TAM 445 Continuum Mechanics - Spring 2024
Homework 9 - Stress

Due: April 23, 2024

1. A bar made of an incompressible material is loaded by a force $\boldsymbol{h} = h\boldsymbol{e}_1$,where the bar's axis is along $\boldsymbol{e}_1$ direction .The bar is uniform along its length and unconstrained in the 2- and 3-directions. The stretch in the 1-direction is α. Assume the responses in the 2- and 3-directions are the same and that no shearing deformation (with respect to the Cartesian coordinate system) takes place in the bar as a result of the uniaxial loading. The cross-sectional area of the bar when it is not loaded is a_0 .

1. Determine the Cauchy stress tensor and of the first and second Piola–Kirchhoff stress tensors in the bar. (The second Piola–Kirchhoff stress is given by $\boldsymbol{S} = \boldsymbol{F}^{-1}\boldsymbol{P}$)

2. Determine the plane of maximum shear stress in the deformed configuration and the value of the Cauchy shear stress on this plane.

3. Determine the material plane in the reference configuration corresponding to the plane of maximum shear stress found above. Plot the angle θ between the normal to this plane and the horizontal axis as a function of the stretch in the 1-direction, α. Which plane does this tend to as $\alpha \to \infty$?

2. The following figure shows two configurations of dams (dimensions are shown in the figure). The width of the dams in the out-of-plane direction is w . The dams are subjected to hydrostatic pressure due to the water on the right, atmospheric pressure p_{at} due to the surrounding air, and gravity which acts downwards. The density of the water is ρ_w and the density of the dam material is ρ_d . Compute the total force (body and surface, not including the reactions where the ground supports the dams) acting on the dam for both configurations. Hint: The stress in water is of the form $-p\boldsymbol{I}$, where p is the hydrostatic pressure. The hydrostatic pressure increases linearly with depth below the water surface and is proportional to $\rho_w g$, where g is the gravitational acceleration.

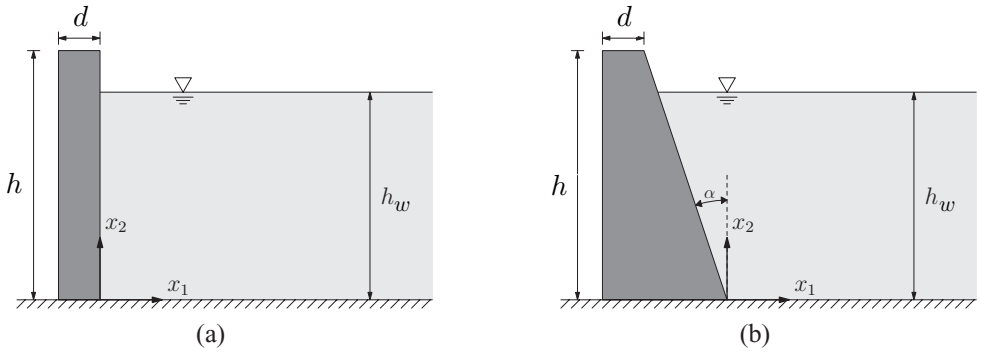

Figure 1: Two configurations of dams (dark gray) with water (light gray) on the right. The dams and water are surrounded by air at atmospheric pressure. The dimensions of the dam and the level of the water are indicated. The width of the dams in the out-of-plane direction is w.