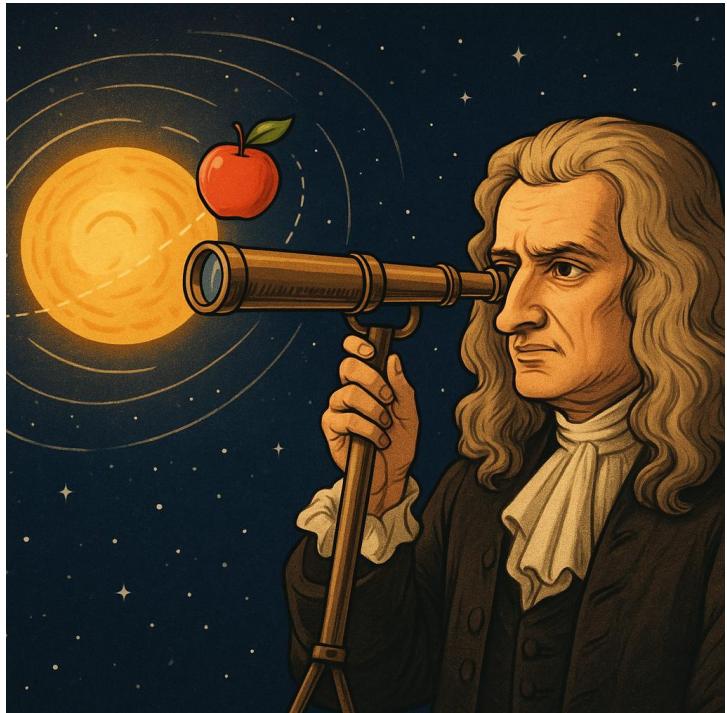


# Forces and Newton's Laws of Motion

*Chapter Four*

# Introduction



- In the previous chapters, we explored how objects move — how to describe their motion using position, velocity, and acceleration. Now, we ask a deeper question: ***What causes motion?***
- Introduce **Newton's Three Laws of Motion**, the foundation of classical mechanics.
- Explore **force as a vector** — including how to add and resolve forces in 2D.
- Analyze forces like **tension**, **normal force**, and **friction**, and how they govern real-world systems.
- Examine both **equilibrium** (net force zero) and **non-equilibrium** (net force  $\neq 0$ ) situations using **Newton's Second Law**.

# Isaac Newton

**Formulated the Three Laws of Motion** — the foundation of classical mechanics

**Discovered the Universal Law of Gravitation** — explaining the motion of the planets and moons

**Invented Calculus** (independently of Leibniz) to solve the mathematics of motion and change

**Unified terrestrial and celestial mechanics** — showing that the same physics governs apples and planets

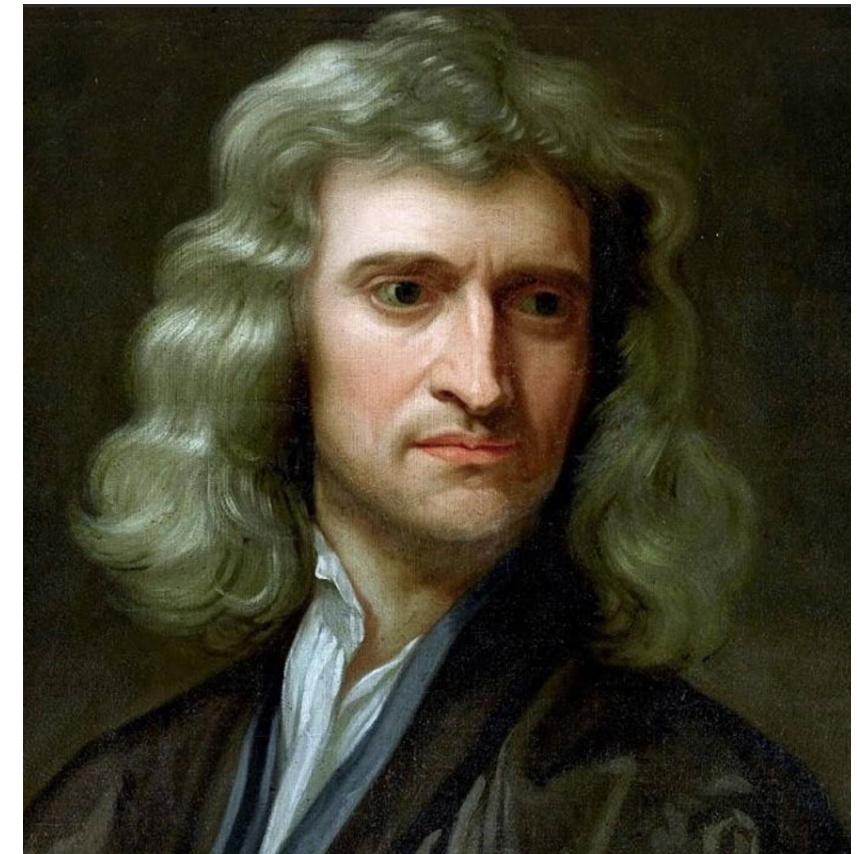
**Developed the first reflecting telescope** — advancing observational astronomy

**Wrote *Philosophiæ Naturalis Principia Mathematica* (1687)** — one of the most important scientific books ever published

**Analyzed light and color** — through experiments with prisms, laying the groundwork for modern optics

**Master of the Royal Mint** — helped reform the English currency and combat counterfeiting

“If I have seen further, it is by standing on the shoulders of giants.”  
— Isaac Newton



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# **Class Activity: What is Mass?**

Let's see what our shared intuition tells us. Before we define anything, what *is* mass to you? Think physically, personally, weirdly — whatever comes to mind.





# Class Activity: What is Force?

Let's see what our shared intuition tells us. We talk about force all the time. Let's find out what we really think it means.

# Newton's First Law of Motion

**The Law of Inertia:** Consider an object that has no forces acting on it. If it is at rest, it will remain at rest. If it is in moving, it will continue to move in a straight line at a constant speed.

## It Defines Inertial Frames

- This law **isn't just about motion**; it sets the **stage** for all Newtonian mechanics.
- It implicitly defines what we mean by a **non-accelerating reference frame** — an *inertial frame* is one in which this law holds.
- Without it, Newton's Second Law has no grounding.
- If a body appears to accelerate with no force, you're not in an inertial reference frame.

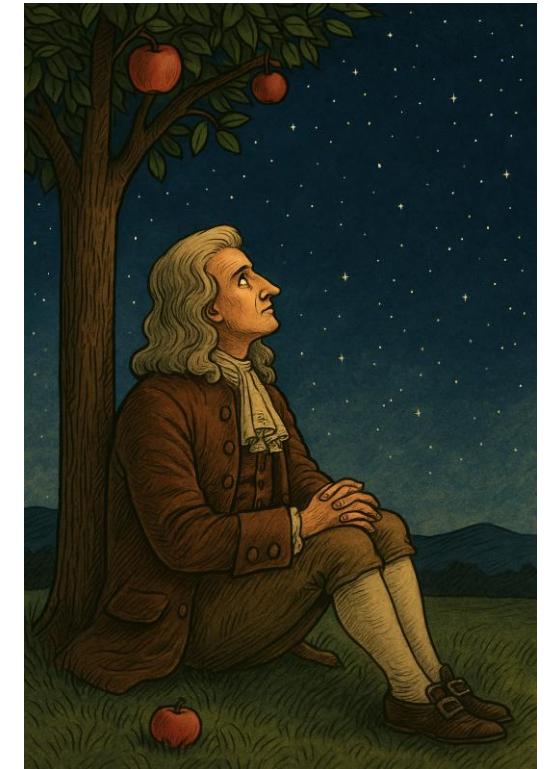
*"Lex I. Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare."*

## It Rejected Aristotelian Physics

- Before Newton, people believed **constant force was needed to sustain motion**.
- Newton: force causes a **change** in motion, not motion itself.
- This insight required a conceptual revolution — that motion is *natural* and needs no cause unless it changes.

## It Introduces Force by Negation

- The law tells us **what happens in the absence** of a net force — this makes it a **baseline**.
- All other forces are deviations from this state. In other words, **forces are only meaningful in terms of their ability to alter this default behavior**.



# Team Activity: Concept Check 4.1

A passenger is sitting in a car at a red light. When the light turns green, the driver steps on the gas and the car rapidly accelerates forward. The passenger feels like they are being pushed backward into their seat.

**Which of the following best explains this sensation?**

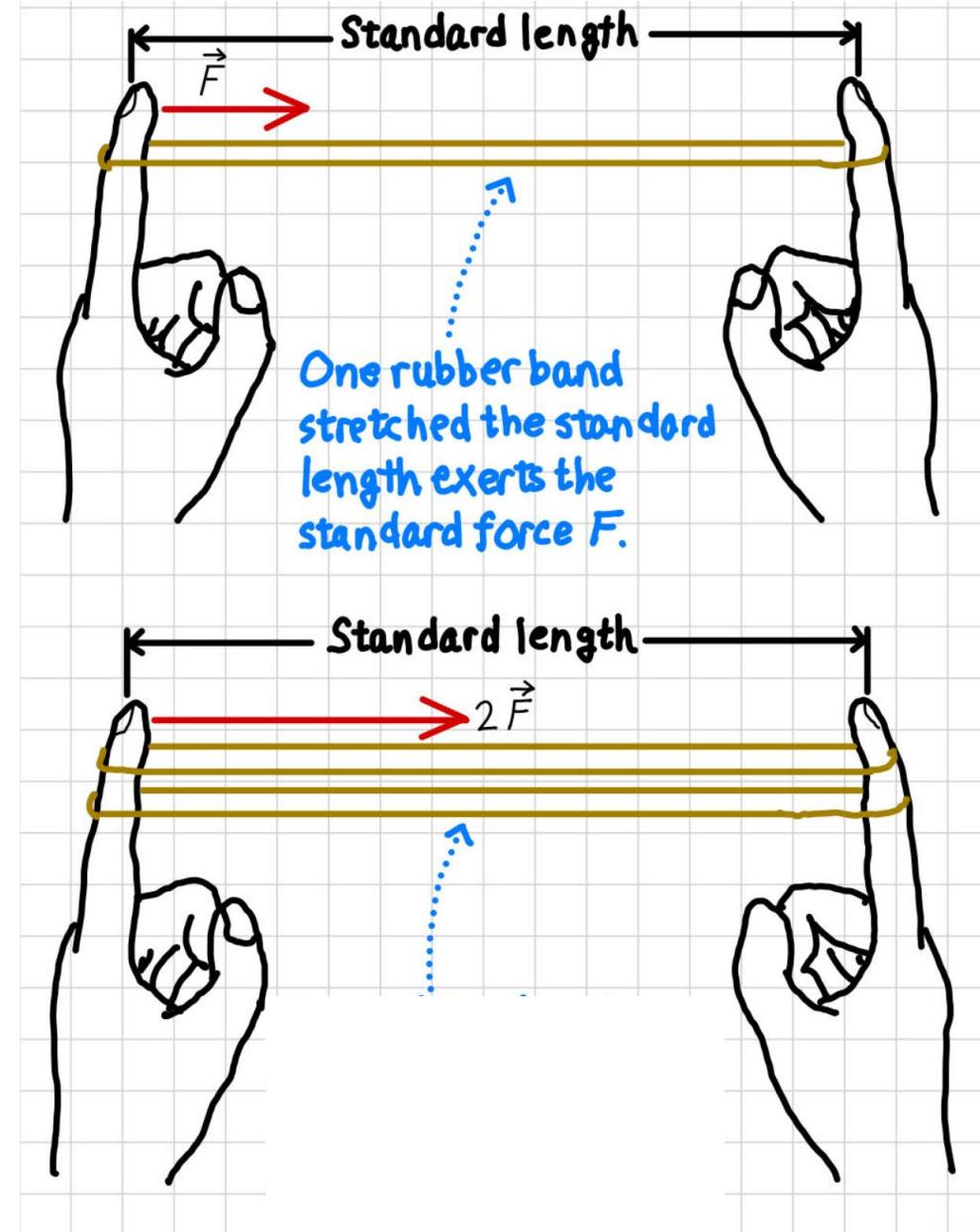
- a) A force is pushing the passenger backward.
- b) The car accelerates forward, but the passenger's body resists the change in motion due to inertia.
- c) The seat exerts a backward force on the passenger.
- d) The passenger was at rest and remains at rest because of a gravitational force.



