# PHYS 213: Thermal Physics—Final Exam

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**Problem 1** (Law of Atmospheres)**.**

$$g_{\text{Pluto}} = -\frac{\ln\left(\frac{n(h)}{n_0}\right) \cdot R \cdot T}{M \cdot h}$$

where:

$g_{\text{Pluto}} = $ local acceleration due to gravity on Pluto

$n(h) = $ concentration of nitrogen at height $h$

$n_0 = $ concentration of nitrogen at the surface

$R = $ universal gas constant (8.314 J/(mol·K))

$T = $ temperature in Kelvin (50 K)

$M = $ molar mass of nitrogen ($N_2$) in kg/mol (0.028 kg/mol)

$h = $ height above the surface in meters (10,000 m)

**Problem 2** (Boiling)**.**

$$dT = \frac{RT}{\Delta S_{\text{molar}}} \frac{dp}{P}$$

where:

$dT = $ change in boiling point temperature

$R = $ universal gas constant (8.314 J/(mol·K))

$T = $ boiling point temperature at atmospheric pressure (373.15 K)

$\Delta S_{\text{molar}} = $ entropy difference between liquid and gas per mole

$\quad = \Delta S \times M$

$\Delta S = $ entropy difference per kilogram ($6.05 \times 10^3$ J/kg·K)

$M = $ molar mass of water (0.018 kg/mol)

$dp = $ small pressure change ($3.00 \times 10^{-1}$ Pa)

$P = $ atmospheric pressure (101300 Pa)

**Problem 3** (Paramagnets)**.**

1.

$$\frac{N_{\text{down}}}{N_{\text{up}}} = e^{-\frac{2\mu B}{k_B T}}$$

where:

$N_{\text{down}}$ = number of electrons in the down state

$N_{\text{up}}$ = number of electrons in the up state

$\mu$ = magnetic moment of the electron $(9.27 \times 10^{-24} \text{ J/T})$

$B$ = magnetic field strength $(2 \text{ T})$

$k_B$ = Boltzmann constant $(1.38 \times 10^{-23} \text{ J/K})$

$T$ = temperature $(5 \text{ K})$

2.

If $kT \ll \mu B$ :

$M = N\mu$ (since nearly all electrons are in the up state)

Answer: (c) $M = N\mu$

3.

If $kT \gg \mu B$ :

$C_V = 0$ (since the magnetic moments are randomized)

Answer: (c) $C_V = 0$

**Problem 4** (Heat Capacity).

1.

$$N = \frac{C}{3k_B}$$

where:

$N$ = approximate number of atoms in the aluminum block

$C$ = heat capacity of the aluminum block $(30 \text{ J/K})$

$k_B$ = Boltzmann constant $(1.38 \times 10^{-23} \text{ J/K})$

2.

$Q_{\text{lost by aluminum}} = Q_{\text{gained by gas}}$

$C_{\text{Al}}(T_f - T_{i,\text{Al}}) = C_{\text{gas}}(T_f - T_{i,\text{gas}})$

where:

$C_{\text{Al}}$ = heat capacity of the aluminum block $(30 \text{ J/K})$

$T_{i,\text{Al}}$ = initial temperature of the aluminum $(300 \text{ K})$

$T_f$ = final temperature of both aluminum and gas $(345 \text{ K})$

$T_{i,\text{gas}}$ = initial temperature of the gas $(400 \text{ K})$

$C_{\text{gas}} = nc_{\text{v, gas}} = (1 \text{ mol}) \, c_{\text{v, gas}}$

$c_{\text{v, gas}}$ = molar heat capacity of the gas at constant volume

3.

$$c_{v,\text{ gas}} = \frac{C_{\text{Al}}(T_{\text{f}} - T_{\text{i,Al}})}{n(T_{\text{i,gas}} - T_{\text{f}})}$$

Substitute values:

$$c_{v,\text{ gas}} = \frac{30 \times (345 - 300)}{1 \times (400 - 345)} = 4.5 \,\text{J/K/mol}$$

Since $c_{v,\text{ gas}} = \frac{5}{2}R$ for polyatomic gases (like $NH_3$), we find:

Answer: (c) $NH_3$

**Problem 5** (Heat Engine Cycle).

1.

$$W_{\text{max}} = Q_H \left( 1 - \frac{T_C}{T_H} \right)$$

where:

$W_{\text{max}} =$ maximum work the engine can do in one cycle

$Q_H =$ heat drawn from the hot reservoir (1000 J)

$T_C =$ temperature of the cold reservoir in Kelvin $(25°C = 298\,\text{K})$

$T_H =$ temperature of the hot reservoir in Kelvin $(220°C = 493\,\text{K})$

2.

$$W_{\text{max}} = 1000 \left( 1 - \frac{298}{493} \right) \approx 395 \,\text{J}$$

Answer: (d) 395 J

3.

