Package 'lestat'

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Author Petter Mostad <mostad@chalmers.se></mostad@chalmers.se>
Maintainer Petter Mostad <mostad@chalmers.se></mostad@chalmers.se>
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Description

lestat-package

The package contains a number of simple functions which can be combined to implement simple Bayesian computations.

LEarning STATistics using Bayesian object oriented computation

Details

anovatable 3

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With this package, functions can be used to create objects representing probability distributions of many different types. These distributions can then be transformed and combined in different ways, representing statistical modelling. The result is an object-oriented way to do Bayesian computation with R.

Author(s)

Petter Mostad < mostad@chalmers.se>

References

Please see more information at www.math.chalmers.se/~mostad/

Examples

```
prior <- normalexpgamma() #Generate a two-parameter flat prior
full <- linearpredict(prior, rep(1, 7)) #Extend with normal distribution
data <- runif(7) #Generate data
posterior <- conditional(full, 1:7, data) #Condition on parameters
credibilityinterval(marginal(posterior, 1)) #Investigate posterior</pre>
```

anovatable

Computes ANOVA table given data and design

Description

Given data and a matrix describing a design for a linear model, the function creates an ANOVA table, using sums of squares based on a subdivision of the columns of the design matrix given as the third argument for the function.

```
anovatable(data, design, subdivisions = c(1, dim(design)[2] - 1))
```

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Arguments

data A vector with data values

design A matrix with the same number of rows as there are data values. The matrix

represents the design matrix for the linear model the ANOVA table is based on.

subdivisions A vector of integers summing to the number of columns in the design matrix.

The number of rows of the ANOVA table will be equal to the length of this

vector.

Value

An ANOVA table.

Author(s)

Petter Mostad < mostad@chalmers.se>

Examples

```
data1 <- simulate(normal(2.7, log(0.7)), 3)
data2 <- simulate(normal(4.0, log(0.7)), 5)
data3 <- simulate(normal(3.2, log(0.7)), 3)
data4 <- simulate(normal(4.1, log(0.7)), 4)
anovatable(c(data1, data2, data3, data4), designManyGroups(c(3,5,3,4)))</pre>
```

betabinomial

Create an Object Representing a Beta-Binomial Distribution

Description

Create an object representing a Beta-Binomial distribution. This can be used for a Binomial distribution where there is uncertainty about the probability of success, and this uncertainty is represented by a Beta distribution.

Usage

```
betabinomial(n, alpha, beta)
```

Arguments

n the number of trials in the binomial distribution (a positive integer).

alpha the alpha parameter of the underlying Beta distribution.
beta the beta parameter of the underlying Beta distribution.

Value

An object of class "betabinomial" and class "probability distribution".

beta distribution 5

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

betadistribution, binomialdistribution, binomialbeta

Examples

```
dist <- betabinomial(10, 5.5, 3.3)
cdf(dist, 3)</pre>
```

betadistribution

A Beta Distribution

Description

Create an object representing a Beta distribution.

Usage

```
betadistribution(alpha, beta)
```

Arguments

alpha The alpha parameter. beta The beta parameter.

Value

A Beta probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se >

See Also

```
betabinomial, binomialbeta
```

```
dist <- betadistribution(4, 6)
plot(dist)</pre>
```

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binomialbeta

Create an Object Representing a bivariate Binomial Beta Distribution

Description

Create an object representing a bivariate distribution, where the first variable is marginally Beta distributed, and the second variable is binomially distributed with probability given by the first variable.

Usage

```
binomialbeta(n, alpha, beta)
```

Arguments

n the number of trials in the binomial distribution (a positive integer).

alpha the alpha parameter of the Binomial distribution.
beta the beta parameter of the Binomial distribution.

Value

An object of class "binomialbeta" and class "probability distribution".

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

betadistribution, binomialdistribution, betabinomial

```
dist <- binomialbeta(10, 5.5, 12.3)
plot(dist)</pre>
```

binomial distribution 7

binomialdistribution Create an Object Representing a Binomial Distribution

Description

Create an object representing a Binomial distribution

Usage

```
binomialdistribution(ntrials, probability)
```

Arguments

ntrials the number of trials in the binomial distribution (a positive integer).

probability the probability for success in each trial (a number between 0 and 1).

Value

An object of class "binomialdistribution" and class "probability distribution".

Author(s)

Petter Mostad < mostad@chalmers.se>

Examples

```
dist <- binomialdistribution(10, 0.4)
cdf(dist, 3)</pre>
```

cdf

Compute Cumulative Distribution Function

Description

Compute the value of the cumulative distribution function for univariate distributions.

Usage

```
cdf(object, val)
```

Arguments

object A univariate probability distribution.

val The probability less than or equal to val is computed.

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Value

The probability that a variable with distribution object is less than or equal to val.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

invcdf

Examples

```
cdf(normal(3, 2), 1)
```

compose

Building a new probability distribution from an old.

Description

The command can be used to generate a new distribution from an old, which is given as the first argument. The new distribution has the old as the marginal for the first variable. The conditional distribution for the second variable is specified with the remaining arguments.

