# **HARMONIC SERIES**

Primary Interval: 12 half-steps [P8]

Har.#	Harmonics fund.=C2	F2/f1 (f1=fund.)	Log f2/f1	Log f2/f1 x 3986	Cents from f1	Equal tmpr. dif. in cents	Harmonic # (intervals)	Intervals in cents	Intervals + dif. in cents
1	C2	/	/	/	Q	0	/	/	/
2	C1	1	0	0	Q	0	1-2	1200	P8
3	G1	3/2=1.5	0.176091	701.8998	<u>702</u>	2+	2-3	702	P5 (2c+)
4	С	1	0	0	Q	0	3-4	<u>498</u>	P4 (2c-)
5	E	5/4=1.25	0.09691	386.2833	<u>386</u>	14-	4-5	<u>386</u>	M3 (14c-)
6	G	3/2=1.5	0.176091	701.8998	702	2+	5-6	<u>316</u>	m3 (16c+)
7	Bb	7/4=1.75	0.243038	968.7497	<u>969</u>	31-	6-7	<u> 267</u>	m3 (33c-)
8	С	1	0	0	Q	0	7-8	<u>231</u>	M2 (31c+)
9	d	9/8=1.125	0.051153	203.894	<u>204</u>	4+	8-9	204	M2 (4c+)
10	е	10/8=1.25	0.09691	386.2833	<u>386</u>	14-	9-10	182	M2 (18c-)
11	f#	11/8=1.375	0.138303	551.2746	<u>551</u>	49-!	10-11	<u>165</u>	M2 (35c-)
12	g	12/8=1.5	0.176091	701.8998	702	2+	11-12	151	m2 (51c+)!!
13	ab	13/8=1.625	0.210853	840.4615	<u>840</u>	40+!	12-13	138	m2 (38c+)
14	bb	14/8=1.75	0.243038	968.7497	<u>969</u>	31-	13-14	129	M2 (71c-) !!!
15	b	15/8=1.875	0.273001	1088.183	1088	12-	14-15	119	m2 (19c+)
16	c1	1	0	0	Q	0	15-16	112	m2 (12c+)

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [P8] recurrences (numbers indicate the position of the note in the series):
    - c(1), d(9), e(5), f#-(11), g(3), g#+(13), a#-(7), b(15); 8 notes.
      - a) Number of recurrences: c(5x), d, e(2x), f#-, g(3x), g#+, a#-(2x), b.
      - b) Excluding notes deviating more than  $25c^*$ : c, d, e, g, b; 5 notes.
  - B) Excluding octave repetitions (same as 1A above): c, d, e, f#-, g, g#+, a#-, b; 8 notes.
    - a) Number of repetitions: c(5x), d, e(2x), f#-, g(3x), g#+, a#-(2x), b.
    - b) Excluding notes deviating more than 25c\*: c, d, e, g, b; 5 notes.
    - c) Excluding primary interval [m9] recurrences: c, d, e, f#-, g, g#+, a#-, b; 8 notes.
- 2) Chord derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [P8] recurrences: c, e, g; 3 notes.
  - B) Excluding octave repetitions (same as 2A above): c, e, g; 3 notes.
- 3) Tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{C1(I)}$ ,  $\underline{G1(II)}$ ,  $\underline{G1(II)}$ ,  $\underline{C(I)}$ ,  $\underline{E(II)}$ ,  $\underline{G(II)}$ ,  $\underline{c(I)}$ ,  $\underline{e(II)}$ ,  $\underline{g(III)}$ ,  $\underline{c(I)}$ .
  - B) D: 3, 6, 9, 12, 15; <u>G1(I)</u>, <u>G(I)</u>, d(III), <u>g(I)</u>, *b(II)*.
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 120, 70, 50, 39, 32, 27, 23, 20, 18, 16.5, 15, 14, 13, 12, 11.
  - B) Intervals in cents/40: 30, 17.5, 12.5, 9.75, 8, 6.75, 5.75, 5, 4.5, 4, 3.75, 3.5, 3.25, 3, 2.75.

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Augmentation: x13/12

Primary Interval: 13 half-steps [m9]

Serial # (intervals)	Int. in cents x13/12	Half-steps + dif. in cents	<u>intervals</u>	Example fund.=C2	Ser.#	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	<u>1300</u>	13	m <u>9</u>	C2-Db1	2	Q	<u>C#1</u>
2-3	<u>760.5</u>	8 (39.5c-)	m6-	Db1-A1*	3	<u>39.5-</u>	A1.
3-4	<u>539.5</u>	5 (39.5c+)	P4+	A1-D*	4	Q	D
4-5	<u>418.166667</u>	4 (18.16c+)	M3+	D-F#*	5	18.16+	F#
5-6	342.333333	3 (42.33c+)	m3+	F#-A*	6	60.5+/39.5-	A#
6-7	289.25	3 (10.75c-)	m3-	A-c*	7	49.75+	c/c#
7-8	<u>250.25</u>	3 (49.75c-)	m3-	c-eb*	8	Q	d#
8-9	221	2 (21c+)	M2+	eb-f*	9	<u>21+</u>	f
9-10	<u>197.166667</u>	2 (2.84c-)	M2-	f-g*	10	18.16+	g
10-11	<u>178.75</u>	2 (21.25c-)	M2-	g-a*	11	3.09-	a
11-12	163.583333	2 (36.42c-)	M2-	a-b*	12	<u>39.5-</u>	Þ
12-13	149.5	1 (49.5c+)	m2+	b-c1*	13	10+	<u>c1</u>
13-14	139.75	1 (39.75c+)	m2+	c1-db1*	14	49.75+	c#1/d1
14-15	128.916667	1 (28.9c+)	m2+	c#1-d#1*	15	78.7+/21.3-	<u>d#1</u>
15-16	121.333333	1 (21.33c+)	m2+	d#-e1*	16	Q	<u>e1</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [m9] recurrences (numbers indicate the position of the note in the series):
    - c(1), c(13), c/c#(7), d#(15), f(9), f#(5), a-(3), a(11); 8 notes.
      - a) Number of recurrences: c(5x), c, c/c#(2x), d#, f, f#(2x), a, a-(3x).
      - b) Excluding notes deviating more than  $25c^*$ : c, c, d#, f, f#, a; 6 notes.
  - B) Excluding octave repetitions: c, c/c#, c#, c#/d, d, d#, e, f, f#, g, a-, a, a#, b; 14 notes.
    - a) Number of repetitions: c(2x), c/c#, c#, c#/d, d, d#(2x), e, f, f#, g, a-, a, a#, b.
    - b) Excluding notes deviating more than 25c\*: c, c#, d, d#, e, f, f#, g, a, a#, b; 11 notes.
    - c) Excluding primary interval [m9] recurrences: c, c/c#, d#, f, f#, a-, a; 7 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [m9] recurrences: c, f#, a; 3 notes.
  - B) Excluding octave repetitions: c, c#, d, f#, a-, a#; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16; C2(I), C#1(I), A1-(III), D(I), F#(II), A#(III), d#(I), g(II), b(III), e1(I).
  - B) D: 3, 6, 9, 12, 15; <u>A1-(I)</u>, <u>A#(I)</u>, f(III), <u>b(I)</u>, d#(II).
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 130, 76, 54, 42, 34, 29, 25, 22, 20, 18, 16, 15, 14, 13, 12.
  - $B)\ Intervals\ in\ cents/40:\ 30.5,\ 19,\ 13.5,\ 10.5,\ 8.5,\ 7.25,\ 6.25,\ 5.5,\ 5,\ 4.5,\ 4,\ 3.75,\ 3.5,\ 3.25,\ 3.$

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Augmentation: x7/6

Primary Interval: 14 half-steps [M9]

