OR 664 / SYST 664 / CSI 674: Homework Assignment 5

due March 2, 2020, 11:59PM

You may submit on paper or electronically via Blackboard. Please make sure your name is on every page of the assignment, and it is clearly marked which question you are answering. Your response will be graded for correctness and clarity. *Points may be deducted if you do not provide information on how you arrived at your answer. Please submit your R code either as a separate attachment on Blackboard or in your main submission.*

1. This problem concerns the automobile data from Assignments 3 and 4. As in Assignment 4, assume that counts of cars per 15-second interval are independent and identically distributed Poisson random variables with unknown mean Λ. Assume a uniform prior distribution for Λ. (This is a gamma distribution with shape 1 and scale infinity.) Using the posterior distribution from Problem 1 of Assignment 4, find the predictive distribution for the number of cars in the next 15-second interval. Name the family of distributions and the parameters of the predictive distribution. Find the predictive probability that 0, 1, 2, 3, 4, and more than 4 cars will pass the point in the next 15 seconds. Compare with your answer to Problem 1e of Assignment 3. Discuss.

2. In previous years, students in this course collected data on people's preferences in the two Allais gambles from Assignment 2. For this problem, we will assume that responses are independent and identically distributed, and the probability is π that a person chooses both B in the first gamble and C in the second gamble.
    a. Assume that the prior distribution for π is Beta(1, 3). Find the prior mean and standard deviation for π. Find a 95% symmetric tail area credible interval for the prior probability that a person would choose B and C. Do you think this is a reasonable prior distribution to use for this problem? Why or why not?
    b. In 2009, 19 out of 47 respondents chose B and C. Find the posterior distribution for the probability π that a person in this population would choose B and C. Find the posterior mean and standard deviation, and a 95% symmetric tail area credible interval for π. Do a triplot.
    c. Find the predictive distribution for the number of B and C responses in a future sample of 50 people drawn from the same population. Compare with a Binomial distribution using a point estimate of the probability of choosing B and C.
    d. Comment on your results.