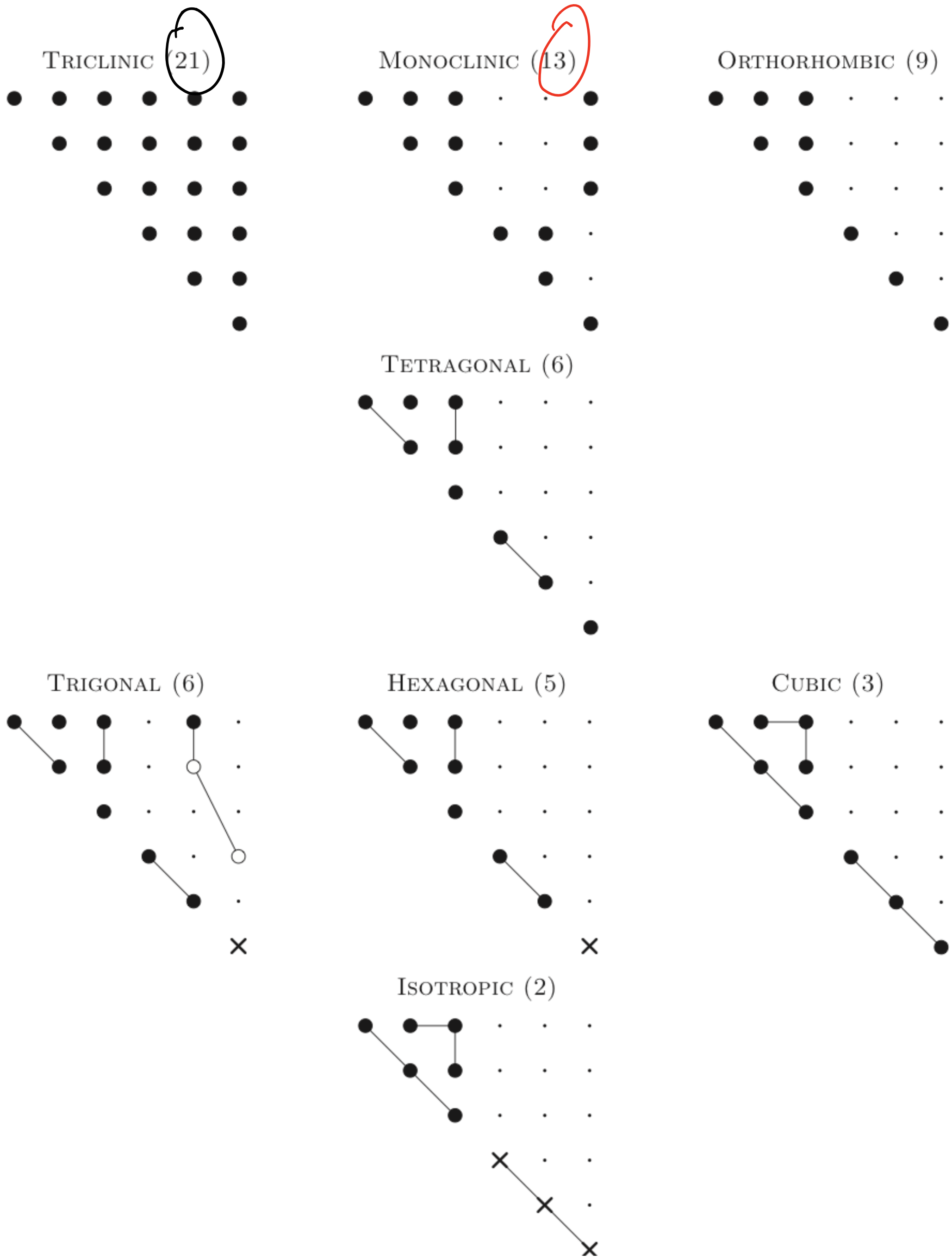
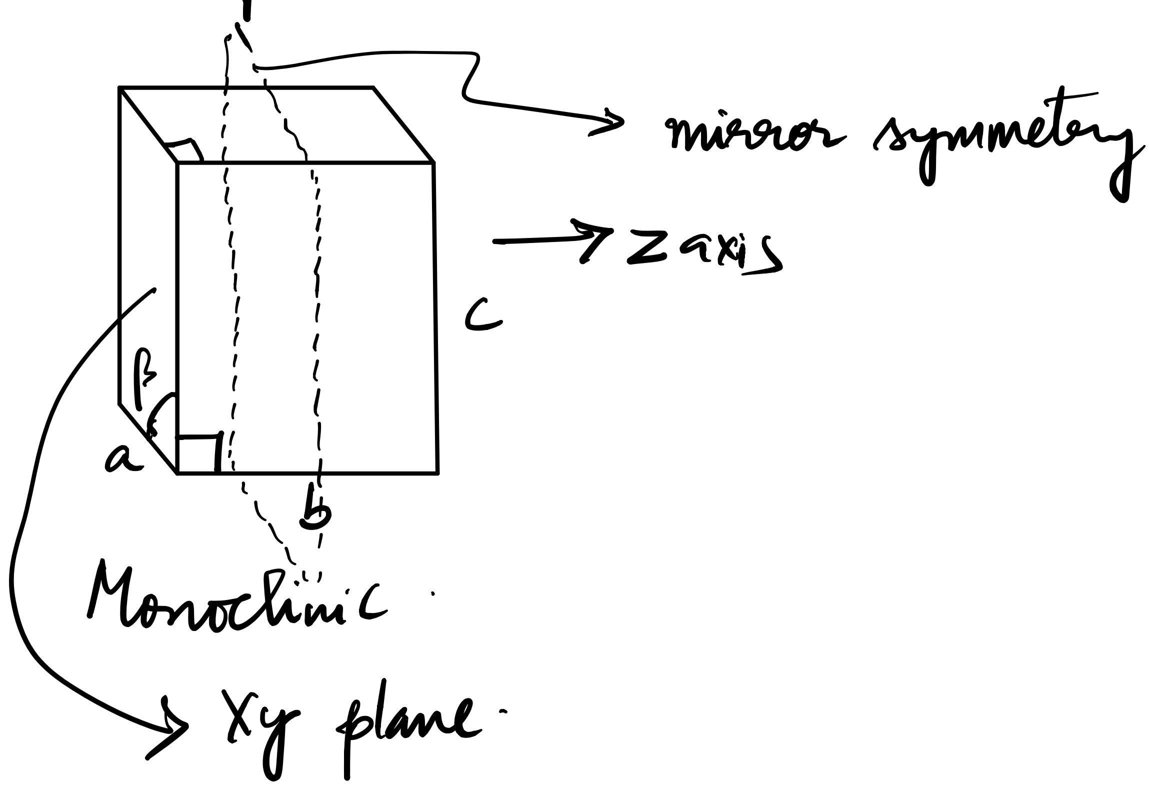
04.30.2020

