TAM 445 Continuum Mechanics - Spring 2024
Homework 2 - Tensors

Due: Feb 09, 2024

In the following problems, let $\boldsymbol{e}_i$ $(i = 1, 2, 3)$ denote three orthonormal basis vectors for a Euclidean vector space $\boldsymbol{V}$ equipped with the standard inner product $\boldsymbol{u} \cdot \boldsymbol{v} = u_i v_i$.

1. Show that the set of nine tensors $\{\boldsymbol{e}_i \otimes \boldsymbol{e}_j : i, j = 1, 2, 3\}$ forms a basis for the real vector space of second-order tensors.

2. (a) Show with an example that the dyadic product is not commutative. In other words,

$$\boldsymbol{u} \otimes \boldsymbol{v} = \boldsymbol{v} \otimes \boldsymbol{u} \qquad \forall \boldsymbol{u}, \boldsymbol{v} \in V \tag{1}$$

is *not* true.

(b) Consider a vector $\boldsymbol{n} \in V$ with $\|\boldsymbol{n}\| = 1$. Such vectors are referred to as *unit vectors*. Examine how the tensor

$$\boldsymbol{I} - \boldsymbol{n} \otimes \boldsymbol{n} \tag{2}$$

operates on vectors. Describe in words, the geometric significance of the above tensor.

(c) Let $\boldsymbol{e}$ and $\boldsymbol{f}$ be orthogonal unit vectors. Describe the geometric nature of the tensor $\boldsymbol{e} \otimes \boldsymbol{e} + \boldsymbol{f} \otimes \boldsymbol{f}$