Usage

```
compose(object, type, ...)
```

Arguments

object A probability distribution

type A text string specifying the type of the conditional distribution given the old

distribution.

... Additional arguments specifying the conditional distribution.

Value

Depends on the input; may be a multivariate discrete distribution, or a Binomialbeta distribution.

Author(s)

Petter Mostad < mostad@chalmers.se >

conditional 9

conditional	The Conditional Distribution

Description

Given a multivariate distribution, the conditional distribution is computed when the variables with the given indices are set to the given values.

Usage

```
conditional(object, v, val)
```

Arguments

object A multivariate probability distribution.

v A vector of the indices of the variables whose values should be fixed.

val A vector, of the same length as v, with the values at which these variables should

be fixed.

Value

An object representing the conditional probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

```
marginal
```

```
prior <- normalexpgamma() #Generate a two-parameter flat prior
full <- linearpredict(prior, rep(1, 7)) #Normal extension
data <- simulate(uniformdistribution(), 7) #Generate data
posterior <- conditional(full, 1:7, data) #Condition on parameters
credibilityinterval(marginal(posterior, 1)) #Investigate posterior</pre>
```

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contrast

Computing the distribution of a Contrast

Description

For some distributions, like the multivariate Normal-ExpGamma and the multivariate Normal-Gamma, a new distribution is constructed from a linear combination of all but the last variables, and the last variable.

Usage

```
contrast(object, v)
```

Arguments

object A multivariate Normal-ExpGamma distribution or multivariate Normal-Gamma

distribution.

v A vector specifying the linear combination.

Value

A Normal-ExpGamma distribution or a Normal-Gamma distribution, depending on the input.

Author(s)

Petter Mostad < mostad@chalmers.se>

Examples

```
data1 <- simulate(normal(13, log(0.4)), 3)
data2 <- simulate(normal(14, log(0.4)), 5)
data3 <- simulate(normal(12, log(0.4)), 6)
dist <- linearmodel(c(data1, data2, data3), designManyGroups(c(3,5,6)))
diff <- contrast(dist, c(0, 1, -1))
credibilityinterval(marginal(diff, 1))
```

credibilityinterval

Compute Credibility Interval for a Univariate Distribution

Description

Given a univariate continuous distribution, a credibility interval is computed. Note that the interval is constructed so that there is an equal probability to be above or below the interval.

```
credibilityinterval(object, prob = 0.95)
```

designBalanced 11

Arguments

object A univariate continuous distribution.

prob The probability inside the credibility interval.

Value

A vector of length two, specifying the interval.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

p.value

Examples

credibilityinterval(normal())

 ${\tt designBalanced}$

Create a Design Matrix for a Balanced Design

Description

The function creates a design matrix suitable for analyzing results from an experiment where a set of factors are analysed in a balanced design: The argument factors lists the number of levels of each factor, and each possible combination of levels of factors is tried out a number of times given by replications.

Usage

```
designBalanced(factors, replications = 1, interactions = FALSE)
```

Arguments

factors A vector of integers, listing the number of levels of each of the factors.

replications An integer: The number of times each combination of factor levels is tried out.

If true, the design matrix will include columns for all possible interactions of the

factors.

Value

A matrix where the number of rows equals the product of factors and replications. The matrix will have only 0's and 1's as values.

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Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

designFactorial, designOneGroup, designTwoGroups, designManyGroups

Examples

```
designBalanced(c(3, 3), 2)
```

designFactorial

Create a Design Matrix for a Factorial Design

Description

The function creates a design matrix suitable for analyzing results from a factorial experiment where all factors have two levels.

Usage

```
designFactorial(nfactors, replications = 1, interactions = FALSE)
```

Arguments

nfactors The number of two-level factors in the experiment.

replications The number of replications at each combination of factor levels.

interactions
If TRUE, columns will be included representing the interactions between all the

factors.

Value

A matrix where the number of rows is $2^n k$, where n is the number of factors and k is the number of replications. The entries are -1's and 1's.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

designBalanced, designOneGroup, designTwoGroups, designManyGroups

```
designFactorial(3,2)
```

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designManyGroups

Create a Design Matrix for Several Groups of Normal Observations

Description

A design matrix is created, to be used for the analysis of data assumed to come from several normal distributions.

Usage

```
designManyGroups(v)
```

Arguments

v

A vector of integers, indicating how many observations there are in each group.

Value

A matrix consisting of 0's and 1's. The number of columns is equal to the length of v. The number of rows is equal to the sum of teh values of v.

Author(s)

Petter Mostad < mostad@chalmers.se >

See Also

designOneGroup, designTwoGroups, designBalanced, designFactorial

```
data1 <- simulate(normal(3.3, log(2)), 9)
data2 <- simulate(normal(4.5, log(2)), 8)
data3 <- simulate(normal(2.9, log(2)), 7)
design <- designManyGroups(c(9,8,7))
posterior <- linearmodel(c(data1, data2, data3), design)
plot(posterior)</pre>
```

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designOneGroup

Create a Design Matrix for One Group of Observations

Description

A design matrix is created, to be used for the analysis of data assumed to come from one normal distribution.