Serial # (intervals)	Int. in cents x7/6	Half-steps + dif. in cents	<u>Intervals</u>	Example fund.=C2	Ser.#	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	1400	14	<u>M9</u>	C2-D1	2	<u>O</u>	<u>D1</u>
2-3	<u>819</u>	8 (19c+)	m6+	D1-Bb1*	3	<u>19+</u>	A#1
3-4	<u>581</u>	6 (19c-)	dim5-	A#1-E*	4	Q	E
4-5	450.333333	5 (49.67c-)	P4-	E-A*	5	49.7-/50.3+	<u>G#/A</u>
5-6	368.666667	4 (31.34c-)	М3-	A-c#*	6	<u>19+/81-</u>	<u>c</u>
6-7	311.5	3 (11.5c+)	m3+	c#-e*	7	<u>30.49+</u>	d#
7-8	<u>269.5</u>	3 (30.5c-)	m3-	e-g*	8	<u>O</u>	f#
8-9	238	2 (38c+)	M2+	g-a*	9	<u>38+</u>	g#
9-10	212.333333	2 (12.33c+)	M2+	a-b*	10	50.33+	<u>a#/b</u>
10-11	192.5	2 (7.5c-)	M2-	b-c1#*	11	<u>42.83+</u>	C
11-12	176.166667	2 (23.84c-)	M2-	c#1-d#1*	12	18.99+	<u>d1</u>
12-13	<u>161</u>	2 (39c-)	M2-	eb1-f1*	13	20.01-	<u>e1</u>
13-14	<u>150.5</u>	2 (49.5c-)	M2-	f1-g1*	14	30.49+/69.51-	f1
14-15	138.833333	1 (38.83c+)	m2+	g1-ab1*	15	69.3+/30.7-	g <u>1</u>
15-16	130.666667	1 (30.66c+)	m2+	g#1-a1*	16	Q	g#1

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [M9] recurrences (numbers indicate the position of the note in the series):
    - c(1), c+(11), d#+(7), e(13), g-(15), g#+(9), g#/a(5), a#(3); 8 notes.
      - a) Number of recurrences: c(5x), c+, d#+(2x), e, g-, g#+, g#/a(2x), a#(3x).
      - b) Excluding notes deviating more than 25c\*: c, e, a#; 3 notes.
  - B) Excluding octave repetitions: c, c+, d, d#+, e, f+, f#, g-, g#, g#+, g#/a, a#, a#/b; 13 notes.
    - a) Number of repetitions: c(2x), c+, d(2x), d#+, e(2x), f+, f#, g-, g#, g#+, g#/a, a#/b.
    - b) Excluding notes deviating more than 25c\*: c, d, e, f#, g#, a#; 6 notes.
    - c) Excluding primary interval [M9] recurrences: c, c/c#, d#, f, f#, a-, a; 7 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [M9] recurrences: c, g#/a, a#; 3 notes.
  - B) Excluding octave repetitions: c, d,e, g#/a, a#; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{D1(I)}$ ,  $\underline{A#1-(III)}$ ,  $\underline{E(I)}$ ,  $\underline{G\#/A(II)}$ ,  $\underline{c(III)}$ ,  $\underline{f\#(I)}$ ,  $\underline{a\#/b(II)}$ ,  $\underline{d1(III)}$ ,  $\underline{g\#1(I)}$ .
  - B) D: 3, 6, 9, 12, 15; <u>A#1(I)</u>, <u>c(I)</u>, g#+(III), <u>d1(I)</u>, *g1-(II)*.
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 140, 82, 59, 45, 37, 31, 27, 24, 21, 19, 18, 16, 15, 14, 13.
  - $B)\ Intervals\ in\ cents/40:\ 35,\ 20.5,\ 14.75,\ 11.25,\ 9.25,\ 7.75,\ 6.75,\ 6,\ 5.25,\ 4.75,\ 4.5,\ 4,\ 3.75,\ 3.5,\ 3.25.$

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Augmentation: x5/4

Primary Interval: 15 half-steps [m10]

Serial # (intervals)	Int. in cents	Half-steps + dif. in cents	<u>Intervals</u>	Example fund.=C2	Ser.#	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	1500	<u>15</u>	m10	C2-Eb1	2	Q	D#1
2-3	<u>877.5</u>	9 (22.5c-)	M6-	Eb1-C*	3	22.5-	C
3-4	622.5	6 (22.5c+)	dim5+	C-F#*	4	Q	E#
4-5	482.5	5 (17.5c-)	P4	F#-B*	5	<u>17.5-</u>	В
5-6	395	4 (5c-)	M3-	B-d#*	6	22.5-	d#
6-7	333.75	3 (33.75c+)	m3+	d#-f#*	7	11.25+	f#
7-8	288.75	3 (11.25c-)	m3-	f#-a*	8	<u>0</u>	a
8-9	<u>255</u>	3 (45c-)	m3-	a-c*	9	<u>45-</u>	C
9-10	227.5	2 (27.5c+)	M2+	c1-d1*	10	<u> 17.5-</u>	d1
10-11	206.25	2 (6.25c+)	M2+	d1-e1*	11	11.25-	<u>e1</u>
11-12	188.75	2 (11.25c-)	M2-	e1-f#1*	12	22.5-	f#1
12-13	172.5	2 (27.5c-)	M2-	f#1-g#1*	13	<u>50-</u>	g1/g#1
13-14	161.25	2 (38.75c-)	M2-	g#1-a#1*	14	11.3+/88.7-	a1
14-15	148.75	1 (48.75c+)	m2+	a#1-b1*	15	60+/40-	<u>b1</u>
15-16	140	1 (40c+)	m2+	b1-c2*	16	Q	<u>c2</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [m10] recurrences (numbers indicate the position of the note in the series):
    - c-(9), c(3), c(1), e(11), f#(7), g/g#(13), b-(15), b(5); 8 notes.
      - a) Number of recurrences: c-, c(3x), c(5x), e, f#(2x), g/g#, b-, b(2x).
      - b) Excluding notes deviating more than  $25c^*$ : c, e, f#, b; 4 notes.
  - B) Excluding octave repetitions: c-, c, d, d#, e, f#, g/g#, a, b-, b; 10 notes.
    - a) Number of repetitions: c-, c(3x), d, d#(2x), e, f#(3x), g/g#, a(2x), b-, b.
    - b) Excluding notes deviating more than  $25c^*$ : c, d, d#, e, f#, a, b; 7 notes.
    - c) Excluding primary interval [m10] recurrences: c-, c, e, f#, g/g#, b-, b; 7 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [m10] recurrences: c, c, b; 3 notes.
  - B) Excluding octave repetitions: c, d#, f#, b; 4 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{D\#1(I)}$ ,  $\underline{C(III)}$ ,  $\underline{F\#(I)}$ ,  $\underline{B(II)}$ ,  $\underline{d\#(III)}$ ,  $\underline{a(I)}$ ,  $\underline{d1(II)}$ ,  $\underline{f\#1(III)}$ ,  $\underline{c2(I)}$ .
  - B) D: 3, 6, 9, 12, 15; C(I), d#(I), c-(III), f#1(I), b1-(II).
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 150, 88, 62, 48, 40, 33, 29, 26, 23, 21, 19, 17, 16, 15, 14.
  - $B)\ Intervals\ in\ cents/40:\ 37.5,\ 22,\ 15.5,\ 12,\ 10,\ 8.25,\ 7.25,\ 6.5,\ 5.75,\ 5.25,\ 4.75,\ 4.25,\ 4,\ 3.75,\ 3.5.$

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Augmentation: x4/3

Primary Interval: 16 half-steps [M10]

