Proofs with structure II

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1 "For all" and implication

We finally get to the for all and implication tools!

Problem 1. Let a be a real number and suppose that for all real numbers x, it is true that $a \leq x^2 - 2x$. Show that $a \leq -1$.

Proof.

$$a \leq 1^2 - 2 \cdot 1 \tag{1}$$

$$= -1 \tag{2}$$

□

In Lean:

```
example {a : \real} (h : \forall x, a \leq x ^ 2 - 2 * x) : a \leq -1 :=
  calc
    a \leq 1 ^ 2 - 2 * 1 := by apply h
    _ = -1 := by numbers
```