Usage

```
designOneGroup(n)
```

Arguments

n

The number of data values.

Value

A matrix consisting only of 1's, with one column and with the number of rows given by n.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

 ${\tt designTwoGroups, designManyGroups, designBalanced, designFactorial}$

Examples

```
data <- simulate(normal(4, log(1.3)), 9)
design <- designOneGroup(9)
posterior <- linearmodel(data, design)
credibilityinterval(marginal(posterior, 1))</pre>
```

designTwoGroups

Create a Design Matrix for Two Groups of Observations

Description

A design matrix is created, to be used for the analysis of data assumed to come from two normal distributions.

```
designTwoGroups(n, m)
```

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Arguments

n The number of data values in the first group.

m The number of data values in the second group.

Value

A matrix consisting of 1's and 0's, with two columns, and with the number of rows given by n+m.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

designOneGroup, designManyGroups, designBalanced, designFactorial

Examples

```
data1 <- simulate(normal(3, log(2)), 7)
data2 <- simulate(normal(5, log(2)), 9)
design <- designTwoGroups(7,9)
posterior <- linearmodel(c(data1, data2), design)
credibilityinterval(marginal(posterior, 1))</pre>
```

difference

Create Object Representing Difference Between Two Distributions

Description

Given two univariate distributions, an attempt is made to create the (approximate) difference between these.

Usage

```
difference(object1, object2)
```

Arguments

object1 A univariate normal or tdistribution. object2 A univariate normal or tdistribution.

Value

A univariate normal or tdistribution, as appropriate.

Author(s)

Petter Mostad <mostad@chalmers.se>

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Examples

```
data1 <- simulate(normal(8, log(1.5)), 6)
posterior1 <- marginal(linearmodel(data1, designOneGroup(6)), 1)
data2 <- simulate(normal(10, log(2.8)), 7)
posterior2 <- marginal(linearmodel(data2, designOneGroup(7)), 1)
posterior <- difference(posterior1, posterior2)
credibilityinterval(posterior)</pre>
```

discretedistribution Create Object Representing a Discrete Distribution

Description

An object representing a discrete distribution is created, based on explicitly given possible values and probabilities for these.

Usage

```
discretedistribution(vals, probs = rep(1, length(vals)))
```

Arguments

vals A vector listing the possible values of the discrete distribution.

probs If given, probs must have the same length as vals, and should list the probabil-

ities of the possible values. If not given, all possible values are assigned equal

probabilities.

Value

A discrete probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

```
dist <- discretedistribution(1:10)
expectation(dist)
variance(dist)</pre>
```

expectation 17

expectation

Compute Expectation

Description

Compute the expectation of a probability distribution.

Usage

```
expectation(object)
```

Arguments

object

A probability distribution.

Value

The expectation of the probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

variance

Examples

```
expectation(normal(3, log(2)))
expectation(binomialdistribution(7, 0.3))
```

expgamma

Create an ExpGamma distribution

Description

Create an ExpGamma Distribution: If a variable has a Gamma distribution with parameters alpha and beta, then its logarithm has an ExpGamma distribution with parameters alpha, beta, and gamma = 1.

```
expgamma(alpha = 1, beta = 1, gamma = -2)
```

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Arguments

alpha The "shape" parameter of the corresponding Gamma distribution.

beta The "rate" parameter of the corresponding Gamma distribution.

gamma The scale parameter for the logarithmhic scale. By default, gamma = -2.

Details

The ExpGamma has probability density function

$$f(x|\alpha, \beta, \gamma) = \exp(\alpha \gamma x - \beta \exp(\gamma x))$$

Value

An ExpGamma distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

```
gammadistribution
```

Examples

```
dist <- expgamma(4, 6)
plot(dist)</pre>
```

fdistribution

Create an F distribution

Description

Create a univariate F distribution.

Usage

```
fdistribution(df1 = 1, df2 = 1)
```

Arguments

df1 The first degree of freedom: Should be a positive number.

df2 The second degree of freedom: Should be a positive number.

Value

An F distribution.

fittedvalues 19

Author(s)

Petter Mostad < mostad@chalmers.se>

Examples

```
dist <- fdistribution(10, 3)
cdf(dist, 4)
expectation(dist)</pre>
```

fittedvalues

Compute Fitted Values for a Linear Model

Description

Given a vector of data values and a design matrix, the fitted values for a linear model is computed.

Usage

```
fittedvalues(data, design)
```

Arguments

data A data vector.

design A design matrix. The number of rows must be equal to the length of the data

vector.

Details

The fitted values represent the expected values all but the last variables in the posterior for the linear model.

Value

A vector of values of length equal to the number of columns in the design matrix.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

```
linearmodel, leastsquares, linearpredict
```

```
xdata <- simulate(uniformdistribution(), 14)
ydata <- xdata + 4 + simulate(normal(), 14)*0.1
plot(xdata,ydata)
design <- cbind(1, xdata)
lines(xdata, fittedvalues(ydata, design))</pre>
```

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 ${\tt gammadistribution}$

Create a Gamma Distribution

Description

Create a Gamma distribution.

