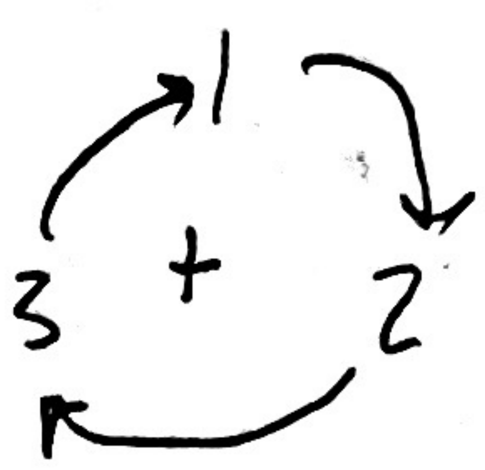
Kronecker delta δ_{ij}

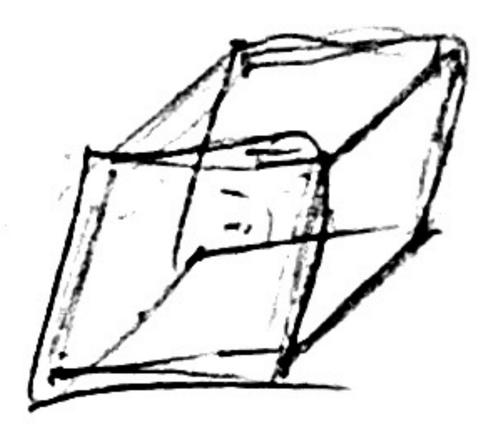
Permutation or Levi-civita symbol ε_{ijk}



$\varepsilon_{231} = \varepsilon_{312} = \varepsilon_{123} = 1$

$\varepsilon_{112} = \varepsilon_{111} = 0$

Useful Identities



1) $\varepsilon_{ijk}\delta_{ij} = \varepsilon_{iik} = 0$

$\varepsilon_{ijk}\varepsilon_{mjk} = 2\delta_{im}$

$\varepsilon_{ijk}\varepsilon_{ijk} = 6$

2) $\varepsilon_{mnp}(\det A) = \varepsilon_{ijk} A_{im} A_{jn} A_{kp}$

$\det A = \varepsilon_{ijk} A_{i1} A_{j2} A_{k3}$

$\hookrightarrow \varepsilon_{ijk} A_{1i} A_{2j} A_{3k}$

Derivative of det A wrt A

$\frac{\partial}{\partial A}(\det A) = A^{-T} \det A$

$\frac{\partial}{\partial A_{ij}}(\det A) = (A^{-T})_{ij} \det A$, $\det A \neq 0$

$(AB)_{ij} = A_{i1}B_{1j} + A_{i2}B_{2j}$

$= A_{ip}B_{pj}$

$\det(AB) = (\det A)(\det A)$

$\det(A+B) = \det A + \det B$

A^{-1} inverse of A

$AA^{-1} = A^{-1}A = I$

A^{-1} exists $\Longleftrightarrow$ $\det A \neq 0$

if and only if

$$(A^{-1})_{kq} = \frac{1}{2\det A} \varepsilon_{ijk} \varepsilon_{mnq} A_{mi} A_{nj}$$

Inverse of A in terms of A

$\varepsilon_{mnq} \times$ (5):

$$\varepsilon_{mnq} \varepsilon_{mnp} \det A = \varepsilon_{mnq} \varepsilon_{ijk} A_{mi} A_{nj} A_{pk}$$

Check free indices and make sure it's the same on both sides

$$2 \delta_{pq} \det A = A_{pk} (\varepsilon_{mnq} \varepsilon_{ijk} A_{mi} A_{nj})$$

Order doesn't matter in terms of notation, the indices imply order

$A_{ij} \longleftrightarrow A$

Individual Product

$$2I \det A = A_{pk} \underbrace{(\varepsilon_{mnq} \varepsilon_{ijk} A_{mi} A_{nj})}_{} B_{kq}$$

$$2 \delta_{pq} \det A = (AB)_{pq}$$

$$I = A \frac{B}{2\det A} A^{-1}$$

$$A^{-1}_{kq} = \varepsilon_{mnq} \varepsilon_{ijk} A_{mi} A_{nj}$$

$$\frac{\partial (\det A)}{\partial A_{rs}} = \frac{\partial}{\partial A_{rs}} \left(\frac{1}{6} \varepsilon_{mnp} \varepsilon_{ijk} A_{im} \cdot A_{jn} A_{kp} \right)$$

$$= \frac{1}{6} \varepsilon_{mnp} \varepsilon_{ijk} \left(\underbrace{\frac{\partial A_{im}}{\partial A_{rs}}}_{= \delta_{ir} \delta_{ms}} A_{jn} A_{kp} + A_{im} \frac{\partial A_{ji}}{\partial A_{rs}} A_{kp} + A_{im} A_{jn} \frac{\partial A_{kp}}{\partial A_{rs}} \right)$$

$= \left(-\frac{1}{6} \varepsilon_{snp} \varepsilon_{ijk} A_{jn} A_{kp} + \right.$

$\epsilon_{ijk} \delta_{ij} = \epsilon_{iij} = 0$

$\epsilon_{ijk} \epsilon_{mjk} = 2\delta_{im}$

$\frac{\partial A_{im}}{\partial A_{rs}} = \delta_{ir} \delta_{ms}$

$\epsilon_{ijk} \epsilon_{ijk} = 6$

$\epsilon_{mnp} (\det A) = \epsilon_{ijk} A_{im} A_{jn} A_{kp}$

$\det(A) = \frac{1}{6} \epsilon_{ijk} \epsilon_{mnp} A_{im} A_{jn} A_{kp}$

$\delta_{ir} \delta_{ms} A_{jn} A_{kp}$

$\frac{\partial}{\partial A}(\det A) = A^{-T} \det A$

$\varepsilon_{snp} \varepsilon_{rjk}$

$(A^{-1})_{kq} = \frac{1}{2 \det A} \epsilon_{ijk} \epsilon_{mnq} A_{mi} A_{nj}$

$= \frac{1}{2} \varepsilon_{snp} \varepsilon_{rjk} A_{jn} A_{kp}$ ⑧

$\delta_{rt} \frac{\partial (\det A)}{\partial A_{rs}} = \frac{1}{2} \varepsilon_{snp} \varepsilon_{rjk} A_{jn} A_{kp}$

multiply by δ_{rt} $\quad A_{ri} A^{-1}_{it}$

$\delta_{rt} \frac{\partial (\det A)}{\partial A_{rs}} = \boxed{\delta_{rt}} \frac{1}{2} \varepsilon_{snp} \varepsilon_{rjk} A_{jn} A_{kp}$

$= \frac{1}{2} \varepsilon_{snp} \varepsilon_{rjk} A_{jn} A_{kp} A_{ri} A^{-1}_{it}$

* Try to derive the identities by yourself