



**BITS** Pilani  
Pilani Campus

# Social Media Analytics: Graph Essentials

Lecture:7  
Garima Jindal



# Homophily

Movy  
locality  
conference  
people in a park

# Homophily in the Society



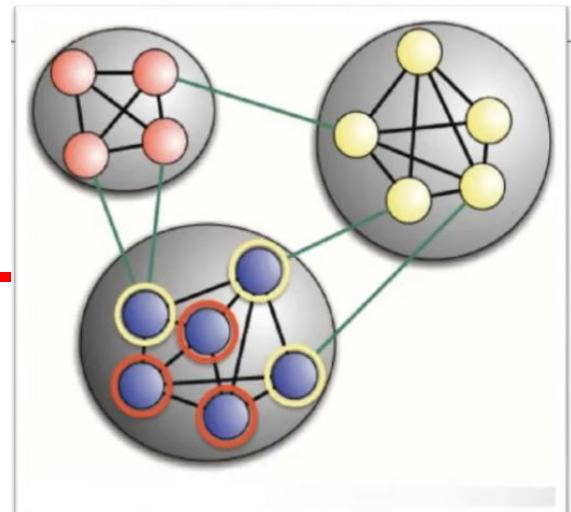
- ❑ Tendency of individuals to associate and bond with similar others
- ❑ Similar nodes tend to attract each other, and dissimilar nodes tend to get away from each other
- ❑ Causes formation of a **community structure** in a social network
- ❑ Homophily occurs against a number of categories:
  - Age
  - Sex and Gender
  - Class: Education, occupation, and Social
  - Religion, Race, and Ethnicity
  - Interests
  - Organizational role, etc.

# Communities in a Network



- ❑ Identifying communities gives an insight about the inherent network structure
- ❑ Community detection is *an ill-defined problem*
  - ❑ What we mean by a 'community' is often not concrete
  - ❑ Often hard to reliably define a ground-truth annotation for communities
  - ❑ No standard measure to assess the performance
- ❑ Diverse approaches to the problem depending on how we define a community structure in the network

