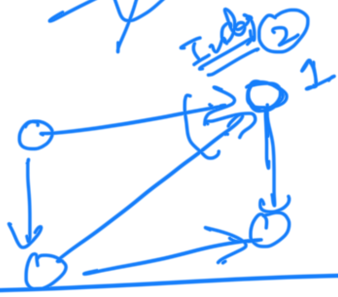
Lecture 26

$G(V, E)$

Degree: no. of edges incident on a node

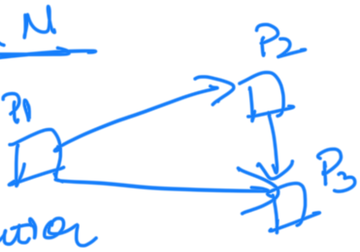


Degree

Indeg — Out degree

Directed graph

citation N



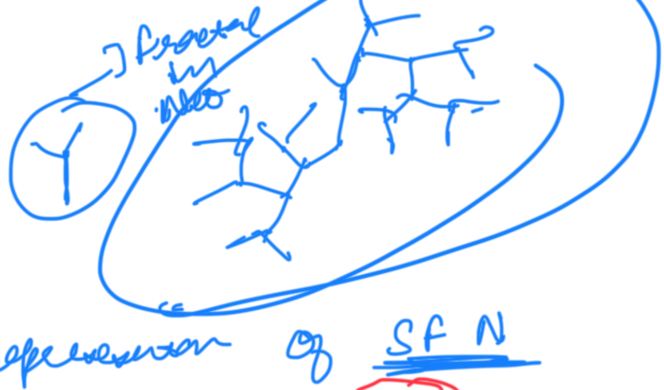
Degree Distribution

Power law

→ The fraction / no of nodes in the network that have degree K.

The fraction decays as $K^{-\alpha}$ for various real world network.

PL → S.F → self repeating topology



Mathematical representation of SF N

If you have function $f(x)$ & a linear scaling is applied on the independent variable

... function form

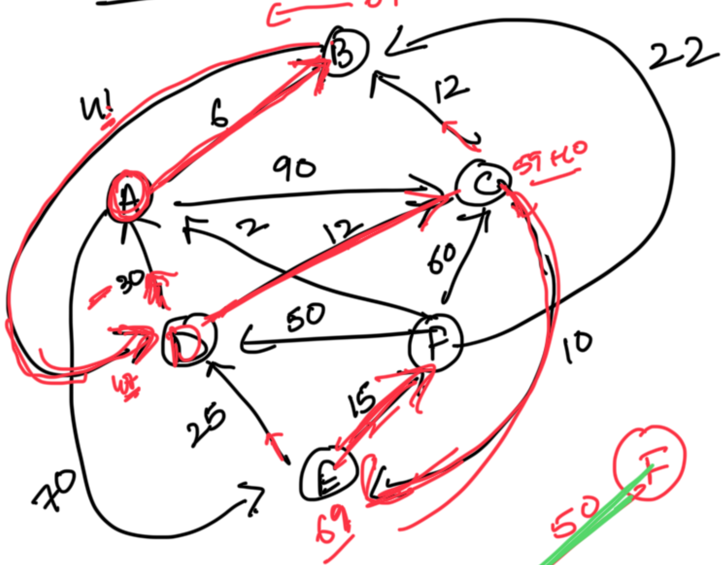
$x \to 'a'x$ the the function form does not change.

$$f(x) = A x^{-\alpha} \quad \text{------} \quad (1)$$

where A, α are constants

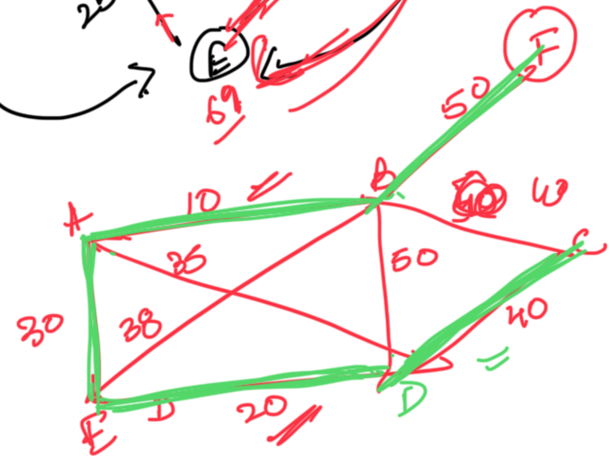
$$f(ax) = A(ax)^{-\alpha}$$
$$= \underbrace{A\, a^{-\alpha}}\, x^{-\alpha}$$
$$= A' x^{-\alpha} \quad \text{------} \quad (2)$$

Lecture - 5:

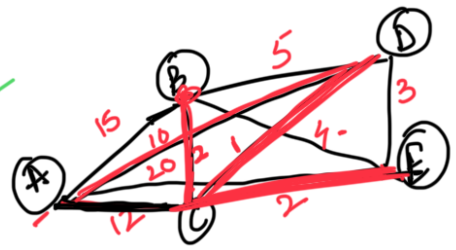


	A	B	C	D	E	F
A	0 / N	∞ / N	∞ / N	∞ / N	∞ / N	∞ / N
B		6 / A	90 / A	∞ / N	70 / A	∞ / N
D			90 / A	47 / B	70 / A	
C			59 / D		70 / A	∞ / N
E					69 / C	84 / E

eg

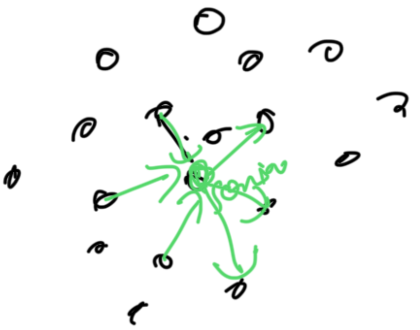


eg



	A	B	C	D	E
A	0 / N	∞ / N	∞ / N	∞ / N	∞ / N
D		15/A	12/A	10/A	20/A
C		5/D	1/D		3/D
B		2/C			2/C

Lecture : 6



Hubs

Auth.

