TAM 445 Lecture 3 1/23/24

Vector spaces

$\underset{\sim}{v} = \alpha_1 \underset{\sim}{e}_1 + \alpha_2 \underset{\sim}{e}_2 + \dots + \alpha_n \underset{\sim}{e}_n$

$\langle \underset{\sim}{u}, \underset{\sim}{v} \rangle = \alpha_1 \beta_1 + \dots \alpha_n \beta_n$

- α_i's are the "coordinates" of the vector $\underset{\sim}{v}$ w.r.t. the basis $\{e_1, \dots, e_n\}$
- Sometimes, I will use $u_1, \dots u_n$ as coordinates of $\underset{\sim}{u}$.

$\langle \underset{\sim}{u}, \underset{\sim}{v} \rangle = u_1 v_1 + \dots + u_n v_n$

<u>Euclidean Vector Space</u>: A finite dimensional real vector space with an inner product.

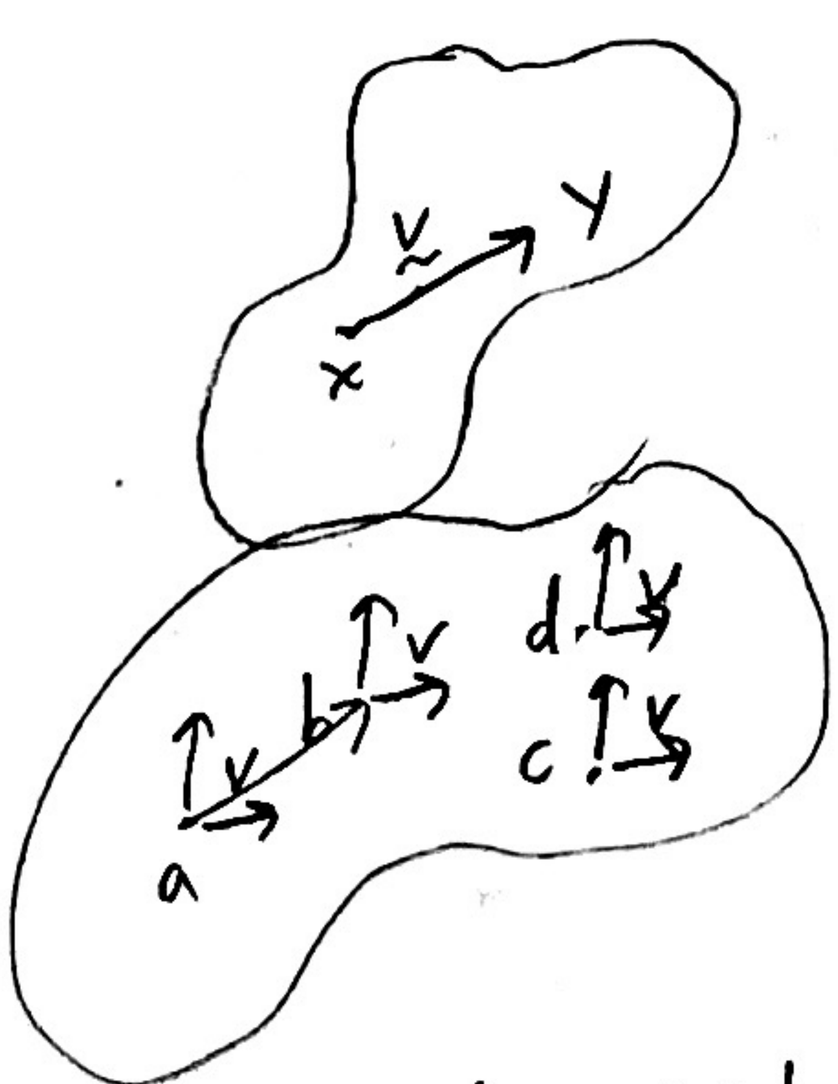
An example is the real coordinate space $\mathbb{R}^n$ equipped with the inner product

$\langle \underset{\sim}{u}, \underset{\sim}{v} \rangle = u_1 v_1 + \dots + u_n v_n$

$\underset{\sim}{u} = (u_1, \dots u_n) \in \mathbb{R}^n$, $\underset{\sim}{v} = (v_1, \dots v_n) \in \mathbb{R}^n$

~~Generalizes the notion~~ of angles between vectors

<u>Affine point space ($\mathcal{E}$)</u> is a collection of points and a vector space $\mathcal{V}$ such that for any two points $x, y \in \mathcal{E}$, there exists a unique vector $\underset{\sim}{v} \in \mathcal{V}$ which translates point x to y.