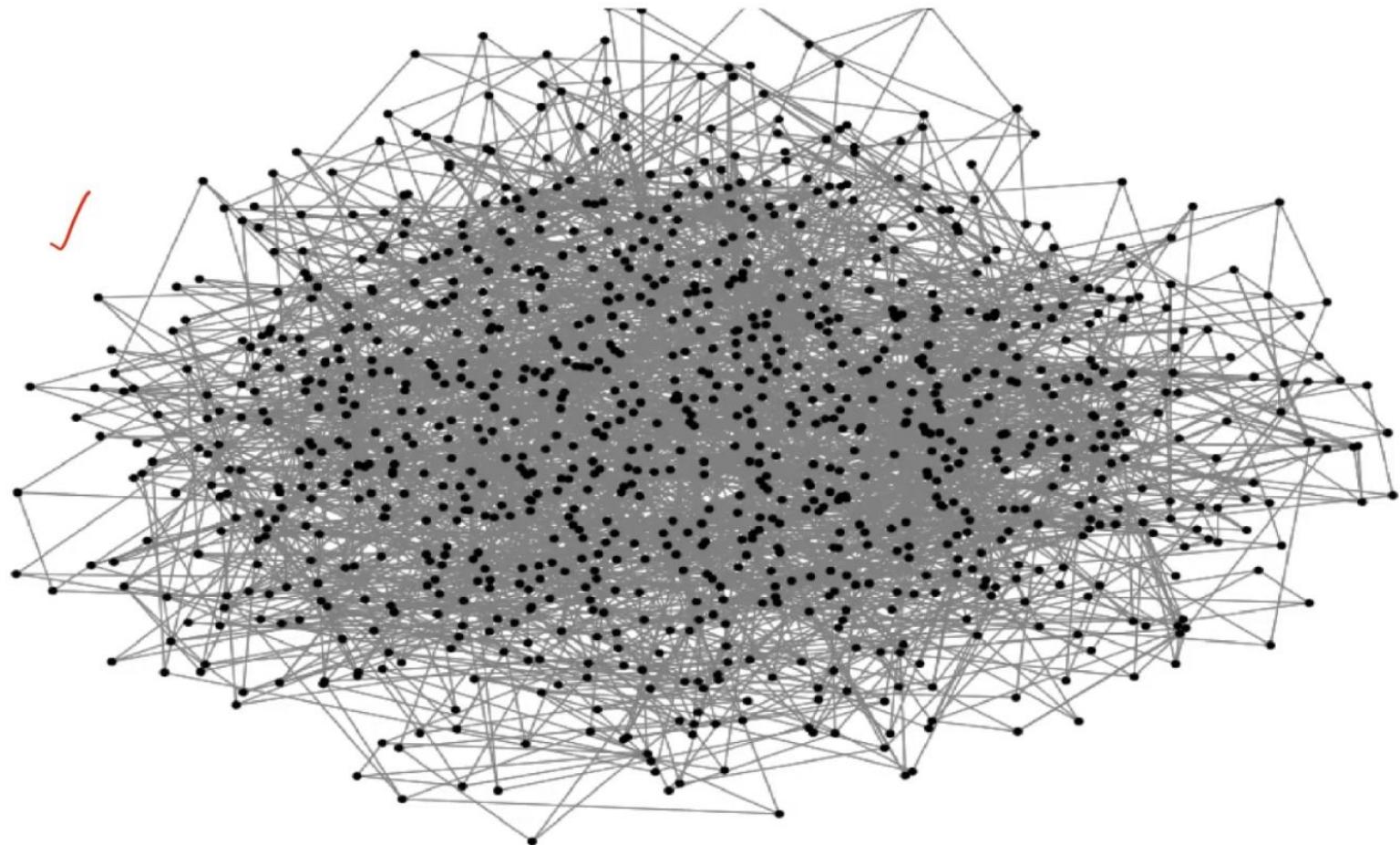
<http://bit.ly/3jZls60>



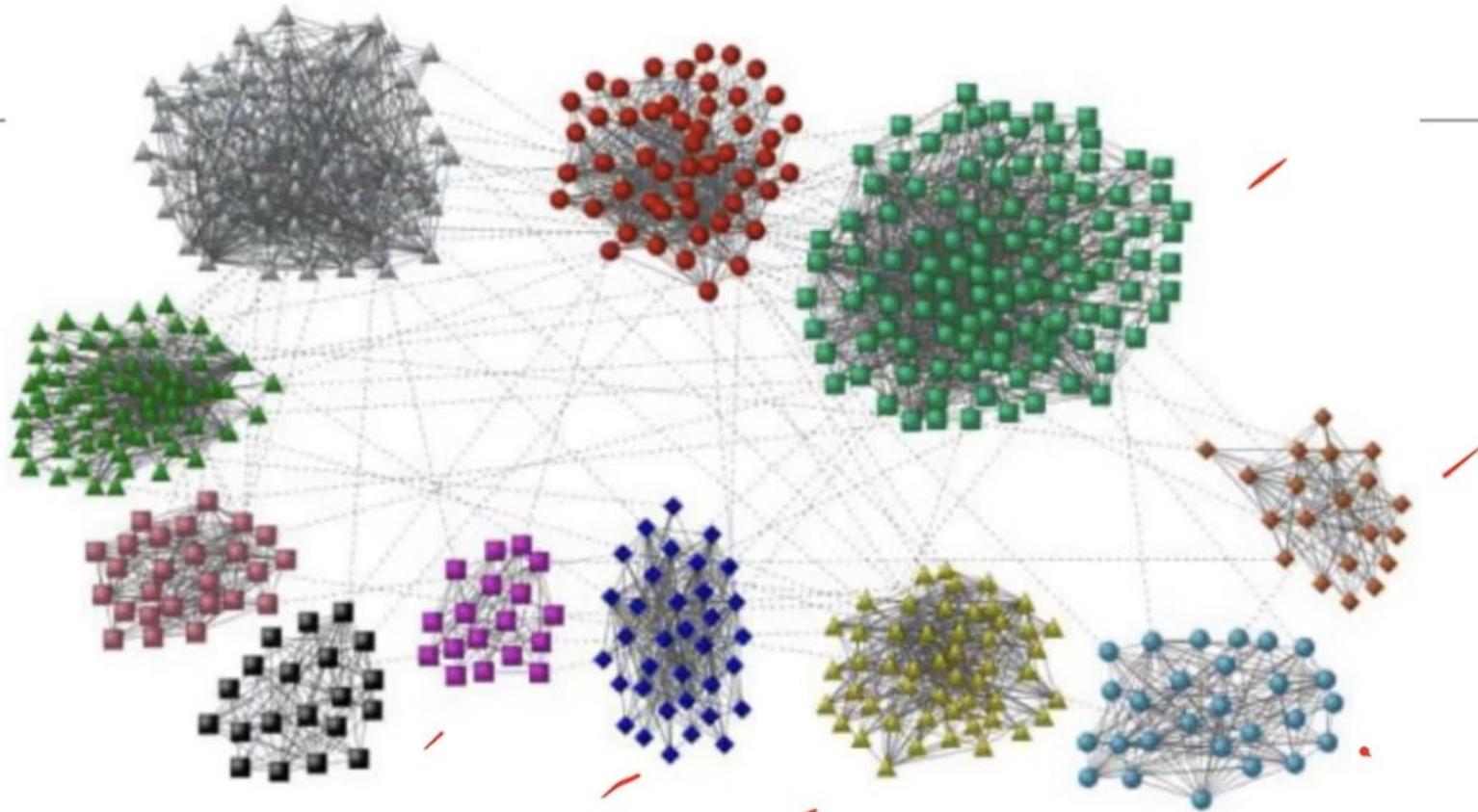
# Community Detection in Networks: Applications

- ❑ Performance enhancement of the similarity-based [link prediction](#) algorithms
- ❑ Improving recommendation quality in [Recommender systems](#) by separating like-minded people
- ❑ Controlling [information diffusion](#) within a network by identifying community memberships
- ❑ Designing better [marketing strategy](#) by identifying position of the target group within the network
- ❑ Restricting [epidemic propagation](#) by suitably isolating and immunizing the vulnerable population
- ❑ Better [anomaly detection](#) in nodes, especially in evolving networks
- ❑ Studying [evolution of communities](#)
- ❑ Applications in [criminology](#) and detecting terrorist groups