Serial #	Int. in cents	Half-steps +	<u>Intervals</u>	Example	Ser.#	Corrections	Example with
(intervals)	<u>x4/3</u>	dif. in cents		fund.=C2		(in cents)	corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	<u>1600</u>	<u>16</u>	M10	C2-E1	2	Q	E1
2-3	936	9 (36c+)	M6+	E1-C#*	3	<u>36+</u>	C#
3-4	<u>664</u>	7 (36c-)	P5-	C#-G#*	4	Q	G#
4-5	<u>514.666667</u>	5 (14.6c+)	P4+	G#-c#*	5	14.6+	C#
5-6	421.333333	4 (21.3c+)	M3+	c#-e#*	6	<u>36+</u>	f
6-7	356	4 (44c-)	M3-	f-a*	7	<u>8-</u>	a
7-8	308	3 (8c+)	m3+	a-c1*	8	Q	<u>c1</u>
8-9	272	3 (28c-)	m3-	c1-eb1*	9	<u>28-</u>	<u>d#1</u>
9-10	242.666667	2 (42.66c+)	M2+	eb1-f1*	10	14.6+	f1
10-11	220	2 (20c+)	M2+	f1-g1*	11	34.6+	g1
11-12	201.333333	2 (1.33c+)	M2+	g1-a1*	12	<u>36+</u>	a1
12-13	184	2 (16c-)	M2-	a1-b1*	13	<u>20+</u>	<u>b1</u>
13-14	172	2 (28c-)	M2-	b1-c#2*	14	<u>8-</u>	<u>c#2</u>
14-15	158.666667	2 (41.34c-)	M2-	c#2-d#2*	15	49.34+	d2/d#2
15-16	149.333333	1 (49.3c+)	m2+	d#2-e2*	16	Q	<u>e2</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [M10] recurrences (numbers indicate the position of the note in the series):
    - c(1), c#(5), c#+(3), d#-(9), g+(11), a(7), b(13), d/d#(15); 8 notes.
      - a) Number of recurrences: c(5x), c#(2x), c#+(3x), d#-, g+, a(2x), b, d/d#.
      - b) Excluding notes deviating more than 25c\*: c, c#, a, b; 4 notes.
  - B) Excluding octave repetitions: c, c#, c#+, d/d#, d#-, e, f, f+, g+, g#, a, a+, b; 13 notes.
    - a) Number of repetitions: c(2x), c#(2x), c#+, d/d#, d#-, e(2x), f, f+, g+, g#, a, a+, b.
    - b) Excluding notes deviating more than  $25c^*$ : c, c#, e, f, g#, a, b; 7 notes.
    - c) Excluding primary interval [M10] recurrences: c, c#, c#+, d#-, g+, a, b, d/d#; 8 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [M10] recurrences: c, c#, c#+; 3 notes.
  - B) Excluding octave repetitions: c, c#, c#+, e, f, g#; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{E1(I)}$ , C#+(III), G#(I), c#(II), f+(III),  $\underline{c1(I)}$ , f1(II), a1+(III),  $\underline{e2(I)}$ .
  - B) D: 3, 6, 9, 12, 15; C#+(I), f+(I), d#1-(III), a1+(I), d2/d#2(II).
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 160, 94, 66, 51, 42, 36, 31, 27, 24, 22, 20, 18, 17, 16, 15.
  - $B)\ Intervals\ in\ cents/40:\ 40,\ 23.5,\ 16.5,\ 12.75,\ 10.5,\ 9,\ 7.75,\ 6.75,\ 6,\ 5.5,\ 5,\ 4.5,\ 4.25,\ 4,\ 3.75.$

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Augmentation: x17/12

Primary Interval: 17 half-steps [P111]

Serial # (intervals)	Int. in cents x17/12	Half-steps + dif. in cents	<u>Intervals</u>	Example fund.=C2	Ser.#	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	<u>1700</u>	17	P11	C2-F1	2	Q	E1.
2-3	994.5	10 (5.5c-)	m7-	F1-Eb*	3	<u>5.5-</u>	D#
3-4	705.5	7 (5.5c+)	P5+	Eb-Bb*	4	Q	A#
4-5	546.833333	5 (46.83c+)	P4+	Bb-eb*	5	46.83+	d#
5-6	<del>44</del> 7.666667	4 (47.66c+)	M3+	eb-g*	6	94.5+/5.5-	g#
6-7	378.25	4 (21.75c-)	M3-	g-b*	7	27.25-	<u>c1</u>
7-8	327.25	3 (27.25c+)	m3+	b-d1*	8	Q	d#1
8-9	289	3 (11c-)	m3-	d1-f1*	9	11-	f#1
9-10	257.833333	3 (42.17c-)	m3-	f1-ab1*	10	<u>46.8+/53.2-</u>	g#1
10-11	233.75	2 (33.75c+)	M2+	g#1-a#1*	11	80.6+/19.4-	<u>b1</u>
11-12	213.916667	2 (13.92c+)	M2+	a#1-b#1*	12	<u>5.5-</u>	<u>c#2</u>
12-13	195.5	2 (4.5c-)	M2-	c2-d2*	13	<u>10-</u>	<u>d#2</u>
13-14	182.75	2 (17.25c-)	M2-	d2-e2*	14	<u> 27.25-</u>	<u>f2</u>
14-15	168.583333	2 (31.42c-)	M2-	e2-f#2*	15	41.3+/58.7-	<u>f#2</u>
15-16	158.666667	2 (41.34c-)	M2-	f#2-g#2*	16	Q	g#2

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [P11] recurrences (numbers indicate the position of the note in the series):
    - $c\text{-}(7),\,c(1),\,d\#(3),\,d\#(13),\,d\#+(5),\,f\#(9),\,f\#+(15),\,a\#(11);\,8\,\,\text{notes}.$ 
      - a) Number of recurrences: c-(2x), c(5x), d#(3x), d#, d#+(2x), f#+, f#+, a#.
      - b) Excluding notes deviating more than  $25c^*$ : c, d#, d#, f#, a#; 5 notes.
  - B) Excluding octave repetitions: c-, c, c#, d#, d#+, f-,f, f#, f#+, g#, g#+, a#; 13 notes.
    - a) Number of repetitions: c-, c, c#, d#(3x), d#+, f-,f, f#, f#+, g#(2x), g#+, a#(2x).
    - b) Excluding notes deviating more than 25c\*: c, c#, d#, f, f#, g#, a#; 7 notes.
    - c) Excluding primary interval [P11] recurrences: c-, c, d#, d#+, f#, f#+, a#; 7 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [P11] recurrences: c, d#, d#+; 3 notes.
  - B) Excluding octave repetitions: c, d#, d#+, f, g#, a#; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{E1(I)}$ ,  $\underline{D\#+(II)}$ ,  $\underline{A\#(I)}$ ,  $\underline{d\#+(II)}$ ,  $\underline{g\#(II)}$ ,  $\underline{g\#1(I)}$ ,  $\underline{g\#1+(II)}$ ,  $\underline{g\#2(I)}$ .
  - B) D: 3, 6, 9, 12, 15;  $\underline{D\#+(I)}$ ,  $\underline{g\#(I)}$ , f#1(III),  $\underline{c\#2(I)}$ , f#2+(II).
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 170, 99, 70, 55, 45, 38, 33, 29, 26, 23, 21, 19, 18, 17, 16.
  - B) Intervals in cents/40: 42.5, 24.75, 17.5, 13.75, 11.25, 9.5, 8.25, 7.25, 6.5, 5.75, 5.25, 4.75, 4.5, 4.25,

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Augmentation: x3/2

Primary Interval: 18 half-steps [aug.11]

