TAM-445 HW 8
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**Problem 1.**

1.

$$\mathbf{a}_s = \frac{\partial \mathbf{v}_s}{\partial t} + \text{grad}\mathbf{v}_s\mathbf{v}_s(\mathbf{x}, t) \tag{1}$$

$$= \begin{bmatrix} -ce^{-ct+x_3}\cos(\omega t) - \omega e^{-ct+x_3}\sin(\omega t) \\ -ce^{-ct+x_3}\sin(\omega t) + \omega e^{-ct+x_3}\cos(\omega t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & c^{-ct+x_3}\cos(\omega t) \\ 0 & 0 & e^{-ct+x_3}\sin(\omega t) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e^{x_3-ct}\cos(\omega t) \\ e^{x_3-ct}\sin(\omega t) \\ c \end{bmatrix} \tag{2}$$

Computing $\mathbf{a}_s \cdot \mathbf{v_s}$ where:

$$\mathbf{v}_s = \begin{bmatrix} e^{x_3-ct}\cos(\omega t) \\ e^{x_3-ct}\sin(\omega t) \\ c \end{bmatrix}$$

$$\mathbf{a}_s \cdot \mathbf{v}_s = 0$$

2.

$$\mathbf{a}_s = \begin{bmatrix} -\omega e^{-ct+x_3}\sin(\omega t) \\ \omega e^{-ct+x_3}\cos(\omega t) \\ 0 \end{bmatrix} \tag{3}$$

3.

$$D(\mathbf{x}, t) = \frac{1}{2}(\text{grad}\mathbf{v}_s + \text{grad}\mathbf{v}_s^{\mathrm{T}}) \tag{4}$$

$$= \begin{bmatrix} 0 & 0 & \frac{1}{2}e^{-ct+x_3}\cos(\omega t) \\ 0 & 0 & \frac{1}{2}e^{-ct+x_3}\sin(\omega t) \\ \frac{1}{2}e^{-ct+x_3}\cos(\omega t) & \frac{1}{2}e^{-ct+x_3}\sin(\omega t) & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} \tag{5}$$

$$= \begin{bmatrix} \frac{1}{4}e^{-ct}\cos(\omega t) \\ \frac{1}{4}e^{-ct}\sin(\omega t) \\ \frac{1}{4}e^{-ct}\cos(\omega t) \end{bmatrix} \tag{6}$$

Taking the dot product to obtain the scalar value:

$$D \cdot \text{dir} \tag{7}$$

$$= \frac{1}{2}e^{-ct+x_3}\cos(\omega t) \tag{8}$$

Integrating the third component of $\mathbf{v}_s$:

$$x_3 = c(t+1) + X_3 \tag{9}$$
$$\tag{10}$$

Substituting the above into $\mathbf{v}_s$:

$$\mathbf{v}_s = \begin{bmatrix} e^{ct+c-ct+X_3}\cos(\omega t) \\ e^{ct+c-ct+X_3}\sin(\omega t) \\ c \end{bmatrix} \tag{11}$$
$$= \begin{bmatrix} e^{X_3+c}\cos(\omega t) \\ e^{X_3+c}\sin(\omega t) \\ c \end{bmatrix} \tag{12}$$

Integrating the components of $\mathbf{v}_s$:

$$\mathbf{x} = \begin{bmatrix} e^{X_3+c}\sin(\omega t) \\ e^{X_3+c}\cos(\omega t) \\ c(t+1) \end{bmatrix} \tag{13}$$

**Problem 2.**

$$\lambda_3 \to n = v_3 \tag{14}$$
$$|N| = |T\mathbf{n}\cdot\mathbf{n}| \leq |T\mathbf{v}_3 \cdot v_3| \tag{15}$$
$$= |\lambda\mathbf{v}_3 \cdot \mathbf{v}_3| \tag{16}$$
$$\implies \parallel T\mathbf{n}\cdot\mathbf{n} \parallel \leq |\lambda_3| \tag{17}$$

$$|T\mathbf{n}\cdot\mathbf{n}| = \tag{18}$$
$$|T(k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k_3\mathbf{v}_3)\cdot(k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k_3\mathbf{v}_3)| \tag{19}$$
$$= |k_1T\mathbf{v}_1 + k_2T\mathbf{v}_2 + k_3Tv_3 \cdot k_1T\mathbf{v}_1 + k_2T\mathbf{v}_2 + k_3Tv_3| \tag{20}$$
$$= |\lambda k_1\mathbf{v_1} + \lambda_2k_2\mathbf{v_2} + \lambda_3k_3\mathbf{v_3} \cdot \lambda_1k_1\mathbf{v_1} + \lambda_2k_2\mathbf{v_2} + \lambda_3k_3\mathbf{v_3}| \tag{21}$$
$$= |\lambda_1k_1^2 + \lambda_2k_2^2 + \lambda_3k_3^2| \tag{22}$$

$$\lambda_1\cos^2\theta_1 + \lambda_2\cos^2\theta_2 + \lambda_3\cos^2\theta_3 \leq |\lambda_3|(\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3) \leq |\lambda_3| \tag{23}$$