TAM 445 Continuum Mechanics

Lecture 27



For the simple case of monoclinic we will now see how restrictions on the elasticity tensors arise:



Recall the restriction on $\mathbb{C}$ due to material symmetry:

$$\mathbb{C}_{IJKL} = Q_{IP} Q_{JQ} Q_{KR} Q_{LS} \mathbb{C}_{PQRS} \quad \forall\, Q \in M_{sg} \quad (1)$$

If $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ → reflection about the xy plane (2)

Substituting ② into ①, we obtain

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ & & c_{33} & c_{34} & c_{35} & c_{36} \\ & & & c_{44} & c_{45} & c_{46} \\ & & & & c_{55} & c_{56} \\ & & & & & c_{66} \end{bmatrix} =$$

$= 0$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & -c_{14} & -c_{15} & c_{16} \\ & c_{22} & c_{23} & -c_{24} & -c_{25} & c_{26} \\ & & c_{33} & -c_{34} & -c_{35} & c_{36} \\ & & & c_{44} & c_{45} & -c_{46} \\ & & & & c_{55} & -c_{56} \\ & & & & & c_{66} \end{bmatrix}$$

Question : How can we get all possible material classes ? Two approaches:

a) Visual inspection like above
   ↳ pain staking
   → Results in 10 distinct symmetry classes

   INCORRECT

b) Mathematical approach using group theory ⟶ CORRECT.

There are only 8 distinct symmetry classes !

This was shown not so long ago by Forte and Vianello in 1996 —
   "Symmetry Classes for elasticity tensors"
Journal of Elasticity, 1996.

Boundary value problems

We will restrict our discussion to solids.

Imagine a solid being subjected to external loads and you would like to measure the deformation of the object. In addition, you would like to measure the stress field to identify places where stress is concentrated. For simplicity, we will assume that the body is in equilibrium, and there are no body forces $\Rightarrow \operatorname{div} T = \underset{\sim}{0}$.