Serial #	Int. in cents	Half-steps +	<u>Intervals</u>	Example	Ser.#	Corrections	Example with
(intervals)	<u>x3/2</u>	dif. in cents		fund.=C2		(in cents)	corrections
1	/	/	/	C2	1	<u>0</u>	<u>C2</u>
1-2	<u>1800</u>	18	aug.11	C2-F#1	2	Q	F#1
2-3	1053	11 (47c-)	M7-	F#1-E#*	3	<u>47-</u>	E
3-4	747	7 (47c+)	P5+	E#-c*	4	Q	<u>c</u>
4-5	<u>579</u>	6 (21c-)	dim5-	c-f#*	5	<u>21-</u>	f#
5-6	474	5 (26c-)	P4-	f#-b*	6	<u>47-</u>	Þ
6-7	400.5	4 (0.5c+)	M3(+)	b1-d#1	7	46.5-	<u>d#1</u>
7-8	<u>346.5</u>	3 (46.5c+)	m3+	d#1-f#1*	8	Q	f#1
8-9	306	3 (6c+)	m3+	f#1-a1*	9	<u>6+</u>	<u>a1</u>
9-10	273	3 (27c-)	m3-	a1-c2*	10	<u>21-</u>	<u>c2</u>
10-11	<u>247.5</u>	2 (47.5c+)	M2+	c2-d2*	11	<u>26.5+</u>	<u>d2</u>
11-12	226.5	2 (26.5c+)	M2+	d2-e2*	12	<u>53+/47-</u>	f2
12-13	207	2 (7c+)	M2+	e2-f#2*	13	<u>40-</u>	g2
13-14	<u>193.5</u>	2 (6.5c-)	M2-	f#2-g#2*	14	<u>46.5-</u>	a2
14-15	<u>178.5</u>	2 (21.5c-)	M2-	g#2-a#2*	15	32+/68-	a#2
15-16	168	2 (32c-)	M2-	bb2-c3*	16	Q	<u>c3</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [aug.11] recurrences (numbers indicate the position of the note in the series):
    - c(1), d+(11), d#-(7), f-(3), f#(5), g-(13), a(9), a#+(15); 8 notes.
      - a) Number of recurrences: c(5x), d+, d#-(2x), f-(3x), f#(2x), g-, a, a#+.
      - b) Excluding notes deviating more than  $25c^*$ : c, f#, a; 3 notes.
  - B) Excluding octave repetitions: c, d+, d#-, f-, f#, g-, a-, a, a#+, b-; 10 notes.
    - a) Number of repetitions: c(4x), d+, d#-, f-(2x), f#(3x), g-, a-, a, a#+, b-.
    - b) Excluding notes deviating more than  $25c^*$ : c, f#, a; 3 notes.
    - c) Excluding primary interval [aug.11] recurrences: c, d+, d#-, f-, f#, g-, a, a#+; 8 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [aug.11] recurrences: c, f-, f#+; 3 notes.
  - B) Excluding octave repetitions: c, f-, f, f#, f#+, b-; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{F\#1(I)}$ , F-(III),  $\underline{c(I)}$ , f#+(II), b-(III), f#1(I), c2(II), f2-(III),  $\underline{c3(I)}$ .
  - B) D: 3, 6, 9, 12, 15; <u>F-(I)</u>, <u>b-(I)</u>, a1(III), <u>f2-(I)</u>, a#2+(II).
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 180, 105, 75, 58, 47, 40, 35, 30, 27, 25, 23, 21, 19, 18, 17.
  - $B)\ Intervals\ in\ cents/40:\ 45,\ 26.25,\ 18.75,\ 14.5,\ 11.75,\ 10,\ 8.75,\ 7.5,\ 6.75,\ 6.25,\ 5.75,\ 5.25,\ 4.75,\ 4.5,\$

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Augmentation: x19/12

Primary Interval: 19 half-steps [P12]

Serial #	Int. in cents	Half-steps +	<u>Intervals</u>	Example	Ser.#	Corrections	Example with
(intervals)	x19/12	dif. in cents		fund.=C2		(in cents)	corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	1900	19	P12	C2-G1	2	Q	<u>G1</u>
2-3	1111.5	11 (11.5c+)	M7+	G1-F#*	3	11.5±	E#
3-4	<u>788.5</u>	8 (11.5c-)	m6-	F#-d*	4	Q	₫
4-5	611.166667	6 (11.17c+)	dim5+	d-ab*	5	11.17±	g#
5-6	500.333333	5 (0.33c+)	P4(+)	g#-c#1	6	11.5±	<u>c#1</u>
6-7	422.75	4 (22.75c+)	M3+	c#1-e#1*	7	<u>34.25+</u>	f1
7-8	<u>365.75</u>	4 (34.25c-)	М3-	f1-a1*	8	Q	<u>a1</u>
8-9	<u>323</u>	3 (23c+)	m3+	a1-c2*	9	<u>23+</u>	<u>c2</u>
9-10	<u>288.166667</u>	3 (11.83c-)	m3-	c2-eb2*	10	11.17±	<u>d#2</u>
10-11	261.25	3 (38.75c-)	m3-	d#2-f#2*	11	<u>27.58-</u>	<u>f#2</u>
11-12	239.083333	2 (39.1c+)	M2+	f#2-g#2*	12	11.5±	g#2
12-13	<u>218.5</u>	2 (18.5c+)	M2+	g#2-a#2*	13	<u>30+</u>	a#2
13-14	204.25	2 (4.25c+)	M2+	a#2-b#2*	14	34.25+	<u>c3</u>
14-15	188.416667	2 (11.58c-)	M2-	c3-d3*	15	22.67±	<u>d3</u>
15-16	177.333333	2 (22.67c-)	M2-	d3-e3*	16	Q	<u>e3</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [P12] recurrences (numbers indicate the position of the note in the series):
    - c(1), c(9), d(15), f+(7), f#-(11), f#(3), g#(5), a#+(13); 8 notes.
      - a) Number of recurrences: c(5x), c, d, f+(2x), f#-, f#(3x), g#(2x), a#+.
      - b) Excluding notes deviating more than  $25c^*$ : c, c, f#, g#; 4 notes.
  - B) Excluding octave repetitions: c, c+, c#, d, d#, e, f+, f#-, f#, g, g#, a, a#+; 13 notes.
    - a) Number of repetitions: c(2x), c+, c#, d(2x), d#, e, f+, f#-, f#, g, g#(2x), a, a#+.
    - b) Excluding notes deviating more than 25c\*: c, c#, d, d#, e, f#, g, g#, a; 9 notes.
    - c) Excluding primary interval [P12] recurrences: c, d, f+, f#-, f#, g#, a#+; 7 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [P12] recurrences: c, f#, g#; 3 notes.
  - B) Excluding octave repetitions: c, c#, d, f#, g, g#; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{G1(I)}$ , F#(III),  $\underline{d(I)}$ , g#(II), c#1(III),  $\underline{a1(I)}$ , d#2(II), g#2(III),  $\underline{e3(I)}$ .
  - B) D: 3, 6, 9, 12, 15; F#(I), c#I(I), c2(III), g#2(I), d3(II).
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 190, 111, 79, 61, 50, 42, 37, 32, 29, 26, 24, 22, 20, 19, 18.
  - $B)\ Intervals\ in\ cents/40:\ 47.5,\ 27.75,\ 19.75,\ 15.25,\ 12.5,\ 10.5,\ 9.25,\ 8,\ 7.25,\ 6.5,\ 6,\ 5.5,\ 5,\ 4.75,\ 4.5.$

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Augmentation: x5/3

Primary Interval: 20 half-steps [m13]

