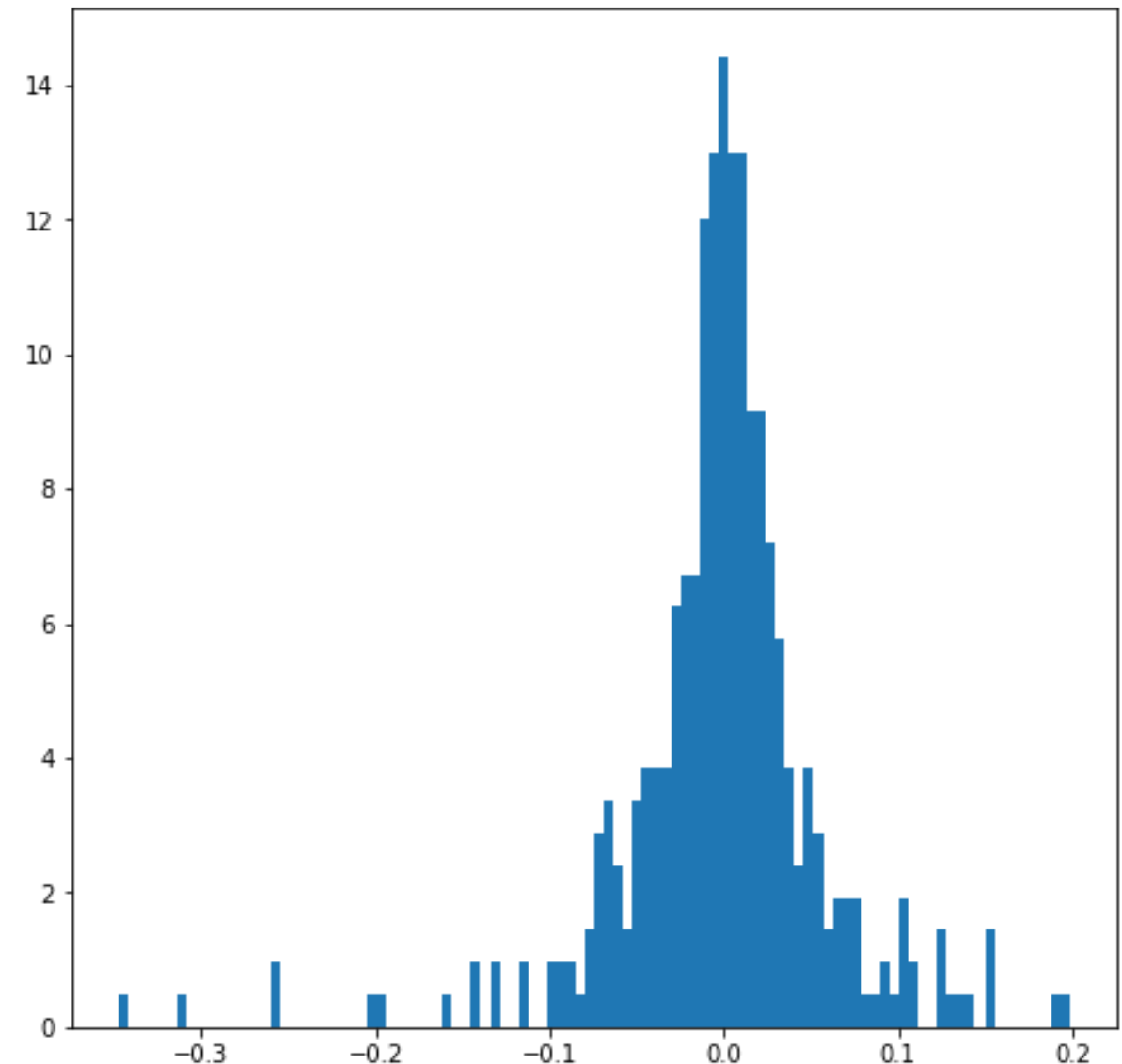
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# The histogram revisited

- **Risk factor distributions**
  - *Assumed* (e.g. Normal, T, etc.)
  - *Fitted* (parametric estimation, Monte Carlo simulation)
  - *Ignored* (historical simulation)
- **Actual data:** histogram
- How to represent histogram by probability distribution?
  - *Smooth* data using **filtering**
  - **Non-parametric estimation**



