TAM 445 - Lecture 1 1/16/24

$$\rho_0 \left[\dot{v} + (\mathrm{grad}\, v) v \right] = \mu \Delta v - \mathrm{grad}\, \pi,$$

$$\mathrm{div}\, v = 0$$

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

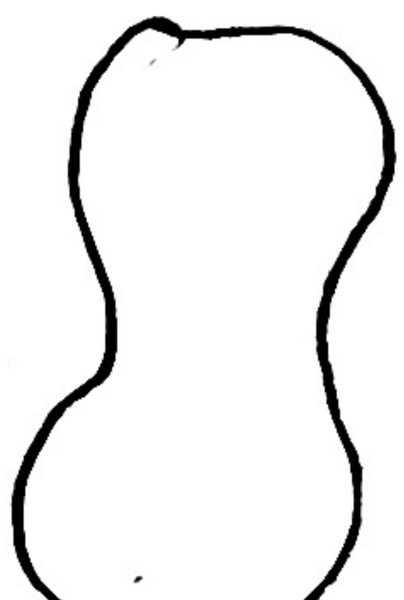
Where did we get these equations?

What is mechanics?

- Study of motion of "bodies" subjected to external loads
- continuum mechanics: bodies are modeled as continuous masses

particles

Mechanics → Classical, Quantum



- language of continuum mechanics
  - a continuum body
  - deformation, aka kinematics
  - forces, stress tensor (Force/Area)
  - balance laws, laws of thermodynamics
  - constitutive laws

→ Elastodynamics, Navier-Stokes, Fourier

1) space, time, & frames of reference

1687 Sir Isaac Newton

- Every body remains in a state
- "resting or moving" - related to what?
- "rate of change"

Frame of reference is a rigid physical object (earth, lab, fixed stars) related to which space is described and a clock to measure time

- There is no absolute frame
- Inertial frames

- You can have multiple frames of references

Einstein vs. Newton, time is dilated vs. time is not

2) Scalars, vectors, tensors

Indicial notation: scheme where a list of real numbers is represented by a single symbol

eg: a_i denotes a_1, a_2, a_3

a_{ij} $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \cdot & & \\ & & \end{bmatrix}_{3\times3}$

Summation & Dummy Indices

$S = a_1x_1 + a_2x_2 + a_3x_3$

$\sum_{i=1}^{3} a_i x_i$

$S = a_i x_i$

whenever an index appears $\boxed{\text{twice}}$ in a $\boxed{\text{product}}$, it is a dummy index over which a sum is applied

$\sigma_{ii} = \sigma_{11} + \sigma_{22} + \sigma_{33}$

a_i $a_i + b_i$

b_i

$A_{ij} y_j$ represents three numbers

$A_{1j}y_j = A_{11}y_1 + A_{12}y_2 + A_{13}y_3$

$A_{2j}x_j =$

$A_{3j}x_j$

Free index: An index that appears only once in a product term of an expression or equation, number of answers is equal to free indices, $3^{(\text{free indices})}$ = # free vari

dummy indices are basically dot products

$$\boxed{D_{ijk} x_k = A_{ij}}$$ 9 values, 9 equations

free indices should appear on the right hand side

2) Scalars, Vectors, Tensors

(1823-1931)

Kronecker Delta

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

↓

identity $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

But not really, more of a tool

$$\frac{\partial x_i}{\partial x_j} = \delta_{ij}$$

1) $a_i \delta_{ij} = a_j$ $\quad a_1 \delta_{11} + \cancel{a_2 \delta_{21}} + \cancel{a_3 \delta_{31}}$

2) $\cancel{\phi}$ 3

3) $A_{ij} \delta_{ij} = A_{11} + A_{22} + A_{33}$

zero free indices, $3^0 = 1$

Ask yourself how many numbers you get

Permutation or Levi Civita symbol

$$\varepsilon_{ijk} = \begin{cases} 1 & \text{if } i,j,k \text{ form an even permutation of } 1,2,3 \\ -1 & \text{" " " " odd " " "} \\ 0 & \text{" " do not form a permutation} \end{cases}$$

$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = 1$

$\varepsilon_{213} = -1$

$\varepsilon_{112} = 0$