# The Network



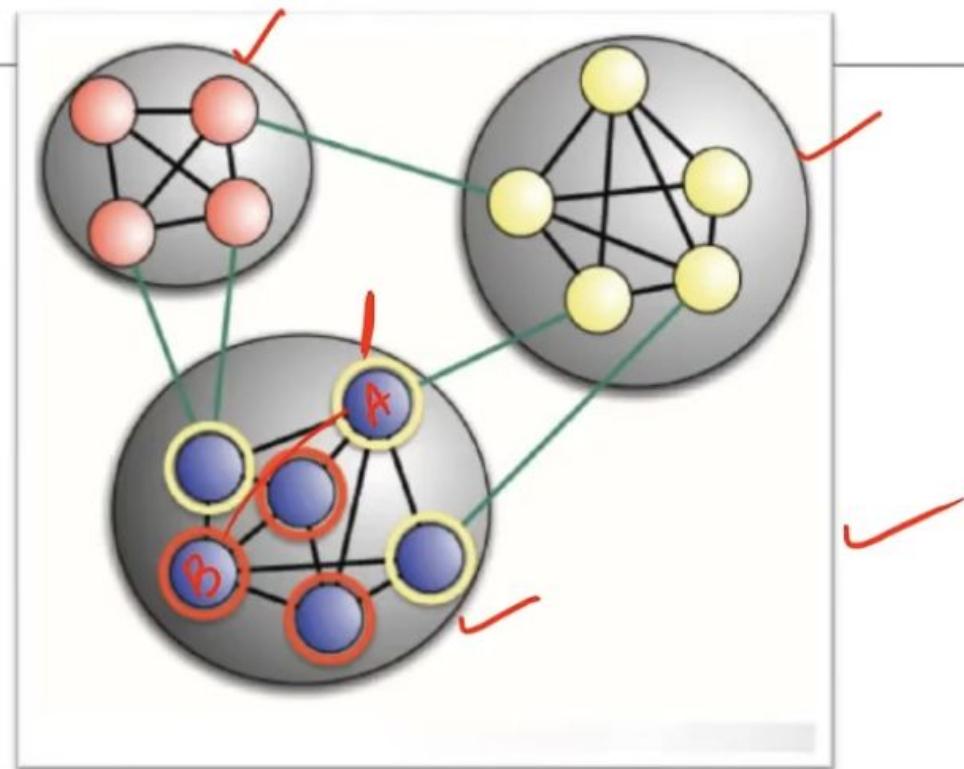
# The Community Structure



# Community Structure

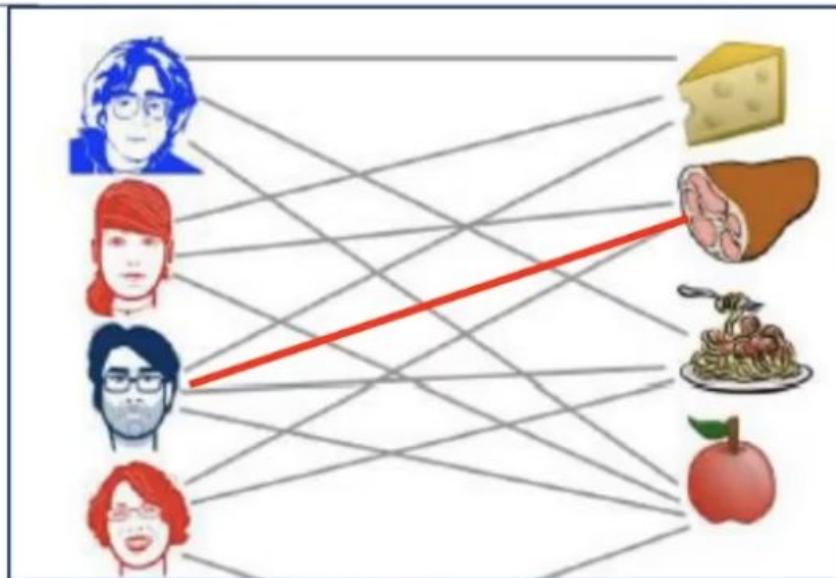
## Theoretical reasons

- Organization
- Node features
- Node classification
- Missing links

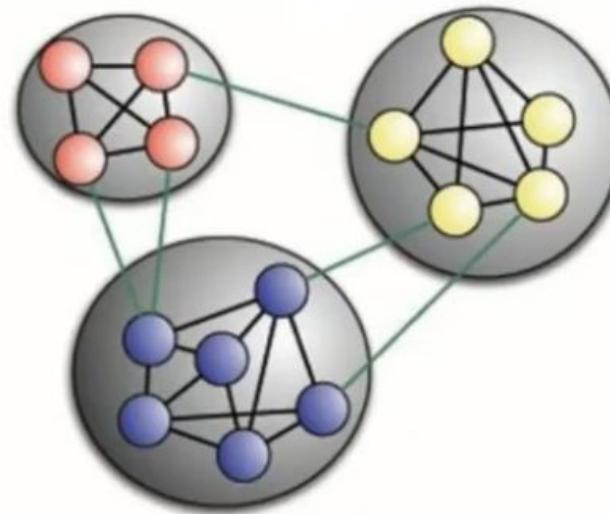


# Community Detection

## Practical Reasons: Recommendation Systems



# Difficult Problem !!

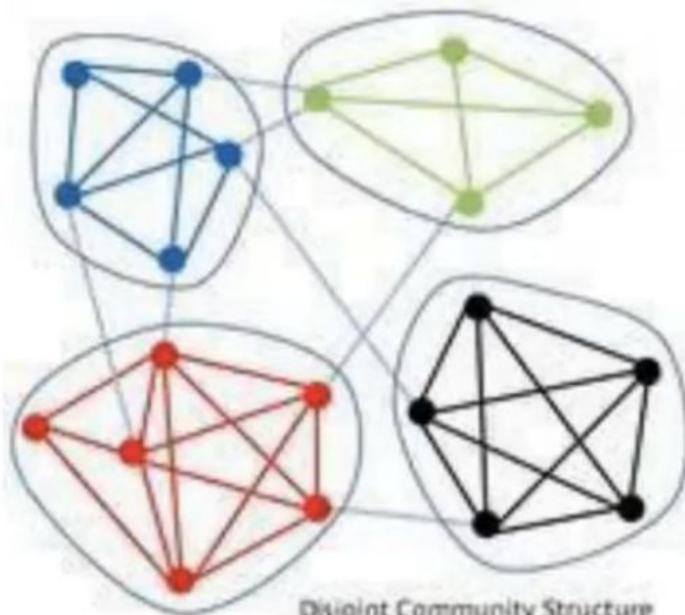


1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1							1	1	1				1	1
2							1	1					1	
3	1									1	1			
4		1							1	1	1			
5			1	1					1	1				
6	1									1				
7		1								1	1			
8			1	1						1				
9				1						1	1			
10	1				1					1	1			
11			1			1				1	1			
12	1				1	1					1			
13		1	1	1	1									
14	1			1	1	1								
15	1		1	1	1	1								