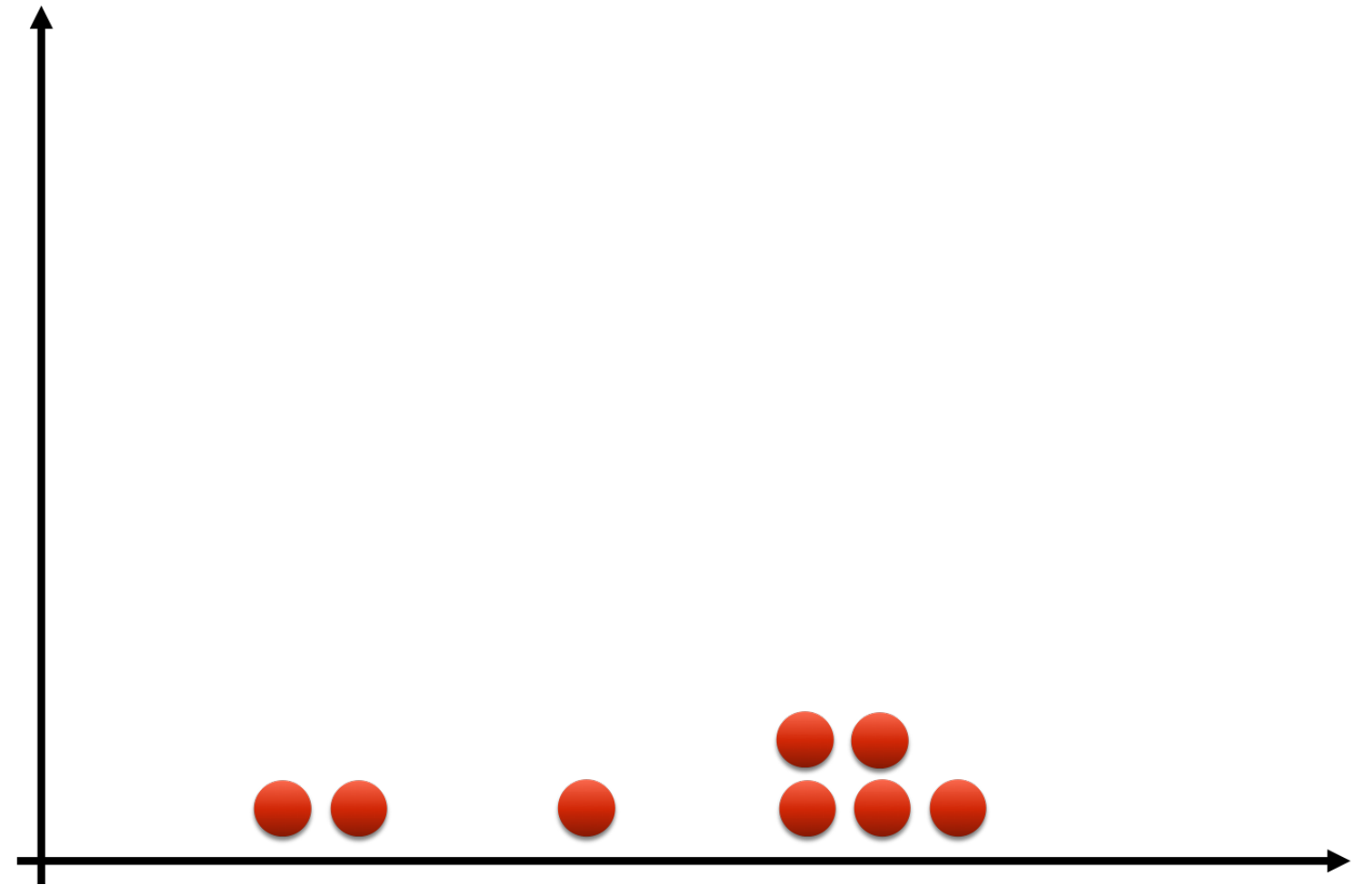
# Data smoothing

- **Filter:** smoothen out 'bumps' of histogram



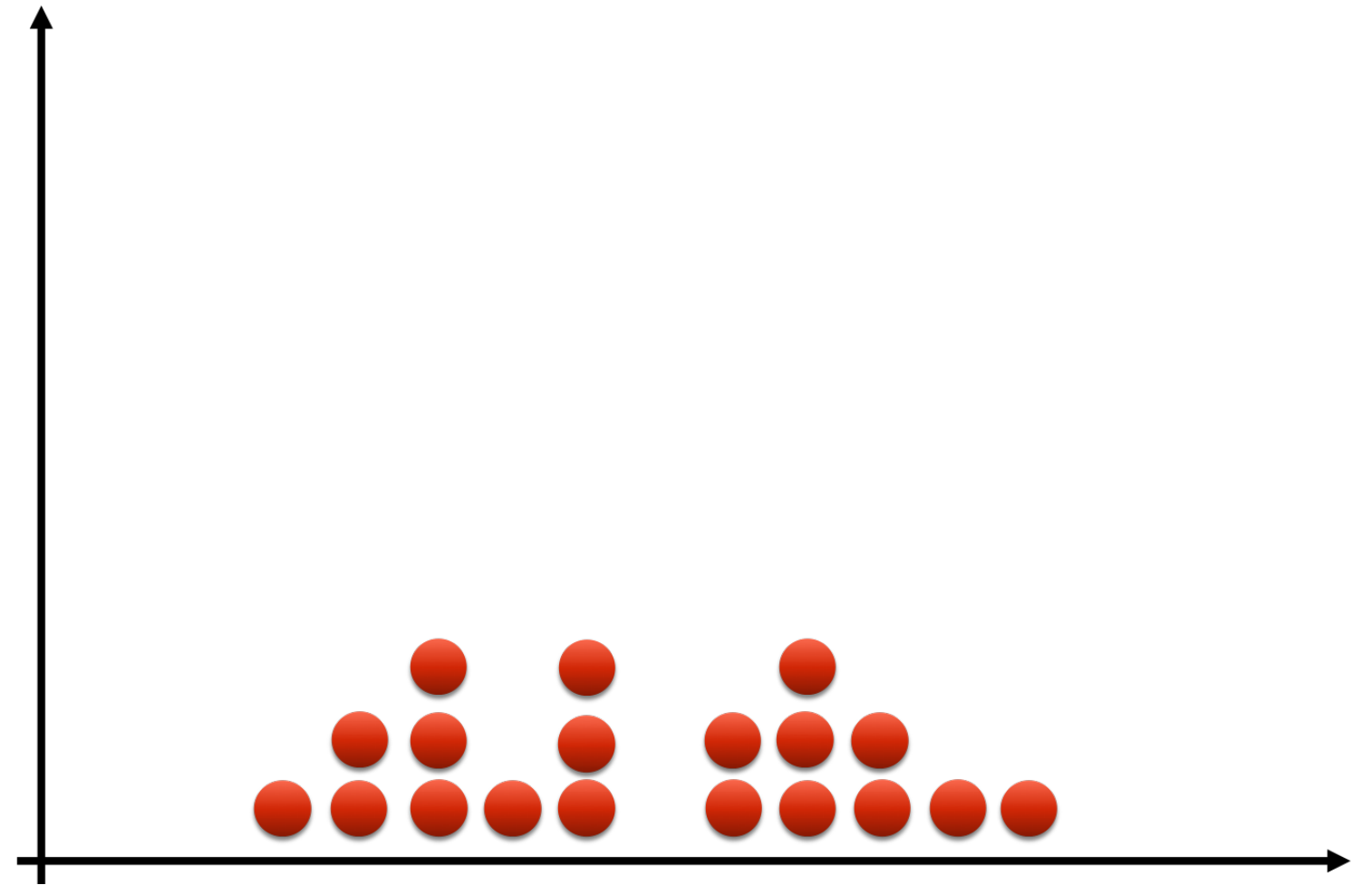
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- Filter: smoothen out 'bumps' of histogram
- Observations accumulate in over time



