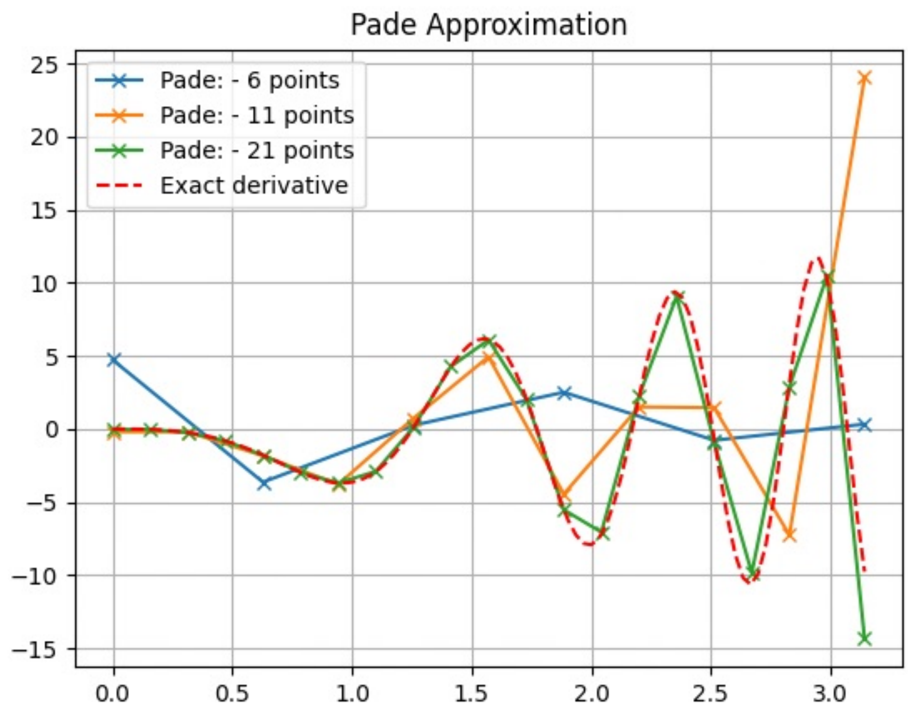
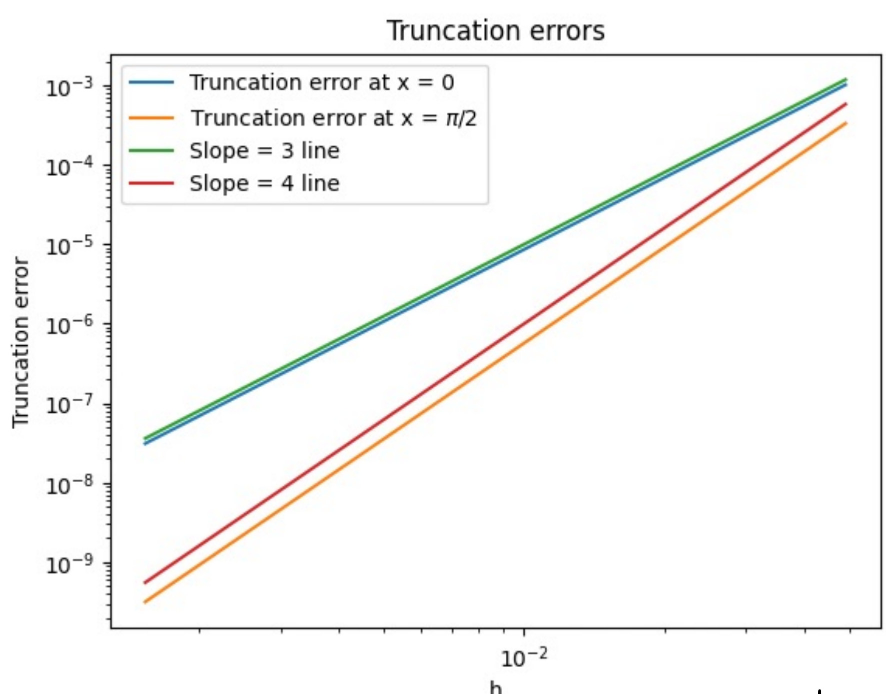
TAM 470 HW 3 solutions

Problem 2

(a)



(b)



The slopes are compared by plotting general h^3 vs h and h^4 vs h. np.polyfit will also gives slope = 3 for $x = 0$ and slope = 4 for $x = \pi/2$.