To achieve maximum theoretical work, the engine must operate reversibly, meaning:

Answer: (a) The cycle is reversible.

**Problem 6** (Harmonic Oscillator).

1.

$$\frac{P_1}{P_0} = e^{-\frac{\epsilon}{k_B T}}$$

where:

$P_1 =$ probability that the oscillator is in the first excited state (energy $E = \epsilon$)

$P_0 =$ probability that the oscillator is in the ground state (energy $E_0 = 0$)

$\epsilon =$ energy difference between levels, given by $\epsilon = hf$

$h =$ Planck's constant $(6.626 \times 10^{-34}\,\text{J s})$

$f =$ frequency of the oscillator $(7 \times 10^{11}\,\text{Hz})$

$k_B =$ Boltzmann constant $(1.38 \times 10^{-23}\,\text{J/K})$

$T =$ temperature in Kelvin

2.

$$T = \frac{\epsilon}{k_B \ln\left(\frac{1}{3}\right)} = \frac{hf}{k_B \ln(3)}$$

Substitute values:

$$T = \frac{6.626 \times 10^{-34} \times 7 \times 10^{11}}{1.38 \times 10^{-23} \times \ln(3)} \approx 30.6\,\text{K}$$

Answer: (e) 30.6 K

3.

$$\text{As } T \to \infty, \frac{P_1}{P_0} \to 1 \quad \text{(since both states become equally populated)}$$

Answer: (b) It tends towards a value of 1.

**Problem 7** (Thermal Equilibrium).

1.

$$\Delta S = mc_{\text{Cu}} \ln\left(\frac{T_{\text{final}}}{T_{\text{initial}}}\right)$$

where:

$$\Delta S = \text{entropy change of the copper sphere}$$
$$m = \text{mass of the copper sphere (5 kg)}$$
$$c_{\text{Cu}} = \text{specific heat of copper (385 J/kg-K)}$$
$$T_{\text{initial}} = \text{initial temperature of the copper sphere in Kelvin (473.15 K)}$$
$$T_{\text{final}} = \text{final temperature of the copper sphere in Kelvin (278.15 K)}$$

2.

$$\Delta S = 5 \times 385 \times \ln\left(\frac{278.15}{473.15}\right) \approx -1010.4\,\text{J/K}$$

Answer: (b) $-1010.4\,\text{J/K}$

**Problem 8** (Latent Heat).

1.

$$\Delta S = \frac{Q}{T} = \frac{mL}{T}$$

where:

$$\Delta S = \text{entropy change when the ice melts}$$
$$Q = \text{heat absorbed during melting}$$
$$m = \text{mass of the ice (9 kg)}$$
$$L = \text{latent heat of fusion for ice (333 kJ/kg)}$$
$$T = \text{temperature at which melting occurs (273.15 K)}$$

2.

$$\Delta S = \frac{9 \times 333000}{273.15} \approx 10972 \, \text{J/K} = 11 \, \text{kJ/K}$$
$$\text{Answer: (c)} \ 11 \, \text{kJ/K}$$

**Problem 9** (Triple Point of Gallium)**.** If pressure is increased, the phase with the highest density is favored in equilibrium. Given densities:

1. $\rho_{\text{solid}} = 5.91 \, \text{g/cm}^3$

2. $\rho_{\text{liquid}} = 6.05 \, \text{g/cm}^3$

3. $\rho_{\text{gas}} = 0.116 \, \text{g/cm}^3$

Since $\rho_{\text{liquid}}$ is the highest, the liquid phase is stabilized at higher pressures. **Answer:** (b) Liquid

