TAM 445 Continuum Mechanics - Spring 2024
Homework 4 - Eigenvectors, eigenvalues, and the polar decomposition

Due: Feb 23, 2024

Notation: Uppercase bold letters denote second-order tensors, lowercase bold letters denote vectors, and greek letters denote scalars.

1. Determine the eigenvalues, eigenvector spaces (also known as *characteristic spaces*), and *a* spectral decomposition for each of the following tensors:

$$\boldsymbol{A} = \alpha \boldsymbol{I} + \beta \boldsymbol{m} \otimes \boldsymbol{m} \tag{1}$$

$$\boldsymbol{B} = \boldsymbol{m} \otimes \boldsymbol{n} + \boldsymbol{n} \otimes \boldsymbol{m}, \tag{2}$$

where $\boldsymbol{m}$ and $\boldsymbol{n}$ are orthogonal unit vectors.

2. Compute the polar decompositions $\boldsymbol{T} = \boldsymbol{R}\boldsymbol{U} = \boldsymbol{V}\boldsymbol{R}$, where $\boldsymbol{U}, \boldsymbol{V} \in \mathrm{Psym}$ and $\boldsymbol{R} \in \mathrm{Orth}$, of a tensor $\boldsymbol{T}$ whose components are given by

$$\begin{bmatrix} \sqrt{3} & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Write down the eigenvectors and eigenvalues of $\boldsymbol{U}$ and $\boldsymbol{V}$, and describe in words the geometric interpretation of the above decompositions.