CSI-674 Jericho McLeod
Homework 9 G00986513

Problem 1:

- Given $(\theta_{1:n_s}, \tau)$, μ is normally distributed with parameters:
    - $mean = \frac{(n*\theta)/\tau^2 + mean_{prior}/{std_{prior}}^2}{n/\tau^2 + 1/{std_{prior}}^2}$
    - $st.d. = (n/\tau^2 + 1/{std_{prior}}^2)^{-1/2}$

- Given $(y_{1:n_s}, \theta_{1:n_s})$, $1/_{\tau^2}$ has a Gamma distribution of parameters
    - $\alpha = \alpha_{prior} + n/2$
    - $\beta = \left(\frac{1}{\beta} + \frac{1}{2}\left(\sum_{s=1}^{n}(\theta_s - \mu)^2\right)\right)^{-1}$

- Given $(\mu, \tau, \sigma, y_{1:ns})$, each $\theta_s$is normally distributed with parameters:
    - $mean = \frac{\sum_{i=1}^{n_s} y_{si}/\sigma^2 + \mu/\tau^2}{\frac{n_s}{\sigma^2} + 1/\tau^2}$
    - $st.d. = (\frac{n_s}{\sigma^2} + \frac{1}{\tau^2})^{-1/2}$

Completing these with the available information:

- Given $(\theta_{1:n_s}, \tau)$, μ is normally distributed with parameters:
    - $mean = \frac{62.896/\tau^2 + 5.718/0.0484}{11/\tau^2 + 1/0.0484}$
    - $st.d. = (11/\tau^2 + 1/0.0484)^{-1/2}$

- Given $(y_{1:n_s}, \theta_{1:n_s})$, $1/_{\tau^2}$ has a Gamma distribution of parameters:
    - $\alpha = 0.5 + \frac{11}{2} = 6$
    - $\beta = \left(\frac{1}{50} + \frac{1}{2}\left(\sum_{s=1}^{n}(\theta_s - \mu)^2\right)\right)^{-1}$

- Given $(\mu, \tau, \sigma, y_{1:n})$, each $\theta_s$is normally distributed with parameters:
    - $mean_s = \frac{\sum_{i=1}^{30} y_{si}/0.0246 + \mu/\tau^2}{1219.512 + 1/\tau^2}$
    - $st.d. = (1219.512 + \frac{1}{\tau^2})^{-1/2}$

- Solving for each θ:
    - $mean_1 = \frac{6990.472+\mu/\tau^2}{1219.512+1/\tau^2}$
    - $mean_2 = \frac{7190.71+\mu/\tau^2}{1219.512+1/\tau^2}$
    - $mean_3 = \frac{6963.774+\mu/\tau^2}{1219.512+1/\tau^2}$
    - $mean_4 = \frac{6957.292+\mu/\tau^2}{1219.512+1/\tau^2}$
    - $mean_5 = \frac{6793.01+\mu/\tau^2}{1219.512+1/\tau^2}$
    - $mean_6 = \frac{7071.808+\mu/\tau^2}{1219.512+1/\tau^2}$
    - $mean_7 = \frac{7155.137+\mu/\tau^2}{1219.512+1/\tau^2}$
    - $mean_8 = \frac{6805.726+\mu/\tau^2}{1219.512+1/\tau^2}$
    - $mean_9 = \frac{6757.813+\mu/\tau^2}{1219.512+1/\tau^2}$
    - $mean_{10} = \frac{7045.878+\mu/\tau^2}{1219.512+1/\tau^2}$
    - $mean_{11} = \frac{6971.088+\mu/\tau^2}{1219.512+1/\tau^2}$

