HOMEWORK 4: DUE THURSDAY OCT 3

13.3(a), 13.4, 13.10(a), 13.12(b).

Problems not from the textbook.

A. Show that a sequence in a metric space (S, d) cannot have more than one limit.

B. (a) Suppose a sequence (s_n) in a metric space (S, d) converges to $s \in S$. Prove that any subsequence of (s_n) converges to s as well.
(b) Is a subsequence of a Cauchy sequence necessarily Cauchy?

C. Suppose (S, d) is a complete metric space, and $E \subset S$. We can view E as a metric space, equipped with the metric inherited from S. Prove that E is complete iff it is a closed subset of S.

D. Suppose (s_n) is a Cauchy sequence in a metric space (S, d), which has a convergent subsequence. Is it true that the sequence (s_n) itself converges?

Bonus problem: is the metric space (B, d) defined in Problem 13.3(a) complete?