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1) Sean $u = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $v = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ $w = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ & $c \in \mathbb{R}$

$$\langle cu+v, w \rangle = \left\langle c\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} \right\rangle$$

$$= \left\langle \begin{pmatrix} cx_1 + x_2 \\ cy_1 + y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} \right\rangle = (cx_1 + x_2)x_3 + 3(cy_1 + y_2)y_3$$

$$= cx_1x_3 + x_2x_3 + 3cy_1y_3 + 3y_2y_3$$

Por otro lado

$$c\langle u, w \rangle + \langle v, w \rangle = c\left\langle \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} \right\rangle + \left\langle \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} \right\rangle$$

$$= c(x_1x_3 + 3y_1y_3) + x_2x_3 + 3y_2y_3$$

$$= cx_1x_3 + x_2x_3 + 3cy_1y_3 + 3y_2y_3$$

$$\therefore \quad \langle cu+v, w \rangle = c\langle u, w \rangle + \langle v, w \rangle$$

Análogamente

$$\langle u, cv+w \rangle = \text{[illegible]}$$

$$= \left\langle \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, c\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} cx_2 + x_3 \\ cy_2 + y_3 \end{pmatrix} \right\rangle$$

$$= x_1(cx_2 + x_3) + 3y_1(cy_2 + y_3)$$

$$= cx_1x_2 + x_1x_3 + 3cy_1y_2 + 3y_1y_3$$

Por otro lado

$$c\langle u, v \rangle + \langle u, w \rangle = c\left\langle \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \right\rangle + \left\langle \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} \right\rangle$$

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$= c(x_1x_2 + 3y_1y_2) + x_1x_3 + 3y_1y_3$

$= cx_1x_2 + x_1x_3 + 3cy_1y_2 + 3y_1y_3$

2) $\|u\| = \langle u, u\rangle^{1/2} = \langle \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \rangle^{1/2}$

$= \left(x_1^2 + 3y_1^2\right)^{1/2}$

$\Rightarrow \|x\| = \left(4 + 3(-3)^2\right)^{1/2} = (4 + 27)^{1/2} = \sqrt{31}$