SYST/STAT 664: Homework Assignment 8
due April 20, 2020

Homework is at the time and date indicated. Please make sure your name is on every page of the assignment, and it is clearly marked which question you are answering. Your response will be graded for correctness and clarity.

1. This problem continues with the HCB pollution data from the Assignments 6 and 7.

Surface	Bottom
3.74	5.44
4.61	6.88
4.00	5.37
4.67	5.44
4.87	5.03
5.12	6.48
4.52	3.89
5.29	5.85
5.74	6.85
5.48	7.16

As with Assignments 6 and 7, assume the observations are normally distributed with unknown depth-specific means Θ_s and Θ_b and precisions P_s and P_b. Assume that experts have provided the following prior information based on previous studies.
- The unknown means Θ_s and Θ_b are independent and normally distributed with mean μ and standard deviation τ. The unknown precisions P_s and P_b are independent of Θ_s and Θ_b and have gamma distributions with shape α and scale β.
- Experts specified a 95% prior credible interval of [3, 9] for Θ_s and Θ_b. A good fit to this credible interval is obtained by setting the prior mean to μ =6 and the prior standard deviation to τ=1.5.
- A 95% prior credible interval of [0.75, 2.0] is given for the unknown standard deviations Σ_s and Σ_b. This translates to a credible interval of [0.25, 1.8] for $P_s = \Sigma_s^{-2}$ and $P_b = \Sigma_b^{-2}$. A good fit to this credible interval is obtained by setting the prior shape to $\alpha = 4.5$ and the prior scale to $\beta = 0.19$.

Find the following conditional distributions.
- The conditional distribution for Θ_s given the other parameters and the observations.
- The conditional distribution for Θ_b given the other parameters and the observations.
- The conditional distribution for P_s given the other parameters and the observations.
- The conditional distribution for P_b given the other parameters and the observations.

2. Using the distributions you found in Part 1, draw 10,000 Gibbs samples of $(\Theta_s, \Theta_b, P_s, P_b)$. Estimate 90% credible intervals for $\Theta_s, \Theta_b, \Sigma_s = P_s^{-1/2}, \Sigma_b = P_b^{-1/2}$, and $\Theta_b - \Theta_s$.

3. Do a traceplot of $\Theta_b - \Theta_s$. Find the autocorrelation function of $\Theta_b - \Theta_s$ and the effective sample size for your Monte Carlo sample for $\Theta_b - \Theta_s$.

4. Comment on your results. Compare with Assignment 6.