**Problem 10** (Boltzmann Distribution and Entropy)**.**

1. The probability $P_1$ that the electron is in state 1—with energy $E_1 = 6.1 \, \text{eV}$—in thermal equilibrium at temperature $T = 1144 \, \text{K}$ is given by:

$$P_1 = \frac{e^{-\frac{E_1}{k_B T}}}{Z}$$

$$Z = \sum_{i=1}^{3} e^{-\frac{E_i}{k_B T}} = e^{-\frac{E_1}{k_B T}} + e^{-\frac{E_2}{k_B T}} + e^{-\frac{E_3}{k_B T}}$$

where:
$$E_1 = \text{energy of state 1 (6.1 eV)}$$
$$E_2 = \text{energy of state 2 (6.3 eV)}$$
$$E_3 = \text{energy of state 3 (6.6 eV)}$$
$$k_B = \text{Boltzmann constant } (8.617 \times 10^{-5} \, \text{eV/K})$$
$$T = \text{temperature (1144 K)}$$

Substitute the values to find $P_1$:

$$Z = e^{-\frac{6.1}{8.617 \times 10^{-5} \times 1144}} + e^{-\frac{6.3}{8.617 \times 10^{-5} \times 1144}} + e^{-\frac{6.6}{8.617 \times 10^{-5} \times 1144}} \approx 1.137$$

$$P_1 = \frac{e^{-\frac{6.1}{8.617 \times 10^{-5} \times 1144}}}{1.137} \approx 0.879$$
$$\text{Answer: (c)} \ 8.79 \times 10^{-1}$$

2. The entropy $S$ of the electron as $T \to 0$ approaches zero because, at absolute zero, the system will be in the ground state with no disorder. This results in $S = 0$ as $T \to 0$.

$$\text{Answer: (a)} \ 0.00 \, \text{eV/K}$$

**Problem 11** (Freezing Point Depression)**.**

1. The addition of sugar to water lowers the chemical potential of the liquid phase. This results in a depression of the freezing point because the solid phase (ice) now has a higher chemical potential relative to the liquid phase. Thus, the system requires a lower temperature to reach the equilibrium where the chemical potentials of the liquid and solid phases are equal.

2. **Answer:** (a) The mixing of the sugar lowered the chemical potential of the liquid water.

**Problem 12** (Phases of Matter).

1. The diagram shows chemical potential $\mu$ versus temperature $T$ for solid, liquid, and gas phases of a substance at a particular pressure. In thermodynamic equilibrium, a substance will minimize its chemical potential. Given that state $Q$ is at a higher chemical potential than the liquid phase at the same temperature, the substance will tend to transition to the liquid phase to lower its chemical potential.

2. **Answer:** (a) It will melt.

**Problem 13** (Phase Diagrams).

1. In a phase diagram, phases with higher entropy are generally favored at higher temperatures, as entropy tends to increase with temperature. Near the phase transition lines:

   - Phase III, being at higher temperatures relative to Phase I and Phase II, likely has more entropy per particle than the other phases.

   - Phase I, typically present at lower temperatures and higher pressures, likely has the highest density.

   Therefore, Phase III has more entropy per particle than Phase II.

2. **Answer:** (b) Phase III has more entropy per particle than Phase II.

**Problem 14** (Latent Heat and Binding Energy).

1. The binding energy $E_{\text{binding}}$ of a nitrogen molecule in the liquid phase is given by:

$$E_{\text{binding}} = \frac{Q}{nN_A}$$

$$n = \frac{\text{mass}}{\text{molar mass}} = \frac{20\,\text{g}}{28\,\text{g/mol}}$$

   where:

$$Q = \text{total heat required to vaporize the liquid nitrogen (4480 J)}$$

$$N_A = \text{Avogadro's number } (6.022 \times 10^{23}\,\text{mol}^{-1})$$

   Substitute the values to find $E_{\text{binding}}$:

$$E_{\text{binding}} = \frac{4480}{\left(\frac{20}{28}\right) \times 6.022 \times 10^{23}} \approx 9.4 \times 10^{-21}\,\text{J}$$

$$\text{Answer: (d) } 9.4 \times 10^{-21}\,\text{J}$$

**Problem 15** (Degrees of Freedom in a Gas).

1. The number of active degrees of freedom $f$ can be determined using the relationship:

$$f = \frac{2Q}{nR\Delta T}$$

   where:

$$Q = \text{heat added to the gas } (1.45 \times 10^4\,\text{J})$$

$$n = \text{number of moles (6 moles)}$$

$$R = \text{gas constant } (8.314\,\text{J/(mol K)})$$

$$\Delta T = T_{\text{final}} - T_{\text{initial}} = 383\,\text{K} - 300\,\text{K} = 83\,\text{K}$$