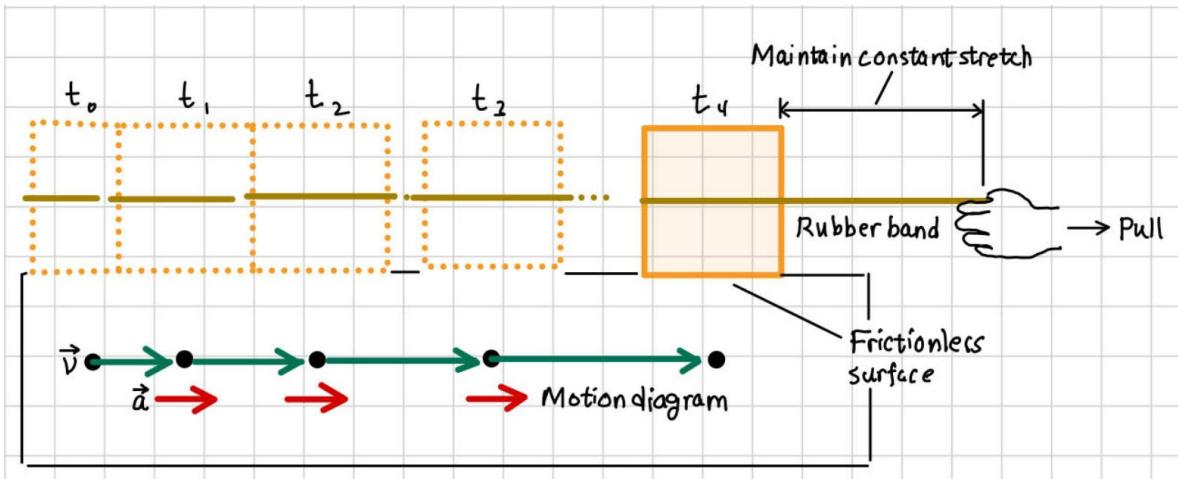
# What do Forces Do? 1/4

A quick experiment:

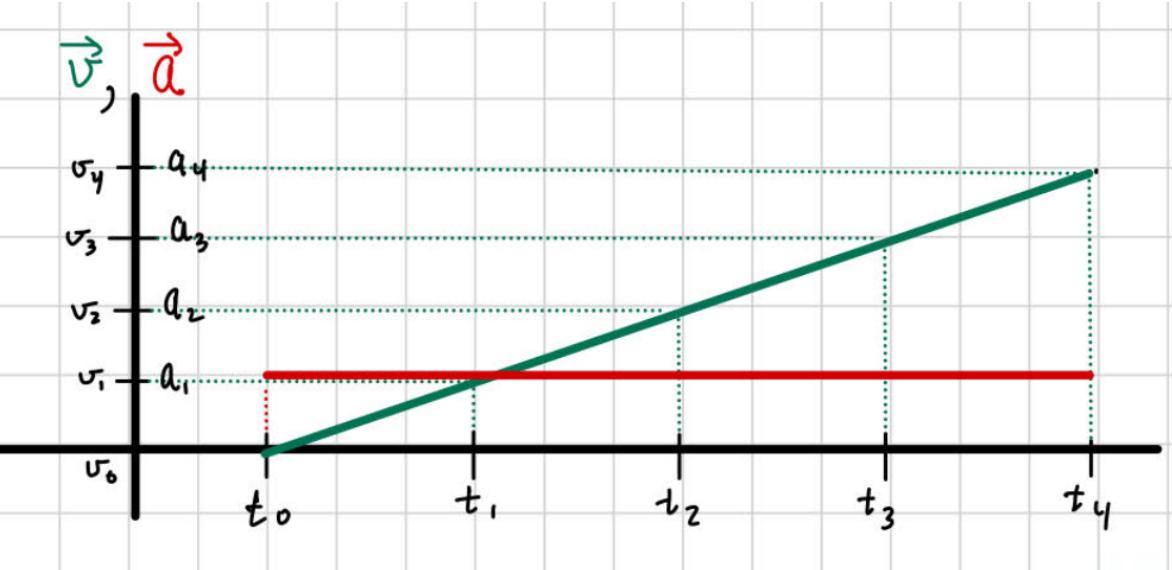
- Use your fingers to stretch a rubber band to a specific length – say 10 cm, measured with a ruler. Call this the “standard length”.
- You feel your fingers pull toward one another – due to a force exerted by the rubber band.
- Call this force, the standard force,  $\vec{F}$ .
- What if we use 2 identical rubber bands? What will the standard force be? 3 rubber bands?
  
- 2 rubber bands =  $2\vec{F}$
- 3 rubber bands =  $3\vec{F}$
- We conclude that  $N$  rubber bands =  $N\vec{F}$



# What do Forces Do? 2/4

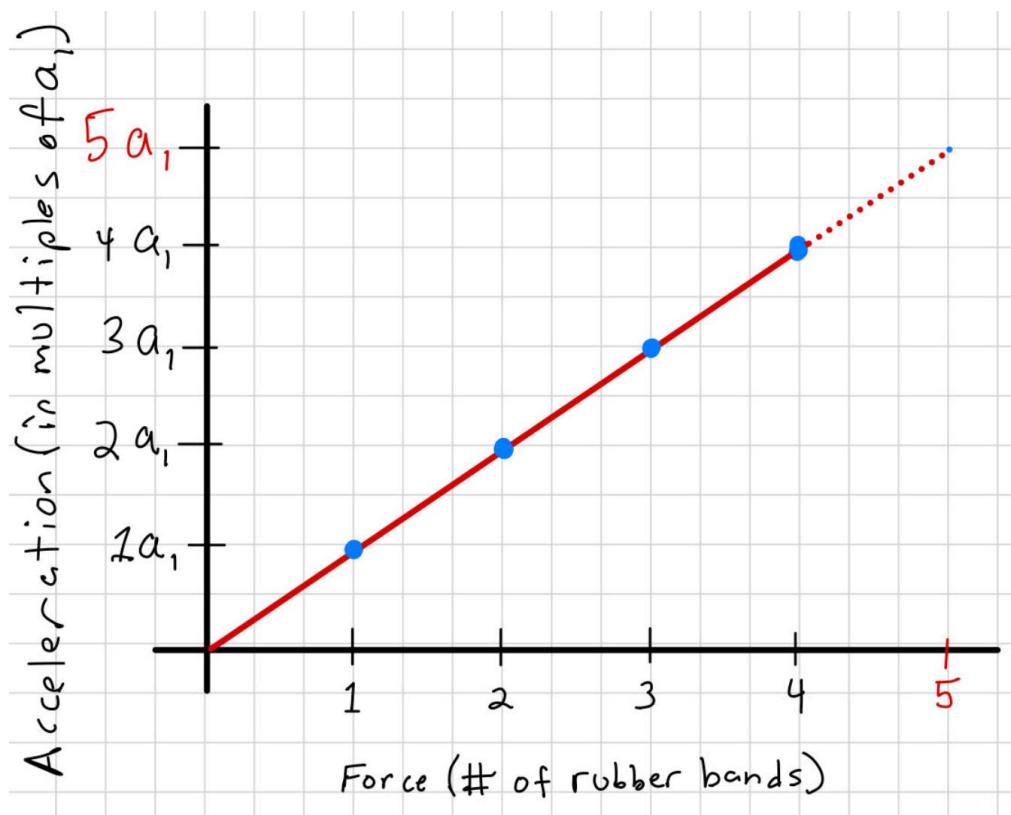


- Place a 1kg block on a frictionless surface.
- Attach a rubber band to the block and stretch the band to standard length while holding the block.
- Release the block. What happens?
- The block experiences the same force that your finger did.
- Repeat, but this time continue to pull the rubber band to maintain a constant stretch – this means that your hand will have to always move at the same velocity of the block.
- If we took measurements of velocity and acceleration at each instant of time and plotted the results on a  $v$  vs.  $t$  and a  $a$  vs.  $t$  plot



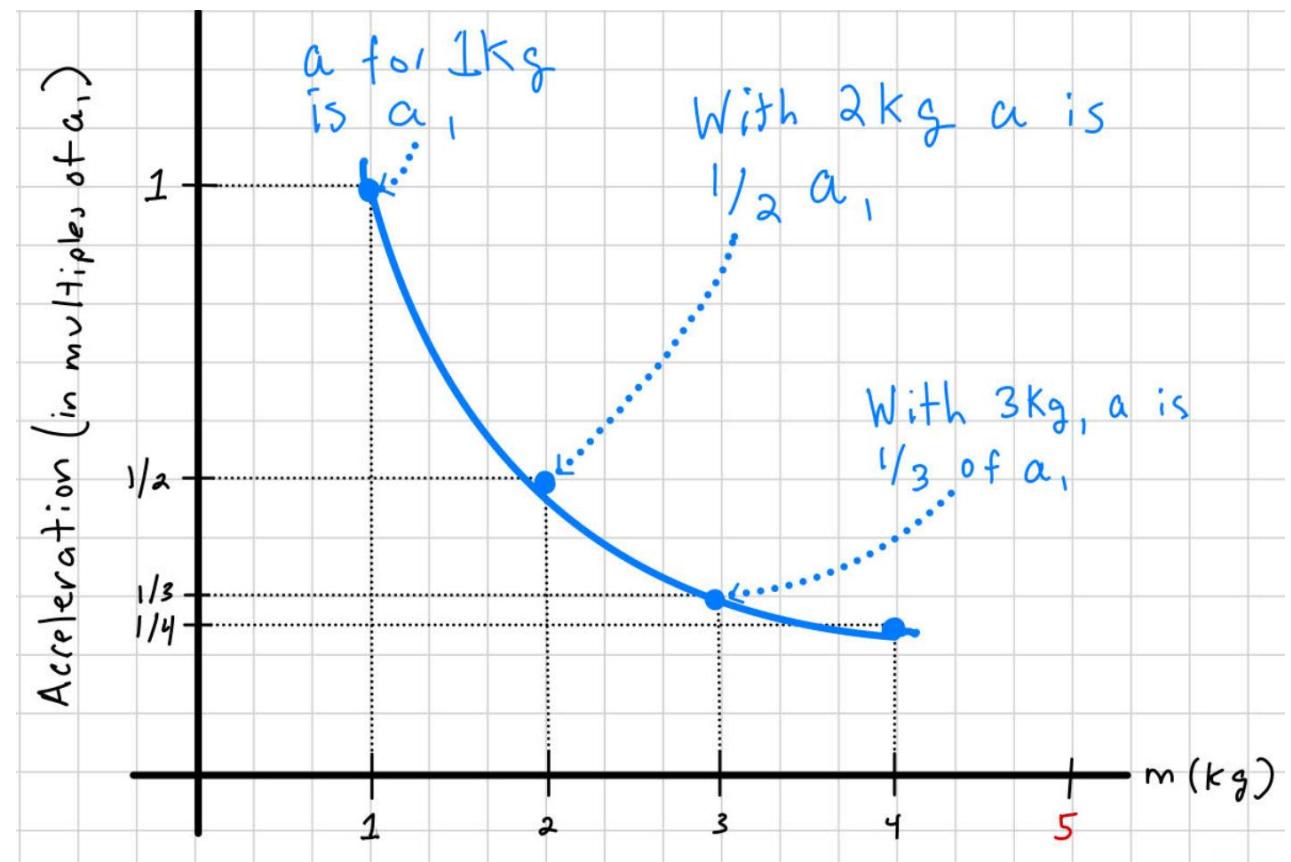
# What do Forces Do? 3/4

- We found in our first experiment, that 1 rubber band yields and acceleration with magnitude  $a_1$ .
  - That is a coordinate,  $(F, a_1)$  where  $F$  is the magnitude of our standard force that we defined earlier.
- Let's repeat the experiment but with 2 rubber bands rather than 1. Recall that 2 rubber bands =  $2F$ . Something interesting happens – when we measure our acceleration we get  $2a_1$ .
- We keep going,
  - $(F, a_1), (2F, 2a_1), (3F, 3a_1)$ , and  $(4F, 4a_1)$
- We have a trend? There seems to be a linear relationship between Force and acceleration  $a \propto F$
- Can we make a prediction?
  - What multiple of  $a_1$  do we think will correspond to 5 rubber bands?



