OR 664 / SYST 664 / CSI 674: Homework Assignment 3

due February 17, 2020 at 11:59PM

You may submit on paper or electronically via Blackboard. Please make sure your name is on every page of the assignment, and it is clearly marked which question you are answering. Your response will be graded for correctness and clarity. You are encouraged to submit R code so the grader can understand what you did, but your submission should stand alone and be clear and understandable without reading code.

1. Times were recorded at which 41 vehicles passed a fixed point on the M1 motorway in Bedfordshire, England on March 23, 1985.[1] The times were subtracted to form 40 intervals between successive cars. These interarrival times, rounded to the nearest second, are:

   12, 2, 6, 2, 19, 5, 34, 4, 1, 4, 8, 7, 1, 21, 6, 11, 8, 28, 6, 4, 5, 1, 18, 9, 5, 1, 21, 1, 1, 5, 3, 14, 5, 3, 4, 5, 1, 3, 16, 2

   a. A common model for interarrival times is a random sample from an exponential distribution. Do you think an exponential distribution provides a good model for the interarrival times? Justify your answer.
   b. When interarrival times are randomly sampled from an exponential distribution, the counts of events per unit time are a random sample from a Poisson distribution. Using a time unit of 15 seconds, find the number of cars passing in each 15-second block of time after the initial car. (The initial car is used to bound the recording interval, so the total car count in your data set should be 40.) Do you think a Poisson distribution provides a good model for the count data? Justify your answer.
   c. Assume that Λ, the rate parameter of the Poisson distribution has a discrete uniform prior distribution on 20 equally spaced values between (0.2, 0.4, …, 3.8, 4.0) cars per 15-second interval. Find the posterior distribution after observing the first 10 observations of car counts in 15 second intervals. Find the posterior mean, standard deviation, median and 95th percentile of Λ given the first 10 observations.
   d. Using the posterior distribution from part c as the prior distribution, find the new posterior distribution after observing the remaining observations. Find the posterior mean, standard deviation, median and 95th percentile of Λ given all observations.
   e. Find the predictive distribution for the number of cars in the next 15-second interval. Find the predictive probability that 0, 1, 2, 3, 4, and more than 4 cars will pass the point in the next 15 seconds.
   f. Discuss what your results mean in terms of traffic on this motorway.

[1] These data were taken from Hand, et al., *A Handbook of Small Data Sets*, Chapman and Hall, 1994.