Usage

```
gammadistribution(alpha = 1, beta = 1)
```

Arguments

alpha The first parameter of the Gamma distribution: The "shape" parameter.

beta The second parameter of the Gamma distribution: The "rate" parameter.

Details

The density of the distribution is proportional to

$$f(x) = x^{\alpha - 1} \exp(-\beta x)$$

Value

A Gamma probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

expgamma

```
dist <- gammadistribution(4, 2)
plot(dist)</pre>
```

invcdf 21

invcdf

Compute the Inverse Cumulative Distribution Function

Description

Compute the inverse of the cumulative distribution function for a univariate probability distribution.

Usage

```
invcdf(object, val)
```

Arguments

object A univariate probability distribution.

val A value between 0 and 1.

Value

A value v such that the probability that $x \leq v$ is given by val.

Author(s)

Petter Mostad <mostad@chalmers.se>

See Also

cdf

Examples

```
invcdf(normal(), 0.975)
invcdf(binomialdistribution(10, 0.4), 0.5)
```

leastsquares

Find the Least Squares Solution in a Linear Model

Description

Given a vector of data and a design matrix, the least squares estimates for a linear model is computed.

```
leastsquares(data, design)
```

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Arguments

data A data vector.

design A design matrix. The number of rows must be equal to the length of the data

vector.

Details

The fitted values represent the expected values all but the last variables in the posterior for the linear model.

Value

A vector of values of length equal to the number of columns in the design matrix.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

```
linearmodel, fittedvalues, linearpredict
```

Examples

```
xdata <- simulate(uniformdistribution(), 14)
ydata <- xdata + 4 + simulate(normal(), 14)*0.1
plot(xdata,ydata)
design <- cbind(1, xdata)
leastsquares(ydata, design)</pre>
```

linearmodel

Compute the Posterior Distribution for a Linear Model

Description

Given a vector of data and a design matrix, describing how these data are thought to relate to some predictors in a linear model, the posterior for the parameters of this linear model is found, using a flat prior.

Usage

```
linearmodel(data, design)
```

Arguments

data A vector with data values.

design A design matrix. The number of rows must be equal to the length of the data

vector. The number of columns corresponds to the number of explanatory vari-

ables.

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Details

If y_i is the i'th data value and β_j is the j'th unknown parameter, and if x_{ij} is the value in the i'th row and j'th column of the design matrix, then one assumes that y_i is normally distributed with explectation

$$x_{i1}\beta_1 + x_{i2}\beta_2 + \ldots + x_{ik}\beta_k$$

and logged standard deviation λ . The computed probability distribution is then the posterior for the joint distribution of

$$(\beta_1, \beta_2, \ldots, \beta_k, \lambda)$$

.

Value

If k is the number of columns in the design matrix and if k > 1, then the output is a multivariate Normal-ExpGamma distribution representing the posterior for the corresponding k values and the logged scale parameter in the linear model. If k = 1, the output is a Normal-ExpGamma distribution representing the posterior.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

fittedvalues, leastsquares, linearpredict

Examples

```
data1 <- simulate(normal(3.3, log(2)), 9)
data2 <- simulate(normal(4.5, log(2)), 8)
data3 <- simulate(normal(2.9, log(2)), 7)
design <- designManyGroups(c(9,8,7))
posterior <- linearmodel(c(data1, data2, data3), design)
plot(posterior)</pre>
```

linearpredict

Create a Linear Extension of a Distribution

Description

Extends the given probability distribution with new variables which are (multivariate) normally distributed with parameters based on the values of the given probability distribution and values given to the function.

```
linearpredict(object, ...)
```

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Arguments

object

The probability distribution to be extended. Currently, it should be either a (multivariate) normal distribution, or a (multivariate) normal distribution extended with an extra parameter with either a Gamma or an ExpGamma distribution.

A second optional argument may be given, which should then be a matrix with the same number of columns as there are normally distributed variables in the input object. If the matrix has only one row or column, it may be given as a vector. The default is a matrix with one row, consisting of only 1's.

A third optional argument may also be given, which is then the precision matrix of the new normally distributed variables. By default, this matrix is the identity. If the input object contains a Gamma-distributed variable, its value is multiplied with the precision matrix. If the input object contains an ExpGamma-distributed variable y, the value e^{-2y} is multiplied with the precision matrix.

Details

The input is either a (multivariate) variable x with a normal distribution, or a joint distribution consisting of a Gamma- or ExpGamma-distributed variable y, and conditionally on this a (multivariate) normally distributed x. The output is a joint distribution for (z,x) or (z,x,y), where the marginal distribution for x or (x,y) is unchanged, while the conditional distribution for z given x or (x,y) is (multivariate) normal. The expectation and precision for this conditional distribution is $X\mu$ and $P\tau$, respectively. Here, μ is the expectation of x, while X is the optional second argument. The matrix P is the optional third argument, while τ is either equal to y, when y has a Gamma distribution, or equal to e^{-2y} , when y has an ExpGamma distribution.