# Data smoothing

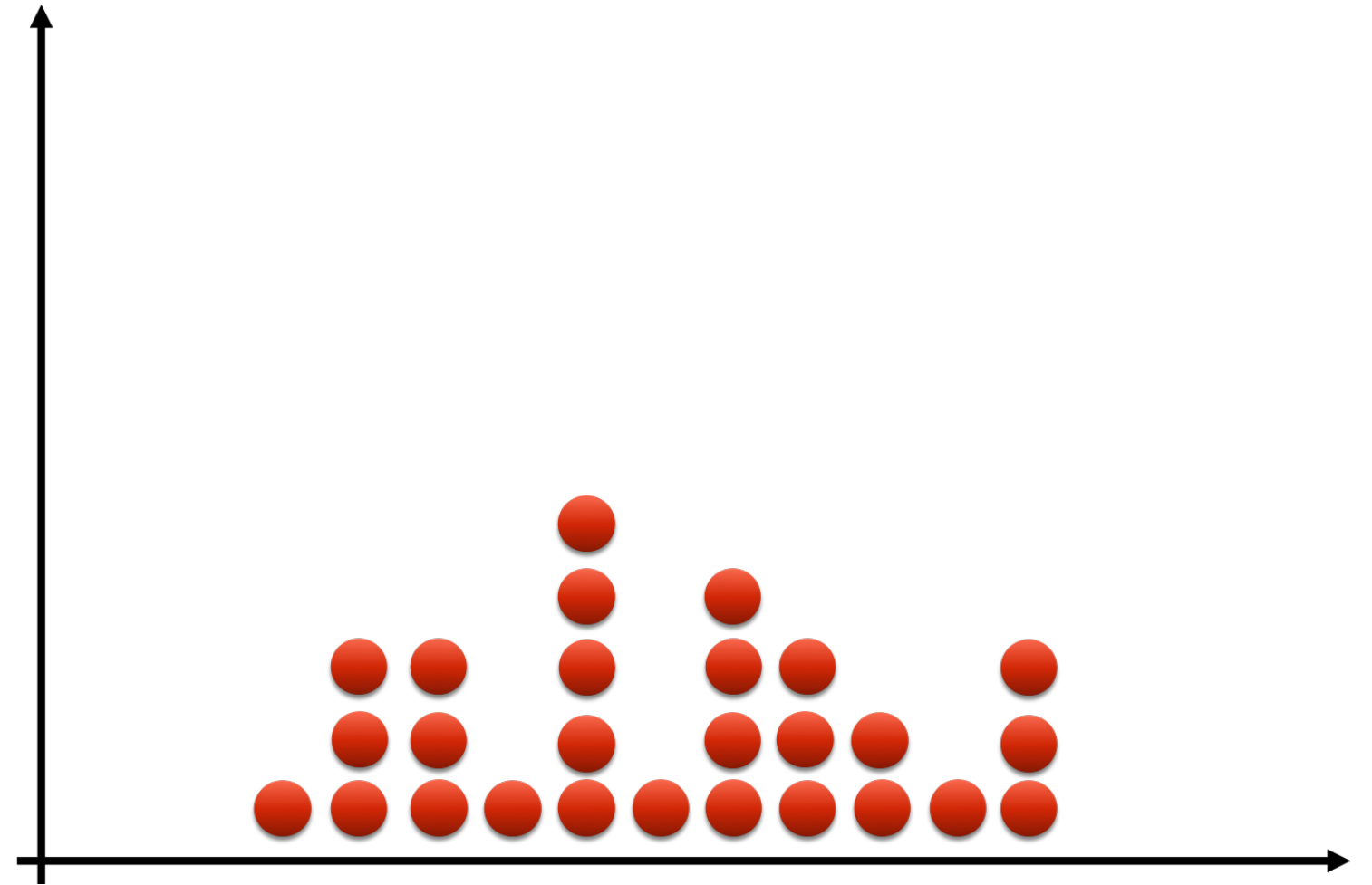
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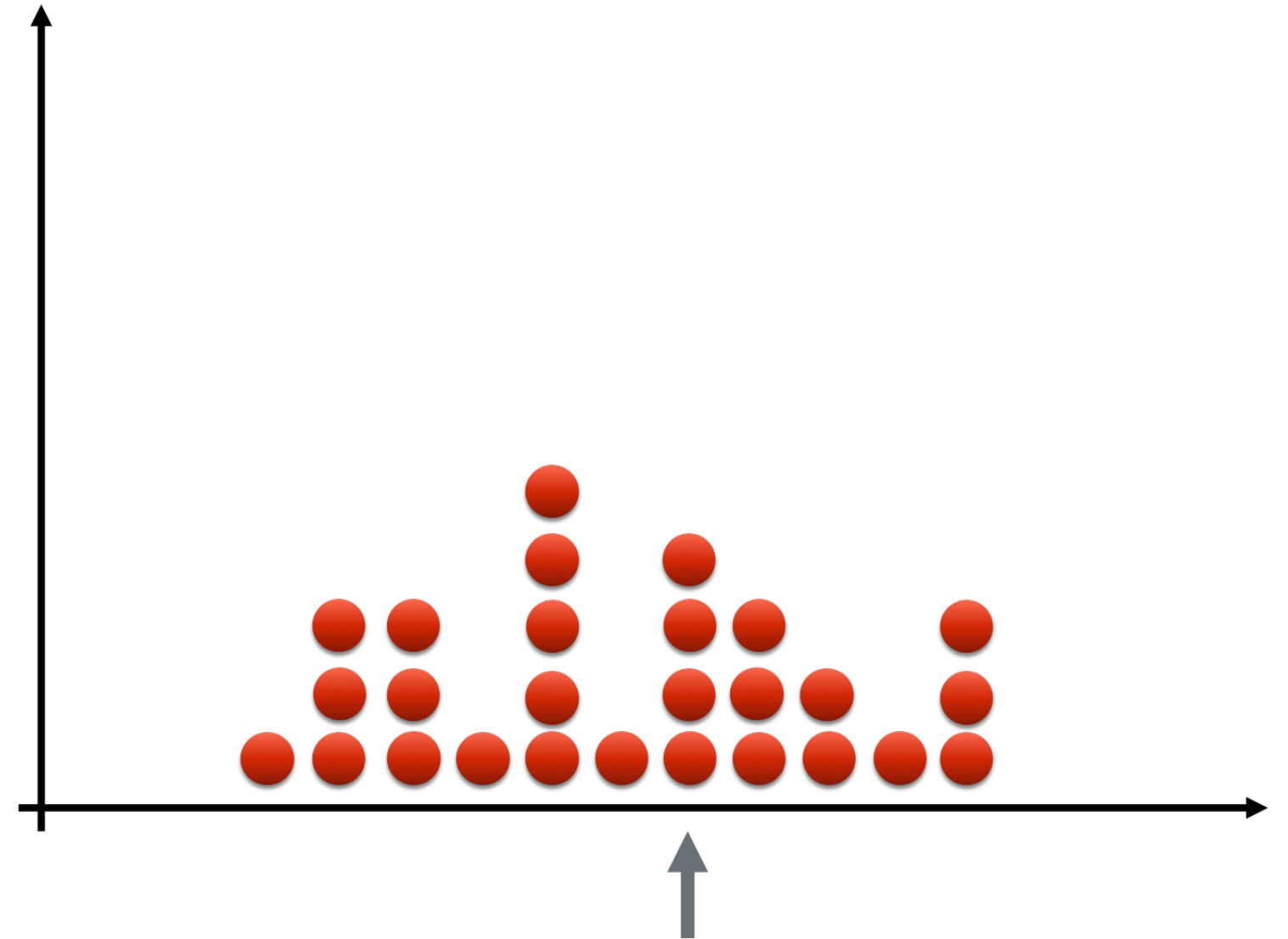
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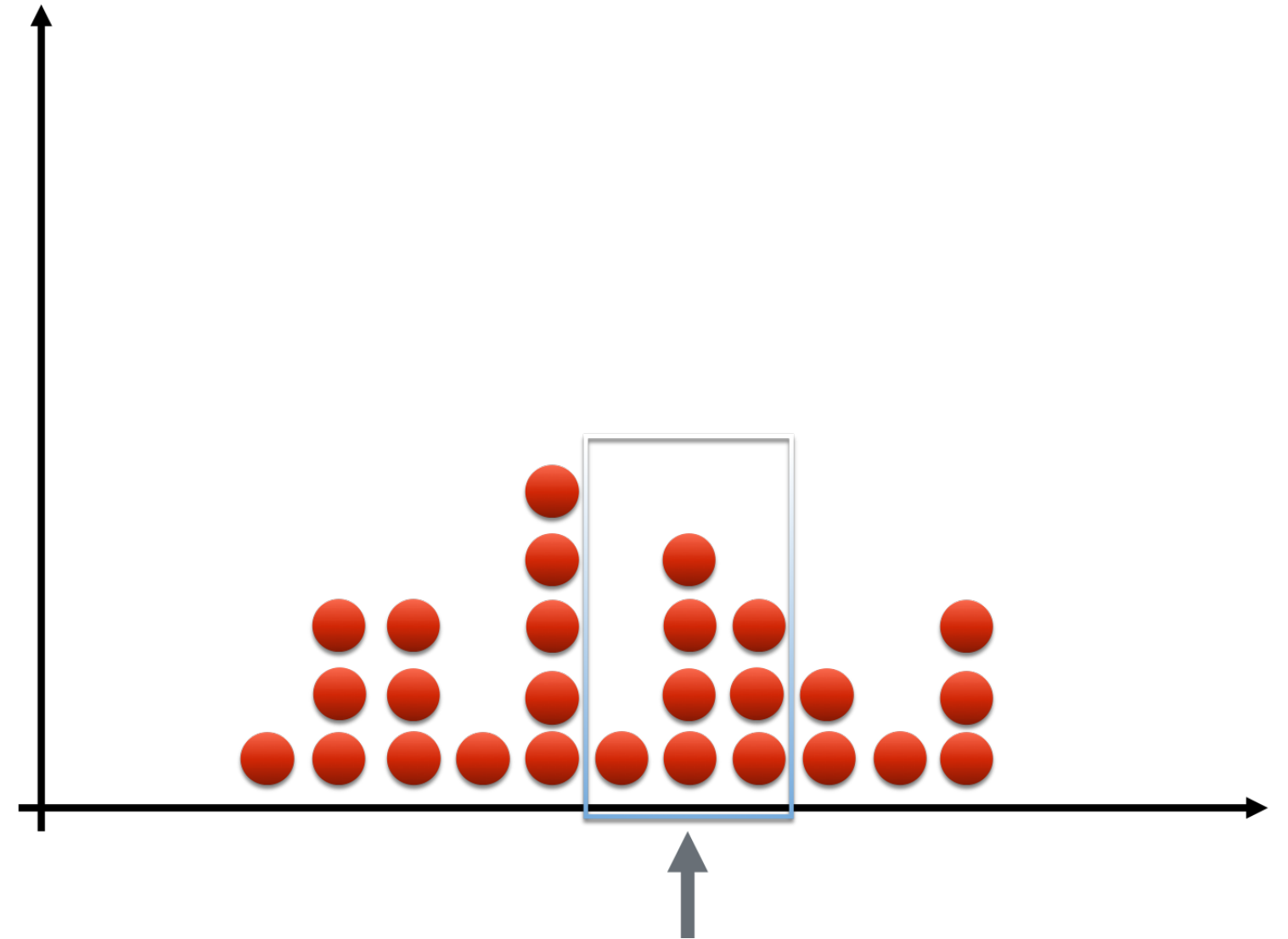
# Data smoothing

- Filter: smoothen out 'bumps' of histogram
- Observations accumulate in over time
- Pick particular portfolio loss



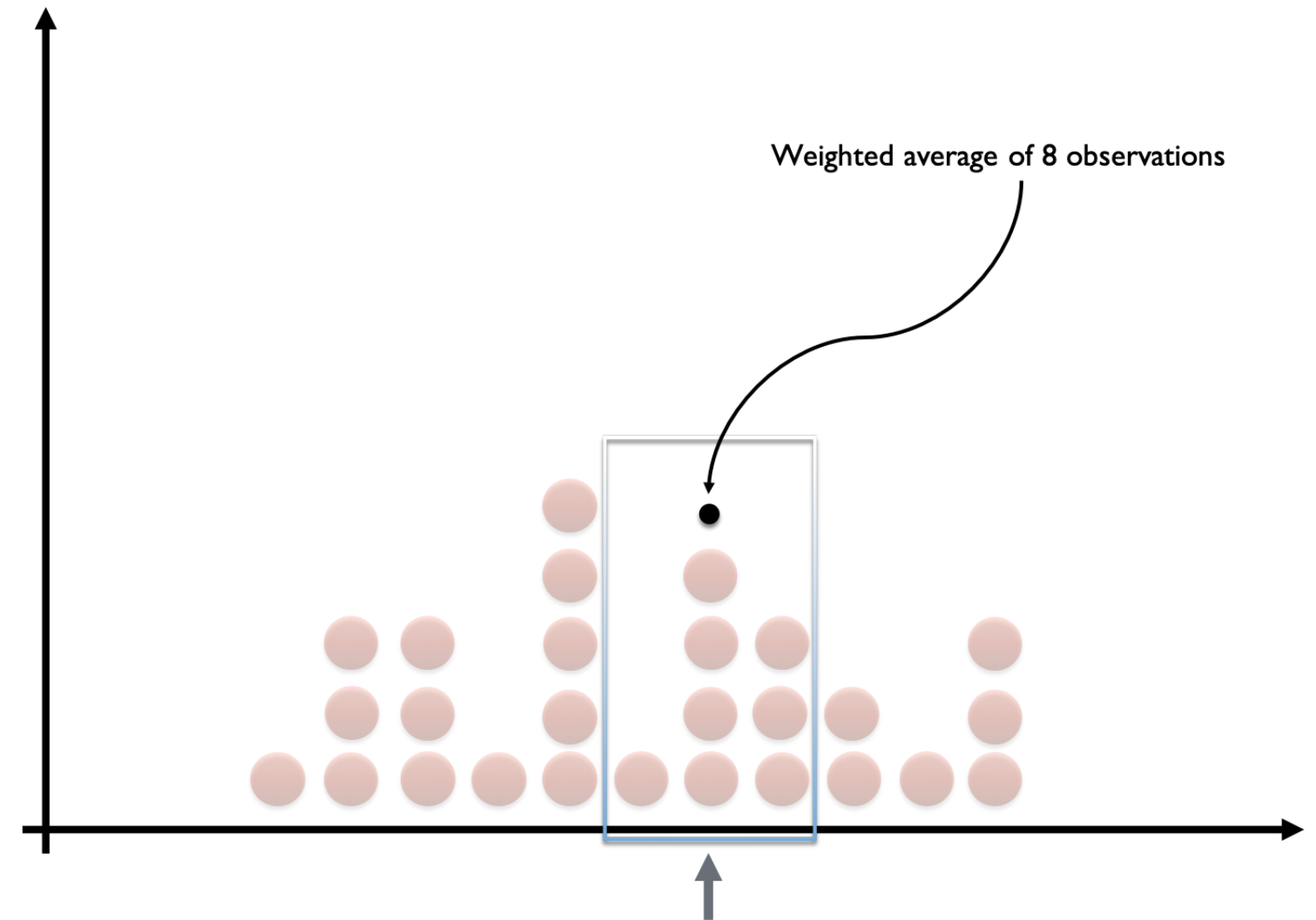
# Data smoothing

- Filter: smoothen out 'bumps' of histogram
- Observations accumulate in over time
- Pick particular portfolio loss
  - Examine nearby losses



