Proofs with structure

Jerich Lee

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We now are able to create our own hypotheses—check it out:

Problem 1. Let a and b be real numbers and suppose that $a - 5b = 4$ and $b + 2 = 3$. Show that $a = 9$.

Solution 1. *Proof.* Since $b + 2 = 3$, we have $b = 1$. Therefore:

$$a = (a - 5b) + 5b \tag{1}$$

$$= 4 + 5 \cdot 1 \tag{2}$$

$$= 9 \tag{3}$$

□

Writing this in Lean:

```
example {a b : \real} (h1 : a - 5 * b = 4) (h2 : b + 2 = 3) : a = 9 := by
  have hb : b = 1 := by addarith [h2]
  calc
    a = a - 5 * b + 5 * b := by ring
    _ = 4 + 5 * 1 := by rw [h1, hb]
    _ = 9 := by ring
```

Pay attention to the *have* command on line 2 of the above; we can create our own givens and use it in our proofs. Coolio. We can use lemmas, or previously proved statements to

help prove our current statement. Check it out:

Problem 2. Let x be a rational number, and suppose that $3x = 2$. Show that $x \neq 1$.

Solution 2. *Proof.* It suffices to prove that $x < 1$:

$$x = \frac{3x}{3} \tag{4}$$

$$= \frac{2}{3} \tag{5}$$

$$< 1 \tag{6}$$

□

Lean:

```
example {x : \rat} (hx : 3 * x = 2) : x \neq 1 := by
    apply ne_of_lt
    calc
      x = 3 * x / 3 := by ring
      _ = 2 / 3 := by rw [hx]
      _ < 1 := by numbers
```