# What do Forces Do? 4/4

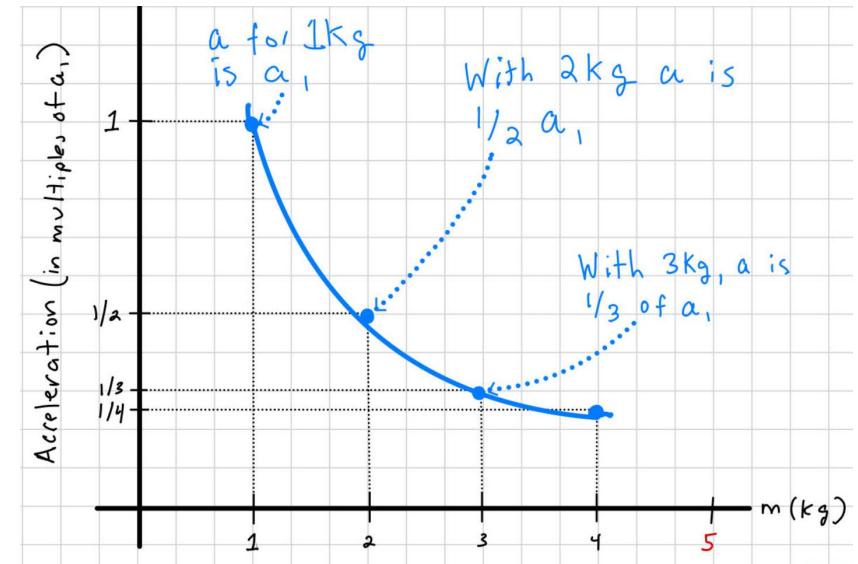
- We have one more variable under our control in this experiment – **mass**.
- We also have  $1\text{kg}$ ,  $2\text{kg}$ ,  $3\text{kg}$ , and  $4\text{kg}$  blocks – all equivalent volume.
- Let's repeat our experiment with a single rubber band but each time lets switch to a more massive block.
- We get the following measurements:  
 $(1\text{kg}, a_1), (2\text{kg}, \frac{1}{2}a_1), (3\text{kg}, \frac{1}{3}a_1), (4\text{kg}, \frac{1}{4}a_1)$
- Acceleration and mass appear to be related inversely:  $a \propto \frac{1}{m}$ .



# Team Activity: Concept Check 4.2

In science we develop **hypotheses**, and we **test** them, and if they pass, we upgrade the hypothesis to a **theories**. Theory, in science, does not mean “guess” or “idea” as it often does in common vernacular. A theory is a model that makes accurate predictions about the world around us. If we come to an observation that our theory can’t explain, the ideal is to find a new hypothesis that not only explains everything that our previous theory did, but also our new observation. If it does, we upgrade it to the new reigning theory – especially if it makes further predictions that turn out to be true. We say that our new theory has more **explanatory power** than our old one.

We seem to have a working model for the relationship between acceleration and mass  $a \propto \frac{1}{m}$ . **Using this model, predict what  $a$  will be when  $m = 5\text{kg}$ .**



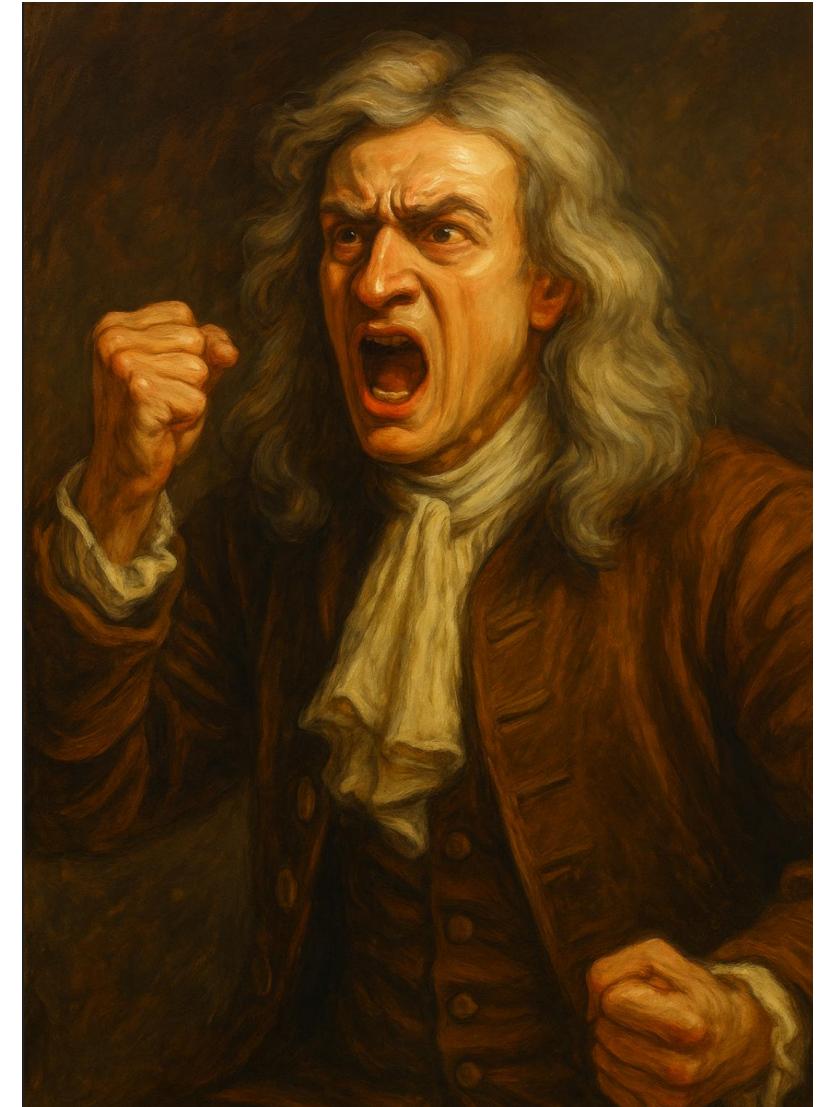
# Put It All Together...

$$a \propto F$$

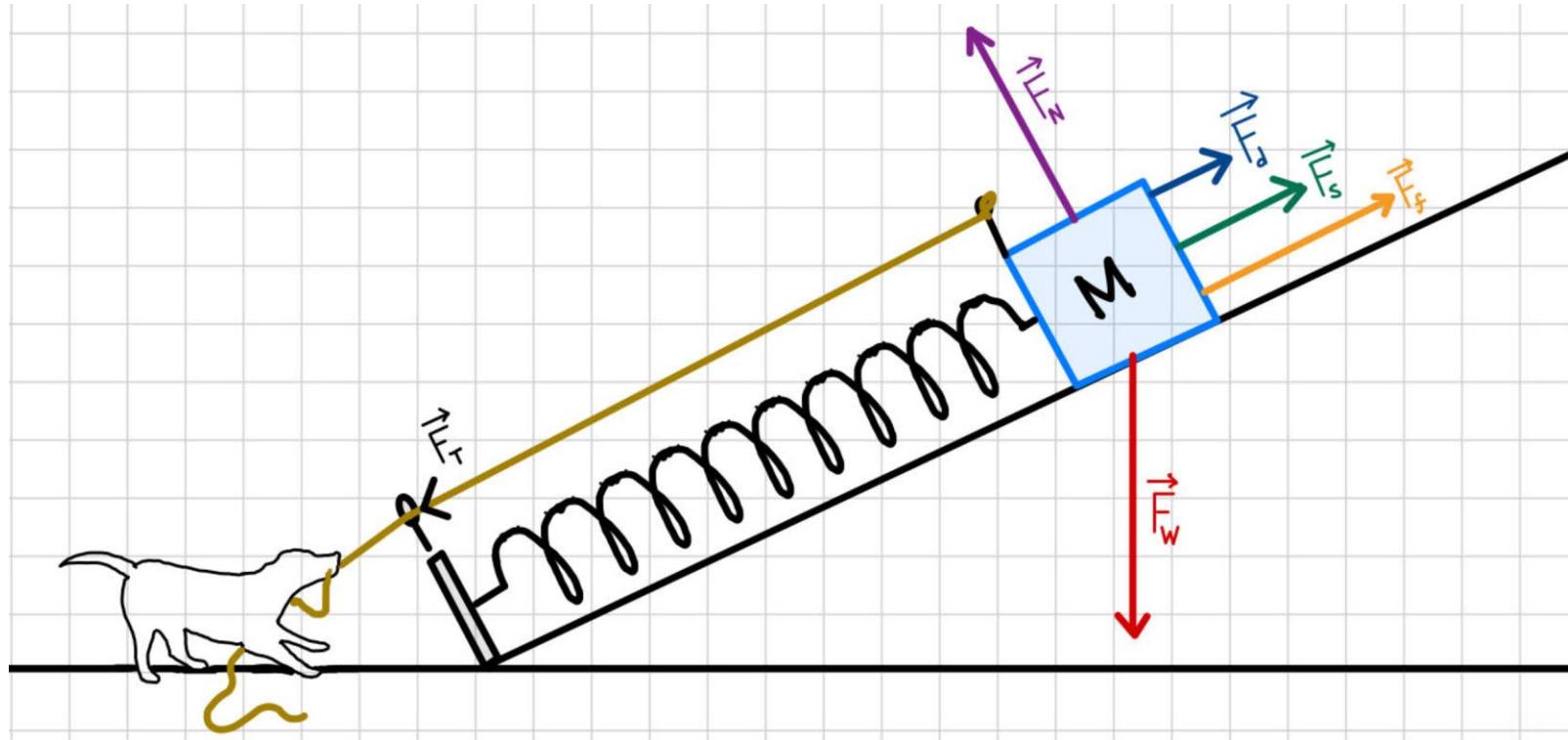
$$a \propto \frac{1}{m}$$

Newton's 2<sup>nd</sup> Law of Motion

$$\vec{a} = \frac{\vec{F}}{m}$$



# Net Force



$$\vec{F}_{net} = \sum \vec{F} = \vec{F}_T + \vec{F}_N + \vec{F}_d + \vec{F}_s + \vec{F}_f + \vec{F}_w$$

# Newton's Second Law of Motion

An object of mass  $m$  subjected to forces  $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_N$  will undergo an acceleration  $\vec{a}$  given by,

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

where the net force  $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_N$  is the vector sum of all forces acting on the object. The acceleration vector,  $\vec{a}$ , points in the same direction as the net force vector,  $\vec{F}_{net}$ .

Newton's Second Law is more than just a formula—it is a monumental shift in how we understand the universe. Before Newton, there was no clear framework for predicting how or why objects changed their motion. With this single law, Newton unified motion and force into a precise, quantitative relationship: force is not just something that causes motion—it causes a *change* in motion, and the amount of change depends on an object's mass. This law gave us a way to predict the future behavior of physical systems from present conditions. It made physics predictive, not just descriptive. It laid the groundwork for everything from launching rockets to understanding planetary orbits—and it's the first time humans had a universal rule to connect cause (force) with effect (acceleration).

# Example: Connecting Kinematics to Newton's 2<sup>nd</sup> Law

A Boeing 737 has a mass of 51,000 kg. It starts from rest and then accelerates down the runway. After traveling 940 m, the plane reaches its take-off speed of 70 m/s and leaves the ground. What thrust (a force) of each engine?

**Solution:** We know we can relate thrust to acceleration via Newton's second law – but how do we get  $a$ ? Kinematic equations!

$$v_x^2 - v_{ox}^2 = 2a_x(x - x_0) \rightarrow a_x = \frac{v_x^2 - v_{ox}^2}{2(x - x_0)} = \frac{\left(70 \frac{m}{s}\right)^2}{2(940 \text{ m})} = 2.61 \text{ m/s}^2$$

Now use the 2<sup>nd</sup> Law:

$$\begin{aligned} F_{net} &= ma_x = (51000 \text{ kg}) \left(2.61 \frac{\text{m}}{\text{s}^2}\right) \\ &= 133000 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 1.33 \times 10^5 \text{ N} \end{aligned}$$

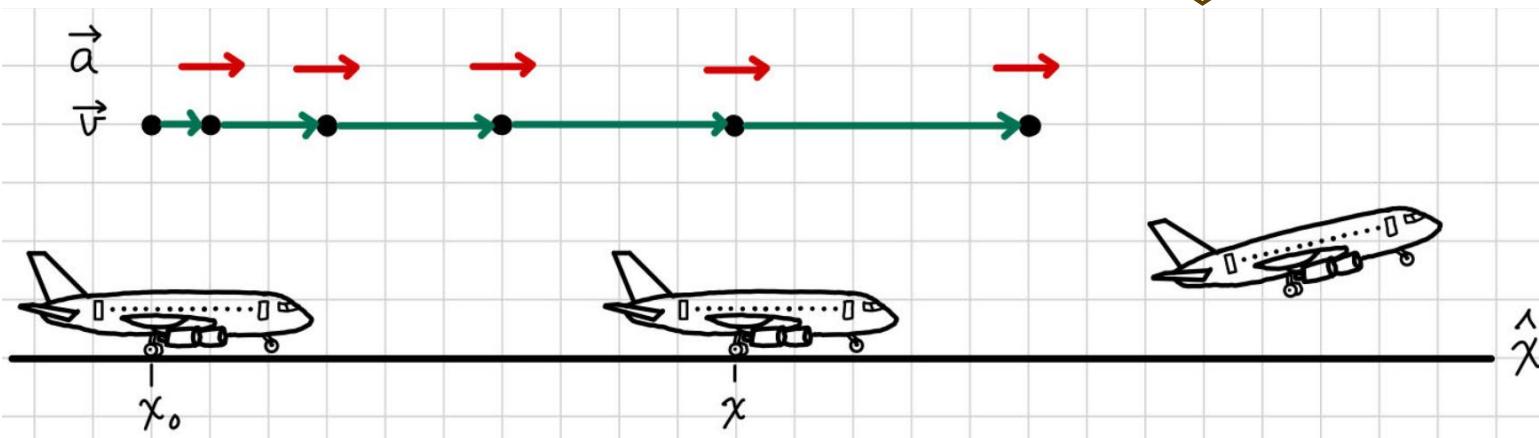
where  $1 \text{ N} = \text{kg} \cdot \text{m/s}^2$

$$F_{net} = 2F_{engine} \rightarrow F_{engine} = \frac{1.33 \times 10^5 \text{ N}}{2} = 6.70 \times 10^4 \text{ N}$$

In vector form,

$$\vec{F}_{engine} = 6.70 \times 10^4 \text{ N} \hat{x}$$

Notice how Newton's Law doesn't give us the acceleration directly—it tells us how to compute force once we *know* acceleration. We use motion to get  $a$ , then Newton to get  $F$ .