6	2	8	14	5	7	13	11	9	12	10	4	1	3	15
6			1	1	1									
2	1		1	1	1									1
8	1	1	1			1								
14	1	1	1			1								
5				1	1	1	1	1						
7					1	1	1	1	1					
13					1	1	1	1	1					
11						1	1	1	1					
9						1	1	1	1					
12						1	1	1	1	1				
10							1	1	1	1	1			
4								1	1	1	1	1		
1								1	1	1	1	1		
3									1	1	1	1		
15									1	1	1	1		

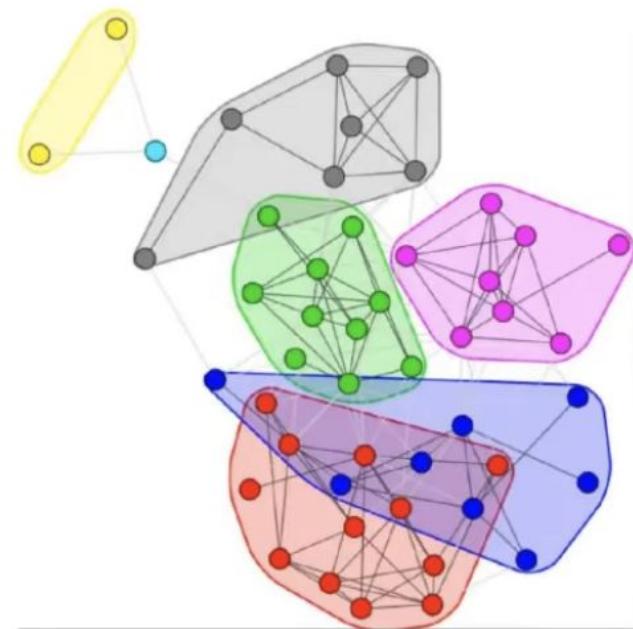
# Types of Communities: Disjoint Communities



- ❑ Also referred to as **flat communities**
- ❑ Each node in the network can belong to at most one community
- ❑ Differs from **disconnected components**:
  - ❑ nodes in two different communities can still have connecting edges
  - ❑ referred to as **bridges**
- ❑ Example: Full-time employees of an organization

<http://optnetsci.cise.ufl.edu/research/disjoint-overlapping-communities/>

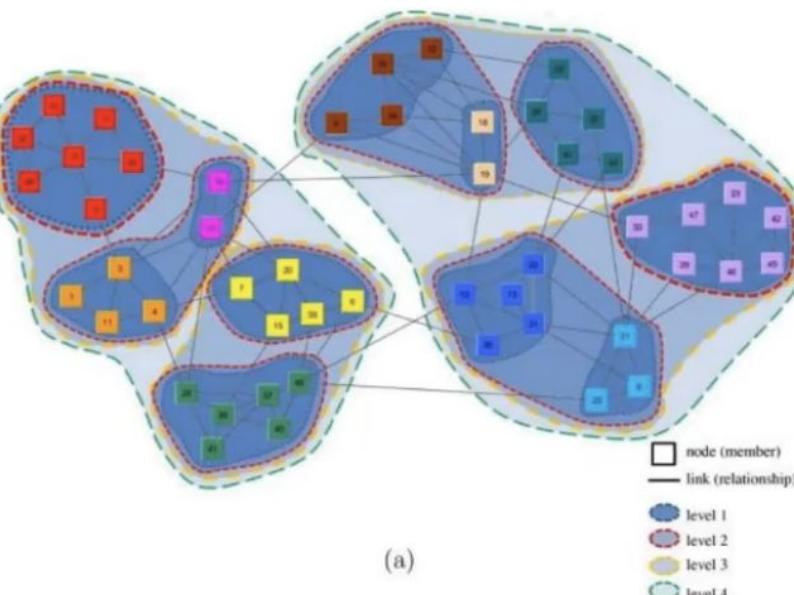
# Types of Communities: Overlapping Communities



- Members can belong to more than one community at a time
- Communities can even share edges
- Realistic and generic community structure
- Harder to find than flat communities
- Example: Various groups in social networks

<https://stackoverflow.com/questions/51102350/python-remove-overlapping-communities-in-igraph-plot>

# Types of Communities: Hierarchical Communities



- Outcome of merging two or more flat or overlapping communities in a network
- Can be linked to other hierarchical, overlapping, or flat communities
- Example: various city-level communities merged to form a state-level community