good node will point to good node

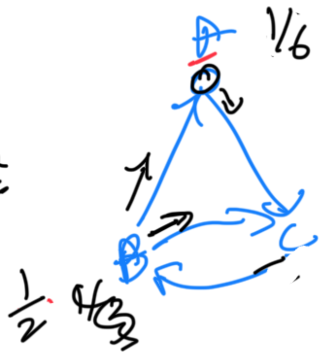
Hub

$h = a_1 + a_2 + a_3 + a_4 - - a_6$

mean, geeps

$a = h_1 + h_2 - - h_6$

1 resource



$\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

	A	B	C
I	1/3	1/3	1/3
II	1/6	1/3	1/2
III	1/6	1/2	1/3

Eign. Centrality

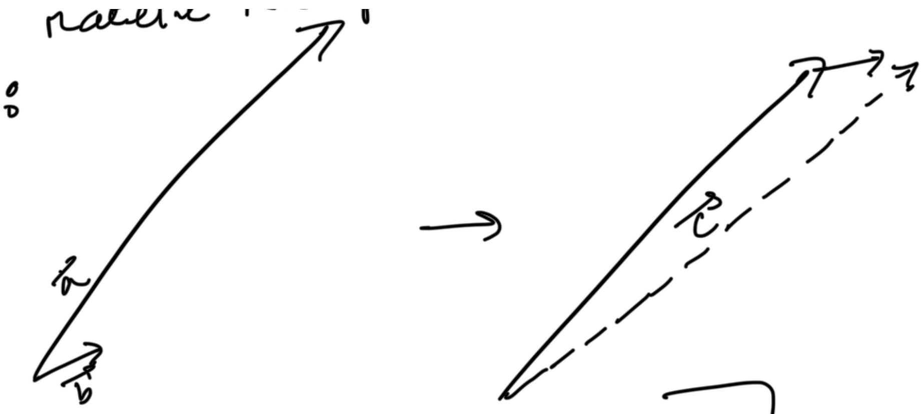
* The basic idea is that the popularity of a node depends on the popularity of it neighbors which further depend on the popula— of its neighbors & so one

---

* Recursive

... Multiplication with vector.

* 1 Preq: Rotate ...
* 2 Preq:



* 3 Preq:
  1. $A\underline{v} = \underline{\lambda}\underline{v}$
  2. $[\ \ ]_{2\times 2} = \underline{2}$ Indedent
  3. $Z = \alpha v_1 + \beta v_2$

$Av = A(\alpha \underline{v_1} + \beta \underline{v_2})$

node   Any vect

Let the initial popularity of node $i = x_i(0)$

After 1 step

popularity = sum of neighbour popularity

$$x_i(1) = \sum A_{ij}\, x_j(0) \quad \text{---} \quad (1)$$

$x_j(0)$

Diy

$$x_i(1) = \sum_j A_{ij}\, x_j(0)$$

Let us assume that value converge at t

$$x(t) = Ax(t-1)$$
$$= A\,A\,x(t-2) = A^2 x(t-2)$$
$$= A^3 x(t-3$$
$$\vdots$$

Power method by nothing

$$x(t) = A^t\, x(0) \quad \text{---} \quad eq2$$

Now Let x_0 be the linear combination of ... vectors of matrix A.

Eign vectors of

$$x_0 = \begin{bmatrix} x_1(0) \\ x_2(0) \\ \vdots \\ x_n(0) \end{bmatrix} = c_1 \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} + c_2 \begin{bmatrix} v_1 \\ \vdots \\ v_2 \end{bmatrix}$$

$$x_0 = \sum c_i v_i \quad \text{—— (3)}$$

$$\underline{x_t} = A^t \sum c_i \underline{v_i}$$

$$= \sum c_i \underbrace{A^t v_i}$$

$$= \sum c_i \lambda_i^t v_i = \ldots$$

$$= c_1 \lambda_1^t v_i + \lambda_1 \sum c_i \left(\frac{\lambda_i}{\lambda_1}\right)^t v_i$$

≈ 0

$\lambda_1 > \lambda_2$

eg $3 > 2$

$9 \to 4$

$27 \to 8$

3^{100} 2^{100}

$2^{100} / 3^{100} \approx 0$

$$x_t = \underline{c_1 \lambda_1^t v_1}$$

popularity corresponds to the principle eig n vector.

$$[\ ] \approx \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_n \end{bmatrix} \to \text{Ink can wet}$$

---

one problem

Directed Network.

in degree of a node can be $\underline{0}$.

Then 0 popularity can be propagted

- in network.

$$x_i = N \sum A_{ij} x_j + \beta$$

$$x_i = \alpha \sum_j \quad =$$

β = small initial value of cons

α = re adjustment constant.

Pag Rank

$$\boxed{x_i = \alpha \sum_j A_{ij} \frac{x_j}{k_j^{out}} + \beta}$$