**THE METRIC SYSTEM IS THE TOOL OF  
THE DEVIL!**

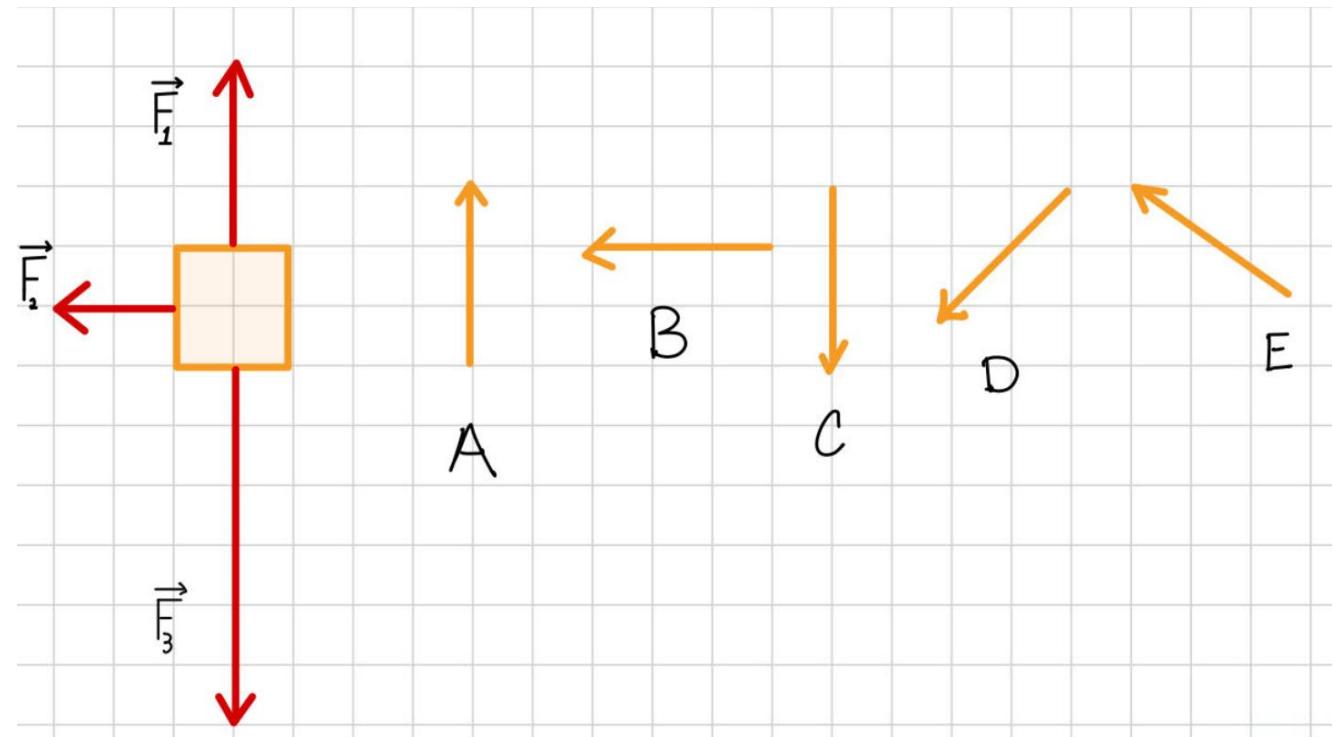


## More on units of force...

- We have already seen Newtons:  $1N = kg \cdot m/s^2$ 
  - This is the unit of force in the SI system (most used in science and engineering and by the rest of the world)
- More familiar to Americans is the pound-force (lbf):
$$1 \text{ lbf} = 1 \text{ slug} \cdot \frac{ft}{s^2}$$
  - In everyday vernacular, when Americans give a weight in lbs, they really mean lbf.
  - lbs is technically lbf (pound-force) and is to slugs what g is to kg – just a smaller unit mass.
- You can convert from lbf to N:  $1 \text{ lbf} = 1 \text{ lb} = 4.45 \text{ N}$

# Team Activity: Concept Check 4.3

Three forces act on an object. In which direction does the object accelerate?



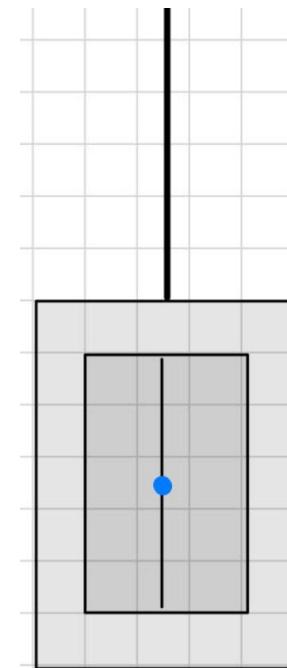
# Free Body Diagrams

How to draw a free body diagram:

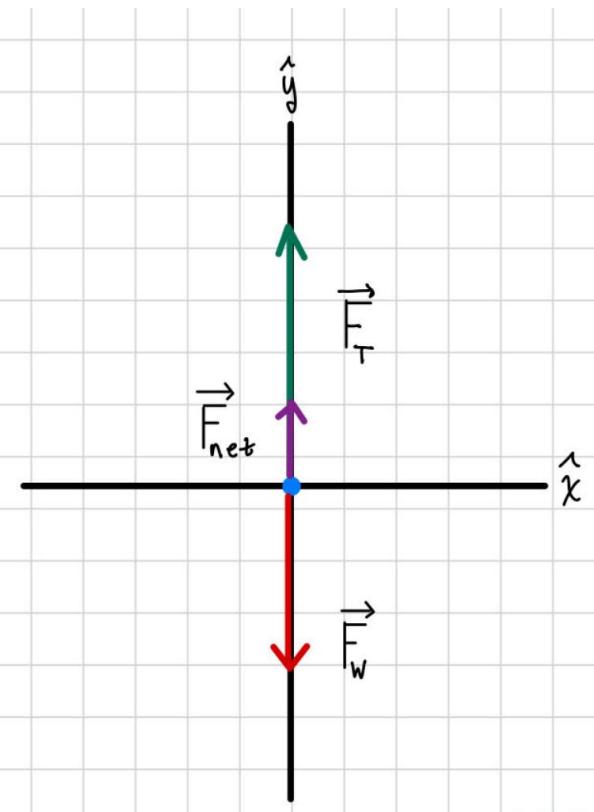
1. Identify all forces acting on an object.
2. Draw a coordinate system.
3. Represent the object as a dot at the origin of the coordinate axis.
4. Draw vectors representing each of the identified forces.
5. Draw and label the net force vector  $\vec{F}_{net}$ .

Example: An elevator suspended by a cable, speeds up as it moves upward from the ground floor. Draw a free-body diagram of the elevator.

Notice that  $\vec{F}_T$  vector is longer than  $\vec{F}_w$  vector?



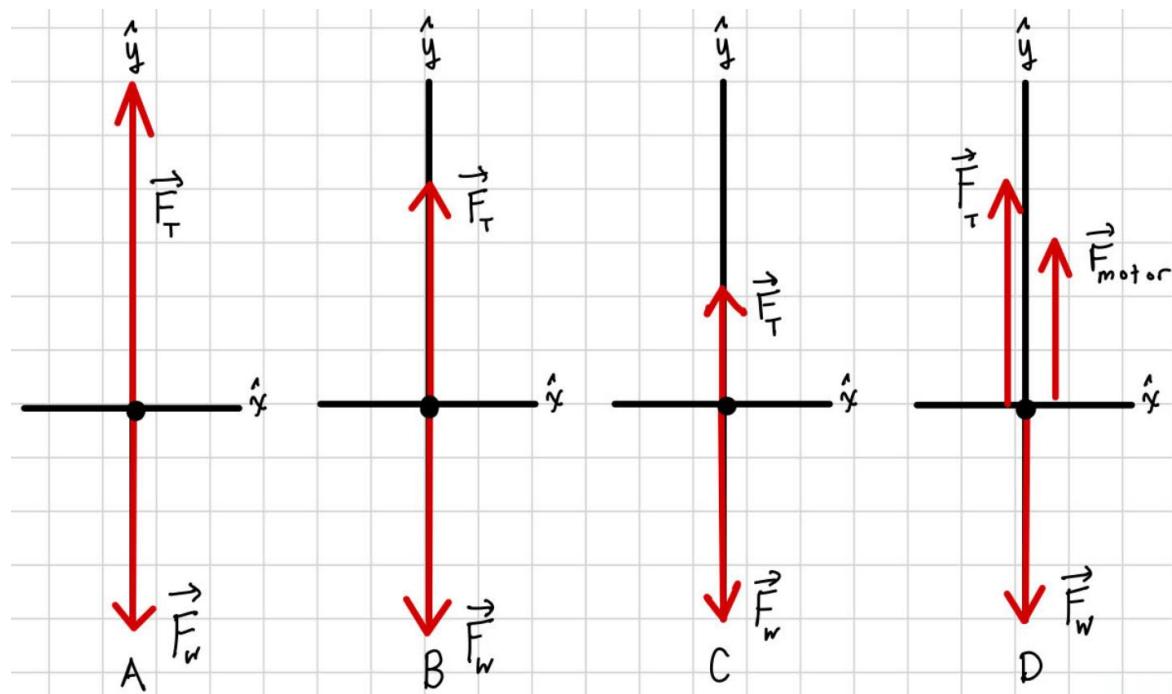
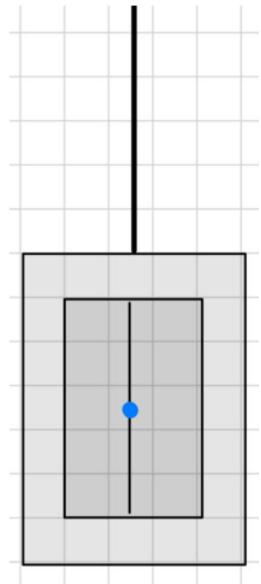
Elevator Diagram



Free-Body Diagram

# Team Activity: Concept Check 4.4

An elevator suspended by a cable is moving upward and slowing to a stop. Which free-body diagram is correct?

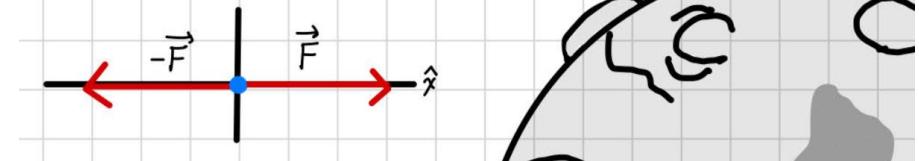
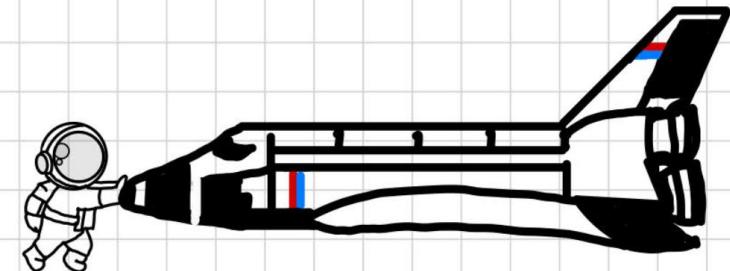


# Newton's Third Law of Motion

## You Push the Wall — the Wall Pushes Back

- Ever tried pushing on a wall as hard as you can?
- You don't move it — but *you* feel pushed backward.
- That's the wall pushing **you** with equal force.

The astronaut pushes on the spacecraft with a force  $\vec{F}$  and the spacecraft simultaneously pushes back with  $-\vec{F}$ .