<https://www.sciencedirect.com/science/article/abs/pii/S0020025514011463>

# Types of Communities: Local Communities

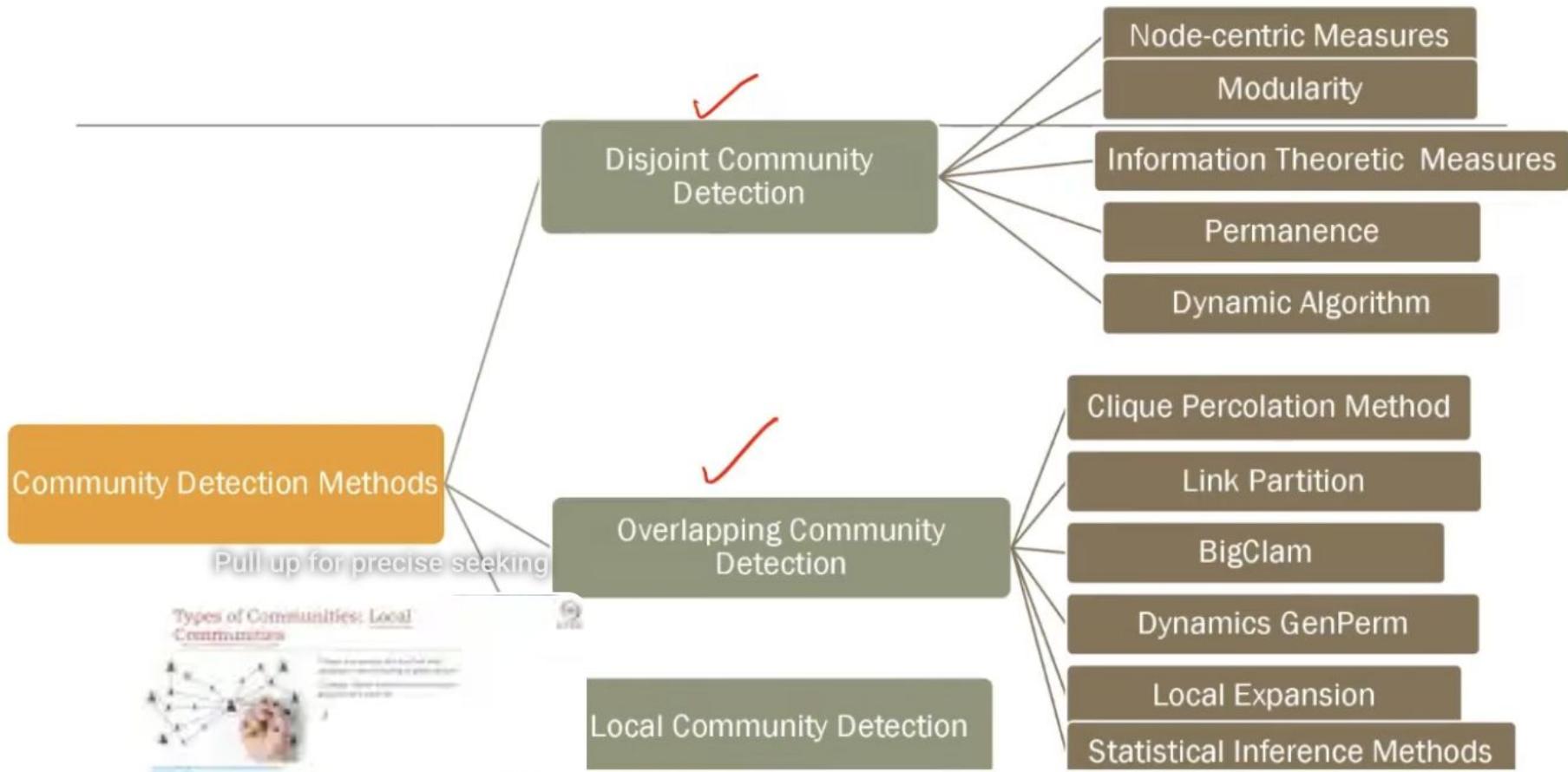


- ❑ Shows a community structure from local perspective without focusing on global structure
- ❑ Example: citation network formed by research groups inside a university



<https://www.digitaltrends.com/features/the-history-of-social-networking/>

# Community Detection Methods: A Taxonomy



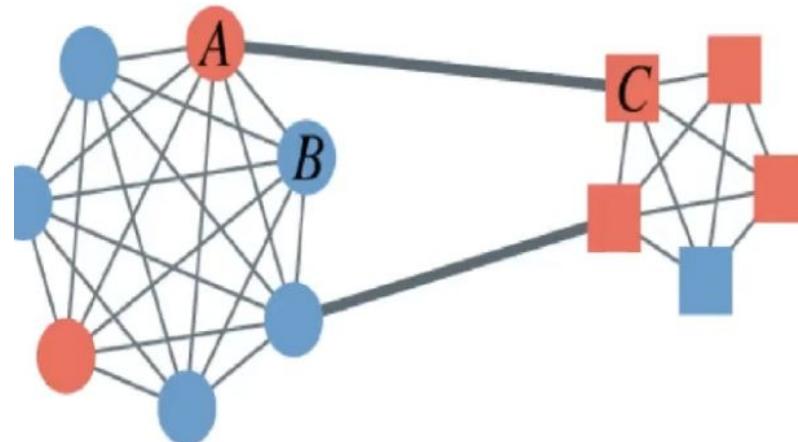
# Node-centric Community Detection

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- Use the property of the nodes to find community structure in the network
- Exploits node-centric features in a number of ways:
  - Complete Mutuality
    - Cliques
    - Reachability of Members
  - K-cliques
  - K-clan
  - K-club
  - Node Degree
    - K-plex
    - K-core

# Node-centric Community Detection: Finding Cliques

Clique 1

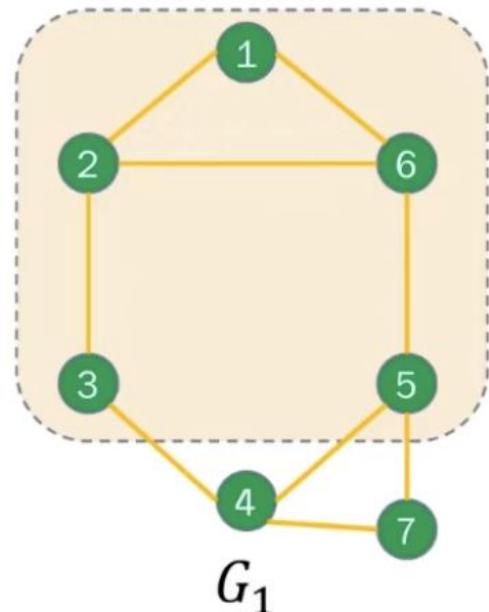


Clique 2

- A subgraph of a graph is a clique if every vertex-pair in the subgraph are adjacent
- Has diameter of 1
- Can be considered as communities
  