Value

A multivariate normal, multivariate Normal-Gamma, or multivariate Normal-ExpGamma distribution, depending on the input.

Author(s)

Petter Mostad < mostad@chalmers.se >

See Also

contrast

```
prior <- normalgamma()
full <- linearpredict(prior, rep(1, 7))
data <- simulate(normal(), 7)
posterior <- conditional(full, 1:7, data)
plot(posterior)</pre>
```

marginal 25

marginal

A Marginal of a Multivariate Distribution

Description

Given a multivariate distribution, one of its marginal distributions is computed.

Usage

```
marginal(object, v)
```

Arguments

object The multivariate probability distribution whose marginal should be computed.

A vector of indices, indicating which parts of the object should be kept after

marginalisation.

Details

The index or indices of the parameter(s) whose marginal distribution is computed is given in v.

Value

A probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

conditional

```
data <- simulate(normal(3, log(3)), 11)
posterior <- linearmodel(data, designOneGroup(11))
credibilityinterval(marginal(posterior, 1))
credibilityinterval(marginal(posterior, 2))</pre>
```

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mdiscretedistribution Create Object Representing a Multivariate Discrete Distribution

Description

An object representing a multivariate discrete distribution is created, based on explicitly given possible values and probabilities for these.

Usage

```
mdiscretedistribution(probs, nms=NULL)
```

Arguments

probs This must be a matrix, or more generally an array with the same number of di-

mensions as the desired variable. The values in the matrix must be non-negative

and represent the probabilities of the variable.

nms If given, nms should be a list with the same number of items as there are dimen-

sions of probs. Each item in the list should be a vector with the names of the possible values of the variable representing this dimension of the multivariate

variable. If not given, integers are used as variable names.

Value

A multivariate discrete probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se >

Examples

```
dist <- mdiscretedistribution(array(1:24, c(2,3,4)))
expectation(dist)
variance(dist)</pre>
```

mnormal

A Multivariate Normal Distribution

Description

Creates an object representing a multivariate normal distribution.

```
mnormal(expectation = c(0,0), P = diag(length(expectation)))
```

mnormalexpgamma 27

Arguments

expectation A vector of length at least 2 specifying the expectation of the distribution. By

default, the vector (0,0).

P A matrix of size $k \times k$, where k is the length of the expectation vector. P specifies

the precision matrix, i.e., the inverse of the covariance matrix.

Details

If μ is the expectation vector and P is the precision matrix, then the probability density function is proportional to

$$f(x) = \exp(-0.5(x - \mu)^t P(x - \mu))$$

Value

A multivariate normal probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

normal

Examples

```
plot(mnormal())
plot(mnormal(c(1,2,3)))
plot(mnormal(c(1,2), matrix(c(1, 0.5, 0.5, 1), 2, 2)))
```

mnormalexpgamma

A Multivariate Normal-ExpGamma Distribution

Description

Creates an object representing a multivariate Normal-ExpGamma distribution. If (x, y) has a multivariate Normal-ExpGamma distribution, then the marginal distribution of y is an ExpGamma distribution, and the conditional distribution of x given y is multivariate normal.

```
mnormalexpgamma(mu=c(0,0), P, alpha, beta)
```

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Arguments

mu The mu parameter. It must be a vector of length at least 2. The default value is (0,0).

P The P parameter.

alpha The alpha parameter.

beta The beta parameter.

Details

If (x,y) has a multivariate Normal-ExpGamma distribution with parameters μ , P, α , and β , then the marginal distribution of y has an ExpGamma distribution with parameters α , β , and -2, and conditionally on y, x has a multivariate normal distribution with expectation μ and precision matrix $e^{-2y}P$. The probability density is proportional to

$$f(x,y) = \exp(-(2\alpha + k)y - e^{-2y}(\beta + (x - \mu)^t P(x - \mu)/2))$$

where k is the dimension.

Value

A multivariate Normal-ExpGamma probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se >

See Also

gamma,normal,expgamma, normalgamma,normalexpgamma mnormal,mnormalgamma

Examples

```
plot(mnormalexpgamma(alpha=3, beta=3))
```

mnormalgamma

A Multivariate Normal-Gamma Distribution

Description

Creates an object representing a multivariate Normal-Gamma distribution. If (x, y) has a multivariate Normal-Gamma distribution, then the marginal distribution of y is an Gamma distribution, and the conditional distribution of x given y is multivariate normal.

```
mnormalgamma(mu=c(0,0), P, alpha, beta)
```

mtdistribution 29

Arguments

mu	The mu parameter. It must be a vector of length at least 2. The default value is $(0,0)$.
Р	The P parameter.
alpha	The alpha parameter.
beta	The beta parameter.