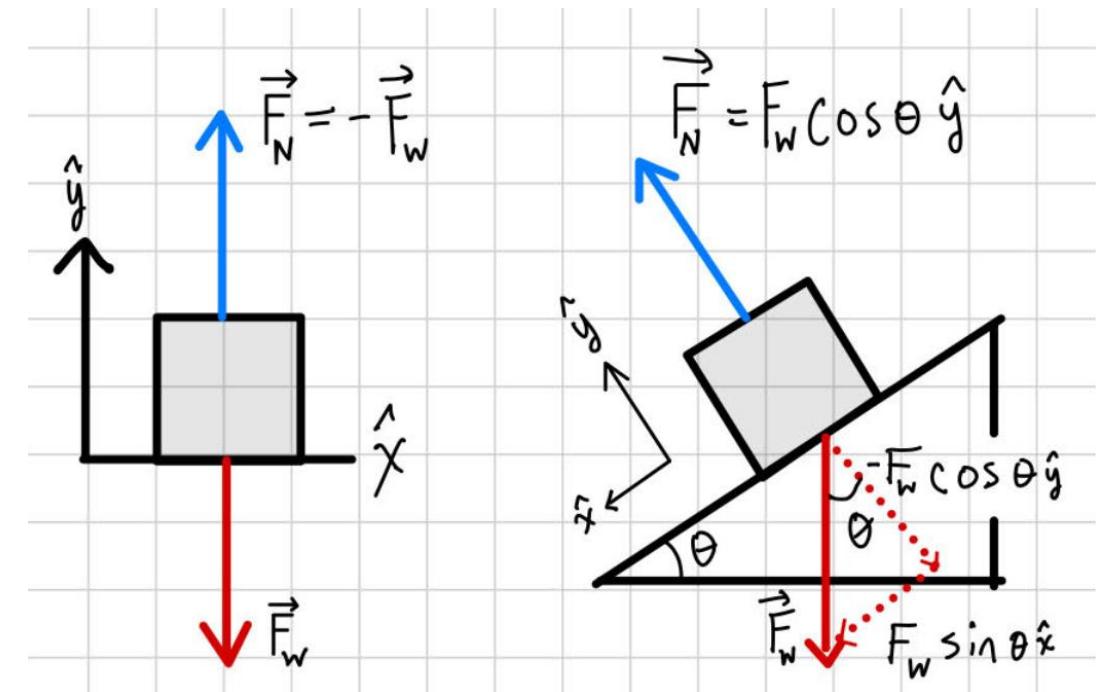
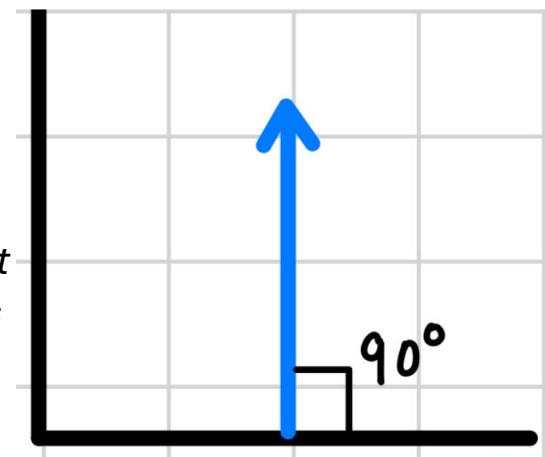
Whenever one object exerts a force on a second object, the second object exerts an oppositely directed force of equal magnitude on the first object.

$$\vec{F}_{12} = -\vec{F}_{21}$$

# The Normal Force: Surface Reaction

- When an object rests on a flat surface, the surface pushes back **perpendicular (normal)** to it.
- This upward force is called the **normal force**, and it balances the object's weight when there's no vertical acceleration.
- This is a consequence of Newton's 3<sup>rd</sup> Law!
- It doesn't always equal weight- only the part of the weight that presses into the surface.

*What does normal mean? It's not the opposite of weird in the physics context. It means that a vector is 90° to the surface.*

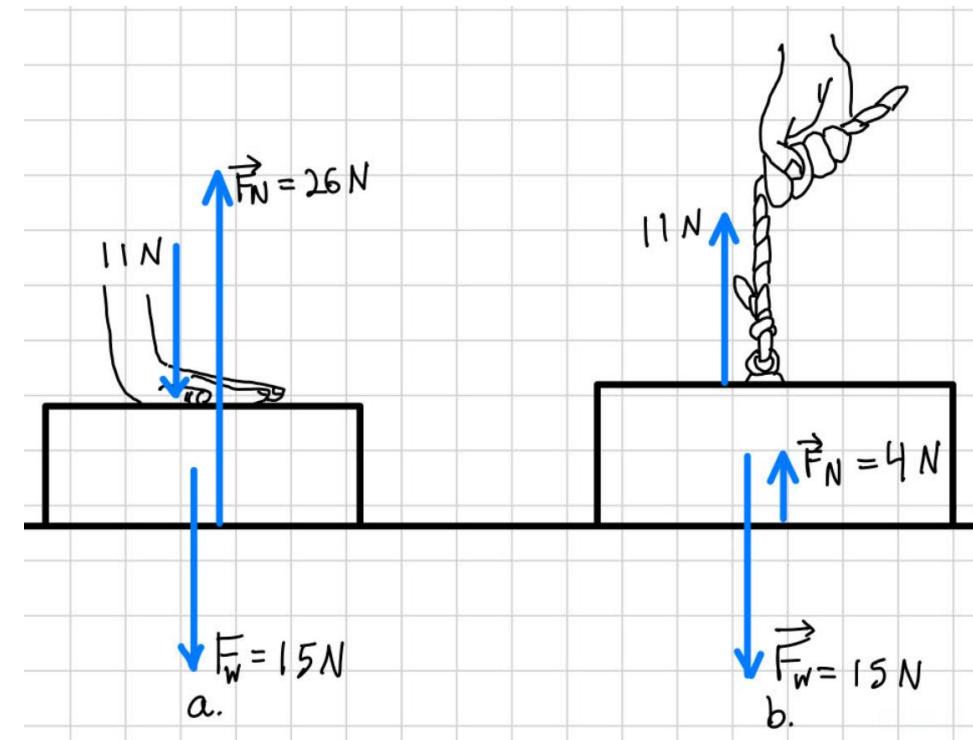


A block rests on a horizontal surface. The vector normal to the surface:  $\vec{F}_N = -F_w \hat{y}$

A block rests on an inclined surface with angle  $\theta$ . The vector normal to the surface:  
 $\vec{F}_N = -F_w \cos \theta \hat{y}$

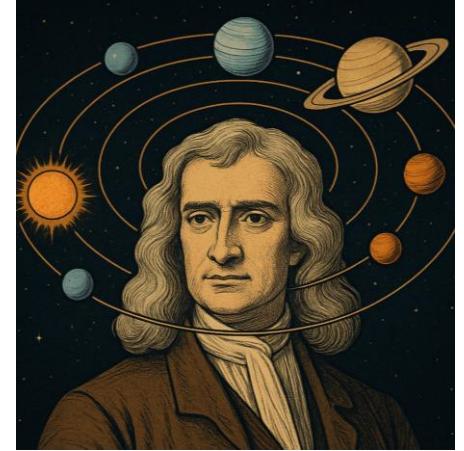
# Team Activity: Concept Check 4.5

A person pushes down with a force of 11 N on a box resting on a horizontal surface. The normal force is 26 N in the y direction. Then they attach a rope with negligible weight to the box and pull it up with a force of 11 N – the normal force is 4N. In both scenarios the weight of the block exerts a 15 N force downward. Why are they different?



# A Mysterious Force

- (Our First Force at a Distance)
- In the 1600s, scientists were puzzled:
  - Why do planets move the way they do?
  - They followed paths in the sky—loops and ellipses—but no one knew why.
- Three of the brightest minds in England—Edmund Halley, Christopher Wren, and Robert Hooke—were trying to figure it out.
  - They guessed the planets were being pulled by the Sun, and that the strength of the pull got weaker with distance.
  - But none of them could prove it.
- Halley made a trip to see a reclusive genius in Cambridge: Isaac Newton. He asked:
  - “What kind of path does something follow if it’s always pulled toward the center with a force that gets weaker as
  - Newton replied without hesitation: **“An ellipse.”**



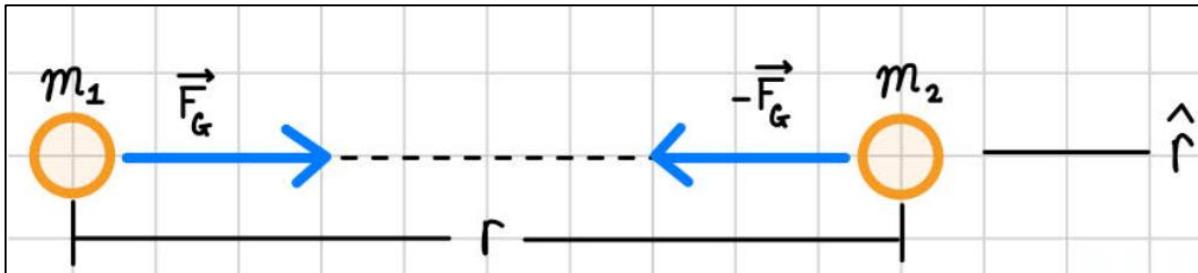
- Halley was stunned. Newton had solved the problem years ago—and wrote it all down.
- That answer became the foundation of a book that would change the world:
  - Principia Mathematica.
- **Why This Still Matters**
  - Gravity was the first force ever discovered that acts at a distance—no contact needed.
  - It holds the planets in orbit... and pulls an apple to the ground.
  - Even today, we don’t fully understand it. Gravity is simple, mysterious, and fundamental.

# Newton's Law of Universal Gravitation

For two particles that have masses  $m_1$  and  $m_2$  and are separated by a distance  $r$ , the force that each exerts on the other is directed along the line joining the particles and has a magnitude given below:

$$\vec{F}_G = G \frac{m_1 m_2}{r^2} \hat{r}$$

where  $G = 6.674 \times 10^{-11} N \cdot m/kg^2$  is the universal gravitational constant.



According to Newton's 2<sup>nd</sup> Law,  $\vec{F} = m\vec{a}$ ,

$$\vec{F}_G = m_1 \vec{a} = G \frac{m_1 m_2}{r^2} \hat{r}$$
$$\vec{a} = G \frac{m_2}{r^2} \hat{r}$$

Let's consider  $m_2 = m_E$  = Mass of the Earth and take the magnitude only, and use the radius of the Earth,  $r_E$ :

$$a = G \frac{m_E}{r_E^2} = \left( 6.674 \times 10^{-11} N \cdot \frac{m}{kg^2} \right) \frac{(5.972 \times 10^{24} kg)}{(6.71 \times 10^6 m)^2}$$
$$\approx 9.81 \frac{m}{s^2} = g$$

Therefore  $g = 9.81 m/s^2$  is only valid near the Earth surface. For any radius from the center of Earth:

$$g = G \frac{m_E}{r^2}$$



# Team Activity: Concept Check 4.5

We found an expression for the acceleration due to gravity,

$$\vec{a} = G \frac{m_2}{r^2} \hat{r}.$$

On the moon's surface this becomes,

$$\vec{a} = G \frac{m_m}{r_m^2} \hat{r}$$

where  $r_m$  and  $m_m$  are the radius and mass of the Moon, respectively. Now think back to the famous video of an astronaut dropping a feather and a hammer at the same time on the Moon. With no air resistance, they hit the ground at the same moment. ***Using this expression for gravitational acceleration, can you explain why?***

# Example: Newton's Law of Gravitation

The Earth orbits the Sun at an average distance of  $1.496 \times 10^{11} \text{ m}$  (1 AU). The Sun has a mass of  $m_{\odot} = 1.989 \times 10^{30} \text{ kg}$ , and the Earth has a mass of  $m_E = 5.972 \times 10^{24} \text{ kg}$ . VY Canis Majoris, the largest known star, is a red hypergiant star with a mass of about 17 times the mass of the Sun. **(a)**. At what distance would Earth need to be to feel the same gravitational force that it feels with respect to the Sun? **(b)**. VY Canis Majoris is also stupendous in size, with a radius of about 1,420 times the Sun's radius. Would Earth's orbit at the distance you found lie outside the star or inside it? The radius of the sun,  $R_{\odot} = 6.96 \times 10^8 \text{ m}$ .

**(a).** Earth - Sun scenario:  $F_1 = G \frac{m_E m_{\odot}}{r_1^2}$

Earth – VY Canis Majoris scenario:  $F_2 = G \frac{m_E M}{r_2^2}$

$$F_1 = F_2$$

$$G \frac{m_E m_{\odot}}{r_1^2} = G \frac{m_E M}{r_2^2}$$

$$r_2 = r_1 \sqrt{\frac{M}{m_{\odot}}} = r_1 \sqrt{\frac{17m_{\odot}}{m_{\odot}}} = r_1 \sqrt{17}$$
$$= (1.496 \times 10^{11} \text{ m}) \sqrt{17} = \boxed{6.10 \times 10^{11} \text{ m}}$$

**(b).** The radius of VY Canis Majoris,

$$R_{CM} = 1420 R_{\odot} = 9.88 \times 10^{11} \text{ m}$$

That is more than 1.42 times bigger than the orbit we calculated in (a). For the Earth to feel the same gravitational force from VY Canis Majoris that it feels from the Sun, its distance from VY Canis Majoris would put it firmly inside the red hypergiant.



