Proofs By Calculation

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Cool, so I am starting from the beginning beginning. Let's run it (LRI)—

Problem 1. Let a and b be rational numbers and suppose that $a - b = 4$ and $ab = 1$. Show that $(a + b)^2 = 20$.

Solution 1. *Proof.*

$$(a+b)^2 = (a-b)^2 + 4ab \tag{1}$$

$$= 4^2 + 4 \cdot 1 \tag{2}$$

$$= 20 \tag{3}$$

□

Cool, now how do we do this in Lean? Check this out:

```
example {a b : \rat} (h1 : a - b = 4) (h2 : a * b = 1) : (a + b) ^ 2 = 20 :=
  calc
    (a + b) ^ 2 = (a - b) ^ 2 + 4 * (a * b) := by ring
    _ = 4 ^ 2 + 4 * 1 := by rw [h1, h2]
    _ = 20 := by ring
```

Ok let's do a new proof:

Proof.

$$d(af - be) = (ad)f - dbe \tag{4}$$

$$= (bc)f - dbe \tag{5}$$

$$= b(cf) - dbe \tag{6}$$

$$= b(de) - dbe \tag{7}$$

$$= 0 \tag{8}$$

□

Woah. Let's do it in Lean!

```
example {a b c d e f : \int} (h1 : a * d = b * c) (h2 : c * f = d * e) :
   d * (a * f - b * e) = 0 :=
  calc
    d * (a * f - b * e)
      = (a*d)*f-d*b*e := by ring
    _ = (b*c)*f-d*b*e := by rw [h1]
    _ = b*(c*f)-d*b*e := by ring
    _ = b*(d*e)-d*b*e := by rw [h2]
    _ = 0 := by ring
```

Fire. Onward. Yuh. Last easy example:

Problem 2. Let a and b be integers and suppose that $a = 2b + 5$ and $b = 3$. Show that $a = 11$.

Solution 2. *Proof.*

$$a = 2b + 5 \tag{9}$$

$$= 2 \cdot 3 + 5 \tag{10}$$

$$= 11 \tag{11}$$

□

Lean:

```
   example {a b : \int} (h1: a = 2 * b + 5) (h2 : b = 3) : a = 11:=
   calc
     a = 2 * b + 5 := h1
     _ = 2 * 3 + 5 := by rw [h2]
     _ = 11 := by ring
```

So I need to make sure to explicitly type in like *slash*int and such or it won't work...noted.