- A couple of problems with this approach
  - Finding cliques from a network is NP-complete
  - Constraints on cliques are too strict a requirement
  - Large cliques are not present in social networks usually

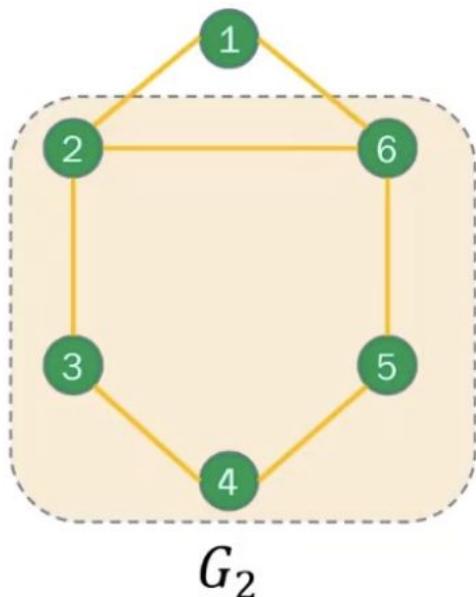
[https://www.researchgate.net/figure/Illustrative-example-of-a-small-two-clique-network-In-this-example-clique-1-is-a\\_fig1\\_337025822](https://www.researchgate.net/figure/Illustrative-example-of-a-small-two-clique-network-In-this-example-clique-1-is-a_fig1_337025822)

# Node-centric Community Detection: K-Cliques



- ❑ The **maximal subset** of vertices of the network such that, for any two nodes belonging to this subset, the shortest distance between them is less than or equal to K
- ❑ 1-clique is normal clique
- ❑ The nodes {1,2,3,5,6} forms a **2-clique** in the network  $G_1$
- ❑ 2-cliques are known as known as **friend of a friend** in social network analysis
- ❑ Issue:
  - ❑ A node not present in K-clique can contribute in formation of the shortest distance in it!!

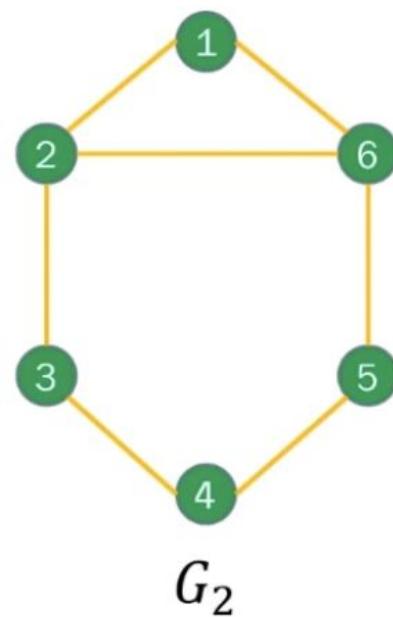
# Node-centric Community Detection: K-clan



- ❑ A stricter version of K-clique
- ❑ Only the nodes present in the set under inspection are used to create the subgraph in which the distance between any two nodes should be less than or equal to K
- ❑ In the network  $G_1$ ,
- ❑ The nodes {1,2,3,4,5,6} forms a **2-clique**, but it is not a **2-clan**
- ❑ The nodes {2,3,4,5,6} forms a **2-clan** in the network  $G_2$
- ❑ **Maximality condition** of K-clique also persists in K-clan

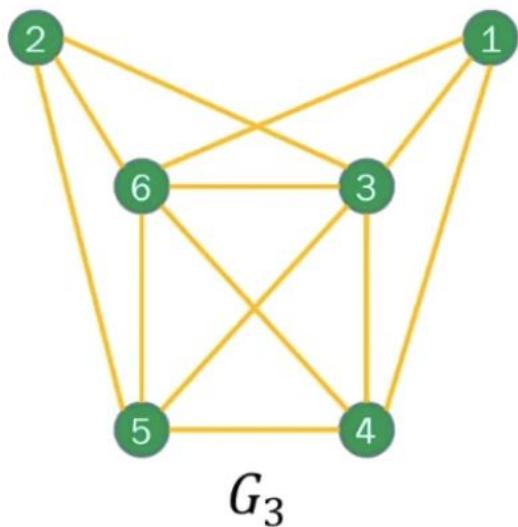
# Node-centric Community Detection: K-club

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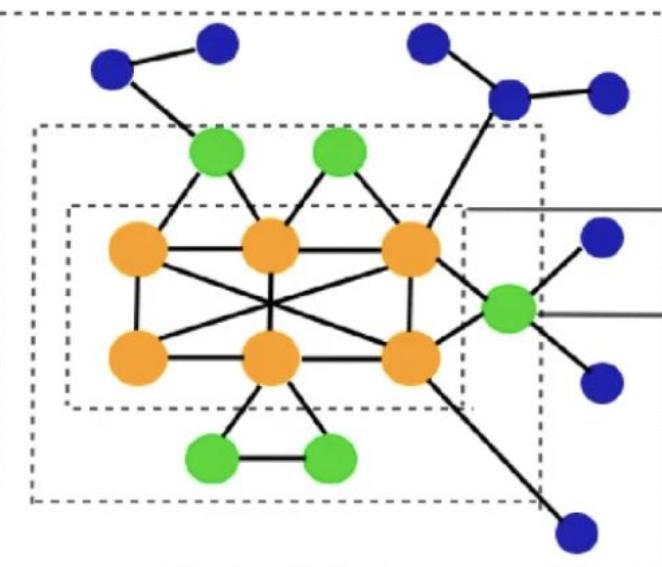
- ❑ K-club is a K-clan minus the maximality condition
- ❑ {2, 3, 4}, {3, 4, 5}, {4, 5, 6}, {5, 6, 2}, and {6, 2, 3} in  $G_2$  are all 2-clubs
  