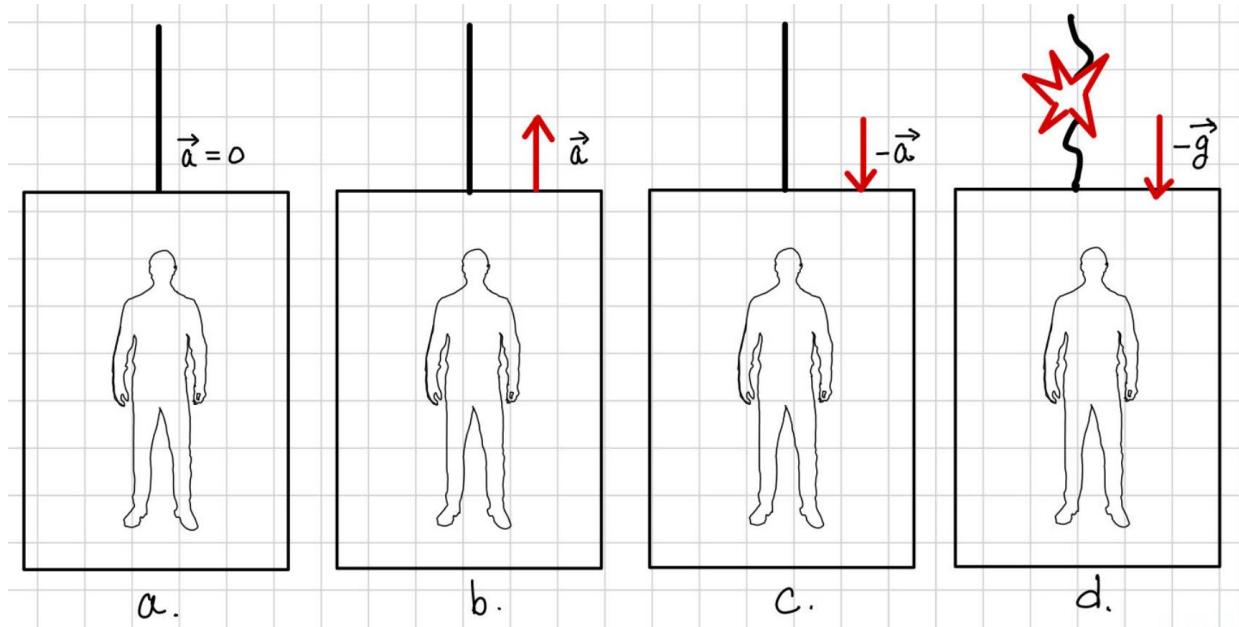
# Apparent Weight

**Weight** is a force—it's the result of gravity acting on a mass. **Apparent weight** is the force an object seems to have when there's additional acceleration involved, such as in an elevator or accelerating vehicle. The man's weight can be determined in each scenario by summing up the forces,

$$F_{net} = N - mg = ma \rightarrow N = m(a + g) = \frac{F_w}{g}(g + a)$$

Consider the following:

- a) A man in an elevator that is not in motion has a weight of  $F_w = 700 \text{ N}$ .
- b) Accelerating upward  $a = 2 \text{ m/s}^2$
- c) Accelerating downward  $a = -2 \text{ m/s}^2$
- d) Free Fall  $a = -g$



Case	Acceleration ( $\frac{\text{m}}{\text{s}^2}$ )	Apparent Weight (N)
a	0	700
b	2	843
c	-2	557
d	-g	0

# Team Activity: Concept Check 4.6

You're standing on a scale in an elevator.

The scale briefly reads zero.

Which of the following must be true?

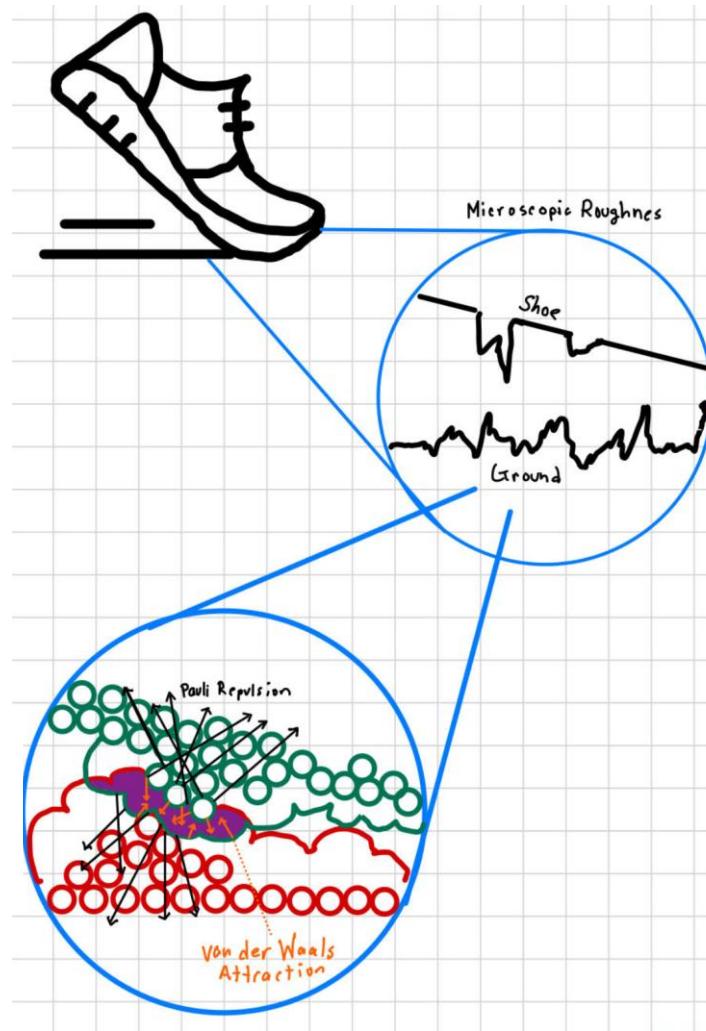
- a) The elevator is at rest.
- b) The elevator is accelerating downward at  $9.8 \text{ m/s}^2$ .
- c) The elevator is moving downward at constant speed.
- d) The elevator cable is pulling upward with a force equal to your weight.



# From Sliding Shoes to Sliding Atoms: What is Friction, Really?

- We all know friction — it's why your shoes grip the floor, or why brakes stop a car.
- It resists motion — but why?
- Friction isn't just about roughness — it's about how atoms and molecules tug on each other when two surfaces meet.

*Atomic Level: Attractive and repulsive forces compete.*



*Microscopic Roughness:  
Surfaces aren't smooth.  
Lots of bumps  
(asperities).*

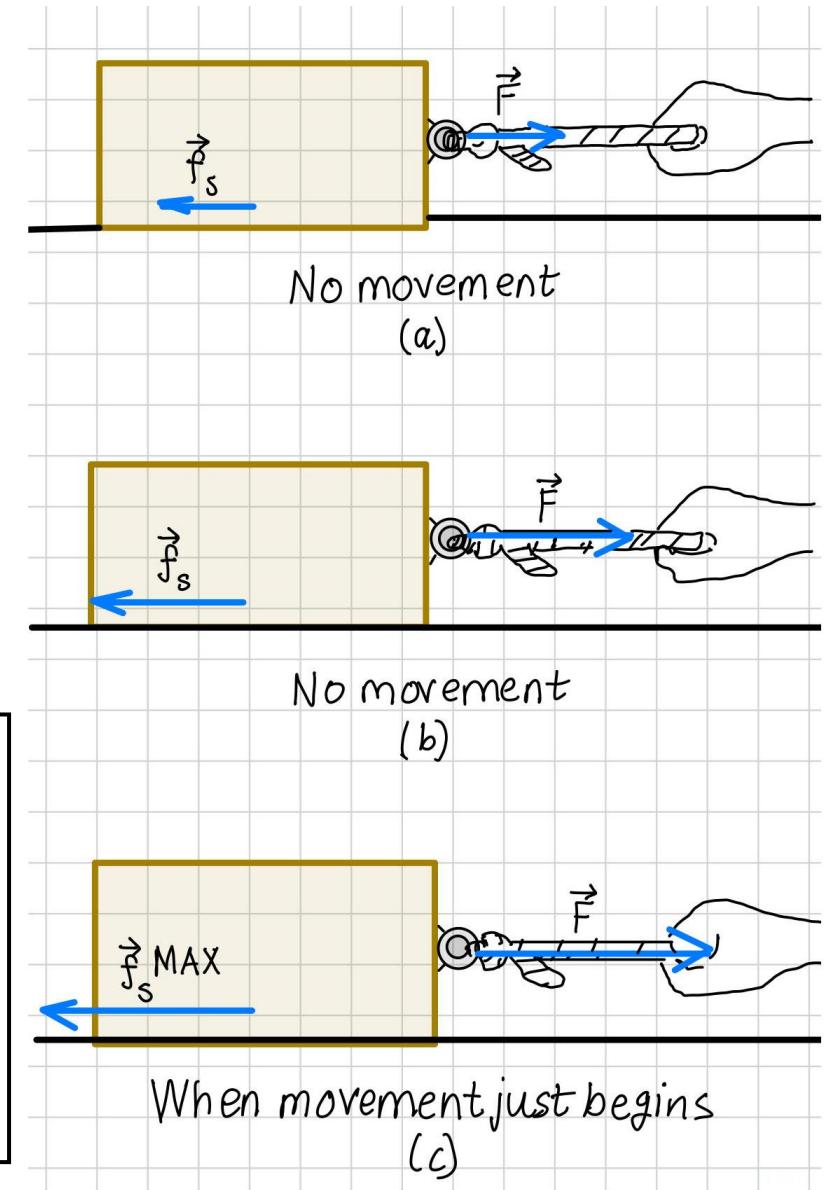
# Static Frictional Force

- (a). Block is initially at rest, then a horizontal force is applied but the block does not move:  $\vec{f}_s = \vec{F}$ .
- (b). A stronger force is applied; static friction force increases to match and block still does not move:  $\vec{f}_s = \vec{F}$ .
- (c). A maximum static friction force is reached and  $\vec{f}_s \leq \vec{F}$ . Block starts to move.

The magnitude  $f_s$  of the static frictional force can have any value from zero up to a maximum value of  $f_s^{max}$ , depending on the applied force.

$$f_s^{max} = \mu_s F_N$$

where  $\mu_s$  is the coefficient of static friction, and  $F_N$  is the magnitude of the normal force.



# Example: Static Frictional Force

A snowboarder is standing motionless on a horizontal patch of snow. She is holding onto a horizontal tow rope, which is about to pull her forward. The snowboarder's mass is 60kg and the coefficient of static friction between her board and the snow is 0.14. What is the magnitude of the maximum force that the tow rope can apply to the skier without causing her to move?