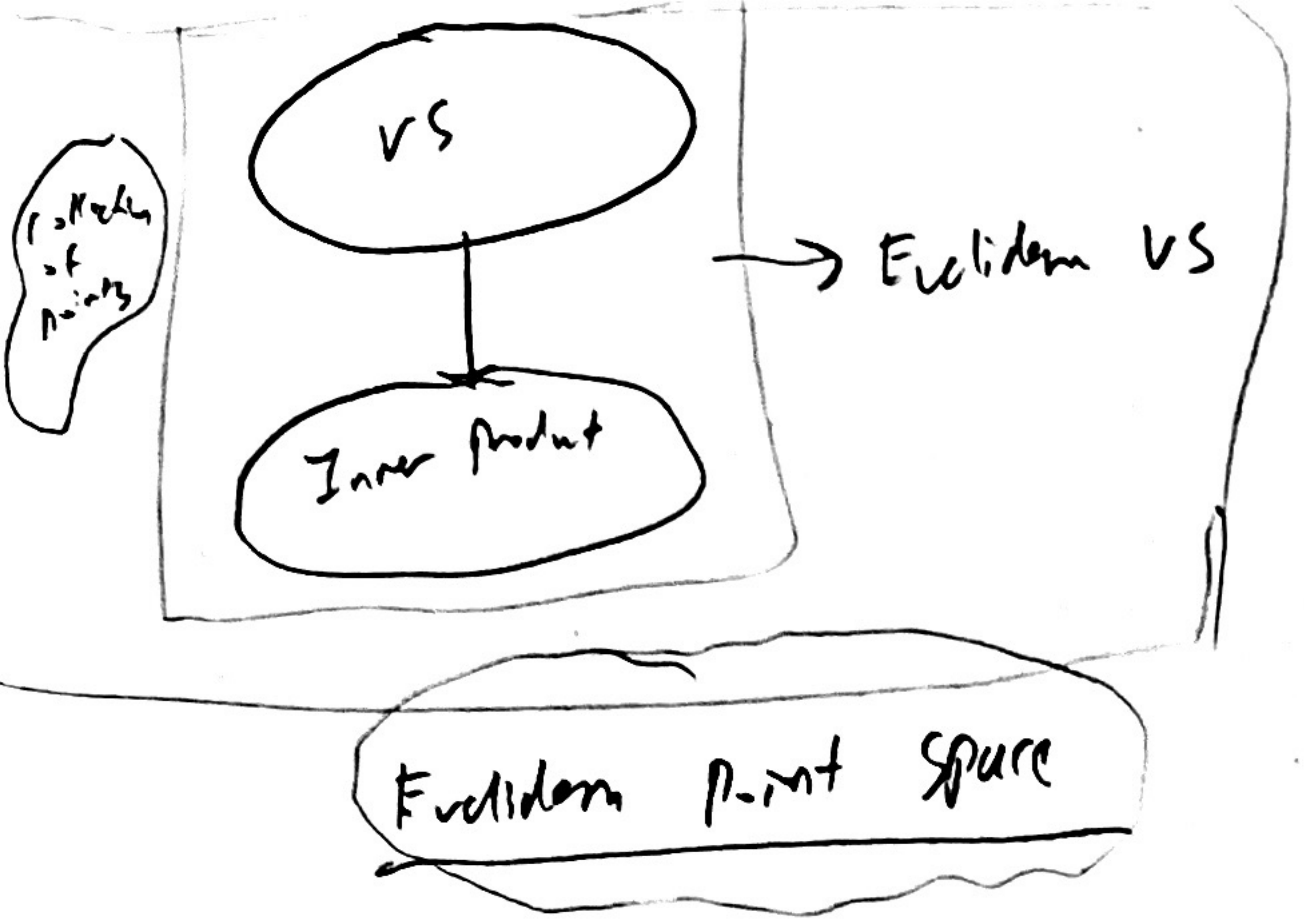


with the properties

a) $x + (\underset{\sim}{v} + \underset{\sim}{w}) = (x + \underset{\sim}{v}) + \underset{\sim}{w}$

b) $x + \underset{\sim}{0} = x$

<u>Euclidean point space</u>: An affine space with Euclidean vector space as its vector space.


collection of points
VS
Inner product
Euclidean VS
Euclidean point space