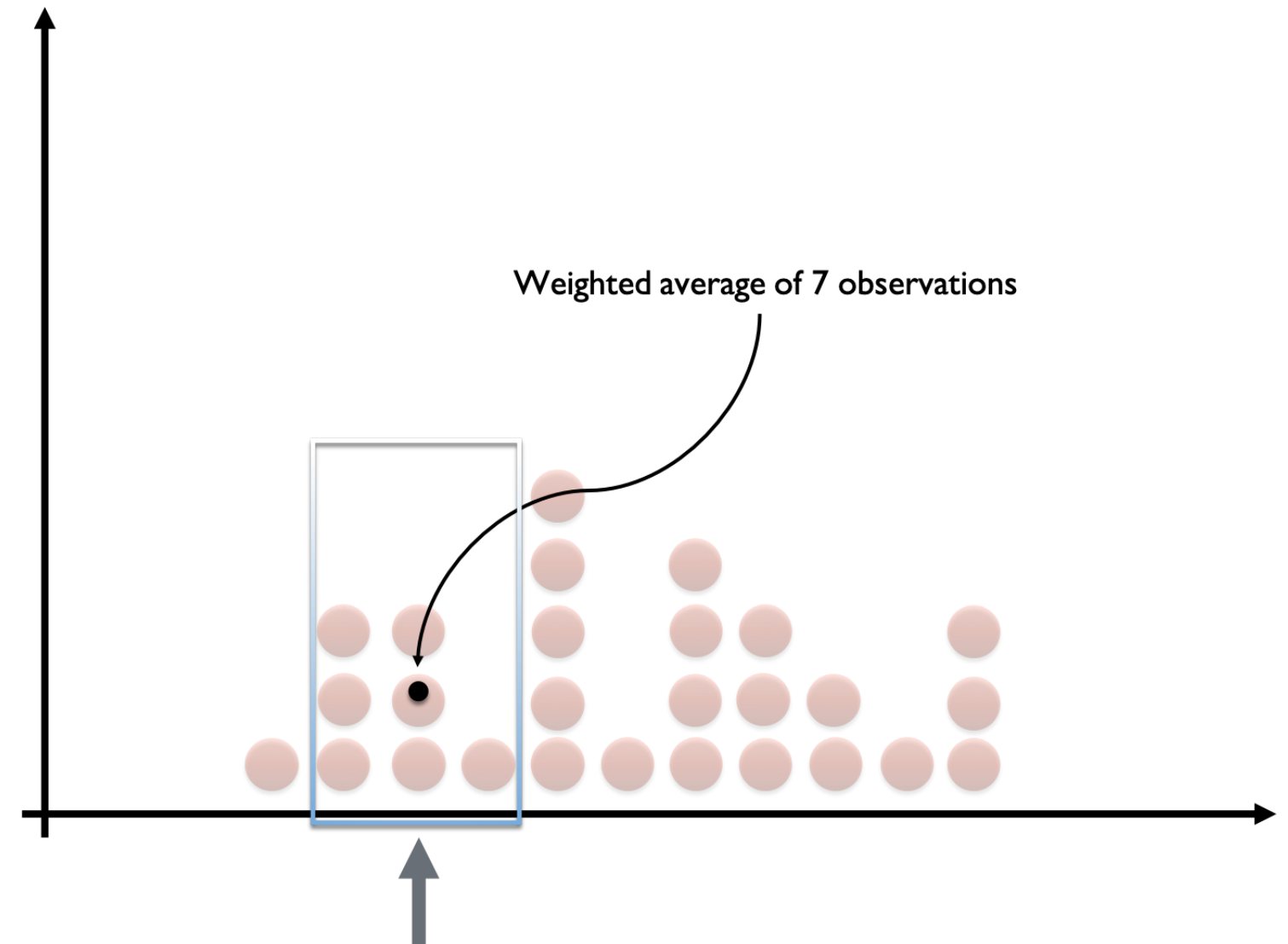
# Data smoothing

- Filter: smoothen out 'bumps' of histogram
- Observations accumulate in over time
- Pick particular portfolio loss
  - Examine nearby losses
  - Form "weighted average" of losses
- **Kernel:** filter choice; determines "window"



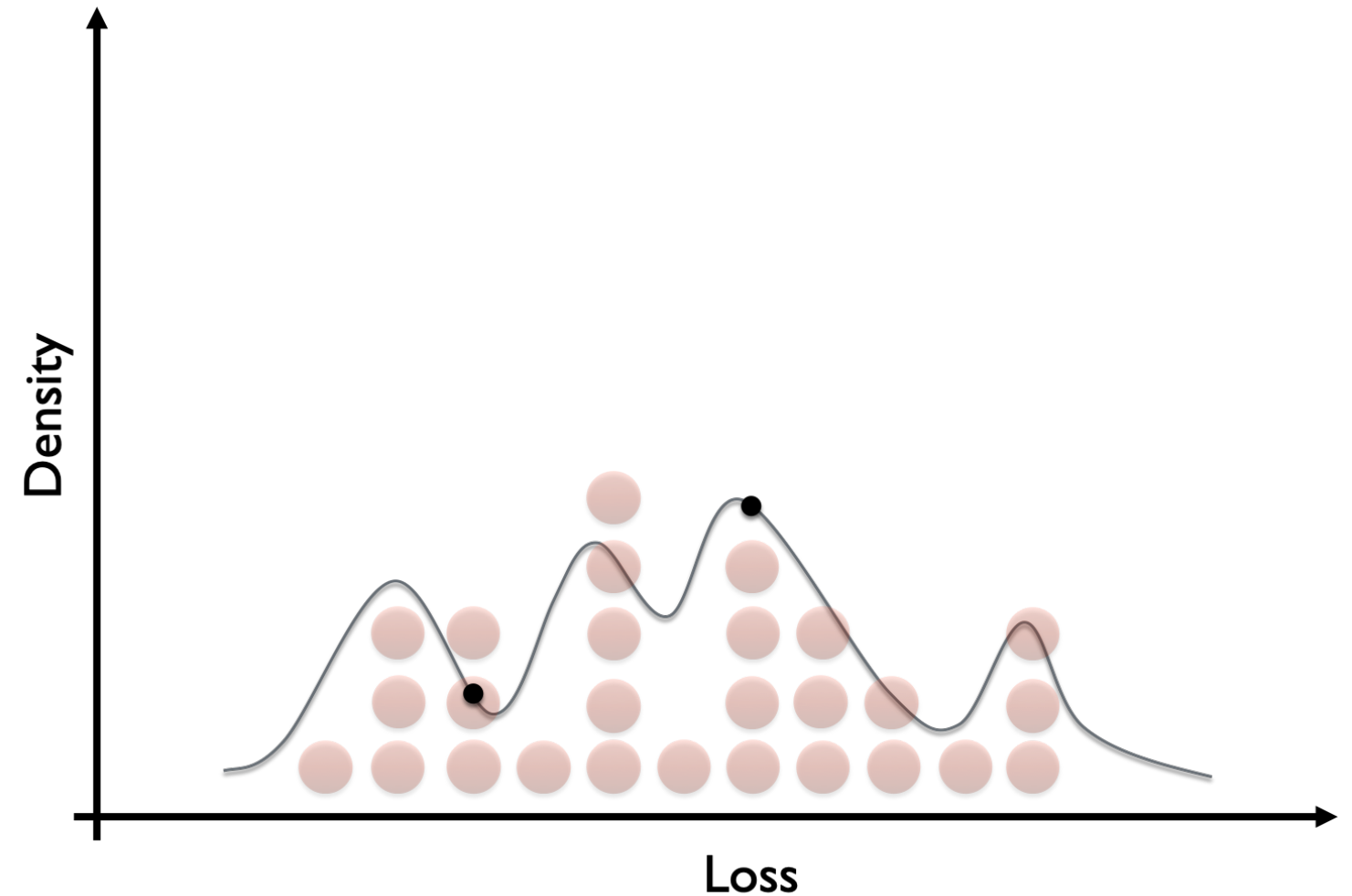
# Data smoothing

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  - Move window to another loss



