OR 664 / SYST 664 / CSI 674: Homework Assignment 2

due February 10, 2020, 11:59PM

You may submit on paper or electronically via Blackboard. Please make sure your name is on every page of the assignment, and it is clearly marked which question you are answering. Your response will be graded for correctness and clarity.

1. Whether certain mice are black or brown depends on a pair of genes, each of which is either *B* or *b*. If both members of the pair are alike, the mouse is said to be homozygous, and if they are different it is said to be heterozygous. Homozygous *bb* mice are brown; heterozygous mice and homozygous *BB* mice are black. The offspring of a pair of mice have two such genes, one from each parent. If the parent is heterozygous, the inherited gene is equally likely to be *B* or *b*.
    a. Suppose two heterozygous mice mate and the offspring is black. We will call this mouse Boff (for black offspring). What is the probability that Boff is homozygous *BB*?
    b. Now suppose Boff is mated with a brown mouse, resulting in seven offspring, all of which are black. Apply Bayes theorem to find the posterior probability that Boff is homozygous *BB*, given that Boff has produced seven black offspring when mated with a brown mouse.
    c. Show that the result to Part b is the same if you redo the previous calculation, processing the seven observations sequentially, using the posterior probability from each observation as the prior probability for the next observation.

2. In an experiment, subjects were given the choice between two gambles:

    Gamble 1:

    A: \$2500 with probability 0.33
    \$2400 with probability 0.66
    \$0 with probability 0.01

    B: \$2400 with certainty

    Suppose that a person is an expected utility maximizer. Set the utility scale so that u(\$0) = 0 and u(\$2500) = 1. Whether a utility maximizing person would choose Option A or Option B depends on the person's utility for \$2400. For what values of u(\$2400) would a rational person choose Option A? For what values would a rational person choose Option B?

    Gamble 2:

    C: \$2500 with probability 0.33
    \$0 with probability 0.67

    D: \$2400 with probability 0.34
    \$0 with probability 0.66

    For what values of u(\$2400) would a rational person choose Option C? For what values would a rational person choose Option D?

    This problem is a version of the famous *Allais paradox*, named after the prominent critic of subjective expected utility theory who first presented it. Kahneman and Tversky[1] found that 82% of subjects preferred B over A, and 83% preferred C over D. Explain why no expected utility maximizer would prefer *both* B in Gamble 1 and C in Gamble 2. (*A utility maximizer might prefer B in Gamble 1. A different utility maximizer might prefer C in Gamble 2. But the same utility maximizer would not prefer both B in Gamble 1 and C in Gamble 2.*) Discuss these results. Why do you think many people prefer B in Gamble 1 and C in Gamble 2? Do you think this is reasonable even if it does not conform to expected utility theory?

[1] Kahneman, D., P. Slovic, et al. (1982). *Judgment under Uncertainty: Heuristics and Biases*. Cambridge University Press.