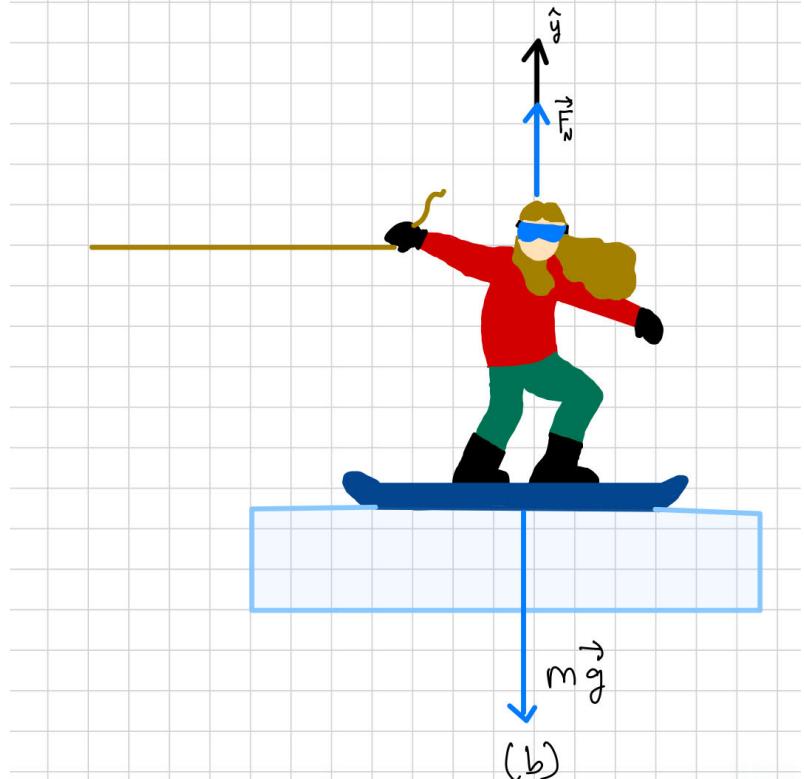
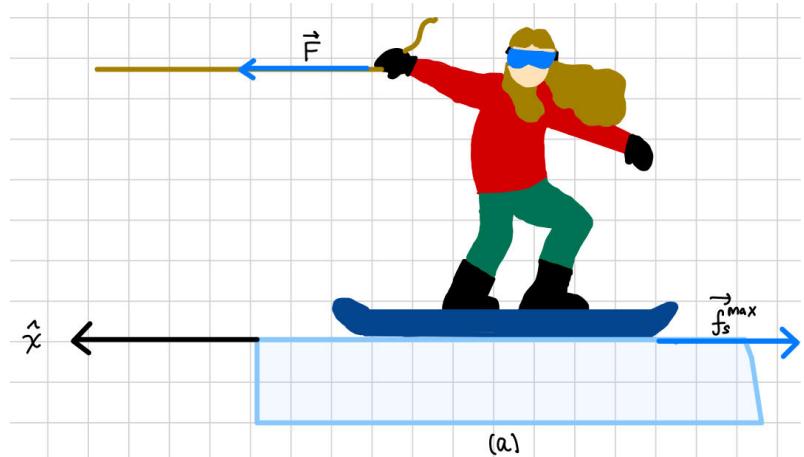
Use Newton's 2<sup>nd</sup> Law to sum the forces:

$$(a). \sum F_x = ma_x = F - f_s^{\max} = 0 \rightarrow F = f_s^{\max} = \mu_s F_N = \mu_s mg$$

$$(b). \sum F_y = ma_y = F_N - mg = 0 \rightarrow F_N = mg$$

$$F = \mu_s mg = (0.14)(60\text{kg})(9.81\text{m/s}^2) = 81 \text{ N}$$

If the force exerted on the tow rope exceeds 81 N the snowboarder will begin to move.



# Kinetic Frictional Force

- Once two surfaces begin sliding over one another, the friction becomes kinetic!
- It takes less force to keep an object sliding than it takes to get it going in the first place,  $\vec{f}_k < \vec{f}_s^{max}$
- Independent of apparent area of contact between the surfaces and the speed (if small).

The magnitude  $f_k$  of the kinetic friction force is given by

$$f_k = \mu_k F_N$$

where  $\mu_k$  is the coefficient of kinetic friction, and  $F_N$  is the magnitude of the normal force.



# Example: Kinetic Fictional Force

Consider a block with mass  $M$  sliding down an incline that is  $\theta$  degrees from the horizontal surface. Derive an expression for the final velocity of the block after it travels a distance  $d$  down the incline once it is in motion.

Sum the forces in y:  $\sum F_y = F_N - mg \cos \theta = 0 \rightarrow F_N = mg \cos \theta$

Sum the forces in x:  $\sum F_x = ma_x = mg \sin \theta - f_k = mg \sin \theta - \mu_k F_N$

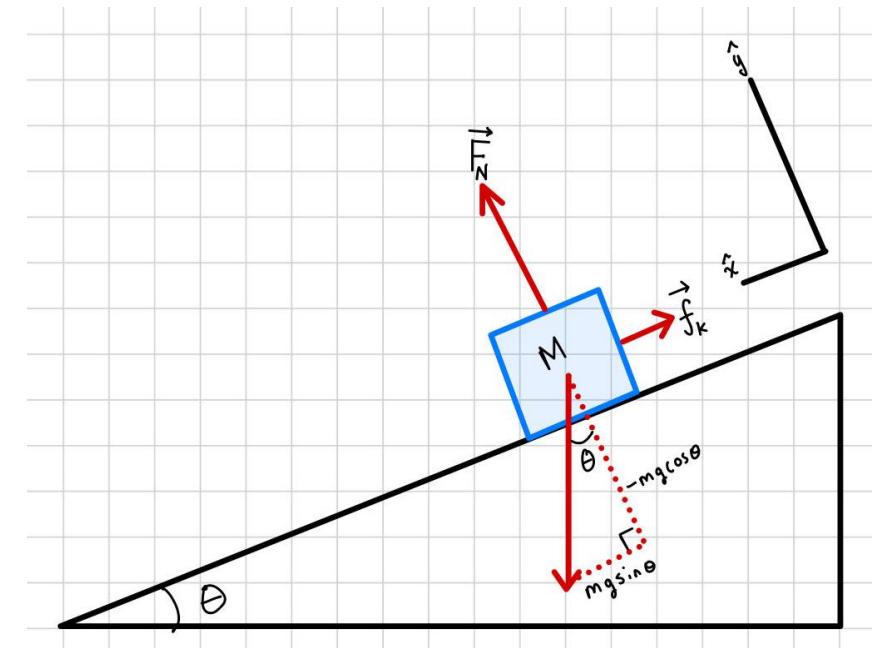
$$ma_x = mg(\sin \theta - \mu_k \cos \theta)$$

$$a_x = g(\sin \theta - \mu_k \cos \theta)$$

Now use kinematic equations,

$$v_x^2 - v_{0x}^2 = 2a_x(x - x_0) \rightarrow v_x = \sqrt{2gd(\sin \theta - \mu_k \cos \theta)}$$

*This is an example of a nonequilibrium system (more on this later).*

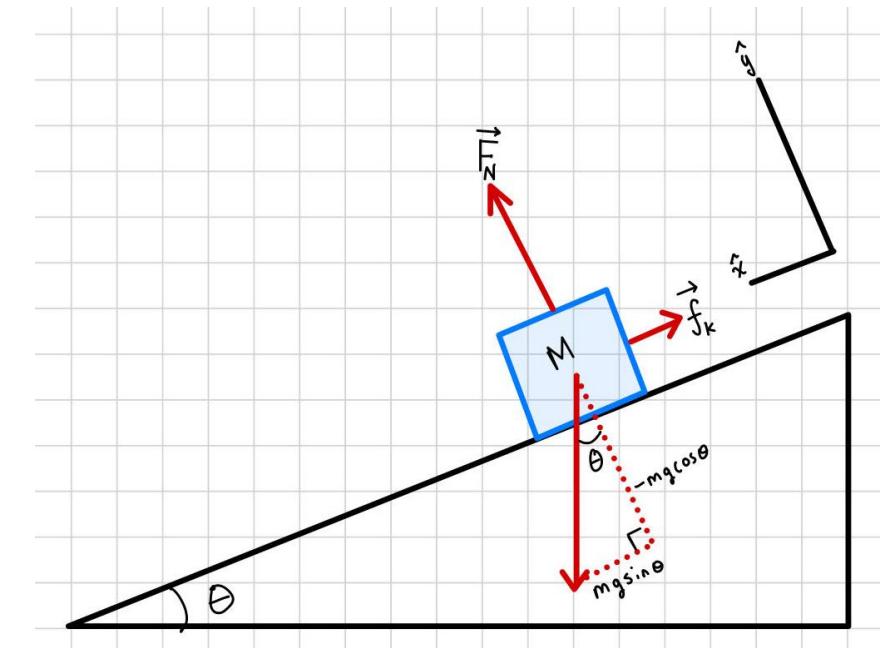


# Team Activity: Concept Check 4.7

On the previous slide we derived an expression for the acceleration of the block down the incline

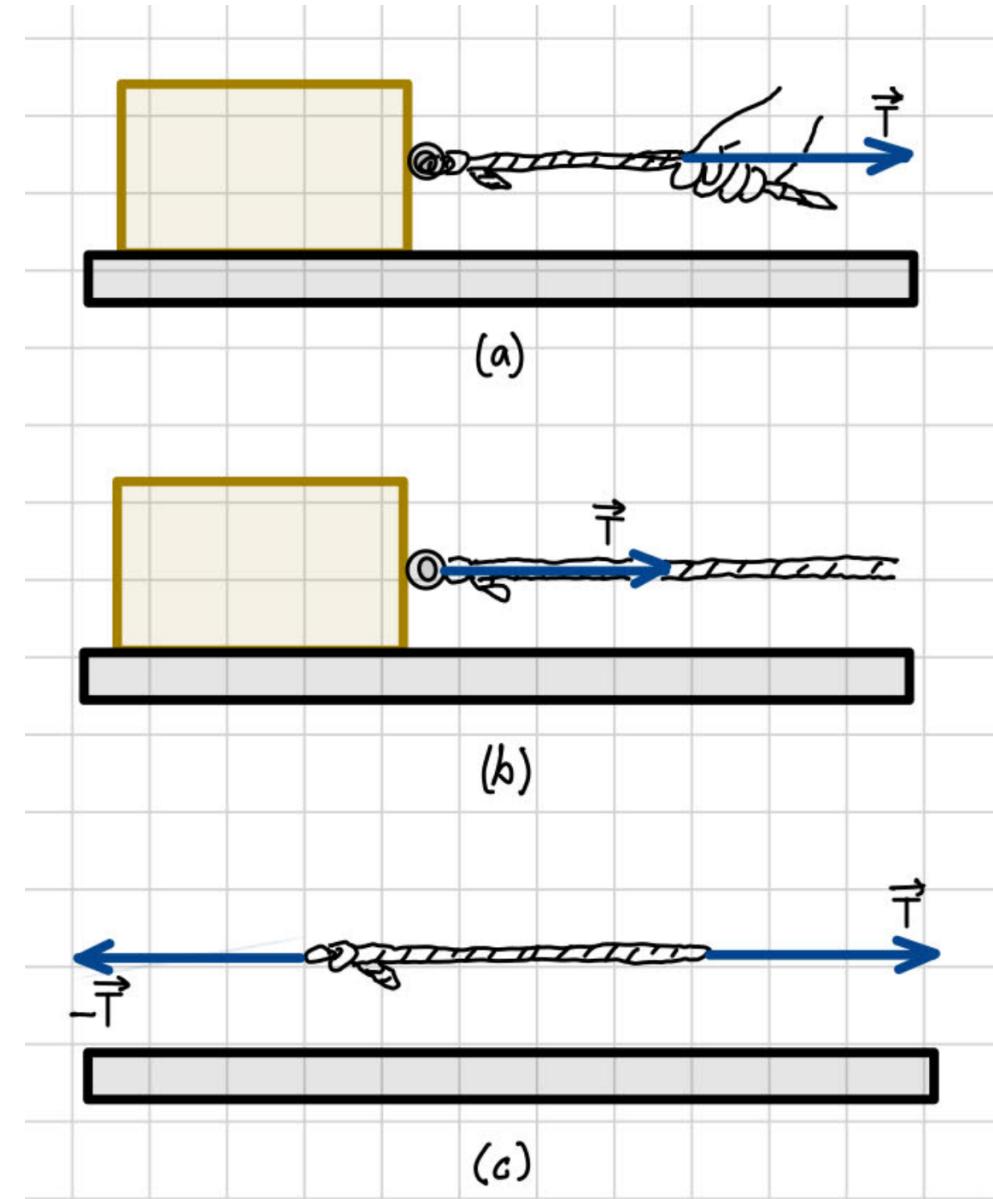
$$a_x = g(\sin \theta - \mu_k \cos \theta)$$

Derive a condition for  $\mu_k$  that must be true in order that must be true for the block to be in motion.



# Tension

- Tension is the name we give to forces applied with things like cables or ropes (we have already seen this).
- In the figure,
  - a) A force that we call  $\vec{T}$  is being applied to the right end of a rope.
  - b) The force is transmitted to the box.
  - c) Via Newton's 3<sup>rd</sup> law we know that forces are applied to both ends of the rope because the box applies a reaction force to the rope.
- We consider the rope massless
  - a massless rope does not exist.
  - We do this to study tension in isolation



# Team Activity: Concept Check 4.8

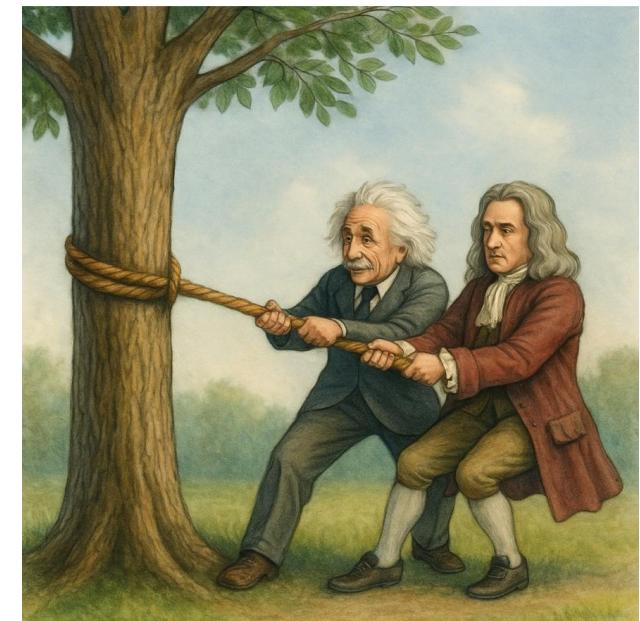
Hooke and Kepler are in a tug-of-war match against Einstein and Newton. Each team pulls on the rope with a force of 1000 N.