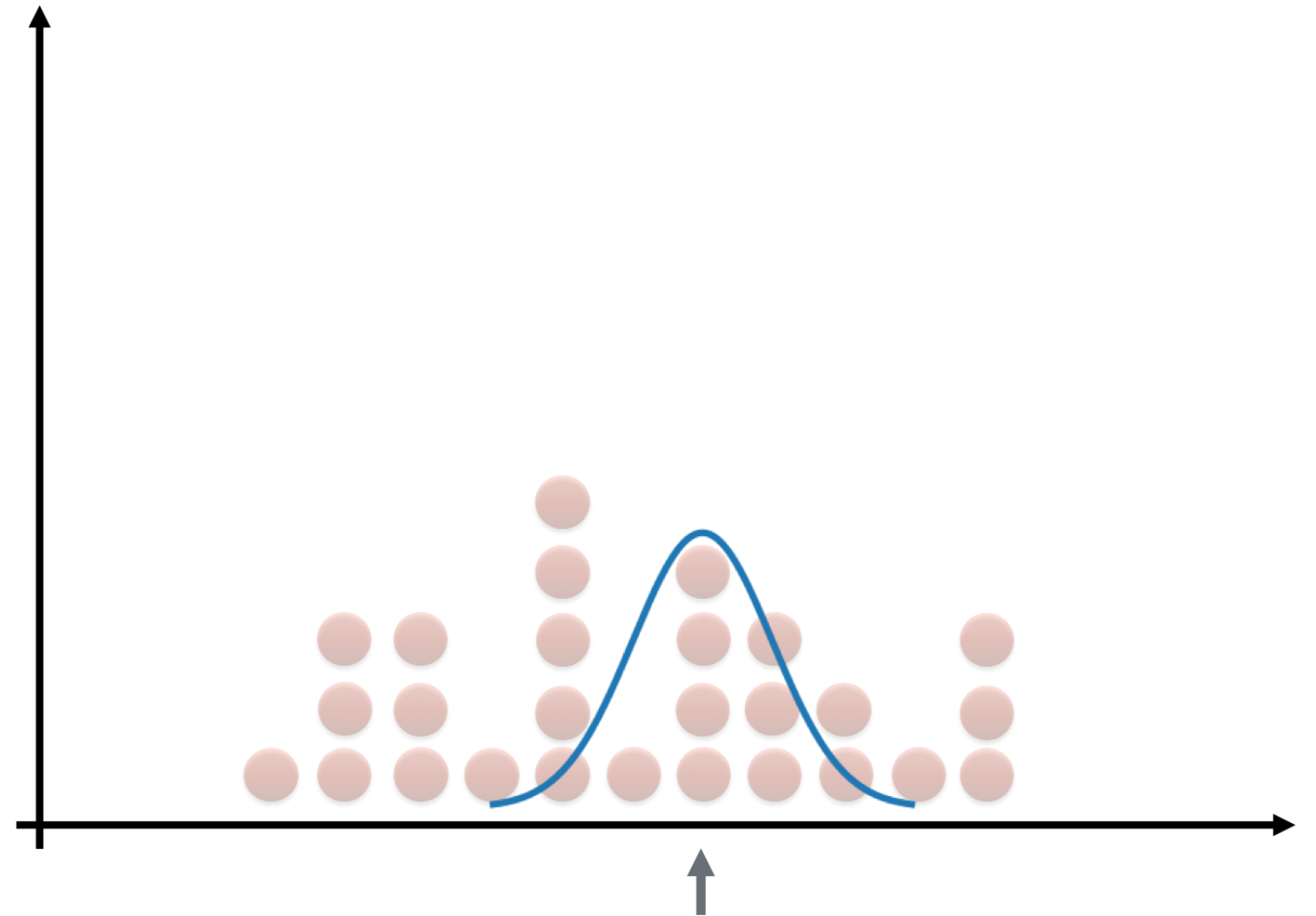
# Data smoothing

- Filter: smoothen out 'bumps' of histogram
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  - Move window to another loss
- **Kernel density estimate**: probability density



# The Gaussian kernel

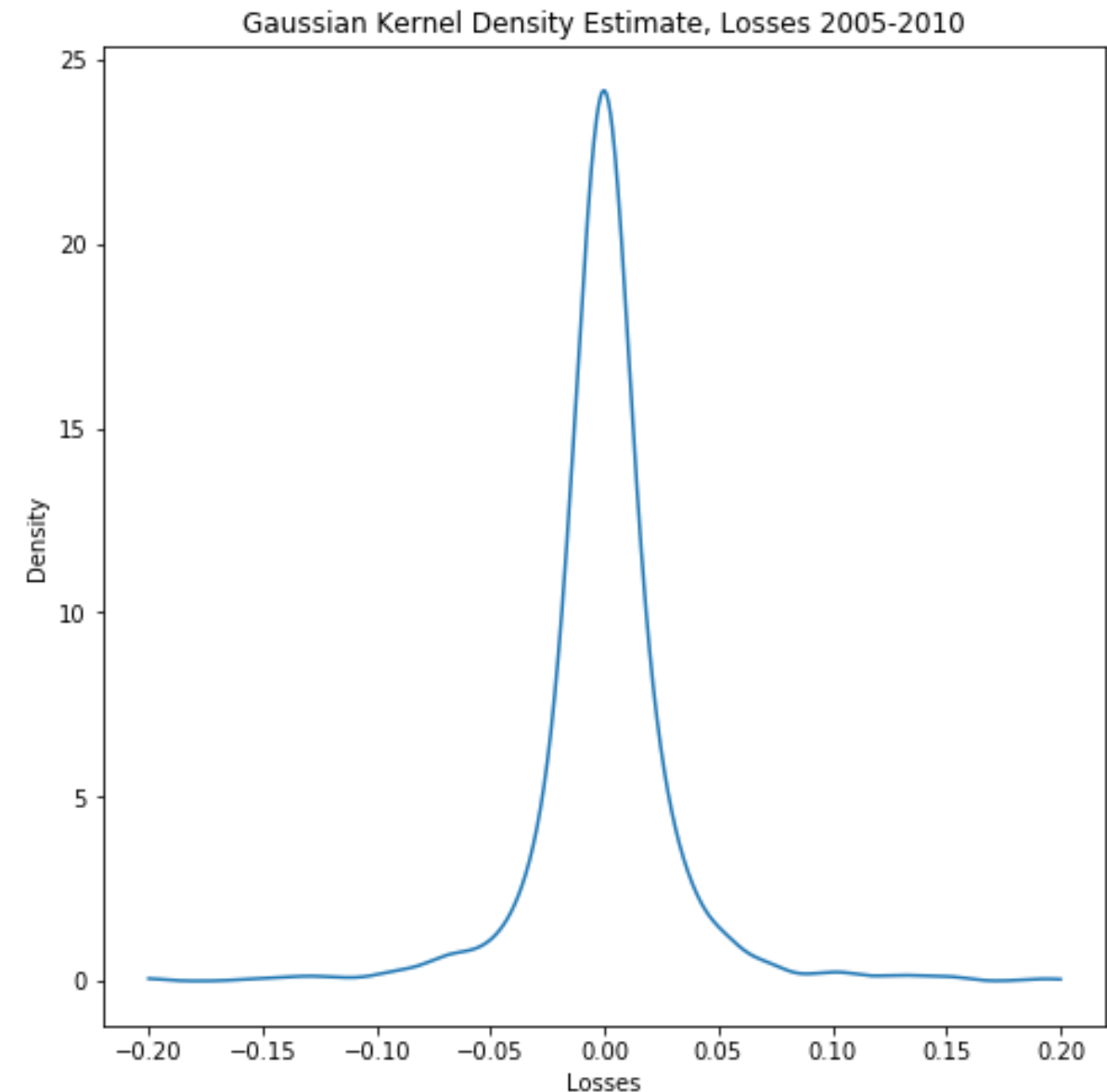
- Continuous kernel
- Weights all observations by distance from center
- **Generally:** many different kernels are available
  - Used in time series analysis
  - Used in signal processing



# KDE in Python

```
from scipy.stats import gaussian_kde
kde = gaussian_kde(losses)
loss_range = np.linspace(np.min(losses),
                          np.max(losses),
                          1000)
plt.plot(loss_range, kde.pdf(loss_range))
```

- Visualization: probability density function from KDE fit





# Finding VaR using KDE

- VaR: use `gaussian_kde` `.resample()` method
- Find quantile of resulting sample
- CVaR: expected value as previously encountered, but
  - `gaussian_kde` has no `.expect()` method => compute integral manually
  - special `.expect()` method written for exercise

```
sample = kde.resample(size = 1000)
VaR_99 = np.quantile(sample, 0.99)
print("VaR_99 from KDE: ", VaR_99)
```

```
VaR_99 from KDE: 0.08796423698448601
```