For solids it is common to have a constitutive law described referentially, i.e $\psi(C)$, as opposed to in fluids where we had $\psi(\rho_s)$. Recall from lecture 20, the momentum balance equation, described referentially, is given by:

$$\mathrm{Div}(P) = 0,$$

where P is the first Piola Kirchhoff stress. From lecture 24, recall that $T = 2\rho_s F \frac{\partial \psi}{\partial C} F^T$. From lecture 20, eq 7, $P = (\det F) T F^{-T}$

$$= 2\rho_0 \frac{\partial \psi}{\partial C}.$$

Therefore, momentum balance says

$$\mathrm{Div}\left(2\rho_0 \frac{\partial \psi}{\partial C}\right) = 0 \quad \text{——} (3)$$

Putting it all together :

Unknown : deformation field $\underset{\sim}{x}(\underset{\sim}{p})$ or $f(\underset{\sim}{p})$.

Derived :

$$F = \nabla f$$

$$C = F^T F$$

Therefore (3) is a partial differential equation (PDE) for $\underset{\sim}{x}(\underset{\sim}{p})$. External loads are described as boundary conditions to the PDE. Boundary conditions are of different kinds :

a) Traction boundary condition

$$P \underset{\sim}{m} = \underset{\sim}{t} \quad \text{on} \quad \partial \mathcal{B}_0$$

surface normal to $\partial \mathcal{B}_0$

given traction field on the surface of $\mathcal{B}$

b) Displacement boundary condition

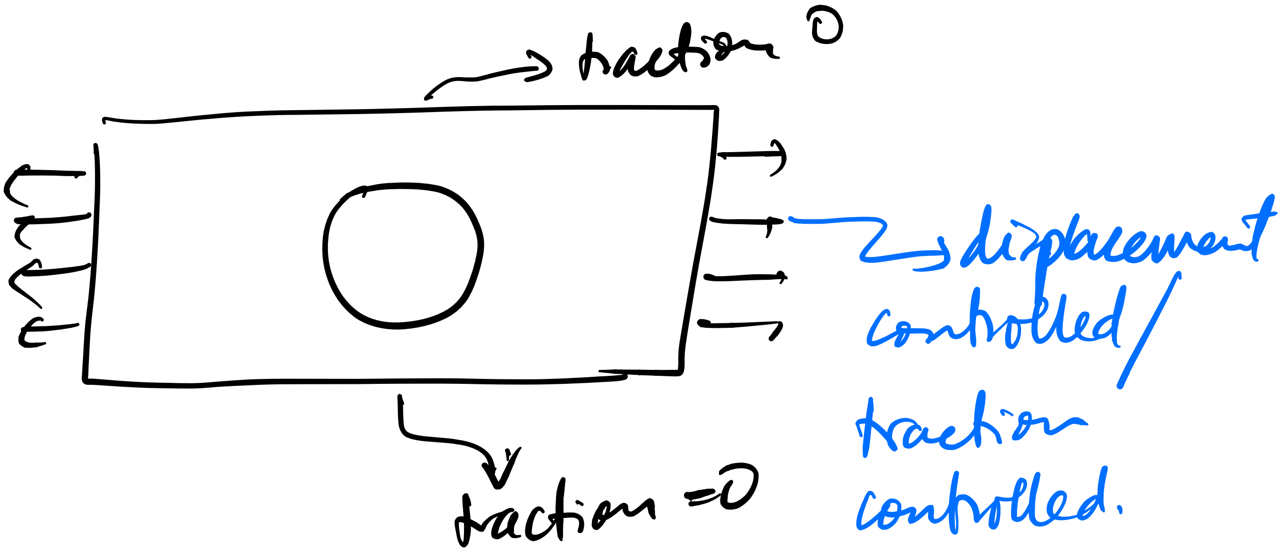
$$u = \underset{\sim}{u_0} \quad \text{on} \quad \partial \Omega$$

c) Mixed :

$$u = u_0 \quad \text{on} \quad \partial \Omega_1$$

$$Tn = t \quad \text{on} \quad \partial \Omega_2$$

where $\partial \Omega = \partial \Omega_1 \cup \partial \Omega_2$



CAUTION: At a point on the boundary, you can either have traction boundary condition or a displacement " condition but not both. You can still have a situation where at a point $\underline{p} \in \partial \mathcal{B}_0$,

$$u_1, \quad (P\underline{n})_2 \quad \text{are specified}$$

(assuming a 2D problem). On the other hand

$$u_1, \ (Pn)_1 \ \text{cannot be specified.}$$

Limitations of continuum mechanics

Some commentary.