Serial # (intervals)	Int. in cents x5/3	Half-steps + dif. in cents	<u>Intervals</u>	Example fund.=C2	Ser.#	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	2000	20	m13	C2-Ab1	2	<u>0</u>	<u>G#1</u>
2-3	1170	12 (30c-)	P8-	G#1-G#*	3	<u>30-</u>	G#
3-4	830	8 (30c+)	m6+	G#-e*	4	Q	e
4-5	643.333333	6 (43.33c+)	dim5+	e-bb*	5	43.33+	a#
5-6	526.666667	5 (26.66c+)	P4	a#-d#1*	6	<del>70+/30-</del>	<u>e1</u>
6-7	445	4 (45c+)	M3+	eb1-g1*	7	15±	g#1
7-8	385	4 (15c-)	М3-	g1-b1*	8	0	<u>c2</u>
8-9	340	3 (40c+)	m3+	b1-d2*	9	40+	<u>d#2</u>
9-10	303.333333	3 (3.33c+)	m3+	d2-f2*	10	43.33+	f#2
10-11	275	3 (25c-)	m3-	e#2-g#2*	11	18.33+	<u>a2</u>
11-12	<u>251.666667</u>	3 (48.34c-)	m3-	g#2-b2*	12	<u>30-</u>	<u>c3</u>
12-13	230	2 (30c+)	M2+	b2-c#3*	13	0	<u>d3</u>
13-14	215	2 (15c+)	M2+	c#3-d#3*	14	15±	<u>e3</u>
14-15	198.333333	2 (1.67c-)	M2-	d#3-e#3*	15	13.33+	<u>f#3</u>
15-16	186.666667	2 (13.33c-)	M2-	f3-g3*	16	Q	g#3

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [m13] recurrences (numbers indicate the position of the note in the series):
    - c(1), d(13), d#+(9), f#(15), g#-(3), g#(7), a(11), a#+(5); 8 notes.
      - a) Number of recurrences: c(5x), d, d#+, f#, g#-(3x), g#(2x), a, a#+(2x).
      - b) Excluding notes deviating more than 25c\*: c, d, f#, g#, a; 5 notes.
  - B) Excluding octave repetitions: c-, c, d, d#+, e-, e, f#, f#+, g#-, g#, a, a#+; 12 notes.
    - a) Number of repetitions: c-, c(2x), d, d#+, e-, e(2x), f#, f#+, g#-, g#(3x), a, a#+.
    - b) Excluding notes deviating more than 25c\*: c, d, e, f#, g#, a; 6 notes.
    - c) Excluding primary interval [m13] recurrences: c, d, d#+, f#, g#-, g#, a, a#+; 8 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [m13] recurrences: c, g#-, a#+; 3 notes.
  - B) Excluding octave repetitions: c, e-, e, g#-, g#, a#+; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{G\#1(I)}$ ,  $\underline{G\#-(III)}$ ,  $\underline{e(I)}$ ,  $\underline{a\#+(II)}$ ,  $\underline{e1-(III)}$ ,  $\underline{c2(I)}$ ,  $\underline{f\#2+(II)}$ ,  $\underline{c3-(III)}$ ,  $\underline{g\#3(I)}$ .
  - B) D: 3, 6, 9, 12, 15; G#-(I), e1-(I), d#2+(III), c3-(I), f#(II).
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 200, 117, 83, 64, 53, 44, 38, 34, 30, 27, 25, 23, 21, 20, 19.
  - B) Intervals in cents/40: 50, 29.25, 20.75, 16, 13.25, 11, 9.5, 8.5, 7.5, 6.75, 6.25, 5.75, 5.25, 5, 4.75.

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Augmentation: x7/4

Primary Interval: 21 half-steps [M13]

Serial # (intervals)	Int. in cents x7/4	Half-steps + dif. in cents	<u>intervals</u>	Example fund.=C2	Ser.#	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	Q	C2
1-2	2100	<u>21</u>	M13	C2-A1	2	Q	<b>A1</b>
2-3	1228.5	12 (28.5c+)	P8+	A1-A*	3	28.5+	Α
3-4	<u>871.5</u>	9 (28.5c-)	M6-	A-f#*	4	Q	f#
4-5	<u>675.5</u>	7 (24.5c-)	P5-	f#-c#1*	5	<u> 24.5-</u>	<u>c#1</u>
5-6	<u>553</u>	6 (47c-)	dim5-	c#1-g1*	6	28.5+/71.5-	f#1
6-7	467.25	5 (32.75c-)	P4-	g1-c2*	7	4.25-	<u>b1</u>
7-8	404.25	4 (4.25c+)	M3+	c2-e2*	8	Q	<u>d#2</u>
8-9	357	4 (43c-)	М3-	e2-g#2*	9	<u>43-</u>	g <u>2</u>
9-10	<u>318.5</u>	3 (18.5c+)	m3+	g#2-b2*	10	<u> 24.5-</u>	<u>a#2</u>
10-11	<u> 288.75</u>	3 (11.25c-)	m3-	b2-d3*	11	<u>35.75-</u>	<u>c#3</u>
11-12	264.25	3 (35.75c-)	m3-	d3-f3*	12	28.5+/71.5-	<u>d#3</u>
12-13	<u>241.5</u>	2 (41.5c+)	M2+	f3-g3*	13	<u>70+/30-</u>	f#3
13-14	225.75	2 (25.75c+)	M2+	g3-a3*	14	<u>4.25-</u>	g#3
14-15	208.25	2 (8.25c+)	M2+	a3-b3*	15	<b>4</b> ±	<u>a#3</u>
15-16	196	2 (4c-)	M2-	b3-c#4*	16	Q	<u>c4</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [M13] recurrences (numbers indicate the position of the note in the series):
    - c(1), c#-(11), c#-(5), f#-(13), g-(9), a#(15), a+(3), b(7); 8 notes.
      - a) Number of recurrences: c(5x), c#-, c#-(2x), f#-, g-, a#, a+(3x), b(2x).
      - b) Excluding notes deviating more than 25c\*: c, a#, b; 3 notes.
  - B) Excluding octave repetitions: c, c#-, d#, d#+, f#-, f#, f#+, g-, g#, a, a+, a#-, a#, b; 14 notes.
    - a) Number of repetitions: c(2x), c#-(2x), d#, d#+, f#-, f#, f#+, g-, g#, a, a+, a#-, a#, b.
    - b) Excluding notes deviating more than  $25c^*$ : c, d#, f#, g#, a, a#, b; 7 notes.
    - c) Excluding primary interval [M13] recurrences: c, c#-, f#-, g-, a#, a+, b; 8 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [M13] recurrences: c, c#-, a+; 3 notes.
  - B) Excluding octave repetitions: c, c#-, f#, f#+, a, a+; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{\text{C2(I)}}$ ,  $\underline{\text{A1(I)}}$ , A+(III),  $\underline{\text{f\#(I)}}$ ,  $\underline{\text{c\#1-(II)}}$ ,  $\underline{\text{f\#1+(III)}}$ ,  $\underline{\text{d\#2(I)}}$ ,  $\underline{\text{a\#2-(II)}}$ ,  $\underline{\text{d\#3+(III)}}$ .
  - B) D: 3, 6, 9, 12, 15;  $\underline{A+(I)}$ ,  $\underline{f#1+(I)}$ ,  $\underline{g2-(III)}$ ,  $\underline{d#3+(I)}$ ,  $\underline{a#3(II)}$ .
- $4) \ Intervals \ of \ the \ series \ expressed \ in \ proportions \ more \ suitable \ for \ possible \ rhythmic/structural \ applications.$ 
  - A) Intervals in cents/10: 210, 123, 87, 68, 55, 47, 40, 36, 32, 29, 26, 24, 23, 21, 20.
  - $B)\ Intervals\ in\ cents/40:\ 52.5,\ 30.75,\ 21.75,\ 17,\ 13.75,\ 11.75,\ 10,\ 9,\ 8,\ 7.25,\ 6.5,\ 6,\ 5.75,\ 5.25,\ 5.$

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Table 11

Augmentation: x11/6

Primary Interval: 22 half-steps [m14]