- ❑ Every K-clan is a K-club as well as a K-clique
- ❑ Challenges:
  - ❑ These algorithms are still computationally expensive for large K
  - ❑ Deciding appropriate K is difficult

# Node-centric Community Detection: K-plex



- ❑ A subset of vertices  $S$  in a graph is a  $K$ -plex if every vertex of the induced subgraph  $G[S]$  has degree at least  $|S| - K$
- ❑ A measure based on the degree of the nodes
- ❑ In the network  $G_3$ ,
- ❑ The subset  $\{3,4,5,6\}$  is a **1-plex**, i.e., a regular clique
- ❑ The subset  $\{1,3,4,5,6\}$  is a **2-plex**, but not a 1-plex
- ❑ The subset  $\{1,2,3,4,5,6\}$  is a **3-plex**, but not a 2-plex

# Node-centric Community Detection: K-core



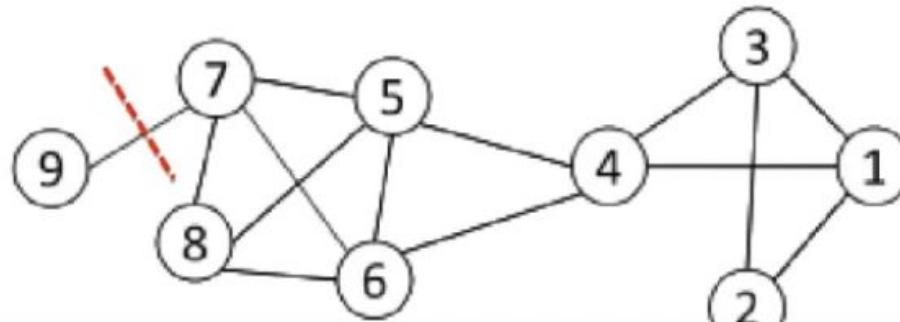
- ❑ Another degree-centric measure
- ❑ A subgraph  $G'$  of a graph  $G$  in which each node has degree greater than or equal to  $K$
- ❑  $K+1$  core subgraph can be created from the current  $K$  core subgraph by recursively removing nodes of degree  $K$ .
- ❑ This above should be repeated until there is no node of degree  $K$  in the current subgraph.
- ❑ Issues:
  - ❑ Checking whether a given network is  $K$ -core or  $K$ -plex is computationally easy
  - ❑ Finding maximal  $K$ -core/ $K$ -plex is NP-complete!!

# Cut ✓

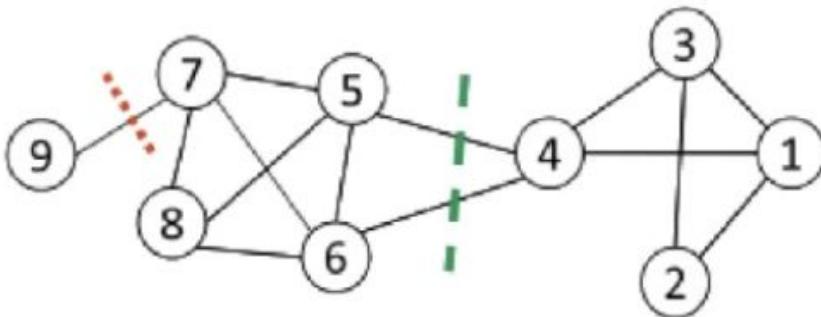
- Most interactions are within group whereas interactions between groups are few.
- Community detection → **Minimum cut problem**

**Cut:** A partition of vertices of a graph into two disjoint sets

**Minimum cut problem:** find a graph partition such that the number of edges between the two sets is minimized



# Ratio Cut & Normalized Cut



- Minimum cut often returns an imbalanced partition, with one set being a singleton, e.g. node 9

Change the objective function to consider community size

$$\text{Ratio Cut}(\pi) = \frac{1}{k} \sum_{i=1}^k \frac{cut(C_i, \bar{C}_i)}{|C_i|}, \quad \begin{aligned} C_i &: \text{a community} \\ |C_i| &: \text{number of nodes in } C_i \\ vol(C_i) &: \text{sum of degrees in } C_i \end{aligned}$$

$$\text{Normalized Cut}(\pi) = \frac{1}{k} \sum_{i=1}^k \frac{cut(C_i, \bar{C}_i)}{vol(C_i)}$$

# Ratio Cut & Normalized Cut Example

For partition in red:  $\pi_1$

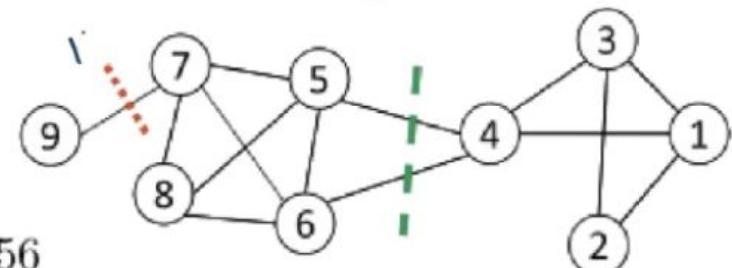
$$\text{Ratio Cut}(\pi_1) = \frac{1}{2} \left( \frac{1}{1} + \frac{1}{8} \right) = 9/16 = 0.56$$