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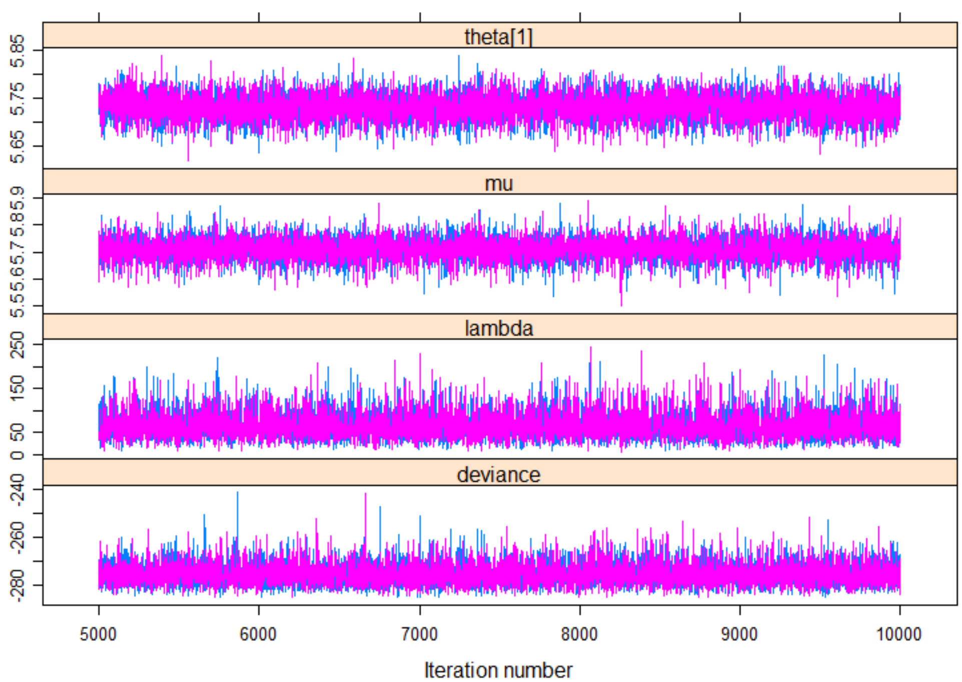
Problem 2:

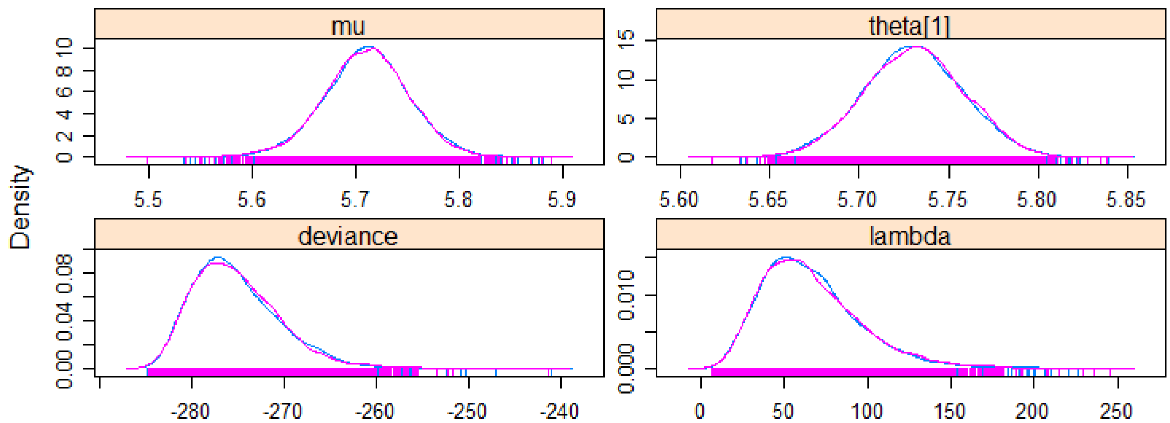
The 95% credible interval for Mu, Tau, and the Theta values are shown in the following table:

	2.50%	**97.50%**
Tau	0.086	0.213
Mu	5.627	5.791
Theta 1	5.675	5.786
Theta 2	5.831	5.942
Theta 3	5.657	5.766
Theta 4	5.65	5.76
Theta 5	5.522	5.632
Theta 6	5.739	5.849
Theta 7	5.804	5.914
Theta 8	5.533	5.643
Theta 9	5.494	5.605
Theta 10	5.72	5.83
Theta 11	5.661	5.772

Looking at a sampling of the trace-plots, the model appears to have run effectively.



And examining the density gives values in the expected range. Note that 'Lambda' was my notation for for 1/tau^2.