Serial # (intervals)	Int. in cents x11/6	Half-steps + dif. in cents	<u>Intervals</u>	Example fund.=C2	<u>Ser.#</u>	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	2200	22	m14	C2-Bb1	2	Q	A#1
2-3	1287	13 (13c-)	P8+m2-	A#1-b1*	3	<u>13-</u>	В
3-4	913	9 (13c+)	M6+	b1-g#*	4	Q	g#
4-5	707.666667	7 (7.66c+)	P5+	g#-d#1*	5	7.66+	<u>d#1</u>
5-6	579.333333	6 (20.67c-)	dim5-	d#1-a1*	6	<u>13-</u>	a1
6-7	489.5	5 (10.5c-)	P4-	a1-d2*	7	23.5-	d2
7-8	423.5	4 (23.5c+)	M3+	d2-f#2*	8	Q	f#2
8-9	374	4 (26c-)	М3-	f#2-a#2*	9	<u> 26-</u>	a#2
9-10	333.666667	3 (33.66c+)	m3+	a#2-c#3*	10	7.66+	<u>c#3</u>
10-11	302.5	3 (2.5c+)	m3+	c#3-e3*	11	10.16+	<u>e3</u>
11-12	276.833333	3 (23.17c-)	m3-	e3-g3*	12	<u>13-</u>	<u>g3</u>
12-13	253	3 (47c-)	m3-	g3-bb3*	13	40+/60-	<u>a3</u>
13-14	236.5	2 (36.5c+)	M2+	a#3-b#3*	14	76.5+/23.5-	<u>c4</u>
14-15	218.166667	2 (18.17c+)	M2+	c4-d4*	15	<u>5.33-</u>	d4
15-16	205.333333	2 (5.33c+)	M2+	d4-e4*	16	Q	<u>e4</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [m14] recurrences (numbers indicate the position of the note in the series):
    - c(1), d-(15), d(7), d#(5), e(11), a+(13), a#-(9), b(3); 8 notes.
      - a) Number of recurrences: c(5x), d-, d(2x), d#(2x), e, a+, a#-, b(3x).
      - b) Excluding notes deviating more than  $25c^*$ : c, d, d#, e, b; 5 notes.
  - B) Excluding octave repetitions: c, c#, d, d#, e, f#, g, g#, a, a+, a#-, a#, b; 13 notes.
    - a) Number of repetitions: c(2x), c#, d(2x), d#, e(2x), f#, g, g#, a, a+, a#-, a#, b.
    - b) Excluding notes deviating more than  $25c^*$ : c, c#, d, d#, e, f#, g, g#, a, a#, b; 11 notes.
    - c) Excluding primary interval [m14] recurrences: c, d-, d, d#, e, a+, a#-, b; 8 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [m14] recurrences: c, d#, b; 3 notes.
  - B) Excluding octave repetitions: c, d#, g#, a, a#, b; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{A\#1(I)}$ ,  $\underline{B(III)}$ ,  $\underline{g\#(I)}$ ,  $\underline{d\#1(II)}$ ,  $\underline{a1(III)}$ ,  $\underline{f\#2(I)}$ ,  $\underline{c\#3(II)}$ ,  $\underline{g3(III)}$ ,  $\underline{e4(I)}$ .
  - B) D: 3, 6, 9, 12, 15; <u>b1(I)</u>, <u>a1(I)</u>, a#2-(III), <u>g3(I)</u>, <u>d4(II)</u>.
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 220, 129, 91, 71, 58, 49, 42, 37, 33, 30, 28, 25, 24, 22, 21.
  - $B)\ Intervals\ in\ cents/40:\ 55,\ 32.25,\ 22.75,\ 17.75,\ 14.5,\ 12.25,\ 10.5,\ 9.25,\ 8.25,\ 7.57,\ 6.25,\ 6,\ 5.5,\ 5.25.$

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Table 12

Augmentation: x23/12

Primary Interval: 23 half-steps [M14]

Serial # (intervals)	Int. in cents x23/12	Half-steps + dif. in cents	<u>intervals</u>	Example fund.=C2	Ser.#	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	<u>2300</u>	<u>23</u>	M14	C2-B1	2	Q	<u>B1</u>
2-3	1345.5	13 (45.5.c+)	P8+m2+	B1-c*	3	45.5+	<u>c</u>
3-4	<u>954.5</u>	10 (45.5c-)	m7-	c-bb*	4	Q	<u>a#</u>
4-5	739.833333	7 (39.83c+)	P5+	bb-f1*	5	39.83+	f1
5-6	605.666667	6 (5.66c+)	dim5+	f1-cb2*	6	45.5+	<u>b1</u>
6-7	511.75	5 (11.75c+)	P4+	b1-e2*	7	<u>57.3+/42.7-</u>	f2
7-8	442.75	4 (42.75c+)	M3+	e2-g#2*	8	Q	<u>a2</u>
8-9	<u>391</u>	4 (9c-)	M3-	g#2-b#2*	9	<u>9-</u>	<u>c#3</u>
9-10	348.833333	3 (48.83c+)	m3+	с3-еь3*	10	39.83+	<u>e3</u>
10-11	316.25	3 (16.25c+)	m3+	d#3-f#3*	11	56.1+/43.9-	g#3
11-12	289.416667	3 (10.58c-)	m3-	f#3-a3*	12	<u>45.5+/54.5-</u>	<u>a#3</u>
12-13	<u>264.5</u>	3 (35.5c-)	m3-	a3-c4*	13	10±	<u>c#4</u>
13-14	247.25	2 (47.25c+)	M2+	c4-d4*	14	<u>57.3+/42.7-</u>	<u>e4</u>
14-15	228.083333	2 (28.09c+)	M2+	d4-e4*	15	14.66-	f#4
15-16	214.666667	2 (14.66c+)	M2+	e4-f#4*	16	Q	g#4

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [M14] recurrences (numbers indicate the position of the note in the series):
    - c(1), c+(3), c#(9), c#(13), f-(7), f+(5), f#(15), g#-(11); 8 notes.
      - a) Number of recurrences: c(5x), c+(3x), c#, c#, f-(2x), f+(2x), f#, g#-.
      - b) Excluding notes deviating more than 25c\*: c, c#, c#, f#; 4 notes.
  - B) Excluding octave repetitions: c, c+, c#, e-, e+, f-, f+, f#, g#-, g#, a, a#, a#+, b, b+; 15 notes.
    - a) Number of repetitions: c, c+, c#(2x), e-, e+, f-, f+, f#, g#-, g#, a, a#, a#+, b, b+.
    - b) Excluding notes deviating more than 25c\*: c, c#, f#, g#, a, a#, b; 7 notes.
    - c) Excluding primary interval [M14] recurrences: c, c+, c#, f-, f+, f#, g#-; 7 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [M14] recurrences: c, c+, f+; 3 notes.
  - B) Excluding octave repetitions: c, c+, f+, a#, b, b+; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{B1(I)}$ ,  $\underline{c(III)}$ ,  $\underline{a\#(I)}$ ,  $\underline{f1+(II)}$ ,  $\underline{b1+(III)}$ ,  $\underline{a2-(I)}$ ,  $\underline{e3+(II)}$ ,  $\underline{a\#3+(III)}$ ,  $\underline{g\#4(I)}$ .
  - B) D: 3, 6, 9, 12, 15; c+(I), b1+(I), c#3(III), a#3+(I), f#4(II).
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 230, 135, 95, 74, 61, 51, 44, 39, 35, 32, 29, 26, 25, 23, 21.
  - B) Int. in cents/40: 57.5, 33.75, 23.75, 18.5, 15.25, 12.75, 11, 9.75, 8.75, 8, 7.25, 6.5, 6.25, 5.75, 5.25.