Attached is a code snippet for plotting:-

```
##
#Plots for part b
k = np.arange(6,12,1)
n_pts = 2**k + 1 #creating an array. Need to always have odd number of points to capture value at pi/2.

#Intitialize arrays for errors at x =0,pi/2 and storing h values.
err0_vec = np.zeros(len(n_pts))
h_vec = np.zeros(len(n_pts))
errmid_vec = np.zeros(len(n_pts))

for i in range(len(n_pts)):
    N = n_pts[i]
    Nmid = int((N-1)/2)#Mid point
    x = np.linspace(0,np.pi,N)#Generate a grid with N number of points
    f = np.cos(2*x**2) #Compute f array
    df = pade(x,f) #Call pade function
    h = x[1] - x[0] # Compute h for this grid.

    err0_vec[i] = np.abs(0 - df[0]) #Error at x = 0.
    errmid_vec[i] = np.abs(-4*np.pi/2*np.sin(2*(np.pi/2)**2) - df[Nmid]) # Error at x = pi/2.
    h_vec[i] = h
```

Problem 3

$$I = \int_0^{\pi/2} x^{1.5} \cos(x^2)\, dx \quad , \text{ let } f(x) = x^{1.5} \cos(x^2)$$

(a) Trapezoidal rule (n = 4, number of panels)

$$I_{trapezoidal} = h\left[\frac{1}{2} f_0 + \frac{1}{2} f_n + \sum_{i=1}^{n-1} f_i\right]$$

$$= \underline{0.10779}.$$

(b) Midpoint rule.

The given intervals are $\left\{ \left[0, \frac{\pi}{8}\right], \left[\frac{\pi}{8}, \frac{\pi}{4}\right], \left[\frac{\pi}{4}, \frac{3\pi}{8}\right], \left[\frac{3\pi}{8}, \frac{\pi}{2}\right] \right\}$

Midpoints, $x_{mid} = \left[\frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}, \frac{7\pi}{16}\right]$

Evaluate the function $f(x)$ at each of there midpoints, $h = \frac{\pi}{8}$.

$$I_{mid} = \sum_{i=0}^{N_{mid}-1} h\, f(x_{mid})_i = \underline{0.220946}$$

(c) Trapezoidal with end correction.

$$I_{trap,end} = I_{trapezoidal} - \frac{h^2}{12}\left(f'\left(\frac{\pi}{2}\right) - f'(0)\right)$$

$$= \underline{0.1762866}$$

(d) Simpson's rule

$$I_{Simpson} = \frac{h}{3}\left(f_0 + f_n + 4\sum_{i=1}^{n-1} f_i + 2\sum_{i=2}^{n-2} f_i\right)$$

$$= \frac{h}{3}\left(f_0 + f_4 + 4(f_1 + f_3) + 2f_2\right)$$

$$= \underline{0.1964066}$$

Scipy quadrature :- $I_{quad} = 0.182137$

Relative errors w.r.t I_{quad}.

(a) Trapezoidal — 40.81%

(b) Midpoint — 21.30%

(c) Trapezoid, end — 3.212%

(d) Simpson — 7.834%

Trapezoidal with end correction (c) is closest of I_{quad}.

Problem 4

h	$\tilde{I}$
0.2	12.045
0.1	11.801

Let $\tilde{I}_1 = 12.045$, $\tilde{I}_2 = 11.801$

Let I be the more accurate integral.

Simpson is $O(h^4)$ globally.

$$\therefore \quad I = \tilde{I}_1 + c_1 (0.2)^4 + O(h^6) \quad - (3.1)$$

$$I = \tilde{I}_2 + c_1 (0.1)^4 + O(h^6) \quad - (3.2)$$

$16 \times (3.2) \Rightarrow 16 I = 16 \tilde{I}_2 + c_1 (0.0016) + O(h^6) \quad - (3.3)$

(This step will eliminate c_1 terms).

$(3.3) - (3.1)$

$$\Rightarrow 15 I = 16 \tilde{I}_2 - \tilde{I}_1 + O(h^6)$$

$$\Rightarrow I = \frac{16 \tilde{I}_2 - \tilde{I}_1}{15} + O(h^6)$$

$$= \underline{11.784733} + O(h^6)$$

(The new I is 6^{th} order accurate!)