TAM 445 Continuum Mechanics HW-1

1/18/24

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1. a) $a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3$, rank $= 0$, # exp. $= 1$

b) $a_i b_j = \begin{bmatrix} a_1 b_1 + a_1 b_2 + a_1 b_3 \\ + a_2 b_1 + a_2 b_2 + a_2 b_3 \\ + a_3 b_1 + a_3 b_2 + a_3 b_3 \end{bmatrix}$, rank $= 2$, # exp. $= 9$

c) $\sigma_{ik} n_k = \begin{bmatrix} \sigma_{1k} n_k = \sigma_{11} n_k + \sigma_{12} n_2 + \sigma_{13} n_3 \\ \sigma_{2k} n_k = \sigma_{21} n_1 + \sigma_{22} n_2 + \sigma_{23} n_3 \\ \sigma_{3k} n_k = \sigma_{31} n_1 + \sigma_{32} n_2 + \sigma_{33} n_3 \end{bmatrix}$, rank $= 1$, # exp $= 3$

d) $A_{ij} x_i x_j$ (A is symmetric, i.e. $A = A^T$)

$$= A_{ji} x_i x_j = \left[A_{11} x_1 x_1 + A_{22} x_2 x_2 + A_{33} x_3 x_3 \right]$$

$+ A_{12}$ ~~x~~ rank $= 0$
exp $= 1$

e) $\frac{\partial u_i}{\partial z_k} \frac{\partial z_k}{\partial x_j}$

$$= \frac{\partial u_i}{\partial x_j} = \frac{\partial_j u_i}{\partial u_k} \cdot \frac{\partial u_k}{\partial x_j} \begin{bmatrix} \partial_1 u_1 & \partial_1 u_2 & \partial_1 u_3 \\ \partial_2 u_1 & \partial_2 u_2 & \partial_2 u_3 \\ \partial_3 u_1 & \partial_3 u_2 & \partial_3 u_3 \end{bmatrix}$$

rank $= 2$
exp $= 9$

f) $\sigma_{ij,j} + \rho b_i = \rho a_i$, where $\sigma_{ij,j} := \frac{\partial \sigma_{ij}}{\partial x_j} = \partial_j \sigma_{ij}$

$$\partial_j \sigma_{ij} + \rho b_i = \rho a_i = \begin{bmatrix} (\partial_1 \sigma_{11} + \partial_2 \sigma_{12} + \partial_3 \sigma_{13}) + \rho b_1 = \rho a_1 \\ (\partial_1 \sigma_{21} + \partial_2 \sigma_{22} + \partial_3 \sigma_{23}) + \rho b_2 = \rho a_2 \\ (\partial_1 \sigma_{31} + \partial_2 \sigma_{32} + \partial_3 \sigma_{33}) + \rho b_3 = \rho a_3 \end{bmatrix}$$

rank $= 2$
exp. $= 9$

2) a) $\delta_{mm}\delta_{nn}$

$= (3)(3)$

$= 9$

b) $X_I \delta_{IK} \delta_{JK}$

$= X_K \delta_{JK}$

$= X_J$

c) $B_{ij}\,\delta_{ij}$

$= -B_{ji}\,\delta_{ij}$

$= -B_{kk}$

d) $\left[A_{ij}B_{jk} - 2A_{im}B_{mk}\right]\delta_{ik}$

$A_{ij}B_{jk}\delta_{ik} - 2A_{im}B_{mk}\delta_{ik}$

$= A_{ij}B_{ji} - 2A_{im}B_{mi}$

$= A_{ij}B_{ji} - 2A_{ij}B_{ji}$

$= -A_{ij}B_{ji}$

* e) $A_{ij} = B_{ik}C_{kj}$ into $\phi = A_{mk}C_{mk}$

$\phi = A_{ij}C_{ij}$

$\phi = (B_{ik}C_{kj})C_{ij}$

f) $\epsilon_{ijk}a_i a_j a_k = \epsilon_{111}a_1a_1a_1 + \epsilon_{222}a_2a_2a_2 + \epsilon_{333}a_3a_3a_3$

$= 0 + 0 + 0$

$= 0$

$$\frac{\partial A^{-1}_{ip}}{\partial A_{mn}} = -A^{-1}_{im} A^{-1}_{np}$$

$$A^{-1}_{ik} A_{kj} = \delta_{ij}$$

$$\frac{\partial A^{-1}_{ik} A_{kj}}{\partial A_{mn}} = \frac{\partial \delta_{ij}}{\partial A_{mn}}$$

$$\frac{\partial A^{-1}_{ik}}{\partial A_{mn}} A_{kj} + A^{-1}_{ik} \frac{\partial A_{kj}}{\partial A_{mn}} = 0$$

$$\frac{\partial A^{-1}_{ik}}{\partial A_{mn}} = -$$

4.