Substitute the values to find $f$:

$$f = \frac{2 \times 1.45 \times 10^4}{6 \times 8.314 \times 83} \approx 7$$

Answer: (b) 7

**Problem 16** (Three-State System)**.**

1. The probability $P_3$ that the system is in state 3 is given by:

$$P_3 = \frac{e^{-\frac{E_3}{k_B T}}}{Z}$$

$$Z = e^{-\frac{E_1}{k_B T}} + e^{-\frac{E_2}{k_B T}} + e^{-\frac{E_3}{k_B T}}$$

where:

$$E_1 = \text{energy of state 1 } (3.2 \times 10^{-20} \text{ J})$$
$$E_2 = \text{energy of state 2 (equal to } E_1)$$
$$E_3 = \text{energy of state 3 } (4.8 \times 10^{-20} \text{ J})$$
$$k_B = \text{Boltzmann constant } (1.38 \times 10^{-23} \text{ J/K})$$
$$T = \text{temperature } (605 \text{ K})$$

Substitute the values to find $P_3$:

$$Z = e^{-\frac{3.2 \times 10^{-20}}{1.38 \times 10^{-23} \times 605}} + e^{-\frac{3.2 \times 10^{-20}}{1.38 \times 10^{-23} \times 605}} + e^{-\frac{4.8 \times 10^{-20}}{1.38 \times 10^{-23} \times 605}}$$

$$P_3 = \frac{e^{-\frac{4.8 \times 10^{-20}}{1.38 \times 10^{-23} \times 605}}}{Z} \approx 0.0685$$

Answer: (e) 0.0685

2. To make the probability of being in state 1 equal to the probability of being in state 3, the energies $E_1$ and $E_3$ must be equal, which would make the Boltzmann factors for these states equal as well.

3. **Answer: (c)** $E_3 = E_1$

**Problem 17** (Heat Engine Efficiency)**.**

1. The maximum temperature $T_C$ of the cold reservoir is calculated based on the efficiency of the heat engine:

$$\eta = \frac{W}{Q_H} = 1 - \frac{T_C}{T_H}$$

$$T_C = T_H(1 - \eta)$$

where:

$$W = \text{work output of the engine } (600 \text{ W})$$
$$Q_H = \text{heat input from the hot reservoir } (1000 \text{ W})$$
$$T_H = \text{temperature of the hot reservoir } (373 \text{ K})$$

Substitute the values to find $T_C$:

$$\eta = \frac{600}{1000} = 0.6$$

$$T_C = 373 \times (1 - 0.6) = 149.2 \text{ K} \approx -124°\text{C}$$

Answer: (e) $-124°\text{C}$

**Problem 18** (Entropy of Coin Tosses)**.**

1. The entropy is minimized for outcomes that are farthest from the mean, as these are the least likely. For a fair coin flipped 16 times, the mean number of heads is:

$$\mu = n \cdot p = 16 \cdot 0.5 = 8$$

Values of $k$ near the extremes (such as 0 or 16 heads) will have the lowest entropy, as these outcomes have the smallest chance of occurring.

2. **Answer:** (c) 14 (assuming closest to extremes).

**Problem 19** (Average Energy of a Three-State System)**.**

1. The average energy $\langle E \rangle$ of the system as $T \to \infty$ is calculated by considering the degeneracies of each energy state:

$$\langle E \rangle = \frac{\sum_i g_i E_i}{\sum_i g_i}$$

where:
$$g_1 = 1 \quad \text{(degeneracy of state with } E_1 = 0.4 \, \text{eV)}$$
$$g_2 = 2 \quad \text{(degeneracy of state with } E_2 = 0.6 \, \text{eV)}$$

Substitute the values to find $\langle E \rangle$:

$$\langle E \rangle = \frac{(1 \times 0.4) + (2 \times 0.6)}{1 + 2}$$
$$= \frac{0.4 + 1.2}{3} = \frac{1.6}{3} \approx 0.533 \, \text{eV}$$
$$\text{Answer: (c) } 0.533 \, \text{eV}$$