Code used for Problem 2:

R code:
```
library(R2jags)     # running JAGS from r
library(superdiag) # mcmc diagnostics
library(coda)
library(lattice)    # for some of the plots
library(MCMCvis)    # visualization functions

rtimes  <-
read.csv("reactiontimes_long.csv",quote="",comment.char="",stringsAsFactor
s=FALSE)
individual <- rtimes[,1]
times <- rtimes[,2]
times <- log(times)
indMeans <- aggregate(times,by=list(individual),FUN='mean')[,2]n <-
length(unique(individual))
ns <- length(times)math.data <- list('n','ns','individual','times')
math.params <- c('mu','lambda','theta')math.inits <- rt.inits<-
function(){
  list('mu'=c(6),'lambda'=c(25),'theta'=c(indMeans))
}math.fit <- jags(data=math.data, inits = math.inits,math.params,
n.chains=2,

n.iter=10000,n.burnin=5000,model.file='reactiontimes.jags',n.thin=1)reacti
on.times.fit.mcmc<-as.mcmc(math.fit)
summary(reaction.times.fit.mcmc)
plot(reaction.times.fit.mcmc)select.mcmc=
reaction.times.fit.mcmc[,match(c("deviance","lambda","mu",'theta[1]'),
varnames(reaction.times.fit.mcmc))]
xyplot(select.mcmc)densityplot(select.mcmc)
```

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JAGS Model:
```
model {
      for(i in 1:n) {
            theta[i]~dnorm(mu,lambda)
      }
      for(j in 1:ns) {
            times[j]~dnorm(theta[individual[j]],40.58379) # Hard coded
1/sigma^2
      }
      mu ~ dnorm(5.52,1/0.22^2) # uses prec rather than std
      lambda~ dgamma(0.5,1/50) # uses rate not scale, is 1/tau^2
}
```

Problem 3:

Assuming $\sigma = 0.15697$, $\mu = 5.71783$, $\tau = 0.11682$, the posterior distributions for the Theta values are normal. The standard deviation for each theta is the same, as it is not dependent on the Ysj values; it is **0.02783**. The means are:

	Mean
Theta 1	5.731372
Theta 2	5.886247
Theta 3	5.710722
Theta 4	5.705709
Theta 5	5.578644
Theta 6	5.794281
Theta 7	5.858732
Theta 8	5.58848
Theta 9	5.551421
Theta 10	5.774226
Theta 11	5.716379

Comparing this to the Gibbs sampler we get the following table of differences for the 95% credible intervals:

	Gibbs 2.5%	**Post 2.5%**	**Diff 2.5%**	**Gibbs 2.5%**	**Post 2.5%**	**Diff**
Theta 1	5.675	5.67682	-0.00182	5.786	5.785924	7.6E-05
Theta 2	5.831	5.831694	-0.000694	5.942	5.940799	0.001201
Theta 3	5.657	5.65617	0.00083	5.766	5.765275	0.000725
Theta 4	5.65	5.651157	-0.001157	5.76	5.760261	-0.000261
Theta 5	5.522	5.524092	-0.002092	5.632	5.633197	-0.001197
Theta 6	5.739	5.739729	-0.000729	5.849	5.848834	0.000166
Theta 7	5.804	5.80418	-0.00018	5.914	5.913285	0.000715
Theta 8	5.533	5.533928	-0.000928	5.643	5.643032	-3.2E-05
Theta 9	5.494	5.496869	-0.002869	5.605	5.605974	-0.000974
Theta 10	5.72	5.719673	0.000327	5.83	5.828778	0.001222
Theta 11	5.661	5.661827	-0.000827	5.772	5.770932	0.001068

The difference column is (Gibbs – Posterior), and is very small; perhaps not as small as machine precision, but small enough that in practice either would likely be an effective model. Of course, that would be unknown until after comparing results from both.

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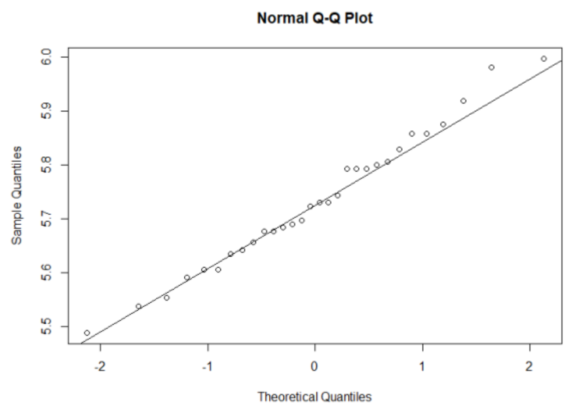
Code used for Problem 3:

```
theta_mean_update <- function(ys,s,m,t){
  th_m <- ( sum(ys)/s^2 + m/t^2 ) / ( length(ys)/s^2 + 1/t^2 )
  return(th_m)
}theta_sd_update <- function(ns,s,t){
  th_sd <- (ns/s^2 + 1/t^2)^-0.5
}
```

