# Bayesian Econometrics Homework 3

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2/13/2019

**This is a Group Assignment. Please work as a group and submit your answers as a group.** ~

Problem 1. a) Show that when U is uniformly distributed on (0,1), 1-U is also uniformly distributed on (0,1). A random variable, U, is uniformly distributed if $P(U \leq x) = x$.

b) Using inverse distribution function method, show that $Y = -\frac{1}{\theta} logU$, where U is uniform random variable on (0,1), is exponentially distributed and use the fact to draw a sample from it given a uniform sample.

Problem 2 It is sometimes possible to write a probability density function in the form of a finite mixture distribution, which is defined as follows:

$$f(x) = \sum_{i=1}^{k} P_i f_i(x)$$

where $\sum_{i=1}^{k} P_i = 1$. For example, finite mixtures of normal distributions with different means and variances can display a wide variety of shapes. Generate samples from the following distribution:

$$f(x) = 1.8e^{-3x} + 0.4e^{-x}$$

a) Show first that the above density is a finite mixture of exponential distributions.

b) Draw 300 sample from the above mixture density by a) drawing each exponential distribution (e.g., see problem 1 for sampling from exponential distribution) and b) using the probability of drawing each exponential distribution (i.e., $(P_i's)$ above).

3) Compare densities (or histograms) of all three distributions (two exponential distributions and the mixture distribution.

Problem 3. Consider the problem of sampling from Beta(3,3) using Accept-Reject sampling method using the following information:

1) Use uniform distribution on (0,1) as the proposal density.
2) Note that the probability density of Beta(3,3) is symmetric around 0.5 with the mode of 1.875.

a) Provide the accept-reject algorithm for the problem.

b) Draw 600 observations using the algorithms. Compare the mean and variance of the sample to the true values.

Problem 4. Consider the following expectation of a function of truncated exponential distribution:

$$E(\frac{1}{1+x^2}), \tag{1}$$

where x $\sim$ $Exp(1)$ truncated to [0, 1]. I.e,, x is truncated exponential distribution from 0 and 1 with parameter=1 (i.e. $\theta = 1$ from problem 1).
a) Show that the above problem is equivalent to the following problem:

$$\frac{1}{1-e^{-1}} \int_0^1 \frac{1}{1+x^2} e^{-x} dx, \tag{2}$$

b) Calculate equation (2) using importance sampling method. Use Beta(2,3) density function as the importance function (i.e., h in the lecture note). First, describe the algorithm of the method for this problem and Second, implement the algorithm in R with the number of simulations=10,000.

Problem 5. Consider the Gibbs sampling example for a standard bivariate normal distribution discussed in the class. Estimate the mean and standard deviation of $y_1$ using Monte Carlo with Gibbs Sampling method using last 100 observations.

Do this with
a) $\rho = 0.1$ and 0.99
b) starting values of $y_{20}$=-1.5 and 1.5
c) Number of simulations=200, 500, 5000,10,000.

Compare your results with the true marginal distribution of $y_1$. What does your result tell you about Gibbs Sampling?

Problem 6. Describe the steps for constructing a random walk Metropolis-Hasting sampler to generate a sample of 10,000 from the distribution with the p.d.f $= 0.5e^{-|x|}$, known as Laplace distribution using the standard normal for $\epsilon$ to generate proposals: $y = x^{i-1} + \epsilon$. I do **not** need a R-script for this, but I recommend you try this in R as well. Please do **not** submit R-script. I will make a R-script for this available when I post the answer keys. FYI, Laplace distribution is not implemented in the base R, although there are several packages which have it