**Problem 20** (Change in Gibbs Free Energy of a Diatomic Ideal Gas)**.**

1. The change in Gibbs free energy $\Delta G$ for a diatomic ideal gas expanding isothermally can be calculated using:

$$\Delta G = N k_B T \ln \left( \frac{P_f}{P_i} \right)$$

where:
$$N = \text{number of molecules } (7 \times 10^{23})$$
$$k_B = \text{Boltzmann constant } (1.38 \times 10^{-23} \, \text{J/K})$$
$$T = \text{temperature (600 K)}$$
$$P_i = \text{initial pressure (700 Pa)}$$
$$P_f = \text{final pressure (850 Pa)}$$

Substitute the values to find $\Delta G$:

$$\Delta G = (7 \times 10^{23}) \times (1.38 \times 10^{-23}) \times 600 \times \ln \left( \frac{850}{700} \right)$$

$$= 1125 \, \text{J}$$
$$\text{Answer: (c) } 1125 \, \text{J}$$

**Problem 21** (Vapor Pressure, Boiling Point, and Heat Capacity).

1. The Clausius-Clapeyron relation describes how vapor pressure varies with temperature:

$$\ln(P) = -\frac{L}{R}\frac{1}{T} + \text{constant}$$

   where:
   $$P = \text{vapor pressure}$$
   $$L = \text{latent heat of vaporization}$$
   $$R = \text{gas constant } (8.314 \text{ J/(mol K)})$$
   $$T = \text{temperature in Kelvin}$$

   A plot of $\ln(P)$ versus $\frac{1}{T}$ gives a straight line with slope $-\frac{L}{R}$.

2. To find the boiling point $T_2$ at $P_2 = 2\,\text{atm}$, we use:

$$\ln\left(\frac{P_2}{P_1}\right) = -\frac{L}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$
$$T_1 = 373\,\text{K} \quad \text{(boiling point at 1 atm)}$$
$$P_1 = 1\,\text{atm}, \quad P_2 = 2\,\text{atm}$$
$$L = 40650\,\text{J/mol}$$

   Rearranging and solving:
$$\frac{1}{T_2} = \frac{1}{T_1} - \frac{R}{L}\ln\left(\frac{P_2}{P_1}\right)$$

3. The average energy of a solid (as 3D oscillators) is given by:

$$U = 3N\epsilon\frac{e^{\frac{\epsilon}{k_B T}}}{e^{\frac{\epsilon}{k_B T}} - 1}$$
$$C = \frac{dU}{dT} \quad \text{(heat capacity)}$$

   At low $T$, $C$ increases with $T$; at high $T$, $C$ approaches a constant.

**Problem 22** (Chemical Potential and Phase Equilibrium).

1. Given the initial conditions:

$$N_L = 5\,\text{moles (in liquid phase)}$$
$$N_S = 2\,\text{moles (in solid phase)}$$
$$\mu_L = 9 \times 10^{-20}\,\text{J (chemical potential of liquid)}$$
$$\mu_S = 9.5 \times 10^{-20}\,\text{J (chemical potential of solid)}$$

   Since $\mu_L < \mu_S$, the liquid phase is favored. At equilibrium, all moles will be in the liquid phase.

2. Therefore, the equilibrium value of $N_L$ is:

$$N_L = N_L + N_S = 5 + 2 = 7\,\text{moles}$$
$$\text{Answer: (c) } 7\,\text{moles}$$

**Problem 23** (Heat Pump Work Requirement).

1. To find the minimum work $W$ required, we use the coefficient of performance (COP) of a heat pump:

$$\text{COP} = \frac{Q_H}{W} = \frac{T_H}{T_H - T_C}$$

$$W = \frac{Q_H}{\text{COP}}$$

where:

$$Q_H = Q_{\text{leak}} = 26\,\text{kW} \quad \text{(heat required to maintain house temperature)}$$
$$T_H = \text{inside temperature in Kelvin } (21°\text{C} = 294\,\text{K})$$
$$T_C = \text{outside temperature in Kelvin } (-11°\text{C} = 262\,\text{K})$$