Details

If (x,y) has a multivariate Normal-Gamma distribution with parameters μ , P, α , and β , then the marginal distribution of y has a Gamma distribution with parameters α , β , and conditionally on y, x has a multivariate normal distribution with expectation μ and precision matrix yP. The probability density is proportional to

$$f(x,y) = y^{\alpha+k/2-1} \exp(-y(\beta + (x-\mu)^t P(x-\mu)/2))$$

where k is the dimension.

Value

A multivariate Normal-Gamma probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

gamma,normal,expgamma, normalgamma, normalexpgamma, mnormal,mnormalexpgamma

Examples

```
plot(mnormalgamma(alpha=3, beta=3))
```

mtdistribution

A Multivariate t-Distribution

Description

Creates an object representing a multivariate non-centered t-distribution.

```
mtdistribution(expectation = c(0,0), degreesoffreedom = 10000,
P = diag(length(expectation)))
```

30 muniformdistribution

Arguments

expectation

A vector of length at least 2 specifying the expectation of the distribution. By default, the vector (0,0).

degreesoffreedom

The degrees of freedom parameter.

Ρ

A matrix of size $k \times k$, where k is the length of the expectation vector. P plays a similar role in the multivariate t-distribution as the precision matrix does in the multivariate normal distribution. By default, P is the identity matrix.

Details

If μ is the expectation, ν the degrees of freedom, P is the last parameter, and k the dimension, then the probability density function is proportional to

$$f(x) = \exp(\nu + (x - \mu)^t P(x - \mu))^{-(\nu + k)/2}$$

Value

A multivariate t-distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

tdistribution, mnormal

Examples

```
plot(mtdistribution())
plot(mtdistribution(c(1,2,3), 3))
plot(mtdistribution(c(1,2), 3, matrix(c(1, 0.5, 0.5, 1), 2, 2)))
```

muniformdistribution A Multivariate Uniform Distribution

Description

An object representing a multivariate univariate muniform distribution is created.

```
muniformdistribution(startvec, stopvec)
```

normal 31

Arguments

startvec A vector with the lower bounds for the distribution.

stopvec A vector with the upper bounds for the distribution.

Value

A multivariate uniform probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

Examples

```
dist <- muniformdistribution(rep(0, 5), rep(1, 5)) expectation(dist) variance(dist)
```

normal

A Normal Distribution

Description

Create an object representing a univariate normal distribution.

Usage

```
normal(expectation = 0, lambda, P = 1)
```

Arguments

expectation The expectation of the distribution.

1ambda THE NATURAL LOGARITHM OF THE STANDARD DEVIATION OF THE

DISTRIBUTION. Thus, if the desired standard deviation is s, the second argument should be $\log(s)$. If the desired variance is v, the second argument should be $\log(v)/2$. The default is a standard deviation of 1. An alternative to specify-

ing this argument is to specify the precision parameter P.

P If given, this argument specifies the precision of the distribution, i.e., the inverse

of the variance.

Value

A univariate normal probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

32 normalexpgamma

See Also

mnormal

Examples

```
dist <- normal(3, log(0.7))
variance(dist)
dist <- normal(5, log(0.49)/2)
variance(dist)
dist <- normal(7, P = 2)
variance(dist)</pre>
```

normalexpgamma

A Normal-ExpGamma Distribution

Description

Creates an object representing a Normal-ExpGamma distribution. If (x, y) has a Normal-ExpGamma distribution, then the marginal distribution of y is an ExpGamma distribution, and the conditional distribution of x given y is normal.

Usage

```
normalexpgamma(mu, kappa, alpha, beta)
```

Arguments

mu	The mu parameter.
kappa	The kappa parameter.
alpha	The alpha parameter.
beta	The beta parameter.

Details

If (x,y) has a Normal-ExpGamma distribution with parameters μ , κ , α , and β , then the marginal distribution of y has an ExpGamma distribution with parameters α , β , and -2, and conditionally on y, x has a normal distribution with expectation μ and logged standard deviation $\kappa + y$. The probability density is proportional to

$$f(x,y) = \exp(-(2\alpha + 1)y - e^{-2y}(\beta + e^{-2\kappa}(x - \mu)^2/2))$$

Value

A Normal-ExpGamma probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

normalgamma 33

See Also

gamma, normal, expgamma, normalgamma, mnormal, mnormalgamma, mnormalexpgamma

Examples

```
plot(normalexpgamma(3,4,5,6))
```

normalgamma

A Normal-Gamma Distribution

Description

Creates an object representing a Normal-Gamma distribution. If (x, y) has a Normal-Gamma distribution, then the marginal distribution of y is a Gamma distribution, and the conditional distribution of x given y is normal.

Usage

```
normalgamma(mu, kappa, alpha, beta)
```

Arguments

mu	The mu parameter.
kappa	The kappa parameter.
alpha	The alpha parameter.
beta	The beta parameter.

Details

If (x,y) has a Normal-Gamma distribution with parameters μ , κ , α , and β , then the marginal distribution of y has a Gamma distribution with parameters α and β , and conditionally on y, x has a normal distribution with expectation μ and logged standard deviation $\kappa - log(y)/2$. The probability density is proportional to

$$f(x,y) = y^{\alpha - 0.5} \exp(-y(\beta + e^{-2\kappa}(x - \mu)^2/2))$$

Value

A Normal-Gamma probability distribution.