Normalized Cut( $\pi_1$ ) =  $\frac{1}{2} \left( \frac{1}{1} + \frac{1}{27} \right) = 14/27 = 0.52$

For partition in green:  $\pi_2$

$$\text{Ratio Cut}(\pi_2) = \frac{1}{2} \left( \frac{2}{4} + \frac{2}{5} \right) = 9/20 = 0.45 < \text{Ratio Cut}(\pi_1)$$

$$\text{Normalized Cut}(\pi_2) = \frac{1}{2} \left( \frac{2}{12} + \frac{2}{16} \right) = 7/48 = 0.15 < \text{Normalized Cut}(\pi_1)$$



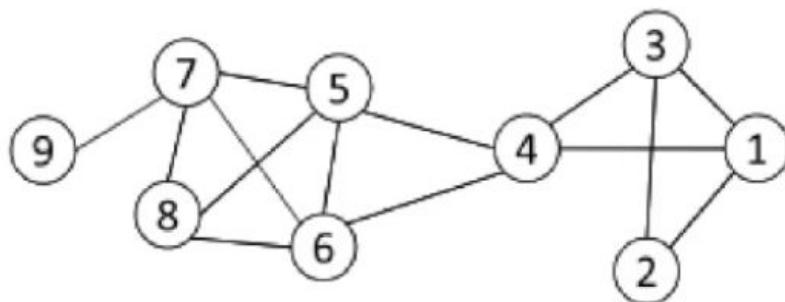
Both ratio cut and normalized cut prefer a balanced partition.

# Edge Betweenness

Girvan & Newman, PNAS, 2002

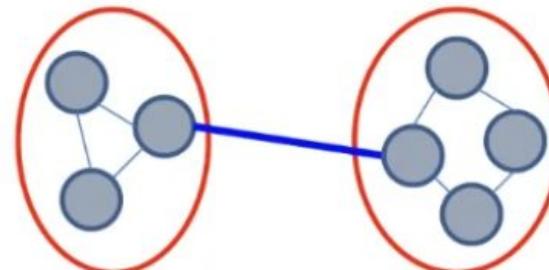
The strength of a tie can be measured by **edge betweenness**

**Edge betweenness:** the number of shortest paths that pass along with the edge

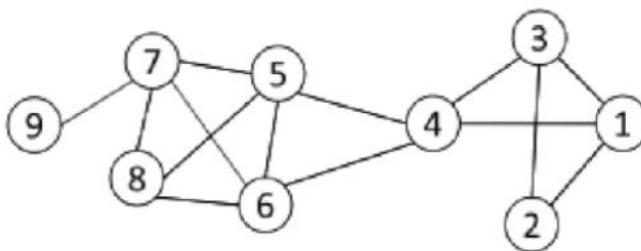


The edge betweenness of  $e(1, 2)$  is 4 ( $=6/2 + 1$ ), as all the shortest paths from 2 to  $\{4, 5, 6, 7, 8, 9\}$  have to either pass  $e(1, 2)$  or  $e(2, 3)$ , and  $e(1,2)$  is the shortest path between 1 and 2.

- The edge with higher betweenness tends to be the **bridge** between two communities.

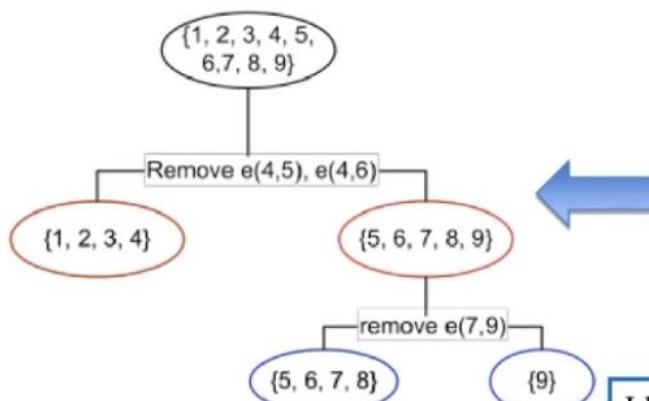


# Divisive Clustering based on Edge Betweenness



Initial betweenness value

		Table 1: Edge Betweenness								
		1	2	3	4	5	6	7	8	9
1	1	0	4	1	9	0	0	0	0	0
	2	4	0	4	0	0	0	0	0	0
3	1	4	0	9	0	0	0	0	0	0
4	9	0	9	0	10	10	0	0	0	0
5	0	0	0	10	0	1	6	3	0	0
6	0	0	0	10	1	0	6	3	0	0
7	0	0	0	0	6	6	0	2	8	0
8	0	0	0	0	3	3	2	0	0	0
9	0	0	0	0	0	0	8	0	0	0



After remove  $e(4,5)$ , the betweenness of  $e(4,6)$  becomes 20, which is the highest;

After remove  $e(4,6)$ , the edge  $e(7,9)$  has the highest betweenness value 4, and should be removed.

Idea: progressively removing edges with the highest betweenness

# Community Detection: Modularity

- ❑ Node-centric methods discussed so far are not very useful when the network is large
- ❑ Modularity comes from the word '**module**'
- ❑ a network-centric metric to determine the quality of a community structure
- ❑ Based on the principle of comparison between
  - ❑ the **actual number of edges** in a subgraph and its **expected number of edges**
  - ❑ the expected number of edges is calculated by assuming a **null model**
- ❑ In the null model,
  - ❑ each vertex is randomly connected to other vertices irrespective of the community structure
  - ❑ However, some of the structural properties are preserved
  - ❑ One popular structural property is the **degree distribution**



# Questions?

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