# Let's practice!

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# Neural network risk management

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# Real-time portfolio updating

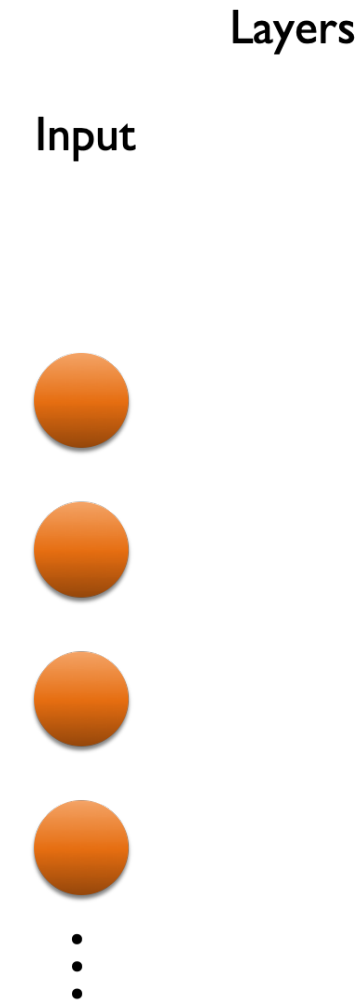
- Risk management
  - **Defined** risk measures (VaR, CVaR)
  - **Estimated** risk measures (parameteric, historical, Monte Carlo)
  - **Optimized** portfolio (e.g. Modern Portfolio Theory)
- *New* market information => *update* portfolio weights
  - **Problem:** portfolio optimization costly
  - **Solution:**  $\text{weights} = f(\text{prices})$
  - *Evaluate*  $f$  in real-time
  - *Update*  $f$  only occasionally

# Neural networks

- **Neural Network:**  $\text{output} = f(\text{input})$ 
  - **Neuron:** interconnected processing node in function
- Initially developed 1940s-1950s
- **Early 2000s:** application of neural networks to "big data"
  - Image recognition, processing
  - Financial data
  - Search engine data
- **Deep Learning:** neural networks as part of Machine Learning
  - **2015:** Google releases open-source Tensorflow deep learning library for Python

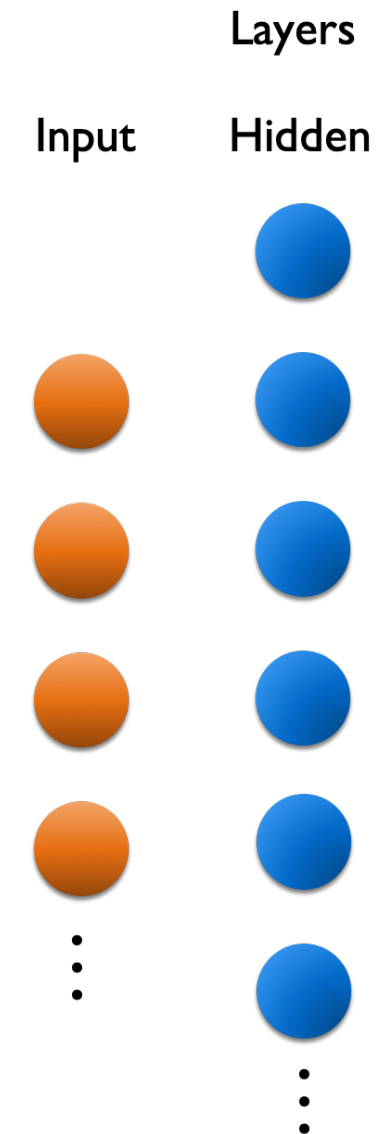
# Neural network structure

- **Layers: connected processing neurons**
  - Input layer



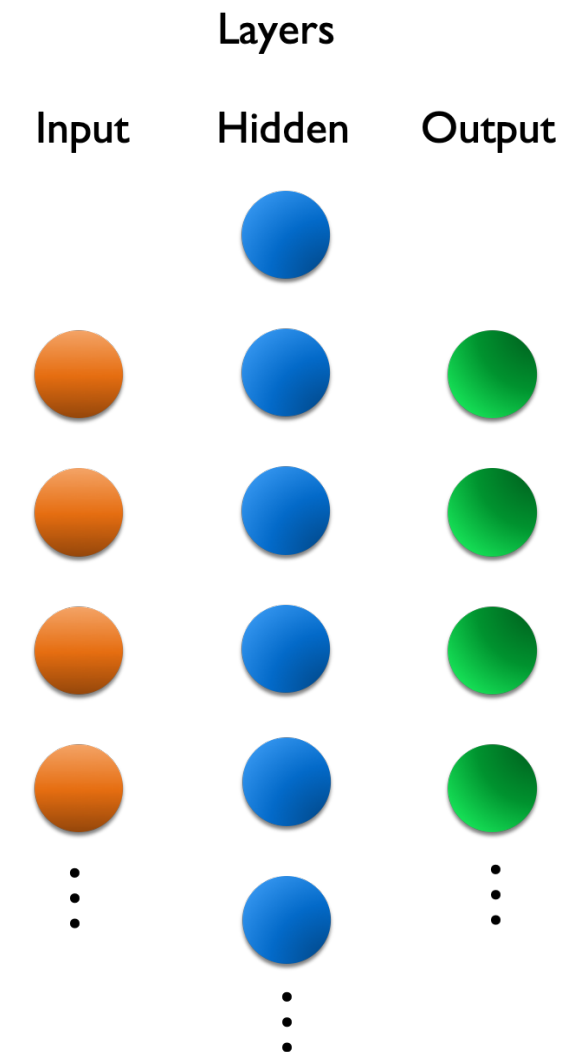
# Neural network structure

- **Neural network structure**
  - Input layer
  - Hidden layer



# Neural network structure

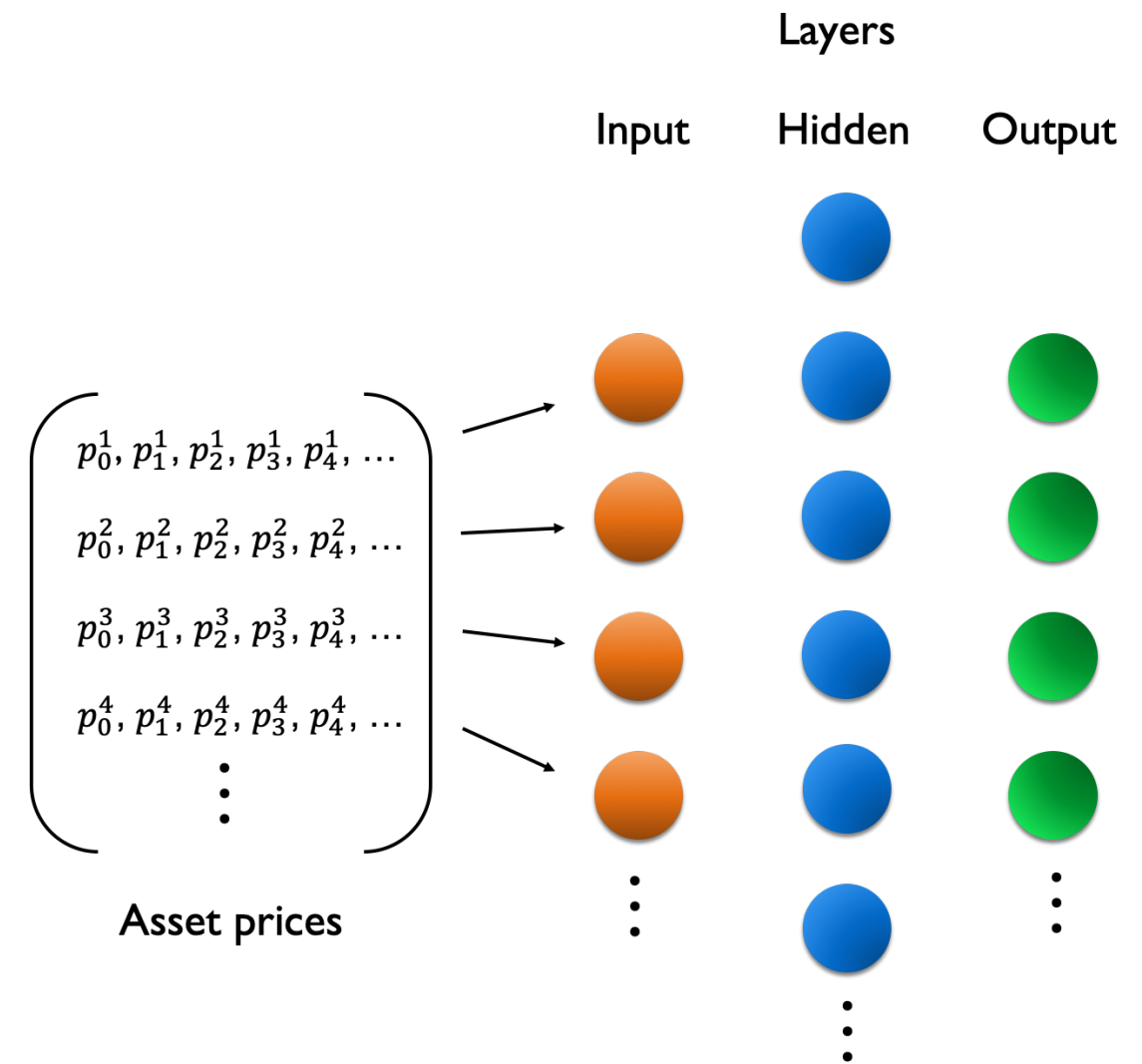
- **Neural network structure**
  - Input layer
  - Hidden layer
  - Output layer
- **Training:** learn relationship between input and output





