Polar Decomposition

$F \in \mathrm{Inv}$ (i.e. $\det F \neq 0$). There exist unique U, V in Psym

and unique $R \in \mathrm{Orth}$ such that

$F = RU = VR$ $\quad U = \sqrt{F^TF}$, $V = \sqrt{FF^T}$

$\det F \gtrless 0 \quad \det R \gtrless 0$

$\sum_{i=1}^{3} \sqrt{\lambda_i}\, \underset{\sim}{u}_i \otimes u_i = \sqrt{F^TF}$ $\quad F^TF\underset{\sim}{v}\cdot \underset{\sim}{v} \overset{?}{>} 0$

> 0 since $F^TF \in$ Psym $\quad \boxed{F\underset{\sim}{v} \cdot F\underset{\sim}{v} > 0}$

Existence of $\boxed{U, V}$ & R Let $R = FU^{-1} \overset{?}{\in}$ Orth $\quad A^{-1} = A^T$

$R^TR^T = (FU^{-1})^T(FU^{-1})$ $\quad U^2 = F^TF \Rightarrow U = F^TFU^{-1} \Rightarrow$

$U = U^{-1}F^TF \Rightarrow I$

$= U^{-T}F^TFU^{-1} = I$

$= FUU^{-1}F^T$

$V = RUR^T \rightsquigarrow T' = \Lambda T \Lambda^T$ Matrix, not tensor

$V^2 = FF^T$

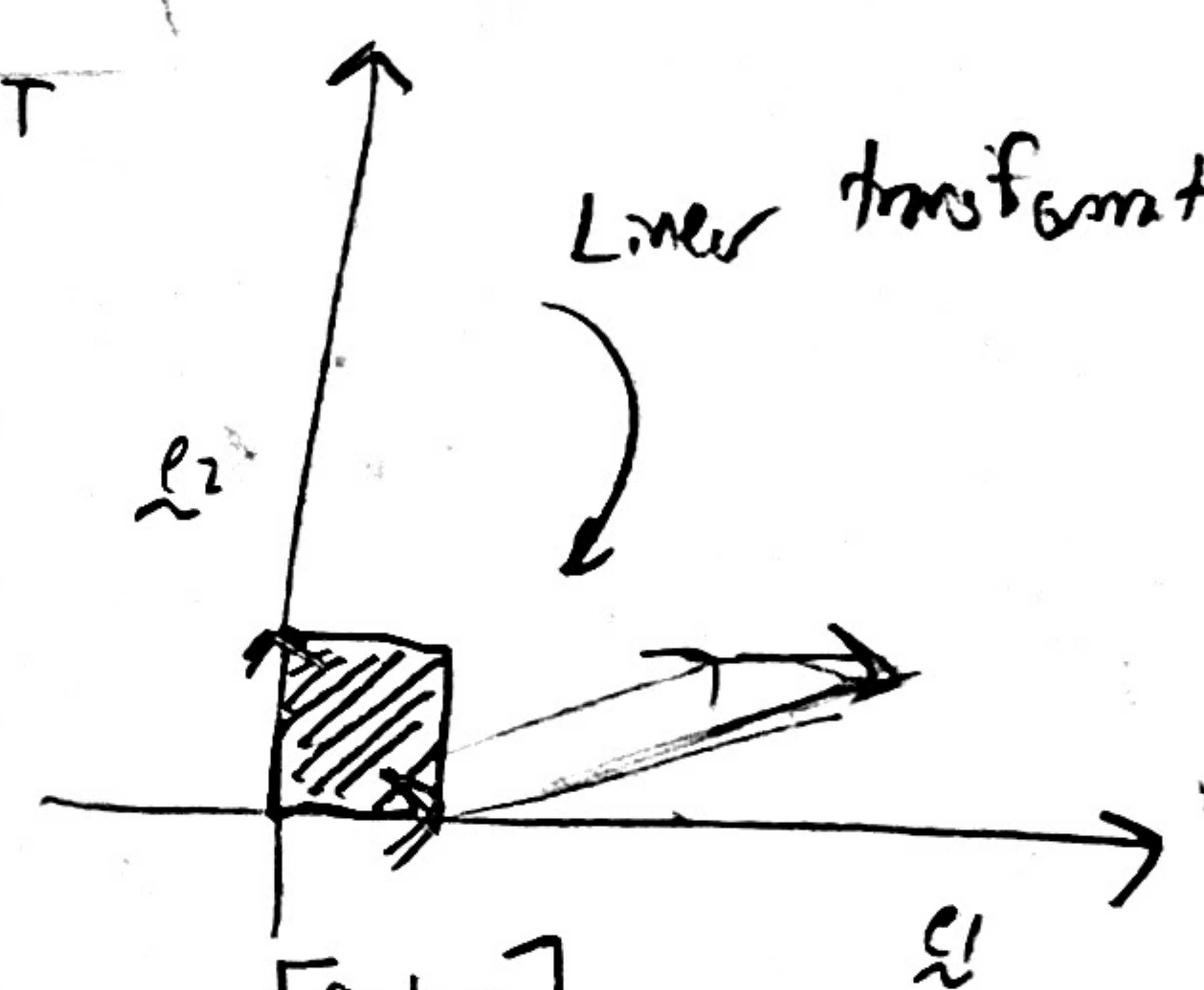
1. Solve for $\sqrt{F^TF}$ given F

2. Use U & solve for V

$R(a \otimes b)R^T = R\underset{\sim}{a} \otimes R\underset{\sim}{b}$

$(RU)\underset{\sim}{v}_j \quad (VR)\underset{\sim}{x}$

$F\underset{\sim}{v} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 + 2v_2 \\ v_2 \end{bmatrix}$



Tensor Fields

$\underset{\sim}{v} : B \rightarrow \mathbb{R}^2$

$B \subset E$ Euclidean point space

$\underset{\sim}{v}(p) \equiv \begin{bmatrix} v_1(p) \\ v_2(p) \end{bmatrix}$

$\underset{\sim}{V}(x)$