$$\boxed{\frac{\partial A^{-1}_{ik}}{\partial A_{mn}} = A^{-1}_{im} A^{-1}_{np}}$$

$$A_{kj} A^{-1}_{ik} = \delta_{ij}$$

$$= \frac{\partial A_{kj} A^{-1}_{ik}}{\partial A_{mn}} = \frac{\partial \delta_{ij}}{\partial A_{mn}}$$

$$= \frac{\partial A_{kj}}{\partial A_{mn}} A^{-1}_{ik} + A_{kj} \frac{\partial A^{-1}_{ik}}{\partial A_{mn}} = 0$$

$$A_{kj} \frac{\partial A^{-1}_{ik}}{\partial A_{mn}} = -\frac{\partial A_{kj}}{\partial A_{mn}} A^{-1}_{ik}$$

$$(A^{-1}_{jp}) A_{kj} \frac{\partial A^{-1}_{ik}}{\partial A_{mn}} = -\delta_{km}\delta_{jn} A^{-1}_{ik} (A^{-1}_{jp})$$

$$\delta_{pk} \frac{\partial A^{-1}_{ik}}{\partial A_{mn}} = -\delta_{km}\delta_{jn} A^{-1}_{ik} (A^{-1}_{jp})$$

$$=$$

g) $(x_m x_m x_i A_{ij})_{,k} =$

rnk: 2,
#exp: 9

$\partial_k x_m x_m x_i A_{ij}$

$= \Big[\partial_1 x_1 x_1 x_1 A_{11} + \partial_1 x_2 x_2 x_2 A_{21} + \partial_1 x_3 x_3 x_3 A_{31}$

$\partial_1 x_1 x_1 x_1 A_{12} + \partial_1 x_2 x_2 x_2 A_{22} + \partial_1 x_3 x_3 x_3 A_{32}$;

3. a) $A_{ii} = A_{11} + A_{22} + A_{33}$

b) $A_{ji} A_{ij} = A^T A$

c) $(A_{ii}) A_{kk} = A_{11}^2 + A_{22}^2 + A_{33}^2$

d) $B_{i1} \frac{\partial c_1}{\partial x_j} + B_{i2} \frac{\partial c_2}{\partial x_j} + B_{i3} \frac{\partial c_3}{\partial x_j}$

$= B_{ik} \frac{\partial c_k}{\partial x_j}$

e) $(u_1^2 + u_2^2 + u_3^2)(v_1^2 + v_2^2 + v_3^2)$

$= u_i u_i v_k v_k$

f)

f) $A_{11} = B_{11} C_{11} + B_{12} C_{21}$

$$A_{ij} = B_{ik} C_{kj} + B_{ik} C_{kj}$$

$$\frac{\partial \lambda}{\partial A_{kl}} = A_{ij} \delta_{ik} \delta_{jl} + \delta_{ik} \delta_{jl} A_{ij}$$

$$= A_{il} \delta_{ik}$$

$$= \delta_{ki} A_{il}$$

$$= A_{kl}$$

4) $\dfrac{\partial A^{-1}_{ip}}{\partial A_{mn}} = -A^{-1}_{im} A^{-1}_{np}$, $A^{-1}_{ik} A_{kj} = \delta_{ij}$

$$\frac{\partial A_{ij}}{\partial A_{kl}} = A_{ij} \delta_{ik} \delta_{jl}$$

$$\frac{\partial A^{-1}_{ip}}{\partial A_{mn}} = -(A_{ip})^{-2} \frac{\partial A_{ip}}{\partial A_{mn}} = -\frac{1}{A_{ip}^2} \delta_{im} \delta_{pn}$$

$$= A^{-1}$$

$$A_{ij} = \delta_{ik} A_{kj}$$

$$-\frac{1}{A_{ip}^2} (A^{-1}_{ik} A_{km}) (A^{-1}_{pl} A_{ln})$$

$$A^{-1}_{kj} A_{ij} = \delta_{ik}$$

$$\cancel{= -A_{ip}^{-2} \delta_{mi} \delta_{np}}$$

$$\cancel{= -\frac{1}{A_{ip}^2} (A^{-1}_{mk} A_{ki}) (A^{-1}_{nl} A_{lp})}$$

$$= \frac{-A^{-1}_{ik}}{A^{-1}_{ki} A_{ip} A_{ik}}$$

$$- A^{-1}_{ip} A^{-1}_{ip} (A^{-1}_{mk} A_{ki}) (A^{-1}_{nl} A_{lp})$$