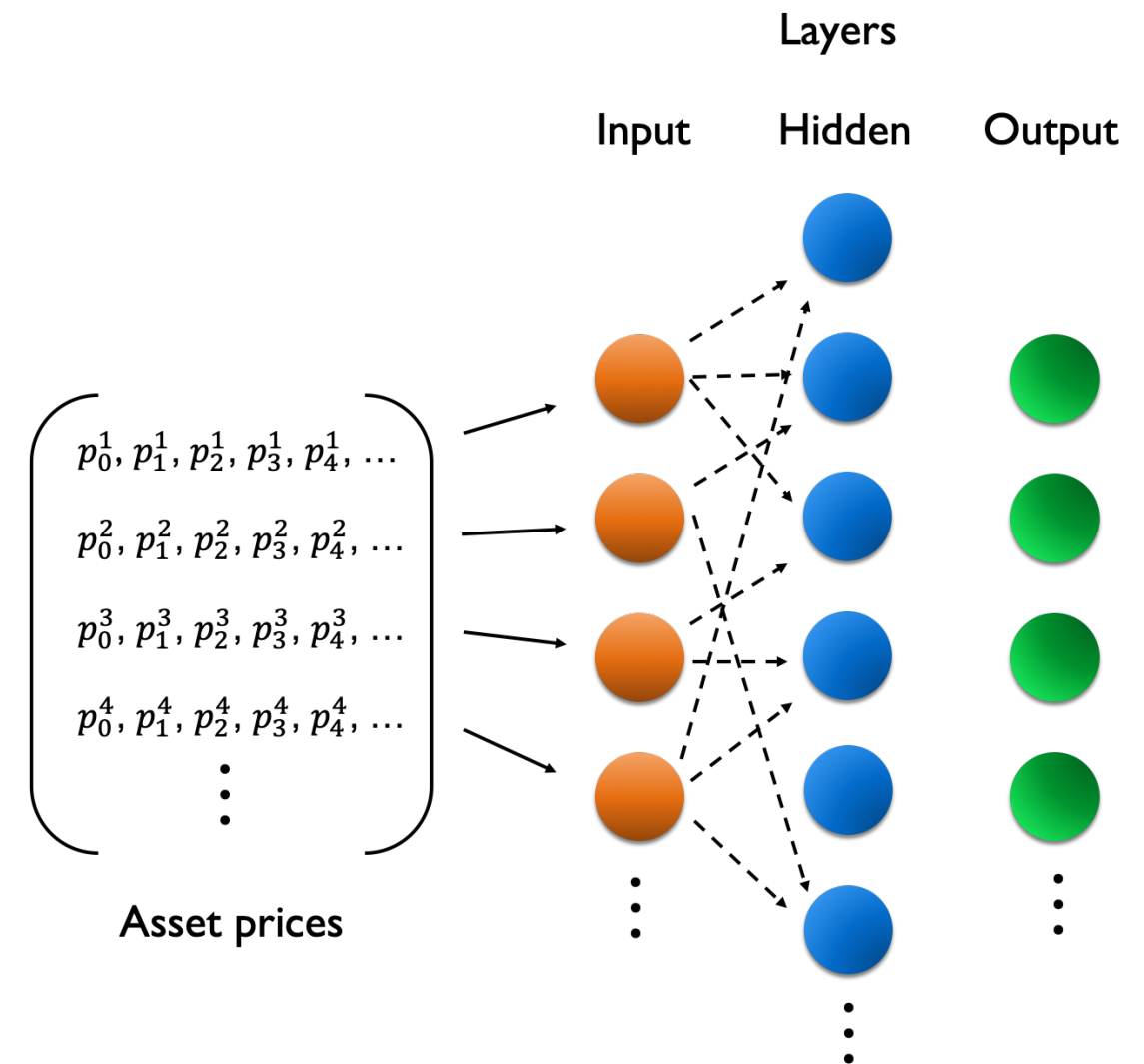
# Neural network structure

- **Neural network structure**
  - Input layer
  - Hidden layer
  - Output layer
- **Training:** learn relationship between input and output
  - Asset prices => Input layer



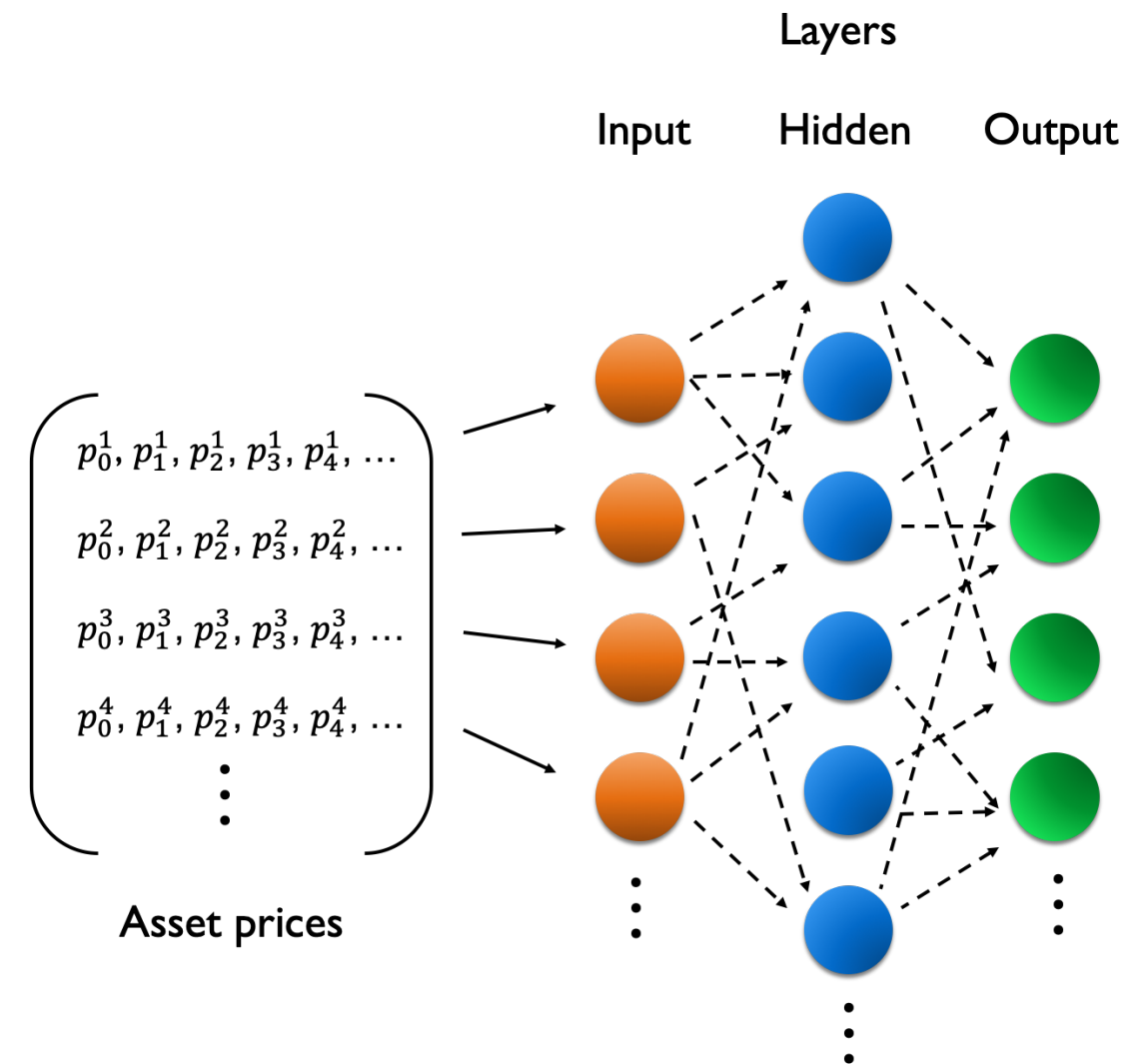
# Neural network structure

- **Neural network structure**
  - Input layer
  - Hidden layer
  - Output layer
- **Training:** learn relationship between input and output
  - Asset prices => Input layer
  - Input + hidden layer processing