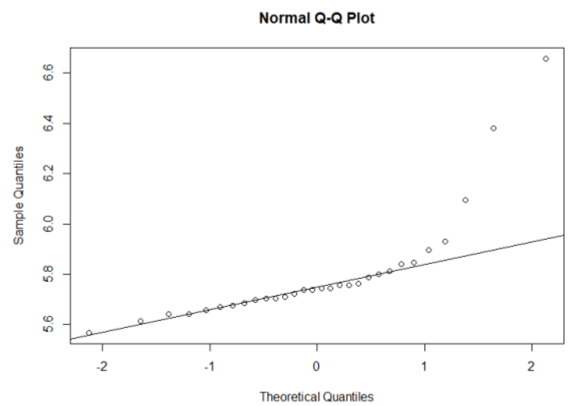
Problem 4:

The assumptions that sigma is common and that thetas are normally distributed do not appear to be reasonable across the set of individuals. The theta values, or means for each individual, can be tested with QQ plots. Some, such as for the first individual, appear to be normal. However, others, such as for the 6th individual, are notably not normally distributed.

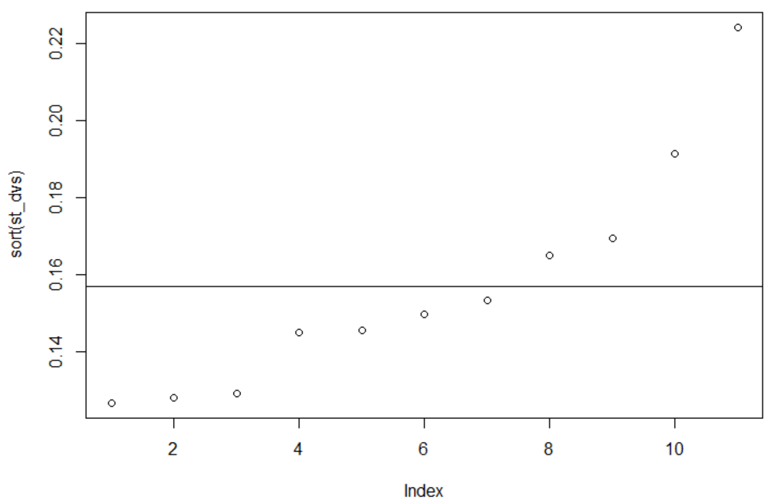
First Individual



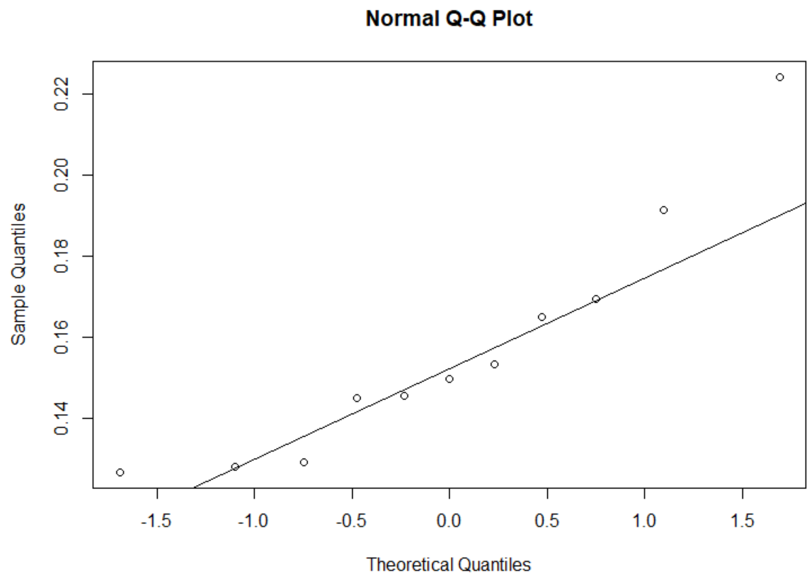
Sixth Individual



Additionally, the standard deviations appear to be distributed around the mean, with the largest being over 50% greater than the smallest.



Nor is the distribution of standard deviations among the individuals normal, though it is closer than the means.



For the individuals, though, the veracity of reaction time testing may be important. It is possible that reaction times are indeed normally distributed, but the individuals being tested were inattentive, resulting in some times that were abnormally high. This may also have impacted the standard deviations within individual times, but is purely conjecture. It is more reasonable to assume that reaction times are right-skewed distributions, as they are near 0 normally and have a much greater space to expand to the right. The observed data supports this, while my conjecture is unsupported logic.