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Table 13

Augmentation: x2

Primary Interval: 24 half-steps [P15]

Serial #	Int. in cents	Half-steps +	<u>Intervals</u>	Example	Ser.#	Corrections	Example with
(intervals)	x2	dif. in cents		fund.=C2		(in cents)	corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	2400	24	P15	C2-C	2	Q	C
2-3	1404	14 (4c+)	8+M2+	C-d*	3	4±	d
3-4	996	10 (4c-)	m7-	d-c1*	4	Q	<u>c1</u>
4-5	772	8 (28c-)	m6-	c1-ab1*	5	28-	g#1
5-6	632	6 (32c+)	dim5+	g#1-d2*	6	<b>4</b> ±	<u>d2</u>
6-7	534	5 (34c+)	P4+	d2-g2*	7	38+	g2
7-8	462	5 (38c-)	P4-	g2-c3*	8	Q	<u>c3</u>
8-9	408	4 (8c+)	M3+	c3-e3*	9	<u>8+</u>	<u>e3</u>
9-10	<u>364</u>	4 (36c-)	М3-	e3-g#3*	10	28-	g#3
10-11	330	3 (30c+)	m3+	g#3-b3*	11	<u>2+</u>	<u>b3</u>
11-12	302	3 (2c+)	m3+	b3-d4*	12	4±	<u>d4</u>
12-13	276	3 (24c-)	m3-	d4-f4*	13	<u>20-</u>	f4
13-14	258	3 (42c-)	m3-	f4-ab4*	14	38+/62-	<u>g4</u>
14-15	238	2 (38c+)	M2+	g#4-a#4*	15	76+/24-	a#4
15-16	224	2 (24c+)	M2+	b64-c5*	16	Q	<u>c5</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [P15] recurrences (numbers indicate the position of the note in the series):
    - c(1), d(3), e(9), f(13), g+(7), g#-(5), a#(15), b(11); 8 notes.
      - a) Number of recurrences: c(5x), d(3x), e, f, g+(2x), g#-(2x), a#, b.
      - b) Excluding notes deviating more than 25c\*: c, d, e, f, a#, b; 6 notes.
  - B) Excluding octave repetitions (same as 1A above): c, d, e, f, g+, g#-, a#, b; 8 notes.
    - a) Number of repetitions: c(5x), d(3x), e, f, g+(2x), g#-(2x), a#, b.
    - b) Excluding notes deviating more than 25c\*: c, d, e, f, a#, b; 8 notes.
    - c) Excluding primary interval [P15] recurrences: c, d, e, f, g+, g#-, a#, b; 8 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [P15] recurrences: c, d, g#-; 3 notes.
  - B) Excluding octave repetitions (same as 2A above): c, d, g#-; 3 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{C(I)}$ ,  $\underline{d(III)}$ ,  $\underline{c1(I)}$ ,  $\underline{g\#1-(II)}$ , $\underline{d2(III)}$ ,  $\underline{c3(I)}$ ,  $\underline{g\#3-(II)}$ , $\underline{d4(III)}$ ,  $\underline{c5(I)}$ .
  - B) D: 3, 6, 9, 12, 15; <u>d(I)</u>, <u>d2(I)</u>, e3(III), <u>d4(I)</u>, *a#4(II)*.
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 240, 140, 100, 77, 63, 53, 46, 41, 36, 33, 30, 28, 26, 24, 22.
  - $B)\ Intervals\ in\ cents/40:\ 60,\ 35,\ 25,\ 19.25,\ 15.75,\ 13.25,\ 11.5,\ 10.25,\ 9,\ 8.25,\ 7.5,\ 7,\ 6.5,\ 6,\ 5.5.$

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Table 14

Augmentation: x25/12

Primary Interval: 25 half-steps [m16]

Serial #	Int. in cents	Half-steps +	<u>Intervals</u>	Example	Ser.#	Corrections	Example with
(intervals)	x25/12	dif. in cents		fund.=C2		(in cents)	corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	<u>2500</u>	<u>25</u>	m16	C2-Db	2	Q	<u>C#</u>
2-3	1462.5	15 (37.5c-)	m10-	C#-e*	3	<u>37.5-</u>	<u>e</u>
3-4	1037.5	10 (37.5c+)	m7+	e-d1*	4	Q	<u>d1</u>
4-5	804.166667	8 (4.17c+)	m6+	d1-b <sub>b</sub> 1*	5	4.17±	<u>a#1</u>
5-6	658.333333	7 (41.67c-)	P5-	bb1-f2*	6	<u>37.5-</u>	<u>f2</u>
6-7	<u>556.25</u>	6 (43.75c-)	aug.4-	f2-b2*	7	18.7+/81.3-	a#2
7-8	<u>481.25</u>	5 (18.75c-)	P4-	b2-e3*	8	Q	<u>d#3</u>
8-9	425	4 (25c+)	M3+	e3-g#3*	9	<u>25+</u>	<u>g3</u>
9-10	379.166667	4 (20.83c-)	M3-	g#3-b#3*	10	4.17±	<u>b3</u>
10-11	<u>343.75</u>	3 (43.75c+)	m3+	b#3-d#4*	11	47.92+	<u>d4</u>
11-12	314.583333	3 (14.58c+)	m3+	d#4-f#4*	12	62.5+/37.5-	f#4
12-13	287.5	3 (12.5c-)	m3-	f#4-a4*	13	<u>50-</u>	g#4/a4
13-14	268.75	3 (31.25c-)	m3-	a4-c5*	14	18.7+/81.3-	<u>b4</u>
14-15	247.916667	2 (47.92c+)	M2+	c5-d5*	15	66.6+/33.4-	<u>d5</u>
15-16	233.333333	2 (33.33c+)	M2+	d5-e5*	16	Q	<u>e5</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [m16] recurrences (numbers indicate the position of the note in the series):
    - c(1), e-(3), g+(9), g#/a(13), a#(7), a#(5), d-(15), d+(11); 8 notes.
      - a) Number of recurrences: c(5x), e-(3x), g+, g#/a, a#(2x), a#(2x), d-, d+.
      - b) Excluding notes deviating more than 25c\*: c, g#/a, a#, a#; 4 notes.
  - B) Excluding octave repetitions: c, c#, d-, d, d+, d#, e, f-, f#-, g+, g#/a, a#, b; 13 notes.
    - a) Number of repetitions: c, c#, d-, d, d+, d#, e(2x), f-, f#-, g+, g#/a, a#(2x), b(2x).
    - b) Excluding notes deviating more than 25c\*: c, c#, d, d#, e, a#, b; 7 notes.
    - c) Excluding primary interval [m16] recurrences: c, e-, g+, g#/a, a#, d-, d+; 7 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [m16] recurrences: c, e-, a#; 3 notes.
  - B) Excluding octave repetitions: c, c#, d, e-, f-, a#; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{C\#(I)}$ ,  $\underline{e}$ -(III),  $\underline{d1(I)}$ , a#1(II), f2-(III),  $\underline{d\#3(I)}$ , b3(II), f#4-(III),  $\underline{e}5(I)$ .
  - B) D: 3, 6, 9, 12, 15; <u>e-(1)</u>, <u>f2-(1)</u>, g3+(III), <u>f#4-(1)</u>, *d5-(II)*.
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 250, 146, 104, 80, 66, 56, 48, 42, 38, 34, 31, 29, 27, 25, 23.
  - $B)\ Intervals\ in\ cents/40:\ 62.5,\ 36.5,\ 26,\ 20,\ 16.5,\ 14,\ 12,\ 10.5,\ 9.5,\ 8.5,\ 7.75,\ 7.25,\ 6.75,\ 6.25,\ 5.75.$

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

Table 15

Augmentation: x13/6

Primary Interval: 26 half-steps [M16]