Substitute the values to find COP and $W$:

$$\text{COP} = \frac{294}{294 - 262} = \frac{294}{32} \approx 9.19$$

$$W = \frac{26\,\text{kW}}{9.19} \approx 2.83\,\text{kW}$$

Answer: (a) $2.8\,\text{kW}$

**Problem 24** (Heat Capacity of a Cold Block).

1. The heat capacity $C$ of the block is calculated using the Carnot efficiency and the relationship between heat, work, and temperature change:

$$\eta = 1 - \frac{T_C}{T_H}$$

$$W = \eta Q = \left(1 - \frac{T_C}{T_H}\right) Q$$

$$Q = C(T_H - T_C)$$

$$C = \frac{W}{\left(1 - \frac{T_C}{T_H}\right)(T_H - T_C)}$$

where:
$W = 80\,\text{kJ} = 80000\,\text{J} \quad \text{(work obtained)}$
$T_H = \text{temperature of air reservoir } (294\text{ K})$
$T_C = \text{initial temperature of block } (77\text{ K})$

Substitute the values to find $C$:

$$\eta = 1 - \frac{77}{294} \approx 0.738$$

$$C = \frac{80000}{0.738 \times (294 - 77)} \approx 452\,\text{J/K}$$

Answer: (a) $4.52 \times 10^2\,\text{J/K}$

**Problem 25** (Heat Capacity of Solid Germanium).

1. To find the heat capacity $C$ for 4 kg of germanium, we use the molar heat capacity under the Dulong-Petit law, assuming equipartition holds:

$$C = n \cdot C_m = \frac{m}{M} \cdot 3R$$

where:

$$m = 4\,\text{kg (mass of germanium)}$$
$$M = 0.07263\,\text{kg/mol (molar mass of germanium)}$$
$$R = 8.314\,\text{J/(mol K) (gas constant)}$$
$$C_m = 3R = 3 \times 8.314\,\text{J/(mol K)} = 24.942\,\text{J/(mol K)}$$

Substitute the values to find $C$:

$$C = \frac{4}{0.07263} \cdot 24.942$$
$$\approx 1374\,\text{J/K}$$
$$\text{Answer: (d) } 1.37 \times 10^3\,\text{J/K}$$

**Problem 26** (Centrifugal Separation of Molecules).

1. To find the number of hemoglobin molecules $N(h_2)$ at height $h_2 = 0.2\,\text{m}$, we use the Boltzmann factor for the distribution of molecules in a gravitational (centrifugal) field:

$$N(h_2) = N(h_1) \cdot e^{-\frac{(mg_{\text{eff}})(h_2 - h_1)}{k_B T}}$$

where:

$$N(h_1) = 20000\,\text{molecules (at } h_1 = 0.03\,\text{m)}$$
$$h_2 = 0.2\,\text{m}$$
$$m = 1.07 \times 10^{-22}\,\text{kg (mass of hemoglobin molecule)}$$
$$g_{\text{eff}} = 3g = 3 \times 9.8 = 29.4\,\text{m/s}^2$$
$$k_B = 1.38 \times 10^{-23}\,\text{J/K (Boltzmann constant)}$$
$$T = 312\,\text{K (temperature)}$$

Substitute the values to find $N(h_2)$:

$$N(h_2) = 20000 \cdot e^{-\frac{(1.07 \times 10^{-22}) \times 29.4 \times (0.2 - 0.03)}{1.38 \times 10^{-23} \times 312}}$$
$$\approx 17700\,\text{molecules}$$
$$\text{Answer: (e) } 1.77 \times 10^4\,\text{molecules}$$

2. Since nitrogen ($N_2$) has a much smaller mass compared to hemoglobin, it will experience less centrifugal force and thus will be more prevalent at the top of the tube.

