TAM 445 Continuum Mechanics - Spring 2024
Homework 8 - Motion, stress

Due: April 15, 2024

1. Given the spatial velocity field

$$(v_{\mathrm{s}})_1 = \exp(x_3 - ct)\cos(\omega t), \quad (v_{\mathrm{s}})_2 = \exp(x_3 - ct)\sin(\omega t), \quad (v_{\mathrm{s}})_3 = c = \text{const}$$

(a) Show that the speed (magnitude of the velocity) of every particle is constant.

(b) Calculate the components of $\boldsymbol{a}_{\mathrm{s}}$. (Note that the previous part implies that $\boldsymbol{a} \cdot \boldsymbol{v} (= \boldsymbol{a}_{\mathrm{s}} \cdot \boldsymbol{v}_{\mathrm{s}}) = 0$.)

(c) Calculate the rate of stretch along the direction $(1/2, 0, 1/2)$ in the deformed configuration at $\boldsymbol{x} = \boldsymbol{0}$.

(d) Integrate the velocity equations to find the motion $\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{X}, t)$. Hint: Integrate the v_3 equation first.

2. Let $\boldsymbol{T}(\boldsymbol{x}, t)$ denote the Cauchy stress tensor field defined on $\mathcal{B}_t$. Consider all planes that pass through a particular point $\boldsymbol{x} \in \mathcal{B}_t$. Let $\boldsymbol{n}$ denote a unit normal vector to such a plane. The normal traction/normal stress acting on the plane is $\boldsymbol{T}\boldsymbol{n} \cdot \boldsymbol{n}$. Show that the magnitude of the normal stress is maximum on the plane with normal equal to an eigenvector of $\boldsymbol{T}$ that has a maximum (in magnitude) eigenvalue.