Author(s)

Petter Mostad <mostad@chalmers.se>

See Also

gamma, normal, expgamma, normalexpgamma, mnormal, mnormalgamma, mnormalexpgamma

p.value

Examples

```
plot(normalgamma(3,4,5,6))
```

p.value

Compute the p-value for a Distribution

Description

The p-value of a distribution is here interpreted as the probability outside the smallest credibility interval or region containing a point; if no point is explicitly given, it is assumed to be zero, or the origin.

Usage

```
p.value(object, point)
```

Arguments

object The probability distribution for which the p-value should be computed.

point The point which should be included in the credibility interval or region.

Value

The probability outside the smallest credibility interval or region containing the point.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

```
credibilityinterval
```

```
data <- simulate(normal(3, log(2)), 10)
posterior <- linearmodel(data, designOneGroup(10))
p.value(marginal(posterior, 1))</pre>
```

plot.normal 35

plot.normal

Plotting a Probability Distribution

Description

A plot is constructed convering the central part of a probability distribution. The purpose is simply to illustrate the properties of the distribution.

Usage

```
## S3 method for class 'normal' plot(x, ...) ## S3 method for class 'binomialdistribution' plot(x, ...)
```

Arguments

x A probability distribution.

. . . Other arguments (not currently in use).

Value

For univariate discrete distributions, a plot is generated showing with a histogram the probabilities of each of the possible values of the distribution. For univariate continuous distributions, a plot is made of roughly the central 99 of the distribution. For multivariate distributions, a combined plot is made, where one can find the marginal distributions along the diagonal, and contour plots for bivariate marginal distributions off the diagonal.

Author(s)

Petter Mostad < mostad@chalmers.se>

```
plot(normal())
plot(mnormal(c(3,4,5), diag(3)))
plot(poissondistribution(3))
```

36 posteriornormal1

poissondistribution

A Poissondistribution

Description

Create an object representing a Poisson distribution.

Usage

```
poissondistribution(rate)
```

Arguments

rate

The rate parameter of the distribution.

Value

An object representing a Poisson distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

binomialdistribution

Examples

```
dist <- poissondistribution(4)
cdf(dist, 3)</pre>
```

posteriornormal1

Compute the Posterior Distribution for Parameters of One Normal Distribution

Description

Given a vector of data, this function computes the bivariate posteror for the expectation parameter and the logged scale parameter of a normal distribution, assuming that the data represents independent observations from the normal distribution. One assumes a flat prior.

```
posteriornormal1(data)
```

posteriornormal2 37

Arguments

data

A vector with data values.

Value

An object representing a bivariate Normal-ExpGamma distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

```
posteriornormal2, linearmodel
```

Examples

```
data <- simulate(normal(3.3, log(2)), 9)
posteriornormal1(data)
linearmodel(data, designOneGroup(length(data))) #Gives same result</pre>
```

posteriornormal2

Compute a Posterior Distribution for Parameters of Two Normal Distributions

Description

Given a vectors data1 and data2 of data, this function assumes data1 is a sample from one normal distribution while data2 is a sample from another, while both distributions are assumed to have the same logged scale. The bivariate posterior for the difference between the expectations of the two distributions and the common logged scale of the distributions is computed, assuming a flat prior.

Usage

```
posteriornormal2(data1, data2)
```

Arguments

data1 A vector with data values. Assumed to be a sample from the first normal distri-

bution.

data2 Another vector with data values. Assumed to be a sample from the second

normal distribution.

Value

An object representing a Normal-ExpGamma distribution.

38 precision

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

```
posteriornormal1, linearmodel
```

Examples

```
data1 <- simulate(normal(3.3, log(2)), 9)
data2 <- simulate(normal(5.7, log(2)), 4)
posteriornormal2(data1, data2)
marginal(linearmodel(c(data1, data2),
designTwoGroups(length(data1), length(data2))), 2:3) #Gives same result</pre>
```

precision

The Precision of a Distribution

Description

Compute the precision (i.e., the inverse of the variance) of a probability distribution.

Usage

```
precision(object)
```

Arguments

object

A probability distribution.

Value

The precision of the probability distribution: Either a number or a matrix.

Author(s)

Petter Mostad <mostad@chalmers.se>

See Also

```
expectation, variance
```

Examples

```
precision(normal(3, log(0.7)))
precision(binomialdistribution(7, 0.4))
```

print.normal 39

print.normal

Printing Probability Distributions

Description

When a probability distribution is printed, its main features are listed.

Usage

```
## S3 method for class 'normal'
print(x, ...)
```

Arguments

x The object to be printed.

. . . Other possible arguments (not currently implemented).

Value

Readable text describing the object.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

summary

Examples

```
print(normal())
```

probability

The Probability at a Value for a Discrete Distribution

Description

Given a possible value for a probability distribution, the probability at that value is computed.

