TAM 445 Continuum Mechanics - Spring 2024
Homework 1 - Indicial Notatoin

Due: Jan 26, 2022

1. Expand the following indicial expressions (all indicess range from 1 to 3). Indicate the rank (number of free indices) and the number of resulting expressions.

(a) $a_i b_i$

(b) $a_i b_j$

(c) $\sigma_{ik} n_k$

(d) $A_{ij} x_i x_j$ ($\boldsymbol{A}$ is symmetric, i.e. $\boldsymbol{A} = \boldsymbol{A}^{\mathrm{T}}$)

(e) $\frac{\partial u_i}{\partial z_k} \frac{\partial z_k}{\partial x_j}$

(f) $\sigma_{ij,j} + \rho b_i = \rho a_i$, where $\sigma_{ij,j} := \frac{\partial \sigma_{ij}}{\partial x_j}$

2. Simplify the following indicial expressions as much as possible (all indices range from 1 to 3).

(a) $\delta_{mm} \delta_{nn}$

(b) $X_I \delta_{IK} \delta_{JK}$

(c) $B_{ij} \delta_{ij}$ ($\boldsymbol{B}$ is anti-symmetric, i.e. $\boldsymbol{B} = -\boldsymbol{B}^{\mathrm{T}}$)

(d) $[A_{ij} B_{jk} - 2 A_{im} B_{mk}] \delta_{ik}$

(e) Substitute $A_{ij} = B_{ik} C_{kj}$ into $\phi = A_{mk} C_{mk}$

(f) $\epsilon_{ijk} a_i a_j a_k$

(g) $(x_m x_m x_i A_{ij})_{,k}$ where $\square_{,k}$ denotes derivative with respect to x_k.

3. Write out the following expressions in indicial notation whenever possible

(a) $A_{11} + A_{22} + A_{33}$

(b) $\boldsymbol{A}^{\mathrm{T}} \boldsymbol{A}$ where $\boldsymbol{A}$ is a 3×3 matrix

(c) $A_{11}^2 + A_{22}^2 + A_{33}^2$

(d) $B_{i1} \frac{\partial c_1}{\partial x_j} + B_{i2} \frac{\partial c_2}{\partial x_j} + B_{i3} \frac{\partial c_3}{\partial x_j}$

(e) $(u_1^2 + u_2^2 + u_3^2)(v_1^2 + v_2^2 + v_3^2)$

(f) $A_{11} = B_{11} C_{11} + B_{12} C_{21} \qquad A_{12} = B_{11} C_{12} + B_{12} C_{22}$
$A_{21} = B_{21} C_{11} + B_{22} C_{21} \qquad A_{22} = B_{21} C_{12} + B_{22} C_{22}$

4. Show that $\partial A_{ip}^{-1} / \partial A_{mn} = -A_{im}^{-1} A_{np}^{-1}$, where $\boldsymbol{A}$ is a square matrix. Hint: Start with the identity $A_{ik}^{-1} A_{kj} = \delta_{ij}$. Use indicial notation in your derivation.