Serial # (intervals)	Int. in cents x13/6	Half-steps + dif. in cents	<u>Intervals</u>	Example fund.=C2	<u>Ser.#</u>	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	Q	<u>C2</u>
1-2	<u>2600</u>	<u>26</u>	M16	C2-D	2	Q	D
2-3	<u>1521</u>	15 (21c+)	m10+	D-f*	3	<u>21+</u>	f
3-4	1079	11 (21c-)	M7+	f-e1*	4	Q	<u>e1</u>
4-5	836.333333	8 (36.33c+)	m6+	e1-c2*	5	36.33+	<u>c2</u>
5-6	<u>684.666667</u>	7 (15.34c-)	P5-	c2-g2*	6	21±	<u>g2</u>
6-7	578.5	6 (21.5c-)	aug.4-	g2-c#3*	7	0.5-	<u>c#3</u>
7-8	500.5	5 (0.5c+)	P4+	c#3-f#3*	8	Q	f#3
8-9	442	4 (42c+)	M3+	f#3-a#3*	9	42+	<u>a#3</u>
9-10	394.333333	4 (5.67c-)	М3-	bb3-d4*	10	36.33+	<u>d4</u>
10-11	<u>357.5</u>	4 (42.5c-)	М3-	d4-f#4*	11	<u>6.17-</u>	f#4
11-12	327.166667	3 (27.17c+)	m3+	f#4-a4*	12	<u>21+</u>	<u>a4</u>
12-13	<u>299</u>	3 (1c-)	m3-	a4-c5*	13	<u>20+</u>	<u>c5</u>
13-14	279.5	3 (20.5c-)	m3-	c5-eb5*	14	<u>0.5-</u>	<u>d#5</u>
14-15	257.833333	3 (42.17c-)	m3-	d#5-f#5*	15	<u>42.67-</u>	f#5
15-16	242.666667	2 (42.67c+)	M2+	f#5-g#5*	16	Q	g# <u>5</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [M16] recurrences (numbers indicate the position of the note in the series):
    - c(1), c(13), c+(5), c#(7), f(3), f#-(15), f#(11), a#+(9); 8 notes.
      - a) Number of recurrences: c(5x), c, c+(2x), c#(2x), f(3x), f#-, f#, a#+.
      - b) Excluding notes deviating more than  $25c^*$ : c, c, c#, f, f#; 5 notes.
  - B) Excluding octave repetitions: c, c+, c#, d, d+, d#, e, f, f#-, f#, g, g#, a, a#+; 13 notes.
    - a) Number of repetitions: c(2x), c+, c#, d, d+, d#, e, f, f#-, f#(2x), g, g#, a, a#+.
    - b) Excluding notes deviating more than 25c\*: c, c#, d, d#, e, f, f#, g, g#, a; 10 notes.
    - c) Excluding primary interval [M16] recurrences: c, c+, c#, f, f#-, f#, a#+; 7 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [M16] recurrences: c, c+, f; 3 notes.
  - B) Excluding octave repetitions: c, c+, d, e, f, g; 6 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16;  $\underline{C2(I)}$ ,  $\underline{D(I)}$ ,  $\underline{f(II)}$ ,  $\underline{e1(I)}$ ,  $\underline{c2+(II)}$ , $\underline{g2(III)}$ ,  $\underline{f\#3(I)}$ ,  $\underline{d4+(II)}$ , $\underline{a4(III)}$ ,  $\underline{g\#5(I)}$ .
  - B) D: 3, 6, 9, 12, 15; <u>f(I)</u>, <u>g2(I)</u>, a#3+(III), <u>a4(I)</u>, *f#5-(II)*.
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 260, 152, 108, 84, 68, 58, 50, 44, 39, 36, 33, 30, 28, 26, 24.
  - B) Intervals in cents/40: 65, 38, 27, 21, 17, 14.5, 12.5, 11, 9.75, 9, 8.25, 7.5, 7, 6.5, 6.

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.

## Augmentation: x3

Primary Interval: 36 half-steps [P22=P8x3]

Serial # (intervals)	Int. in cents	Half-steps + dif. in cents	<u>intervals</u>	Example fund.=C2	Ser.#	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	<u>0</u>	<u>C2</u>
1-2	3600	<u>36</u>	P22=P8x3	C2-c	2	Q	<u>c</u>
2-3	2106	21 (6c+)	M13+	c-a1*	3	<u>6+</u>	<u>a1</u>
3-4	1494	15 (6c-)	m10-	a1-c3*	4	Q	<u>c3</u>
4-5	1158	12 (42c-)	P8-	c3-c4*	5	<u>42-</u>	<u>c4</u>
5-6	948	9 (48c+)	M6+	c4-a4*	6	<u>6+</u>	<u>a4</u>
6-7	801	8 (1c+)	m6+	a4-f5	7	7±	f <u>5</u>
7-8	<u>693</u>	7 (7c-)	P5-	f5-c6*	8	Q	<u>c6</u>
8-9	612	6 (12c+)	aug.4+	c6-f#6*	9	12±	<u>f#6</u>
9-10	<u>546</u>	5 (46c+)	P4+	f#6-b6*	10	<u>58+/42-</u>	<u>c7</u>
10-11	<u>495</u>	5 (5c-)	P4-	b6-e7*	11	<u>47-</u>	fΖ
11-12	<u>453</u>	5 (47c-)	P4-	e7-a7*	12	6+/94-	<u>a7</u>
12-13	414	4 (14c+)	M3+	a7-c#8*	13	<u>20+</u>	<u>c#8</u>
13-14	387	4 (13c-)	M3-	c#8-e#8*	14	7±	<u>f8</u>
14-15	357	4 (43c-)	M3-	f8-a8*	15	<u>36-</u>	<u>a8</u>
15-16	336	3 (36c+)	m3+	a8-c9*	16	Q	<u>c9</u>

- 1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [P15] recurrences (numbers indicate the position of the note in the series):
    - c-(5), c(1), c#(13), f-(11), f(7), f#(9), a-(15), a(3); 8 notes.
      - a) Number of recurrences: c-(2x), c(5x), c#, f-, f(2x), f#, a-, a(3x).
      - b) Excluding notes deviating more than  $25c^*$ : c, c#, f, f#, a; 5 notes.
  - B) Excluding octave repetitions (same as 1A above): c-, c, c#, f-, f, f#, a-, a; 8 notes.
    - a) Number of repetitions: c-(2x), c(5x), c#, f-, f(2x), f#, a-, a(3x).
    - b) Excluding notes deviating more than  $25c^*$ : c, c#, f, f#, a; 5 notes.
    - c) Excluding primary interval [P15] recurrences: c-, c, c#, f-, f, f#, a-, a; 8 notes.
- 2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.
  - A) Excluding primary interval [P15] recurrences: c, c-, a; 3 notes.
  - B) Excluding octave repetitions (same as 2A above): c, c-, a; 3 notes.
- 3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.
  - A) T: 1-6, 8, 10, 12, 16; C2(I), c(I), a1(III), c3(I), c4-(II),a4(III), c6(I), c7-(II),a7(III), c9(I).
  - B) D: 3, 6, 9, 12, 15;  $\underline{a1(1)}$ ,  $\underline{a4(1)}$ , f#6(III),  $\underline{a7(1)}$ , a8-(II).
- 4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.
  - A) Intervals in cents/10: 360, 211, 149, 116, 95, 80, 69, 61, 55, 49, 45, 41, 39, 36, 34.
  - B) Int. in cents/40: 90, 52.75, 37.25, 29, 23.75, 20, 17.25, 15.25, 13.75, 12.25, 11.25, 10.25, 9.75, 9, 8.

<sup>+,-,\*:</sup> Micro-tonal deviations in relation to equal temperament; when analyzing characteristics applied only to deviations larger than 25c.