3. **Answer:** (a) $N_2$

**Problem 27** (Equipartition and Molar Mass of a Solid).

1. To find the molar mass $M$ of the solid, we use the relationship between heat, heat capacity, and temperature change:

$$C = \frac{Q}{\Delta T}$$
$$C_m = 3R = 3 \times 8.314\,\text{J/(mol K)} = 24.942\,\text{J/(mol K)}$$
$$M = \frac{m \cdot C_m}{C}$$

where:

$Q = 700\,\text{J}$    (heat applied)

$\Delta T = 6\,°\text{C}$    (temperature rise)

$m = 1\,\text{kg}$    (mass of the solid)

$R = 8.314\,\text{J}/(\text{mol K})$    (gas constant)

Substitute the values to find $C$ and then $M$:

$$C = \frac{700}{6} \approx 116.67\,\text{J/K}$$

$$M = \frac{1 \times 24.942}{116.67} \approx 214\,\text{g/mol}$$

Answer: (e) $214\,\text{g/mol}$

**Problem 28** (Work Done by an Expanding Ideal Gas).

1. To find the work $W$ done by the gas, we use the first law of thermodynamics:

$$\Delta U = Q - W$$
$$W = Q - \Delta U$$

The change in internal energy $\Delta U$ for a monatomic ideal gas is:

$$\Delta U = \frac{3}{2}nR\Delta T$$

where:

$$n = 9\,\text{moles}$$
$$R = 8.314\,\text{J}/(\text{mol K})$$
$$\Delta T = T_f - T_i = 280\,\text{K} - 250\,\text{K} = 30\,\text{K}$$

Substitute values to calculate $\Delta U$:

$$\Delta U = \frac{3}{2} \times 9 \times 8.314 \times 30$$
$$= 1116.57\,\text{J}$$

Then, calculate $W$:

$$W = Q - \Delta U$$
$$= 3523 - 1116.57$$
$$= 155.83\,\text{J}$$

Answer: $155.83\,\text{J}$

**Problem 29** (Chemical Equilibrium of a Material in a Cylinder).

1. To find the temperature $T$ at which the liquid and gas phases are in equilibrium, we use the chemical potential difference:

$$\mu_L - \mu_G = \Delta = -k_B T \ln\left(\frac{N_L}{N_G}\right)$$

where:

$$\Delta = 0.394\,\text{eV} \quad \text{(difference in chemical potential)}$$
$$k_B = 8.617 \times 10^{-5}\,\text{eV/K} \quad \text{(Boltzmann constant)}$$
$$\frac{N_L}{N_G} = 0.541 \quad \text{(ratio of molecules in liquid to gas phase)}$$

Substitute the values and rearrange to solve for $T$:

$$0 = 0.394 + k_B T \ln(0.541)$$

$$T = \frac{-0.394}{k_B \ln(0.541)}$$

$$= \frac{-0.394}{8.617 \times 10^{-5} \times \ln(0.541)}$$

$$\approx 7442.76\,\text{K}$$

Answer: $7442.76\,\text{K}$

**Problem 30** (Carnot Engine Temperature Ratio).

1. To find the ratio $\frac{T_C}{T_H}$ for a Carnot engine with efficiency $\eta = 0.30$, we use the efficiency formula:

$$\eta = 1 - \frac{T_C}{T_H}$$

where:

$$\eta = 0.30 \quad \text{(Carnot efficiency)}$$
$$T_C = \text{temperature of the cold reservoir}$$
$$T_H = \text{temperature of the hot reservoir}$$

Substitute the given value of $\eta$ and rearrange to solve for $\frac{T_C}{T_H}$:

$$0.30 = 1 - \frac{T_C}{T_H}$$

$$\frac{T_C}{T_H} = 1 - 0.30$$

$$= 0.70$$

Answer: (d) 0.70

**Problem 31** (Heat Capacity of a Cold Block).

1. To find the heat capacity $C$ of the block, we use the Carnot efficiency and the relationship between heat, work, and temperature change:

$$\eta = 1 - \frac{T_C}{T_H}$$

$$W = \eta Q = \left(1 - \frac{T_C}{T_H}\right) Q$$

$$Q = C(T_H - T_C)$$

$$C = \frac{W}{\left(1 - \frac{T_C}{T_H}\right)(T_H - T_C)}$$

2. where:

   (a) $W = 80\,\text{kJ} = 80000\,\text{J}$ (maximum work output)
   (b) $W = 80\,\text{kJ} = 80000\,\text{J}$ (maximum work output)

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(c) $W = 80\,\text{kJ} = 80000\,\text{J}$ (maximum work output)

Substitute the values to find $C$:

$$\eta = 1 - \frac{77}{294} \approx 0.738$$
$$C = \frac{80000}{0.738 \times (294 - 77)}$$
$$\approx 452\,\text{J/K}$$

Answer: (a) $4.52 \times 10^2\,\text{J/K}$