Spectral Methods

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1 What?

1. high degree of accuracy
2. combo of sines + cosines
3. complex functions
4. solving odes + pdes
5. fluid dynamics + turbulence modeling :-O

1.1 Fourier Series (Continuous)

$$f(x) = \sum_{k=-\infty}^{\infty} \hat{f}_k e^{ikx} \tag{1}$$

$$\text{where } e^{ikx} = \cos kx + i \sin kx \tag{2}$$

k is the wave number such that $k \in \mathbb{N}$ The derivative:

$$f^{'}(x) = \sum_{k=-\infty}^{\infty} ik\hat{f}_k e^{ikx} \tag{3}$$

1.2 Discrete Fourier Series

$N-1$ points vs. N points bc the end point is just f_0! bc of periodicity

$$f_j = \sum_{k=1} -\frac{N}{2}^{\frac{N}{2}} \hat{f}_k e^{ikx_j}, \quad j = 0, 1, 2, \ldots, N-1 \tag{4}$$

N is even

1.3 Discrete Orthogonality Property

$$\sum_{j=0}^{N-1} e^{ikx_j} e^{-ik'x_j} = \begin{cases} N, & \text{if } k = k' + mN, m = 0, \pm 1, \pm 2, \ldots; \\ 0, & \text{if } else. \end{cases} \tag{5}$$

1.4 Discrete Fourier Transform

$$\hat{f}_k \frac{1}{N} \sum_{i=0}^{N-1} f_j e^{-(ikx)} \tag{6}$$