Usage

```
probability(object, val)
```

40 probability density

Arguments

object A discrete probability distribution.

val The value at which the probability should be computed.

Value

The probability at val.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

```
probabilitydensity
```

Examples

```
probability(poissondistribution(3), 1)
probability(binomialdistribution(10, 0.24), 2)
```

probabilitydensity

The Probability Density at a Value for a Continuous Distribution

Description

Computes the probability density at a value for a continuous distribution.

Usage

```
probabilitydensity(object, val, log = FALSE, normalize = TRUE)
```

Arguments

object A continuous probability distribution.

The point at which the probability density should be computed.

If TRUE, the logarithm of the probability density is returned.

normalize If FALSE, unnormalized densities are returned.

Value

The probability density at val.

Author(s)

Petter Mostad < mostad@chalmers.se>

simulate.normal 41

Examples

```
probabilitydensity(normal(), 1) probabilitydensity(mnormal(c(0,0), diag(2)), c(1,1))
```

simulate.normal

Simulate values from a Probability Distribution

Description

Simulate independent values from a given probability distribution.

Usage

```
## S3 method for class 'normal'
simulate(object, nsim = 1, ...)
## S3 method for class 'binomialdistribution'
simulate(object, nsim = 1, ...)
## S3 method for class 'discretedistribution'
simulate(object, nsim = 1, ...)
## S3 method for class 'expgamma'
simulate(object, nsim = 1, ...)
## S3 method for class 'fdistribution'
simulate(object, nsim = 1, ...)
## S3 method for class 'gammadistribution'
simulate(object, nsim = 1, ...)
## S3 method for class 'mnormalexpgamma'
simulate(object, nsim = 1, ...)
## S3 method for class 'mnormalgamma'
simulate(object, nsim = 1, ...)
## S3 method for class 'mnormal'
simulate(object, nsim = 1, ...)
## S3 method for class 'mtdistribution'
simulate(object, nsim = 1, ...)
## S3 method for class 'normalexpgamma'
simulate(object, nsim = 1, ...)
## S3 method for class 'normalgamma'
simulate(object, nsim = 1, ...)
## S3 method for class 'poissondistribution'
simulate(object, nsim = 1, ...)
## S3 method for class 'tdistribution'
simulate(object, nsim = 1, ...)
## S3 method for class 'uniformdistribution'
simulate(object, nsim = 1, ...)
```

42 summary.normal

Arguments

object The probability distribution to be simulated from.

nsim The number of simulated values. Default is 1.

... Additional parameters. Currently not in use.

Value

For univariate distributions, a vector of length nsim is produced. For multivariate distributions, a matrix with nsim rows is produced.

Author(s)

Petter Mostad < mostad@chalmers.se >

Examples

```
simulate(normal())
simulate(normal(), 10)
simulate(mnormal(), 10)
```

summary.normal

Summary of a Probability Distribution Object

Description

Lists the main features of a probability distribution object.

Usage

```
## S3 method for class 'normal'
summary(object, ...)
```

Arguments

object The object to be summarized.

. . . Other possible arguments (not currently implemented).

Value

Readable text describing the object.

Author(s)

Petter Mostad <mostad@chalmers.se>

See Also

print

tdistribution 43

Examples

```
summary(normal())
```

tdistribution

A t-distribution

Description

Create an object representing a univariate non-centered t-distribution.

Usage

```
tdistribution(expectation = 0, degreesoffreedom = 1e+20, lambda, P = 1)
```

Arguments

expectation

The expectation of the distribution.

degreesoffreedom

The degrees of freedom parameter.

lambda

The natural logarithm of the scale σ of the distribution. The standard t-distribution

has scale 1, and the default for lambda is log(1) = 0.

Ρ

An alternative to specifying the logged scale $\log(\sigma)$ with lambda is to specify P:

It is defined as $P = 1/\sigma^2$.

Details

The probability density of a t-distribution with expectation μ , degrees of freedom ν , and logged scale λ is proportional to

 $f(x) = (\nu + e^{-2\lambda}(x - \mu)^2)^{-(\nu+1)/2}$

Value

A t-distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

mtdistribution

Examples

```
dist <- tdistribution(3)
plot(dist)</pre>
```

44 variance

uniformdistribution

A Uniform Distribution

Description

An object representing a univariate uniform distribution is created.

Usage

```
uniformdistribution(a = 0, b = 1)
```

Arguments

a The lower bound for the distribution. The default is 0.

b The upper bound for the distribution. The default is 1.

Value

A uniform probability distribution.

Author(s)

Petter Mostad <mostad@chalmers.se>

Examples

```
dist <- uniformdistribution()
expectation(dist)
variance(dist)</pre>
```

variance

The Variance of a Distribution

Description

Compute the variance of a probability distribution.

Usage

```
variance(object)
```

Arguments

object

A probability distribution.

variance 45

Value

The variance of the probability distribution.

Author(s)

Petter Mostad < mostad@chalmers.se>

See Also

```
expectation, variance
```

Examples

```
variance(normal(3, log(0.7)))
variance(binomialdistribution(7, 0.4))
```

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