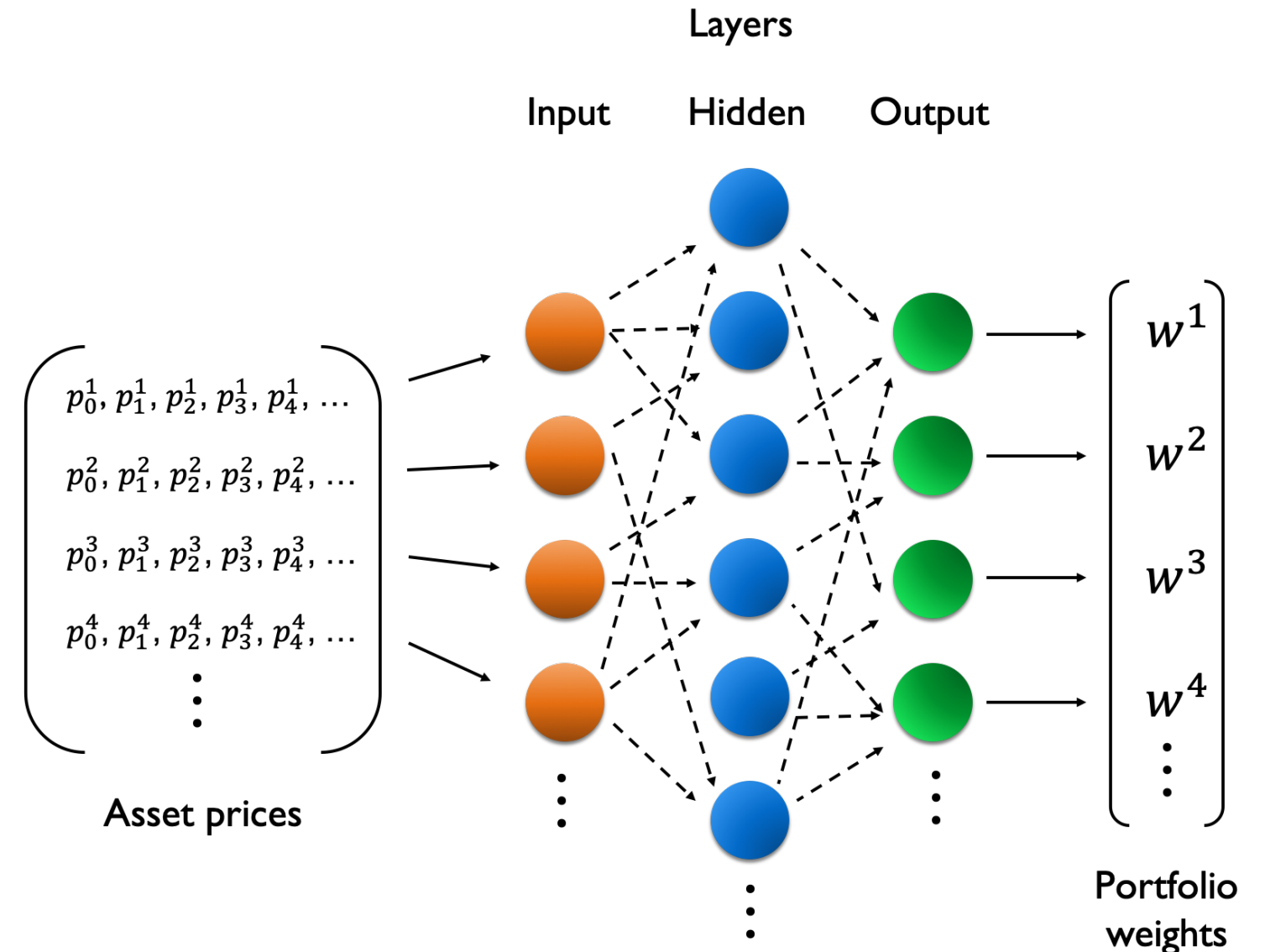
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# Neural network structure

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- **Training:** learn relationship between input and output
  - Asset prices => Input layer
  - Input + hidden layer processing
  - Hidden + output layer processing
  - Output => portfolio weights



# Using neural networks for portfolio optimization

- **Training**

- *Compare* output and pre-existing "best" portfolio weights
- **Goal:** minimize "error" between output and weights
- Small error => network is *trained*

- **Usage**

- **Input:** new, unseen asset prices
- **Output:** predicted "best" portfolio weights for new asset prices
- Best weights = risk management

# Creating neural networks in Python

- **Keras:** high-level Python library for neural networks/deep learning
- Further info: [Introduction to Deep Learning with Keras](#)

```
from keras.models import Sequential
from keras.layers import Dense
model = Sequential()
model.add(Dense(10, input_dim=4, activation='sigmoid'))
model.add(Dense(4))
```

# Training the network in Python

- Historical asset prices: `training_input` matrix
- Historical portfolio weights: `training_output` vector
- **Compile** model with:
  - given error minimization ('**loss**')
  - given optimization algorithm ('**optimizer**')
- **Fit** model to training data
  - **epochs**: number of training loops to update internal parameters

```
model.compile(loss='mean_squared_error', optimizer='rmsprop')  
model.fit(training_input, training_output, epochs=100)
```

# Risk management in Python

- **Usage:** provide new (e.g. real-time) asset pricing data
  - **New vector** `new_asset_prices` given to input layer
- Evaluate network using `model.predict()` on new prices
  - **Result:** `predicted` portfolio weights
- Accumulate enough data over time => **re-train** network
  - Test network on previous data => **backtesting**

```
# new asset prices are in the vector new_asset_prices
predicted = model.predict(new_asset_prices)
```



# Let's practice!

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# Wrap-up and Future Steps

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# Congratulations!

## Chapter I

Risk and  
Return Recap

Return Distribution

Risk Factors

Volatility & Covariance

Modern Portfolio Theory

Efficient Portfolio &  
Efficient Frontier

# Congratulations!

## Chapter 1

Risk and  
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## Chapter 2

Goal-oriented Risk  
Management

Loss Distribution

Value at Risk (VaR)

Conditional VaR

Risk Exposure

Portfolio Hedging

# Congratulations!

Chapter 1	Chapter 2	Chapter 3
Risk and Return Recap	Goal-oriented Risk Management	Estimating & Identifying Risk
Return Distribution	Loss Distribution	Parametric Estimation
Risk Factors	Value at Risk (VaR)	Historical Simulation
Volatility & Covariance	Conditional VaR	Monte Carlo Simulation
Modern Portfolio Theory	Risk Exposure	Structural Breaks
Efficient Portfolio & Efficient Frontier	Portfolio Hedging	Extreme Events

# Congratulations!

Chapter 1	Chapter 2	Chapter 3	Chapter 4
Risk and Return Recap	Goal-oriented Risk Management	Estimating & Identifying Risk	Advanced Risk Management
Return Distribution	Loss Distribution	Parametric Estimation	Extreme Value Theory
Risk Factors	Value at Risk (VaR)	Historical Simulation	Kernel Density Estimation
Volatility & Covariance	Conditional VaR	Monte Carlo Simulation	Neural Networks
Modern Portfolio Theory	Risk Exposure	Structural Breaks	Real-time risk management
Efficient Portfolio & Efficient Frontier	Portfolio Hedging	Extreme Events	

# Tools in your toolkit

Scipy	Statsmodels	PyPortfolioOpt	Keras
<code>scipy.stats</code>	<code>statsmodels.api</code>	<code>pypfopt</code>	<code>keras</code>
<code>norm()</code>	<code>OLS()</code>	<code>risk_models</code>	<code>models</code>
<code>skewnorm()</code>	<code>add_constant()</code>	<code>cla</code>	<code>layers</code>
<code>t()</code>	<code>.fit()</code>	<code>expected_returns</code>	<code>Sequential()</code>
<code>genextreme()</code>		<code>efficient_frontier</code>	<code>Dense()</code>
<code>gaussian_kde()</code>		<code>objective_functions</code>	<code>.add()</code>
<code>anderson()</code>		<code>EfficientFrontier()</code>	<code>.fit()</code>
<code>skewtest()</code>		<code>mean_historical_return()</code>	<code>.predict()</code>
<code>.pdf()</code>		<code>CovarianceShrinkage()</code>	
<code>.ppf()</code>		<code>.negative_cvar()</code>	
<code>.fit()</code>		<code>.CLA()</code>	
<code>.rvs()</code>		<code>.ledoit_wolf()</code>	

# Future steps and reference

- **Upcoming DataCamp courses**
  - Credit Risk Modeling in Python
  - Financial Forecasting in Python
  - Machine Learning for Finance in Python
  - GARCH Models for Finance in Python
- *Quantitative Risk Management: Concepts, Techniques and Tools*, McNeil, Frey & Embrechts, Princeton UP, 2015.



**Best of luck on your  
data science  
journey!**

**QUANTITATIVE RISK MANAGEMENT IN PYTHON**