In round two, Hooke and Kepler tie their end of the rope to a tree, and Einstein and Newton pull on the other end with the same 1000 N force.

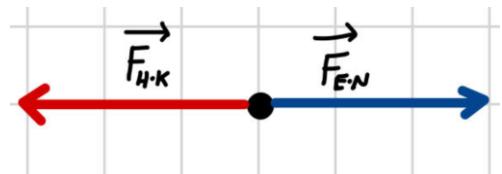
In both rounds, the pulling is steady, and the rope is taut.

Which statement is true?

- a) The tension in the rope in round 1 is greater than in round 2
- b) The tension in the rope in round 1 is less than in round 2
- c) The tension in the rope in round 1 and 2 are equivalent?

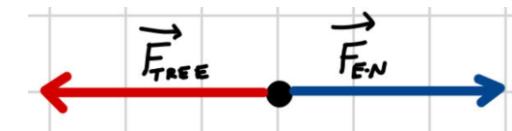
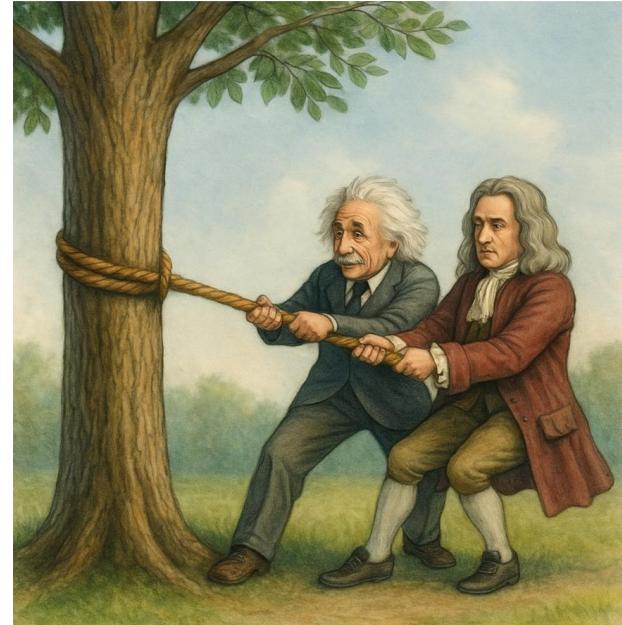


# Equilibrium



$$\begin{aligned}\sum F &= F_{HK} + F_{EN} = ma = 0 \\ F_{HK} &= -F_{EN} = -1000N\end{aligned}$$

*Why is the tension the same?* Use free body diagrams to analyze. In both cases the acceleration of the rope is 0. We call this state **equilibrium**.  
Newton's third law tells us that the tree pulls back with an equal and opposite force. In equilibrium, this is also reflected by Newton's second law. This is no different from the case with Hooke and Kepler!



$$\begin{aligned}\sum F &= F_{TREE} + F_{EN} = ma = 0 \\ F_{TREE} &= -F_{EN} = -1000N\end{aligned}$$

# Equilibrium Application of Newton's Laws of Motion

## Definition of Equilibrium

**An object is in equilibrium when it has zero acceleration.**

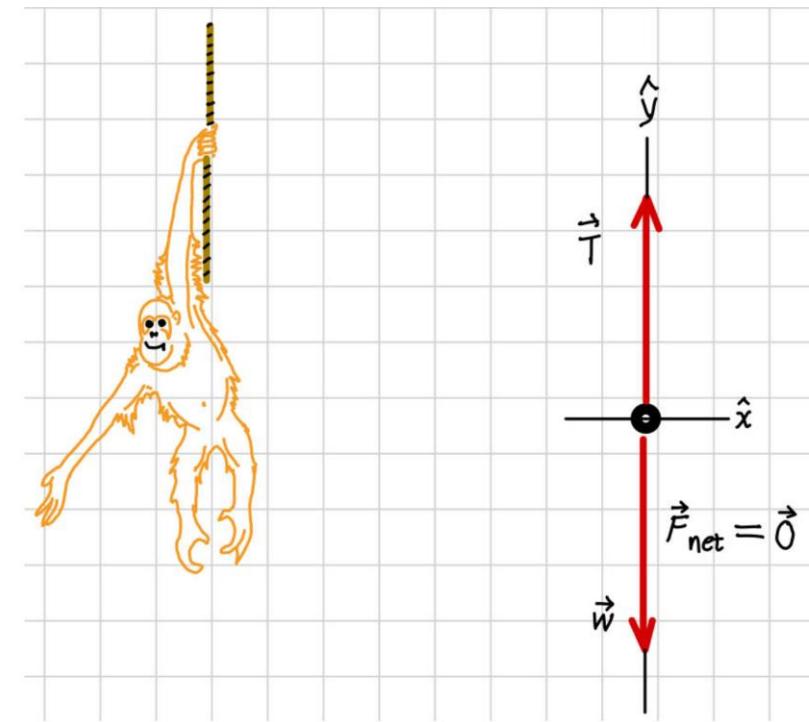
$$\vec{a} = a_x \hat{x} + a_y \hat{y} = 0 \rightarrow a_x = a_y = 0$$
$$\sum F_x = 0 \quad \sum F_y = 0$$

## Example:

An orangutan weighing 500N hangs from a vertical rope. What is the tension in the rope?

$$\sum F_y = T - w = 0$$

$$T = w = 500N$$



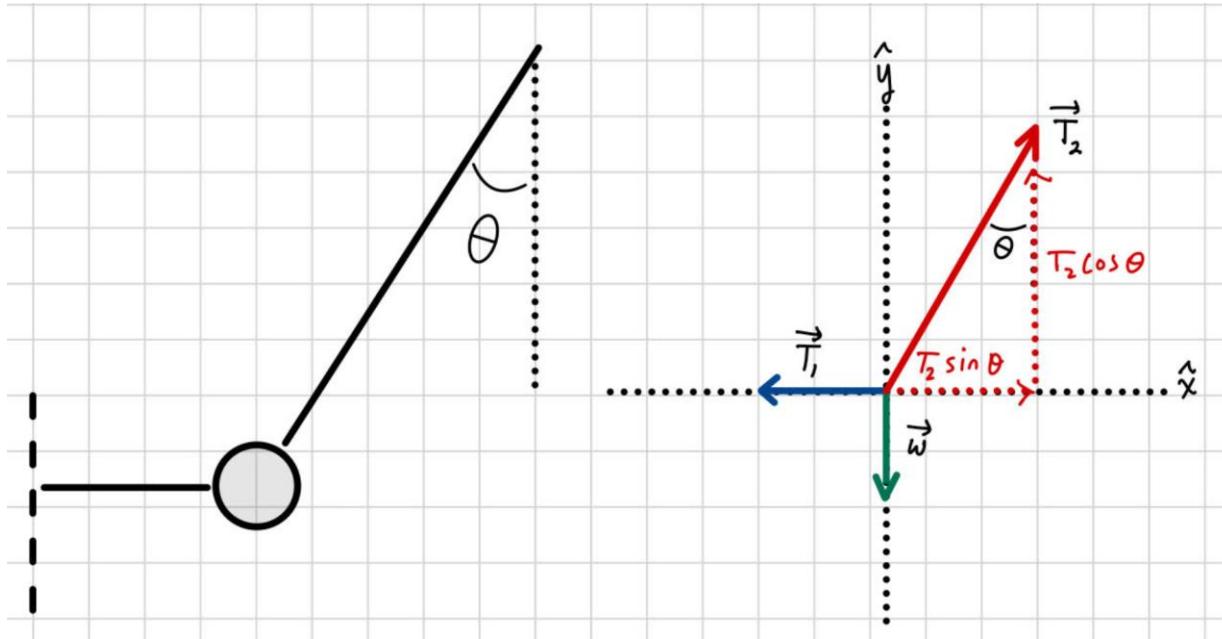
# Example: Equilibrium

A wrecking ball weighing 2500N hangs from a cable. Prior to swinging, it is pulled back to a  $20^0$  angle by a second, horizontal cable. What is the tension in the horizontal cable?

Solution:

$$\sum F_y = T_2 \cos \theta - w = ma_y = 0$$
$$T_2 \cos \theta = w \rightarrow T_2 = w / \cos \theta$$

$$T_1 = T_2 \sin \theta = \left( \frac{w}{\cos \theta} \right) \sin \theta = w \tan \theta = (2500N) \tan 20^0 = 910N$$



# Nonequilibrium Application of Newton's Laws of Motion

## Definition of Nonequilibrium

An object is in equilibrium when it has nonzero acceleration.

$$\vec{a} = a_x \hat{x} + a_y \hat{y} \neq 0$$
$$\sum F_x \neq 0 \text{ and/or } \sum F_y \neq 0$$

Block 1 (mass  $m_1 = 800 \text{ kg}$ ) is moving on a frictionless  $30^\circ$  incline. This block is connected to block 2 (mass  $m_2 = 22.0 \text{ kg}$ ) by a massless cord that passes over a massless and frictionless pulley. Find the acceleration of each block and the tension in the cord.

I.  $\sum F_x = T - m_1 g \sin \theta = m_1 a_x$      $\sum F_y = N - m_1 g \cos \theta = 0$

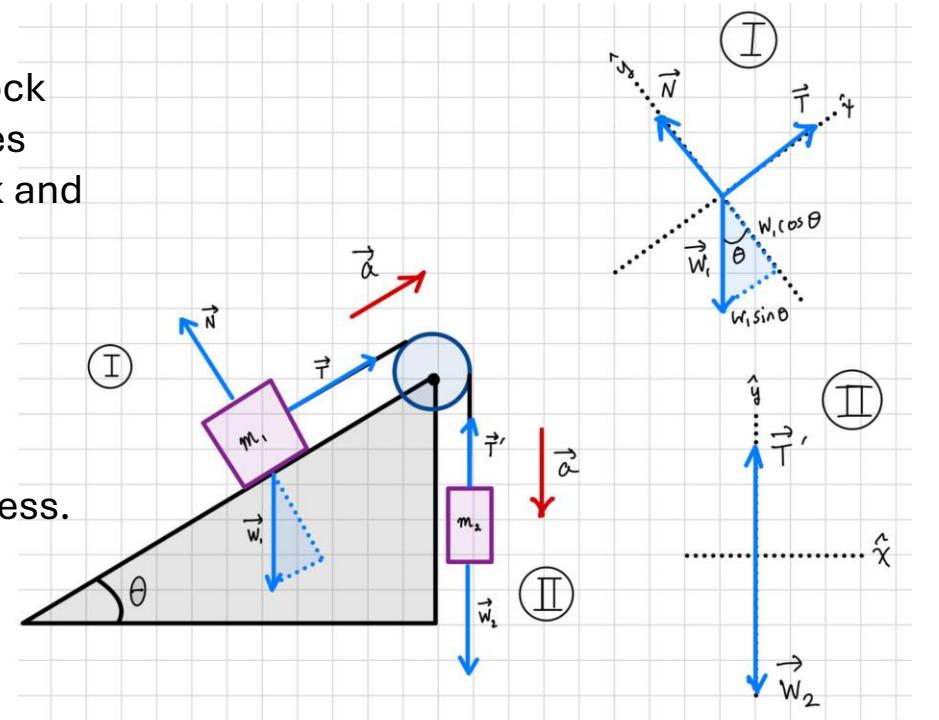
II.  $\sum F_x = 0$      $\sum F_y = T' - m_2 g = -m_2 a_y \rightarrow T' = m_2(g - a_y)$

Note that  $a_x = a_y = a$      $T = T'$  because the pulley is massless and frictionless.

$$m_2(g - a) - m_1 g \sin \theta = m_1 a \quad N = m_1 g \cos \theta$$

$$a(m_1 + m_2) = g(m_2 - m_1 \sin \theta)$$

$$a = \frac{g(m_2 - m_1 \sin \theta)}{m_1 + m_2} = \frac{(9.81 \text{ m/s}^2)(22 - 8 \sin 30^\circ) \text{ kg}}{(8 + 22) \text{ kg}} = 5.89 \text{ m/s}^2$$



$$T = m_2(g - a) = 22 \text{ kg}(9.81 - 5.89) \text{ ms}^{-2} = 86.2 \text{ N}$$